

ON SOLUTION TO MODIFIED UNBALANCED TRANSPORTATION PROBLEM

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Abstract

A new heuristic method of obtaining an initial basic feasible solution (IBFS) to solve modified unbalanced transportation problem is proposed which reduces number of iterations to reach optimality. A given unbalanced transportation problem is converted to a modified unbalanced transportation problem by increasing demand/supply of a origin/destination and the same is solved by the proposed methods. A new algorithm alongwith an illustrative numerical example to find solution to the transportation problem having m origins and n destinations is discussed.¹

1 INTRODUCTION

The transportation problem is famous in operations research due to its wide applications in different walks of life. Many solution procedures have been developed in the literature for solving balanced transportation problem [1,2,3]. They solve an unbalanced transportation problem, after balancing it by creating dummy origin and/or destination as required. Kore and Thakur [4] have proposed another method, which discards the need of balancing the given problem.

In modern times, due to globalization and industrial recession, more transportation problems turn out to be unbalanced. It means that either supply is more or demand is less or vice-a-versa. In this paper a method is proposed to solve such type of problems. The usual method commonly discussed in the literature for solving unbalanced transportation problem suggests the addition of dummy origin /destination with zero costs to balance the given problem and apply known procedure to get Optimal Basic Feasible Solution (OBFS). The OBFS thus obtained naturally contains some positive allocations with no cost. As dummy origin/destination are

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artificial, just introduced to take advantage of the algorithm for solving Balanced Transportation Problem (BTP). The allocations from dummy origins and those to dummy destinations are, in fact, not carried out.

Kore and Thakur [4] proposed another method to solve this problem without balancing it.

In this paper a new method of modifying the given UTP and then obtaining its IBFS is proposed. The optimum solution of modified UTP thus obtained has resulted in increasing the optimum cost. But it means that if we want to satisfy extra demand at destination or supply at origin, then we have to bear extra cost.

Mathematical formulation of our modified UTP under study is given in section 2 and algorithm to solve it, is discussed in section 3. An illustrative numerical example is given in section 4, and section 5 contains concluding remarks.

2 MATHEMATICAL FORMULATION

Let there be m origins O_i having $a_i (i = 1, 2, \dots, m)$ units of supplies respectively, which are to be transported to n destinations D_j s with $b_j (j = 1, 2, \dots, n)$ units of demand respectively. Let C_{ij} be the cost of shipping one unit of commodity from origin i to destination j . If x_{ij} represents the units shipped from origin i to destination j , then problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand conditions. Mathematically the problem can be stated as,

$$\min z = \sum_{i=1}^m \text{suchthat} \sum_{j=1}^n c_{ij}x_{ij}, \sum_{j=1}^n x_{ij} = a_i \quad i = 1, 2, \dots, m \text{(supply constraints)}$$

$$\sum_{i=1}^m x_{ij} = b_j \quad j = 1, 2, \dots, n \text{(demand constraints) and } x_{i,j} \geq 0 \text{ for all } i \text{ and } j,$$

When total supply equals total demand *i.e.* $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$, then the problem is called BTP; otherwise it is an unbalanced transportation problem (UTP)

2.1 Mathematical Models for Modified Unbalanced Transportation Problems

Consider the unbalanced transportation problem, which means that when

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j,$$

compute,

$$\epsilon = \left| \sum_{i=1}^m a_i - \sum_{j=1}^n b_j \right|, \quad S_i = \sum_{j=1}^n c_{ij}, \quad T_j = \sum_{i=1}^m c_{ij}.$$

(A), Total allotment of ϵ :

Case 2.1 : If $\sum_{i=1}^m a_i > \sum_{j=1}^n b_j$, then identify minimum of T_j which occurs at r^{th} destination. Thus demand at r^{th} destination will be increased by ϵ units and order of transportation matrix remains unaltered. This can be stated as,

$$\begin{aligned} \min z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \text{ such that } \sum_{j=1}^n x_{ij} = a_i, \quad i = 1, 2, \dots, m, \\ \sum_{i=1}^m x_{ij} &= b_j, \quad j = 1, 2, \dots, n, \text{ and } x_{ij} \geq 0 \text{ for all } i \text{ and } j, \\ \text{where } b'_j &= b_j \quad \text{for } j \neq r, \\ &= b_j + \epsilon \quad \text{for } j = r. \end{aligned}$$

Case 2.2 : If $\sum_{i=1}^m a_i < \sum_{j=1}^n b_j$, then identify minimum of S_i which occurs at k^{th} origin and corresponding k^{th} origins supply will be increased by ϵ units. This can be stated as follows,

$$\begin{aligned} \min z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \text{ such that } \sum_{j=1}^n x_{ij} = a'_i, \quad i = 1, 2, \dots, m, \\ \sum_{i=1}^m x_{ij} &= b_j, \quad j = 1, 2, \dots, n, \text{ and } x_{ij} \geq 0 \text{ for all } i \text{ and } j, \\ \text{where } a'_i &= a_i \quad \text{for } i \neq k, \\ &= a_i + \epsilon \quad \text{for } i = k. \end{aligned}$$

Case (B)

$\min_{i,j}(a_i, b_j) = \delta$, which occurs at i^{th} origin and j^{th} destination or at more places. The demand / supply at these rows/ columns will be $\delta + \epsilon_j$ where ϵ_j 's are equal proportions of ϵ 's depending upon repetitive occurrence of δ . This can be mathematically stated as follows,

$$\begin{aligned} \min z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \text{ such that } \sum_{j=1}^n x_{ij} = a'_i, \quad i = 1, 2, \dots, m, \\ \sum_{i=1}^m x_{ij} &= b'_j, \quad j = 1, 2, \dots, n, \text{ and } x_{ij} \geq 0 \text{ for all } i \text{ and } j, \end{aligned}$$

Where

$a'_i = \delta + \epsilon_i$ if min occurs at i^{th} origin and $= a_i$ otherwise, and
 $b'_j = \delta + \epsilon_j$ if min occurs at j^{th} destination and $= b_j$ otherwise.

Further $\sum_{i=1}^m \epsilon_i = \epsilon$, $\sum_{j=1}^n \epsilon_j = \epsilon$.

(C) Partial allotment of ϵ :

Case (2.3) When $\sum_{i=1}^m a_i > \sum_{j=1}^n b_j$,

Allocate ϵ to b_j s in such a way that arrange b_j s in ascending order and assign ϵ_j s in descending order such that

$$\sum_{j=1}^n \epsilon_j = \epsilon \text{ and } \epsilon_j \text{s are having positive integral values only.}$$

This can be stated as,

$$\min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad \text{such that } \sum_{j=1}^n x_{ij} = a_i^*, \quad i = 1, 2, \dots, m,$$

$$\sum_{i=1}^m x_{ij} = b_j', \quad j = 1, 2, \dots, n, \quad \text{and}$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j,$$

$$\text{where } a_j^* = a_i + \epsilon_j \quad \text{and} \quad \sum_{j=1}^n \epsilon_j = \epsilon$$

Case (2.4) :When $\sum_{i=1}^m a_i < \sum_{j=1}^n b_j$, allocate ϵ to a_i s in such a way that a_i s have been arranged in ascending order and assigning ϵ_i s in descending order so that

$\sum_{i=1}^m \epsilon_i = \epsilon$ and ϵ_i s are having positive integral values only. This is given as,

$$\min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad \text{such that } \sum_{j=1}^n x_{ij} = a_i^*, \quad i = 1, 2, \dots, m,$$

$$\sum_{i=1}^m x_{ij} = b_j', \quad j = 1, 2, \dots, n, \quad \text{and } x_{ij} \geq 0 \quad \text{for all } i \text{ and } j,$$

$$\text{where } a_i^* = a_i + \epsilon_i \quad \text{and} \quad \sum_{i=1}^m \epsilon_i = \epsilon.$$

3 ALGORITHM

The following algorithm gives initial basic feasible solution for all cases of modified unbalanced problem.

STEP 1: Consider the cost matrix $[C_{ij}]$ of given UTP.

STEP 2: Find the sum of costs for each origin and destination as

$$S_i = \sum_{j=1}^n C_{ij}, \quad T_j = \sum_{i=1}^m C_{ij}, \quad \text{and } \epsilon = \left| \sum_{i=1}^m a_i - \sum_{j=1}^n b_j \right|.$$

STEP 3: Convert the problem to modified UTP depending upon the models presented in section 2. If it belongs to case 2.1 go to step 4. If it belongs to case 2.2 go to step 7. If it belongs to case B go to step 10, if it belongs to case 2.3 or 2.4 go to step 10.

STEP 4: If $\min T_j$ occurs at r^{th} destination then identify $\min C_{ir}$ and allocate there maximum feasible amount by which either supply/demand at i^{th} origin / r^{th} destination or both get exhausted.

STEP 5: Recompute S_i and T_j for reduced matrix after deleting the exhausted origin / destination.

STEP 6: Repeat step 4 and 5 until rim requirements are satisfied.

STEP 7: If $\min S_j$ occurs at k^{th} origin then identify minimum of C_{kj} and allocate maximum feasible amount by which demand /supply at k^{th} origin or j^{th} destination or both get exhausted.

STEP 8: Recompute S_i and T_j for reduced matrix after deleting the exhausted origin or destination.

STEP 9: Repeat steps 7 and 8 until rim requirements are satisfied.

STEP 10: Identify minimum of S_j and T_j . Let it be at k^{th} origin or j^{th} destination, or at more places. Thus k^{th} row / r^{th} column or both enter the basis. If tie occurs then select any row or column arbitrarily.

STEP 11: Identify min of C_{kj} or C_{ir} or both and allocate their maximum feasible amount by which either supply at k^{th} origin / demand at r^{th} Destination or at both get exhausted. If tie occurs while selecting C_{kj} and C_{ir} select them arbitrarily.

STEP 12: Repeat step 10 and 11 to reach rim requirements.
To obtain OBFS use MODI method.

4 NUMERICAL EXAMPLE

Consider the following T. P. having 4 origins and 3 destinations.

	D_1	D_2	D_3	a_i	S_j
F1	3	4	6	100	13
F2	7	3	8	80	18
F3	6	4	5	90	15
F4	7	5	2	120	14
				390	
b_j	110	110	60	280	
T_j	23	16	21		

4.1 Total Allotment of ϵ ; $\epsilon = 110$.

Case A1 : Since $\min T_j = 16$ which occurs at second destination we add ϵ at destinations and apply the method in section 3. OBFS is given by $X_{11} = 100, X_{22} = 80, X_{32} = 90, X_{41} = 10, X_{42} = 50, X_{43} = 60, \text{Min}Z = 1340$.

To reach optimality no iterations are required (*i. e.* IBFS is OBFS).

Case B : $\text{Min}(a_i, b_j) = 60$ at 3 rd destination so ϵ is added to b_3 and applying the method OBFS is given by

$$X_{11} = 100, X_{22} = 80, X_{31} = 10, X_{32} = 30, X_{33} = 50, X_{43} = 120, \text{Min}Z = 1210.$$

Here also IBFS is OBFS.

Case C1 :

(i) If $\epsilon_1 = 22, \epsilon_2 = 22, \epsilon_3 = 66$, then $b_1 = 132, b_2 = 132, b_3 = 126$, with $\text{min} Z = 1210$.

(ii) $\epsilon_1 = 35, \epsilon_2 = 35, \epsilon_3 = 40$, $\text{min} Z = 1290$.

(iii) $\epsilon_1 = 44, \epsilon_2 = 44, \epsilon_3 = 22$, $\text{min} Z = 1382$.

(iv) $\epsilon_1 = 15, \epsilon_2 = 15, \epsilon_3 = 80$, $\text{min} Z = 1230$.

In all these cases IBFS is itself OBFS.

5 CONCLUDING REMARKS

5.1 Proposed method of obtaining IBFS is also useful for getting IBFS of BTP.

5.2 This is the method which takes into account whole cost of each origin, and destination for allotment by which it is possible to reduce the number of iterations to acquire OBFS.

5.3 This method also helps to resolve degeneracy.

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