Journal of Vibration and Control OnlineFirst, published on June 29, 2010 as doi:10.1177/1077546309104879

Damage Detection Method for Composite Structures Based on a Combined Technique of Cross Modal Strain Energy and Niche Genetic Algorithms

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(Received 28 December 2006; accepted 27 February 2009)

Abstract: This paper presents an effective method for structural damage detection using the combined technique of the up-to-date cross modal strain energy (CMSE) and niche genetic algorithms (GAs). The traditional modal strain energy method is not convenient for structural damage detection because it needs to compare the modal information between the same modes of the baseline and damaged structures, and also requires the analytical and measured modes to be consistent in scale, or to be normalized, but no such constraint is required for the CMSE method. However, the current CMSE method is still deficient because it cannot effectively determine the locations of structural damage. In order to overcome the shortcoming of the CMSE method in structural damage detection, a combined method of the CMSE with the niche GAs is proposed in this paper to enhance the capability of identifying structural damage location. Numerical studies of damage detection for the case of a model of an airfoil of composite material are given, and the obtained results show that the method proposed in this paper is successful to detect the damage of complex structures.

Key words: Composite structure, cross modal strain energy, niche genetic algorithms, structural damage detection.

1. INTRODUCTION

Damage in the form of a loss of local stiffness in a structure will alter the dynamic properties of the structure, including the modal frequencies and modal shapes. So, currently, the damage detection methods based on vibration mainly adopt a modal parameter or character picked up from the modal parameter as the indicator for damage detection. Recently, the damage detection methods based on structural vibration modal parameters became a popular strategy (Yan et al., 2007), and many approaches have been developed such as the methods based on natural frequency, modal shape, strain mode shape, etc. But, many of these methods are not very suitable for complex structures. It should be noted that the method for determining damage severity based on the cross modal strain energy was more widely studied. The traditional

Journal of Vibration and Control, **00(0):** 1–11, 2010 DOI: 10.1177/1077546309104879

Figures 1, 2 appear in color online: http://jvc.sagepub.com

modal strain energy methods must compare the modal information between the same modes of the baseline and damaged structures. A kind of cross modal strain energy (CMSE) method was proposed for structural damage detection (Hu et al., 2006). The main advantage of the CMSE method is that there is no need to compare the modal information between the same modes of the baseline and damaged structures, and also it does not require the analytical and the measured modes to be consistent in scale, or to be normalized. The core of the CMSE method is to formulate simultaneous linear equations associated with modal strain energy-like terms that are product terms crossing over the baseline model and the damaged structure, also crossing over various modes. The traditional model strain energy methods for damage detection use an iterative solution procedure or involve rough assumption and immoderate approximation (Kim and Stubbs, 2002; Shi et al., 1998), while the CMSE method is an exact, noniterative solution method. However, the CMSE method only detects the severity of structural damage, and cannot efficiently discover the location of structural damage.

It is equally important to detect structural damage location and severity. As a good method, it should be able to detect structural damage location and severity at the same time. In this paper, we combine the advantage of CMSE in accurately and expediently determining the severity of structural damage with efficiently finding structural damage location based on the optimization and searching functions of niche genetic algorithms (GAs), proposing a new combined method for structural damage detection. For a large and complicated structure, it generally has a tremendous element number in the dynamics finite element model (FEM) and may have many damage elements, and this will cause the egregious calculation workload for searching locations of structural damage in the traditional method of structural damage detection. The combined method can effectively overcome this computational problem. In this paper, we utilize the damage feature index from the CMSE method to construct the fitness function of the niche GAs, and effectively to determine the structural damage location using a globally optimal function of the niche GAs. Detection of the single damage and complex damage for the case of a composite material airfoil model is studied in simulation, and the obtained results show that the combination method of CMSE and the niche GAs is successful for damage detection of complex structures.

2. CMSE METHOD

The CMSE method for structural damage detection formulates a simultaneous linear equation associated with modal strain energy-like terms that are product terms crossing over the baseline structure and the damaged structure, also crossing over different modes. It is significantly different from the ordinary modal strain energy method for damage detection (Kim and Stubbs, 2002; Shi et al., 1998; Ren and Roeck, 2002).

Suppose that $\boldsymbol{M}$ and $\boldsymbol{K}$ are the total mass and stiffness matrices for the intact structural model, and $\boldsymbol{M}^*$ and $\boldsymbol{K}^*$ are the total mass and stiffness matrices for the damaged structure. In this paper, the superscript “*” is used to indicate a damaged status.

Let the stiffness matrix of the damaged structure be written as

$$\boldsymbol{K}^* = \boldsymbol{K} + \sum_{n=1}^{N_\mathrm{d}} \alpha_n \bar{\boldsymbol{K}}_\mathrm{n}, \tag{1}$$

where the subscript n is the counter of the damaged elements; N_d is the total number of the damaged elements; α_n and ln are the damaged extent and the element number of the nth damaged element, respectively. $\bar{\boldsymbol{K}}_\mathrm{n}$ is a symmetric matrix having the same dimension as the stiffness matrix of the structure, which is extended by the element stiffness matrix of the nth damaged element.

Define the structural cross modal strain energy between the ith mode of the intact structure and the jth mode of the damaged structure as

$$C_{ij} = (\Phi_i)^\mathrm{T} \boldsymbol{K} \Phi_j^* \tag{2}$$

and the corresponding elemental cross modal strain energy for the stiffness matrix $\bar{\boldsymbol{K}}_\mathrm{n}$ as

$$C_{n,ij} = (\Phi_i)^\mathrm{T} \bar{\boldsymbol{K}}_\mathrm{n} \Phi_j^*, \tag{3}$$

where Φ_i and Φ_j^* are the ith and jth mode shapes of the intact and damaged structures.

In the modal analysis for the structure, one can get the equation

$$\sum_{n=1}^{N_\mathrm{d}} \alpha_n C_{n,ij} = \left(\frac{\lambda_j^*}{\lambda_i} - 1 \right) C_{ij}, \tag{4}$$

where λ_i and λ_j^* are the squares of the ith and jth frequencies of the intact and damaged structures, respectively. For the detailed process, please refer to Shi et al.'s paper (Shi et al., 2000).

Using a new subscript m to replace ij, equation 4 becomes

$$\sum_{n=1}^{N_\mathrm{d}} \alpha_n C_{n,m} = b_m, \tag{5}$$

where

$$b_m = \left(\frac{\lambda_j^*}{\lambda_i} - 1 \right) C_{ij}. \tag{6}$$

When N_i modes of the intact structure and N_j modes of the damaged structure are available, respectively, one can get $m = ij = 1,\ 2,\ \ldots,\ N_q$, that is $N_q = N_i \times N_j$. Equation 5 can be written in a matrix form as

$$\boldsymbol{C}\boldsymbol{\alpha} = \boldsymbol{b}, \tag{7}$$

where $\boldsymbol{C}$ is an $N_q \times N_\mathrm{d}$ matrix and $\boldsymbol{\alpha}$ and $\boldsymbol{b}$ are column vectors of sizes N_d and N_q, respectively. When N_q is larger than N_d, an unweighted least-squares estimate for α, denoted $\hat{\boldsymbol{\alpha}}$, can be obtained as

$$\hat{\boldsymbol{\alpha}} = \left(\boldsymbol{C}^\mathrm{T} \boldsymbol{C} \right)^{-1} \boldsymbol{C}^\mathrm{T} \boldsymbol{b}. \tag{8}$$

It is worth mentioning that those N_i and N_j modes of the intact and damaged structures can be arbitrary modes. In this case, they are not required to start from the first mode. Actually, it is easy to obtain the analytical modes of the intact structure, but difficult to get the measured modes of the damaged structure; therefore, one may choose a much larger N_i than N_j.

From equation 5 and equation 8, the vector $\boldsymbol{b}$ which includes the real damage information is known because λ_j^* and Φ_j^* can be obtained by experimental measurement. $\boldsymbol{C}$ is a matrix including the damaged message $\bar{\boldsymbol{K}}_{\mathrm{n}}$, which is related to the damage location. Usually, the damage location and damage severity of the structure are unknown, so one cannot get the damage severity $\hat{\boldsymbol{\alpha}}$ from equation 8 using the mode information of the intact and damaged structures.

If the structure is a simple structure, which has few elements, one can analyze each possible damage location to solve the problem. This can be implemented by the following approach.

First, one can calculate $\hat{\boldsymbol{\alpha}}$ with each possible damage location. Then, replacing $\hat{\boldsymbol{\alpha}}$ in equation 7, the corresponding $\hat{\boldsymbol{b}}$ can be written as

$$\hat{\boldsymbol{b}} = \boldsymbol{C}\hat{\boldsymbol{\alpha}}. \tag{9}$$

Define the standard residual of every CMSE equation as

$$e_m = \frac{\hat{b}_m - b_m}{b_m}, \quad m = 1, \dots, N_q. \tag{10}$$

Denoting $\boldsymbol{e}$ to be the vector for all e_m, one calculates the norm of $\boldsymbol{e}$ as

$$\|\boldsymbol{e}\| = \sqrt{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}} = \sqrt{\left(e_1^2\right) + \cdots + \left(e_{N_m}\right)^2}. \tag{11}$$

The quantity $\|\boldsymbol{e}\|$ can be employed as an indicator to quantify the goodness of the "fitting" among all N_m equations. Actually, if the guessed damage scenario is near the true damage scenario, the corresponding $\|\boldsymbol{e}\|$ would be small. Thus, a simple damage localization algorithm is based on finding the particular, among all, guessing scenario that possesses the smallest residue norm. And, an advantage of the CMSE method is that the damage extent estimates are very accurate if the damage locations are true, so the damage extent can be regarded as additional information to judge the correctness of the damage localization.

Apparently, if the structure is complex, and a vast number of elements in the FEM are required, the method has very low efficiency with guessing the damage location. However, compared with other damage detecting methods, the CMSE method also has many advantages. For example, (i) the basic equations constructed to detect damage by the traditional modal strain energy methods must employ the modal information from the same modes of both the intact and damaged structures, but the basic equations formulated to detect damage by the CMSE method are based on cross modes, as shown in equation 10. There is no need for matching modes between the intact and damaged structures. Since many equations can be formulated from a single measured mode (with many analytical modes), minimal modal information from measurements is needed. (ii) The CMSE method uses the "cross" terms between intact and damaged structures; therefore, unlike other methods there is no need in this method for analytical and measured modes to be consistent in scale, or normalized in any particular way. (iii) The CMSE method is a direct (noniterative), exact (without linearization or dropping higher-order terms) solution method. But, almost all other modal strain energy methods either use an iterative solution procedure or involve rough presumptions and significant approximations. (iv) From equation 10, it is clearly seen that the CMSE damage detection has ideally employed both the modal shapes and modal frequencies in each formulated equation.

3. NICHE GENETIC ALGORITHMS

If the structure is a complex one which has thousands upon thousands or even more elements, the number of the damage location scenario may be very large. Using the CMSE method by guessing the damage location or traversing every damage location scenario to detect the damage of the structure is very difficult. This actually is a solution-searching problem. It is known that the GA has a high efficiency and capability to search for the best solution. Hence, it offers a very convenient and viable method to solve the above problem.

The GA is a global optimization self-adapting probabilistic search algorithm, which assimilates an advanced principle from natural selection and genetic evolution of creatures. The GA uses a population hunting technique, developing a new population by performing some genetic operations on the current population such as selection, crossover, or mutation, and lets the population develop to be one having the best or near the best solution. The GA is accepted in many areas because of the simple idea, easy realization and excellent effect. As a question-independent and universal algorithm frame, the GA is an optimization algorithm, which has a stronger stability, even with some large, complex nonlinear systems, and it expresses a unique and superior distinction.

The GA has four basic characteristics; these include: (a) let the decision variable be the operation target, (b) let the object function be the hunting information directly, (c) implied parallelism, (d) using probabilistic search.

The usual procedure of GAs for engineering calculation follows as: (a) determine the decision variable and constraint condition, namely determine the phenotype of the individual and the solution space of the question, (b) establish the optimization model, namely determine the type and the mathematical description form or the quantifying method of the object function (one should obtain the max of the object function or obtain the min of the object function), (c) determine the coded representation of the chromosome, which can express the feasible solution, namely determine the individual genotype X, which is the searching space of the GAs, (d) determine the coded representation, namely determine the corresponding relationship or transform method from individual genotype X to individual phenotype X, (e) determine the quantifying evaluation of the individual fitness, namely determine the transforming rule from the value of the object function $f(x)$ to individual fitness $F(x)$, (f) design the genetic operator, that is to say determine the specific operation methods of selection crossover and the mutation genetic operator, (g) determine the related operational factor in the genetic algorithm, that is to say determine the size of the population, the times of evolution and the probability of crossover and mutation.

In the optimization solving of a multi-peak function using GAs, there are usually only a few of the individual optimal solutions to be found, and they even tend to be the local optimal solution. Sometimes we hope that the algorithm can find all the best solutions, including the local optimal solution and the global optimal solution. The conventional GA is powerless in this context. A lead to the cause of the problem is that the selecting strategy of GAs lacks the mechanisms for biodiversity conservation. In the optimization solving of a multi-peak function, the diversity of individuals in groups ensures that all the optimal solutions of the problems can be found, including the local optimal solution and the global optimal solution. This is the advantage of niche GAs. So, we introduced the niche GAs based on selection previously (Shi et al., 2000). The idea of the niche GAs is that when the filial generation has the higher fitness than the father generation, it will replace the father generation to be the next generation, and otherwise the father generation will be the next generation.

In this paper, the fitness function may have more than one peak or maximum. The conventional GAs may only get some locally optimal solutions. But, we want to get the globally optimal solution. The niche GAs can solve the problem perfectly.

Some details of the algorithm in this paper are:

1. The code scheme: the true unit number of the structure is used to encode.
2. The fitness function: in this paper, the fitness function is designed based on the CMSE. The standard residue in every CMSE equation is

$$e_m = \frac{\hat{b}_m - b_m}{b_m}, \quad m = 1, \ldots, N_q. \tag{12}$$

   The modulus of $\boldsymbol{e}$ can be calculated by

$$\|\boldsymbol{e}\| = \sqrt{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}} = \sqrt{\left(e_1^2\right) + \cdots + \left(e_{N_q}\right)^2}, \tag{13}$$

   since the value of $\|\boldsymbol{e}\|$ can be an indicator to all fitness of N_q equations. In fact, the value of $\|\boldsymbol{e}\|$ will be very small if the guessed damage scenario is near the true damage scenario. So, the objective function is

$$f(x) = \|\boldsymbol{e}\| = \sqrt{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}}. \tag{14}$$

   The fitness function is defined as $F(x) = 100 - 10 \times \log_{10} \|\boldsymbol{e}\|$.
3. The selection operator: the proportional selection is used.
4. The crossover operation: the arithmetic crossover operation is used. The parameter is 0.7.
5. The mutation operator: the uniform mutation operator is used. The parameter is 0.07.

The specific implementation using niche GAs for damage detection in this paper follows as: (a) set the counter of the generation as t, define an array $P0$ to store the population that is in descending order, and initialize them, (b) produce the initial population randomly, the size of the population being M, (c) calculate the fitness F_i of every individual ($i = 1, 2, \ldots, M$), (d) collate the fitness of the individual and store it as an array $P1$, (e) replacement policy of niche GAs: hold the individual that has the higher fitness in $P0$ and $P1$, produce the next population $P(t)'$, (f) crossover: do the crossover operation using the nonuniform arithmetic

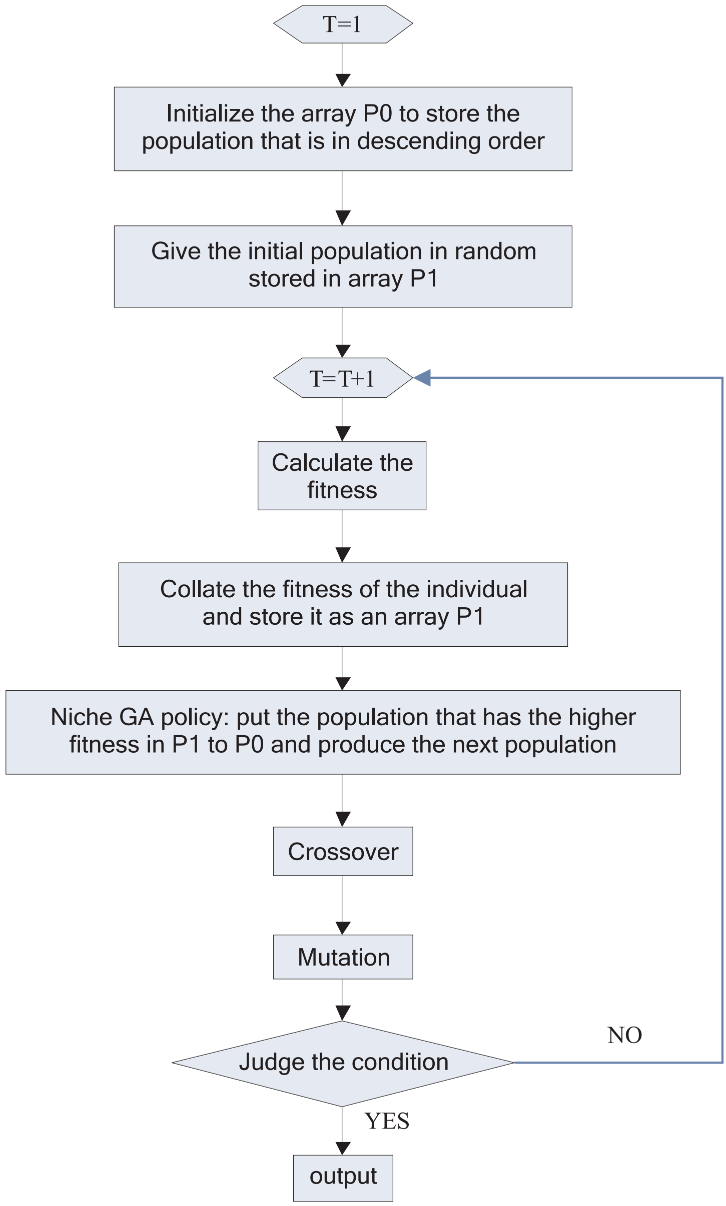

Figure 1. Schematic of niche GAs.

crossover operator and produce the population $P(t)''$, (g) mutation: do the mutation to $P(t)''$ using the uniform mutation operator and get the population $P(t)'''$, (h) judge the end condition: if the condition is not fulfilled, change the generation counter and then do the steps (c)–(h), otherwise end the program and output the result. The procedure of the program is shown in Figure 1.

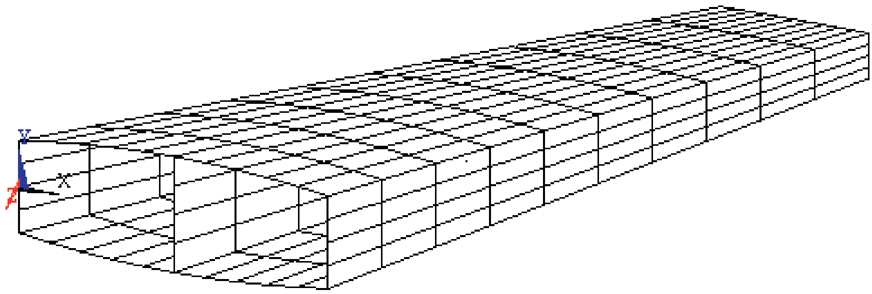

Figure 2. Model of an airfoil case.

4. NUMERICAL STUDY

In order to apply the proposed method to the structural damage detection, this paper provides a numerical study of damage detection in the case of an airfoil model, which is a classical airfoil structure. This study can help to develop techniques for damage detection of working aircraft. The model in this paper is a composite material airfoil case shown in Figure 2.

The case model consists of upper and bottom covers, walls and beams. The section becomes smaller through the Z direction. The cover is made of composite material T300/QY8911. The thickness of every layer is 0.01 mm. The total number of layers is 46. All layer directions are:

$$45/90/-45/0/45/0/-45/0/90/45/-45/-45/0/90/45/-45/90/0/0/-45/45/90/45,$$

where only one-half of the layers are given because of the symmetry. The wall and beam are made of composite material; every layer has the same material as the cover.

The finite-element model of the structure is built using ANSYS software. The cover and the wall are simulated by shell elements and the long truss is simulated by a beam element. The total number of the structural elements is 360. The front 320 elements are covers and walls. The last 40 elements are trusses.

The single damage status and complex damage status are studied. The supposed damage locations are shown in Figure 3. Suppose that the damaged element has some stiffness loss. First, the location of the damaged element is searched for using niche GAs; then, the damage severity is calculated. Three types of single damage status and three types of complex damage status are studied, where the complex status is constructed by two damaged elements. Of course, more complex damage statuses with more damaged elements can also be detected using the present methods.

First, the third and the fourth modes of the intact structure and the first mode of the damaged structure are used. Theoretically, one can use any order mode. However, through more numerical studies, we found that the modes with the similar transmutation except for the torsional modes will benefit the detection effect. The first four modes of the structure are shown in Figure 4.

The results of damage detection for single damage status are shown in Table 1. The results of damage detection for complex damage status are shown in Table 2. The real damage

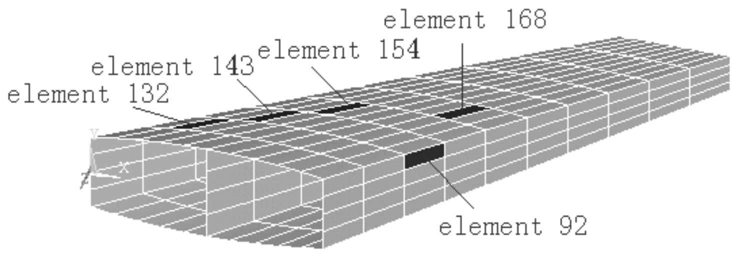

Figure 3. Damage status of the composite airfoil case.

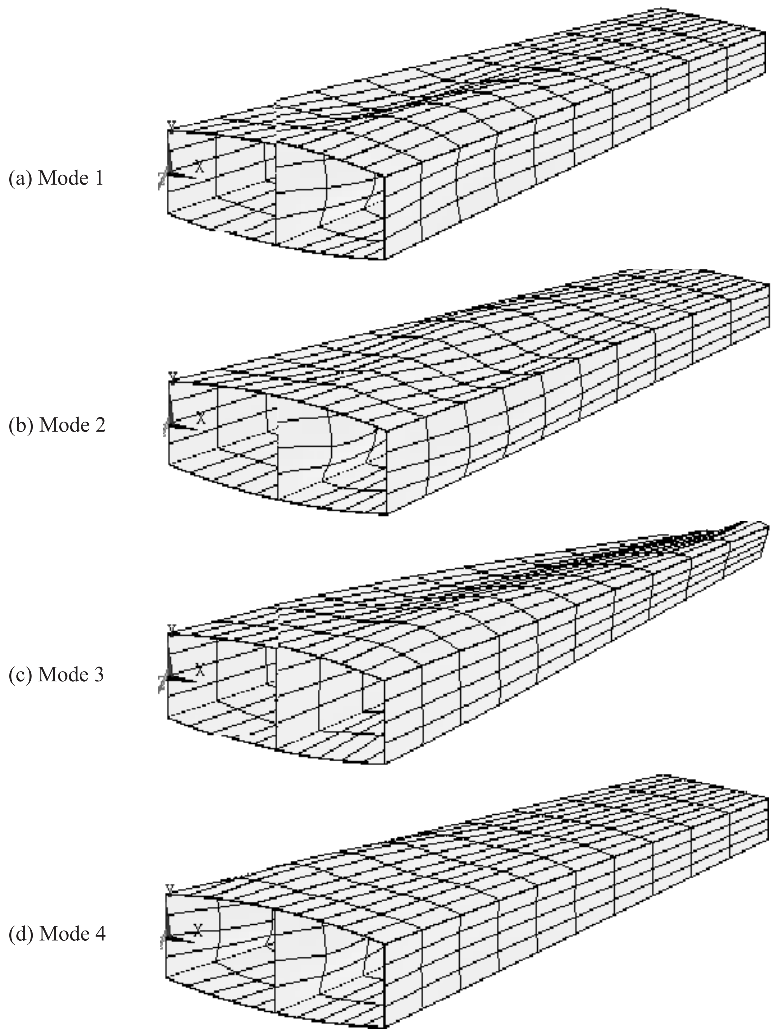

Figure 4. First four models of the composite airfoil case.

Table 1. Results of damage detection for single damage status.

Damage location (element number)	92	143	168
Evolution generations for niche GAs	25	14	10
Given damage degree	0.03%	0.03%	0.03%
Detected damage degree	0.030274%	0.029871%	0.030052%

Table 2. Results of damage detection for complex damage status.

Damage location (element number)	92 and 154	132 and 154	143 and 168
Evolution generations for niche GAs	52	46	37
Given damage degree	0.14%	0.14%	0.14%
Detected damage degree	0.1389%	0.1388%	0.1386%
	0.1390%	0.1390%	0.1388%

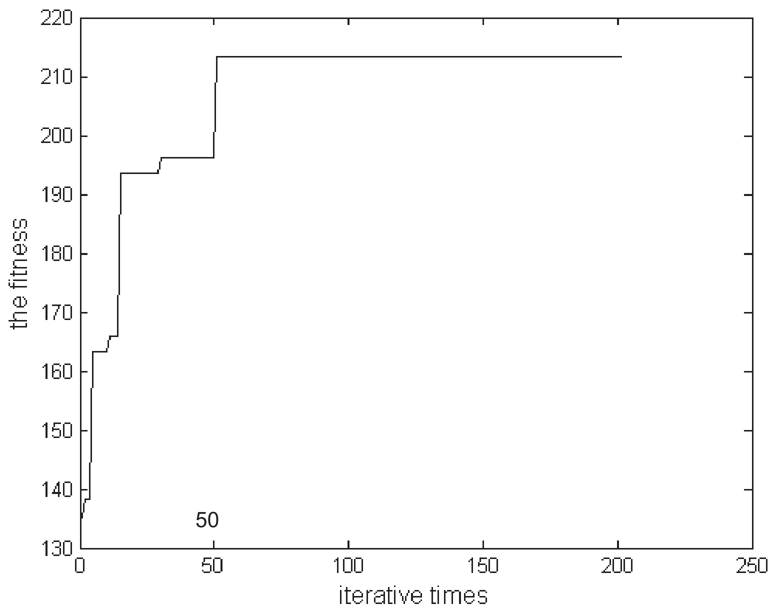

Figure 5. Fitness evolution of objective function for niche GAs in complex damage status.

degree in the tables refers to the stiffness loss of the damaged model. These results show that the damage degree of the structures can be correctly detected by the CSME with the niche GA method. The niche GA operation processing for the first complex damage status is shown in Figure 5.

In order to compare the calculating efficiency between the niche GA method and the direct guess method in searching for structural damage locations, the calculation workloads of two methods are analyzed as follows.

For single damage, the operation number of the niche GAs is $(25 + 16 + 10) \div 3 \times 16 = 272$, and that of the direct guess method is $C_{320}^{1} = 320$.

For complex damage, the operation number of the niche GAs is (52 + 46 + 37) ÷ 3 × 60 = 2700, and that of the direct guess method is $C_{320}^2 + 320 = 51360$.

Obviously, when the damage status is complex, the niche GA method will have much more efficiency than the direct guess method in searching for structural damage locations. Therefore, the CMSE method combined with the niche GAs is a good method for damage detection of complex structures.

5. CONCLUSIONS

In this paper, the damage detection of the airfoil case is studied using the CMSE combined with the niche GAs. Results show that damage location and degree can be easily detected simultaneously; the CMSE also exhibits remarkable advantages. (1) In the detection procedure, only one modal frequency of the damaged structure is used, so it is easy to obtain the information from the damaged structure. (2) A very good accuracy of damage degree detection can be achieved. On the other hand, the niche GAs possess good efficiency in searching for the structural damage location. Therefore, the method using the CMSE combined with the niche GAs proposed in this paper may be widely available in damage detection of complex structures.

Acknowledgements. The authors would like to thank the support by the Natural Science Foundation of China under the grants 50875212 and 50878184, the Specialized Research Fund for the Doctoral Program of Higher Education of China under the grant 200806990019 and the National High Technology Research and Development Program ('863' Program) of China under the grant 2006AA04Z437.

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