Auction-Based Distributed Resource Allocation for Cooperation Transmission in Wireless Networks

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***Abstract*— Cooperative transmission can greatly improve communication system performance by taking advantage of the broadcast nature of wireless channels. Most previous work on resource allocation for cooperation transmission is based on centralized control. In this paper, we propose two share auction mechanisms, the SNR auction and the power auction, to distributively coordinate the resource allocation among users. We prove the existence, uniqueness and effectiveness of the auction results. In particular, the SNR auction leads to a fair resource allocation among users, and the power auction achieves a solution that is close to the efficient allocation (e.g., an average of 95% of the maximum total rate increase in a two-user network).**

I. Introduction

The cooperative communication concept has recently been proposed [1], [2] as a means to take advantage of the broadcast nature of wireless channels by using relays as virtual antennas to provide the advantages of multiple input multiple output (MIMO) transmission. Various cooperative protocols such as amplify-and-forward, decode-and-forward, and estimate-and-forward have been proposed (e.g., [1]–[4]). The work in [5] analyzes cooperative schemes involving dirty paper coding, while energy-efficient transmission is considered for broadcast networks in [6]. In [7], the authors evaluate cooperative-diversity performance when the best relay is chosen according to the average SNR as well as the outage probability of relay selection based on the instantaneous SNR. In [8], the authors propose a distributed relay selection scheme that requires limited network knowledge and is based on instantaneous SNRs. In [9], relay section, power management, and subcarrier assignment are investigated for multiuser OFDM networks.

In order to maximize the performance of the cooperative transmission network, we need to consider the global channel information, including those between source-destination, source-relay, and relay-destination. Most existing work in this area is based on centralized control, which requires considerable overhead for signalling and measurement. In this paper, we focus on designing *distributed* resource allocation algorithms for cooperative networks. In particular, we want to answer the following two questions: 1) "When to relay", i.e., when it is beneficial to utilize the relay? 2) "How to relay", i.e., how the relay allocate resources among multiple competing users?

We answer the two questions by designing an *auction*-based framework for cooperative resource allocation. Auction has recently been introduced to several areas of wireless communications (e.g., time slot allocation [11] and power control [12], [13]). This paper is closely related to the auction mechanisms proposed in [13], where the authors considered distributed interference management in a cognitive radio network without relay. In that case, a user can not obtain positive transmission rate unless it is allocated part of the shared system resource. The problem considered here is significantly different due to the existence of relay and the possibility of achieving positive performance without obtaining for the shared system resource (i.e., relay's transmission power).

We consider two network objectives here: *fairness* and *efficiency*. Both might be difficult to achieve even in a centralized fashion. This is because users' rate increases are non-smooth and non-concave in the relay's transmission power, thus the performance optimization problem is non-convex. We propose two auction mechanism, SNR auction and power auction, which achieve the desired network objectives in a distributed fashion. In both auctions, each user decides "when to relay" based on a simple threshold policy that is locally computable. The question of "how to relay" is answered by a simple weighted proportional allocation among users who use the relay. Simulation results show that the power auction achieves an average of 95% of the maximum rate increase in a two-user network over a wide range of relay locations. The SNR auction achieves a fair allocation among users but leads to a much lower total rate increase. This reflects a fairness-efficiency tradeoff that can be exploited by a system designer.

This paper is organized as follows. The system model and network objectives are given in Section II. Some background of auction mechanisms is given in Section III, including a brief discussion of the Vickrey-Clarke-Groves (VCG) auction. In Section IV, two share auction mechanisms are proposed, their mathematical properties are analyzed, and mechanisms for achieving auction results in a distributed fashion are shown. Simulation results are discussed in Section V and conclusions are drawn in Section VI. Due to space limitations, all proofs are omitted in this conference version of the paper.

II. System Model and Network Objectives

A. System Model

We will employ the amplify-and-forward (AF) cooperation protocol [2] as our system model. Other cooperation protocols can be analyzed in a similar fashion. The system diagrams are shown in Fig. 1, where there are one relay node r and a set $\mathcal{I} = (1, ..., I)$ of source-destination pairs. We also refer to pair i as *user* i, which includes source node s_i and destination node d_i.

For each user i, the cooperative transmission consists of two phases. In Phase 1, source s_i broadcasts its information to both destination d_i and the relay r. The received signals Y_{s_i,d_i} and $Y_{s_i,r}$ at destination d_i and relay r are given by

$$Y_{s_i,d_i} = \sqrt{P_{s_i} G_{s_i,d_i}} X_{s_i} + n_{d_i}, \tag{1}$$

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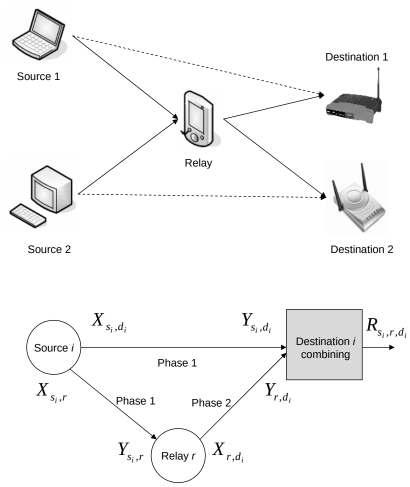

Fig. 1. System model for cooperation transmission

and

$$Y_{s_i,r} = \sqrt{P_{s_i} G_{s_i,r}} X_{s_i} + n_r, \tag{2}$$

where P_{s_i} represents the transmit power of source s_i, X_{s_i} is the transmitted information symbol with unit energy at Phase 1 at source s_i, G_{s_i,d_i} and $G_{s_i,r}$ are the channel gains from s_i to destination d_i and relay r, respectively, and n_{d_i} and n_r are additive white Gaussian noises. Without loss of generality, we assume that the noise level is the same for all of the links, and is denoted by σ^2. We also assume that the channels are stable over each transmission frame.

The signal-to-noise ratio (SNR) that results from source s_i to destination d_i in Phase 1 can be expressed by

$$\Gamma_{s_i,d_i} = \frac{P_{s_i} G_{s_i,d_i}}{\sigma^2}. \tag{3}$$

For amplify-and-forward cooperative transmission, in Phase 2 relay r amplifies $Y_{s_i,r}$ and forwards it to destination d_i with transmitted power P_{r,d_i}. The received signal at destination d_i is

$$Y_{r,d_i} = \sqrt{P_{r,d_i} G_{r,d_i}} X_{r,d_i} + n'_{d_i}, \tag{4}$$

where

$$X_{r,d_i} = \frac{Y_{s_i,r}}{|Y_{s_i,r}|} \tag{5}$$

is the energy-normalized transmitted signal from source s_i to destination d_i at Phase 1, G_{r,d_i} is the channel gain from relay r to destination d_i, and n'_{d_i} is the received noise at Phase 2. Substituting (2) into (5), we can rewrite (4) as

$$Y_{r,d_i} = \frac{\sqrt{P_{r,d_i} G_{r,d_i}} (\sqrt{P_{s_i} G_{s_i,r}} X_{s_i,d_i} + n_r)}{\sqrt{P_{s_i} G_{s_i,r} + \sigma^2}} + n'_{d_i}. \tag{6}$$

Using (6), the relayed SNR at destination d_i for source s_i, which is helped by relay node r, is given by:

$$\Gamma_{s_i,r,d_i} = \frac{P_{r,d_i} P_{s_i} G_{r,d_i} G_{s_i,r}}{\sigma^2 (P_{r,d_i} G_{r,d_i} + P_{s_i} G_{s_i,r} + \sigma^2)}. \tag{7}$$

If source s_i performs only the direct transmission in Phase 1 during the entire transmission frame, it achieves a total information rate of

$$R_{s_i,d_i} = W \log_2 (1 + \Gamma_{s_i,d_i}), \tag{8}$$

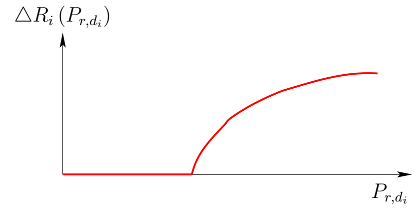

Fig. 2. Rate increase as function of relay transmission power

where W is the signal bandwidth. On the other hand, if source s_i makes use of both Phases 1 and 2, it can then achieve a total information rate at the output of maximal ratio combining as

$$R_{s_i,r,d_i} = \frac{1}{2} W \log_2 (1 + \Gamma_{s_i,d_i} + \Gamma_{s_i,r,d_i}). \tag{9}$$

The coefficient $1/2$ is used to model the fact that cooperative transmission will occupy one out of two phases (e.g., time, bandwidth, code). Since Γ_{s_i,r,d_i} is the extra SNR increase compared with the direct transmission, we also denote

$$\triangle \mathtt{SNR}_i \triangleq \Gamma_{s_i,r,d_i}. \tag{10}$$

Based on (8) and (9), the rate increase that user i obtains by cooperative transmission is

$$\triangle R_i = \max \{R_{s_i,r,d_i} - R_{s_i,d_i}, 0\}, \tag{11}$$

which is nonnegative since the source can always choose not to use the relay and thereby obtain zero rate increase. $\triangle R_i$ is a function of the channel gains of the source-destination, source-relay and relay-destination links, as well as the transmission power of the source and the relay. In particular, $\triangle R_i$ is a non-decreasing, non-smooth, and non-concave function of the relay transmission power P_{r,d_i}, as illustrated in Fig. 2.

We assume that the source transmission power P_{s_i} is fixed for each user i, as well as the relay's total power, P. We allocate the relay power among various users, $\boldsymbol{P}_r \triangleq (P_{r,d_1}, ..., P_{r,d_I})$, such that the total power constraint is not violated, i.e.,

$$\boldsymbol{P}_r \in \mathcal{P}_r \triangleq \left\{ \boldsymbol{P}_r \middle| \sum_i P_{r,d_i} \leq P, P_{r,d_i} \geq 0, \forall i \in \mathcal{I} \right\}. \tag{12}$$

The network objectives that we want to achieve are defined in the next subsection.

B. Network Objectives: Efficiency and Fairness

We consider two different network objectives: *efficiency* and *fairness*. An efficient power allocation $\boldsymbol{P}_r^{\text{efficient}}$ maximizes the total rate increase of all users by solving the following problem,

$$\max_{\boldsymbol{P}_r \in \mathcal{P}_r} \sum_{i \in \mathcal{I}} \triangle R_i (P_{r,d_i}). \tag{13}$$

In many cases, an efficient allocation discriminates against users who are far away from the relay. To avoid this, we also consider a fair power allocation $\boldsymbol{P}_r^{\text{fair}}$, which solves the following problem

$$\max_{\boldsymbol{P}_r \in \mathcal{P}_r, c>0} \sum_{i \in \mathcal{I}} P_{r,d_i} \tag{14}$$

$$\text{subject to } \frac{\triangle R_i (\triangle \mathtt{SNR}_i)}{\partial (\triangle \mathtt{SNR}_i)} = c, \forall i \in \mathcal{I} \text{ where } \triangle \mathtt{SNR}_i > 0.$$

First of all, a fair allocation needs to be Pareto optimal, i.e., no user's rate can be increased without decreasing the rate of another user. More importantly, a fair allocation equalizes the marginal rate increases in terms of $\triangle \mathtt{SNR}_i$ for any user i with a positive rate increase. For example, when the channel gain G_{s_i,d_i} is the same for all user i, the constraint in Problem (14) means that $\triangle \mathtt{SNR}_i$ is the same for all users with positive rate increase. A numerical example of such fair allocation is shown in Section V.

Remark 1: An efficient or fair allocation need not fully utilize the resource at the relay, i.e., $\sum_{i\in\mathcal{I}} P_{r,d_i}$ can be less than P. This could happen, for example, when the relay is far away from all users so that allowing the relay to transmit half of the time will only decrease the total achievable rate. This is very different from most previous network resource allocation problems (including [13]), in which the network performance is maximized only if the resource is fully utilized.

Since $\triangle R_i\left(P_{r,d_i}\right)$ is non-smooth and non-concave, it is well known that Problems (13) and (14) are NP hard to solve even in a centralized fashion. In the rest of the paper, we will propose two auction mechanisms that can (approximately) solve these problems under suitable technical conditions in a distributed fashion.

III. Auction Mechanism Basics and VCG Auction

An auction is a decentralized market mechanism for allocating resources in an economy. An auction consists of three key elements: 1) The *good*, or the resource to be allocated. 2) An *auctioneer*, who determines the allocation of the good according to the auction rules. 3) A group of *bidders*, who want to obtain the good from the auctioneer. The interactions and outcome of an auction are determined by the *rules*, which include four components: 1) The *information* the auctioneer and bidders know before the auction starts. 2) The *bids* submitted to the auctioneer by the bidders. 3) The *allocation* determined by the auctioneer based on the bids. 4) The *payments* payed by the bidders to the auctioneer as functions of bids and allocations.

In the cooperative network considered here, it is natural to design auction mechanisms in which the *good* is the relay's total transmit power P, the auctioneer is the relay, and the bidders are the users. One well known auction mechanism that achieves the efficient allocation is the VCG auction [10].

In a VCG auction, the users are asked to reveal their rate increase functions $\triangle R_i\left(P_{r,d_i}\right)$ (as bids) simultaneously to the relay, based on which the relay can determine the efficient resource allocation by solving Problem (13) and allocating accordingly. The relay then asks the each user i to pay for the "performance loss" of other users due to user i's participation in the auction, which involves solving one additional optimization problem for each user. Details of the VCG auction can be found in [10]. It is well know that it is a (weakly) dominant strategy for the users to bid truths in the VCG auction, i.e., revealing their true rate increase functions. As a result, the VCG auction achieves the efficient allocation.

The limitations of the VCG auction are as follows. First, the users need to submit the complete rate increase functions to the relay, which involves revealing their complete private information (e.g, channel gain G_{s_i,d_i} and transmission power P_{s_i}). This might be overheard by other users and so can lead to security problems. Also, accurately specifying the rate increase functions requires much signaling overhead and communication bandwidth, which may significantly reduce the network performance. Furthermore, it is usually computationally expensive for the relay to solve $I+1$ nonconvex optimization problems.

Due to these concerns, next we propose two simpler share auctions, the *SNR auction* and the *power auction*. Both auctions are similar to the auction mechanisms proposed in [13]. However, due to the unique characteristics of the relay network, especially the non-smooth and non-concave nature of the rate increase function (e.g., Fig. 2), the analysis is more involved and the results are very different from [13].

IV. Share Auction

The main advantages of the two proposed auctions in this section are the simplicities of bids and allocation. The rules of the two auctions are described below, with the only difference being in payment determination.

Share Auction (SNR Auction and Power Auction)

- *Information*: Besides the public and local information (i.e., $W, P, \sigma^2, P_{s_i}, G_{s_i,d_i}$), each user i also knows the channel gains $G_{s_i,r}$ and G_{r,d_i}, either through measurement or explicit feedback from relay r. The relay announces a positive *reserve bid* $\beta > 0$ and a *price* $\pi > 0$ to all users before the auction starts.
- *Bids*: User i submits $b_i \geq 0$ to the relay.
- *Allocation*: The relay allocates transmit power according to
$$P_{r,d_i} = \frac{b_i}{\sum_{j\in\mathcal{I}} b_j + \beta} P. \tag{15}$$
- *Payments*: In an SNR auction, source i pays the relay $C_i = \pi \triangle \mathtt{SNR}_i$. In a power auction, source i pays the relay $C_i = \pi P_{r,d_i}$.

A bidding profile is defined as the vector containing the users' bids, $\boldsymbol{b} = (b_1, ..., b_I)$. The bidding profile of user i's opponents is defined as $b_{-i} = (b_1, ..., b_{i-1,} b_{i+1}, ..., b_I)$, so that $\boldsymbol{b} = (b_i; b_{-i})$. User i chooses b_i to maximize its payoff

$$U_i\left(b_i; b_{-i}, \pi\right) = \triangle R_i\left(P_{r,d_i}\left(b_i; b_{-i}\right)\right) - C_i\left(b_i; b_{-i}, \pi\right). \tag{16}$$

For notational simplicity, we omit the dependence on β and other system parameters.

Remark 2: If the reserve bid $\beta = 0$, then the resource allocation in (15) only depends on the ratio of the bids. A bidding profile $k\boldsymbol{b}$ (for any $k > 0$) leads to the same resource allocation as $\boldsymbol{b}$, which is not desirable in practice. That is why we need a positive reserve bid. However, the value of β is not important as long as it is positive. For example, if we increase β to $k'\beta$, then users can just scale $\boldsymbol{b}$ to $k'\boldsymbol{b}$, which leads to the same resource allocation. For simplicity, we will choose $\beta = 1$ in all the simulations in Section V.

The desirable outcome of an auction is called a *Nash Equilibrium* (NE), which is a bidding profile $\boldsymbol{b}^*$ such that no user wants to deviate unilaterally, i.e.,

$$U_i\left(b_i^*; b_{-i}^*, \pi\right) \geq U_i\left(b_i; b_{-i}^*, \pi\right), \forall i \in \mathcal{I}, \forall b_i \geq 0. \tag{17}$$

Define user i's *best response* (for fixed b_{-i} and price π) as

$$\mathcal{B}_i\left(b_{-i},\pi\right)=\left\{b_i\left|b_i=\arg\max_{\tilde{b}_i\geq 0} U_i\left(\tilde{b}_i;b_{-i},\pi\right)\right.\right\}, \qquad (18)$$

which in general could be a set. An NE is also a fixed point solution of all users' best responses. We would like to answer the following four questions for both auctions: 1) When does an NE exist? 2) When is the NE unique? 3) What are the properties of the NE? 4) How to reach the NE in a distributed fashion?

A. SNR Auction

Let's first determine the users' best responses (e.g., (18)) in the SNR auction, which clearly depend on the price π. For each user i, there are two critical price values, $\underline{\pi}_i^s$ and $\hat{\pi}_i^s$, where

$$\underline{\pi}_i^s \triangleq \frac{W}{2\ln 2\left(1+\Gamma_{s_i,d_i}+\frac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\left(P_{s_i}G_{s_i,r}+PG_{r,d_i}+\sigma^2\right)\sigma^2}\right)}, \qquad (19)$$

and $\hat{\pi}_i^s$ is the smallest positive root of equation $g_i^s(\pi)=0$, where

$$\begin{aligned} g_i^s(\pi) \triangleq\ & \pi\left(1+\Gamma_{s_i,d_i}\right) \\ & -\frac{W}{2}\left(\log_2\left(\frac{2\pi\ln 2}{W}\left(1+\Gamma_{s_i,d_i}\right)^2\right)+\frac{1}{\ln 2}\right) \end{aligned} \qquad (20)$$

has a two positive roots. Both $\underline{\pi}_i^s$ and $\hat{\pi}_i^s$ can be caluclated based on local information. Now we are ready to characterize the best response function of each user i.

Theorem 1: In an SNR auction, the unique best response function is

$$\mathcal{B}_i\left(b_{-i},\pi\right)=f_i^s(\pi)\left(b_{-i}+\beta\right). \qquad (21)$$

If $\hat{\pi}_i^s>\underline{\pi}_i^s$, then

$$f_i^s(\pi)= \left\{\begin{array}{cc} \infty, & \pi\leq\underline{\pi}_i^s \\ \frac{\left(P_{s_i}G_{s_i,r}+\sigma^2\right)\sigma^2}{\frac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\frac{W}{2\pi\ln 2}-1-\Gamma_{s_i,d_i}}-\left(P_{s_i}G_{s_i,r}+PG_{r,d_i}+\sigma^2\right)\sigma^2}, & \pi\in\left(\underline{\pi}_i^s,\hat{\pi}_i^s\right) \\ 0, & \pi\geq\hat{\pi}_i^s \end{array}\right. . \qquad (22)$$

If $\hat{\pi}_i^s<\underline{\pi}_i^s$, then $f_i^s(\pi)=\infty$ for $\pi<\hat{\pi}_i^s$ and $f_i^s(\pi)=0$ for $\pi\geq\hat{\pi}_i^s$.

First consider the case in which $\hat{\pi}_i^s>\underline{\pi}_i^s$, where $\mathcal{B}_i(b_{-i},\pi)$ is illustrated in Fig. 3. The price $\hat{\pi}_i^s$ determines when it is beneficial for user i to use the relay. With any price larger than $\hat{\pi}_i^s$, user i cannot obtain a positive payoff from the auction no matter what bid it submits, thus it should simply use direct transmission and achieve a rate of R_{s_i,d_i}. As a result, $\mathcal{B}_i(b_{-i},\pi)$ is discontinuous at $\hat{\pi}_i^s$. When $\pi\in(\underline{\pi}_i^s,\hat{\pi}_i^s)$, user i wants to participate in the auction, and its best response depends how much other users bid (b_{-i}). When the price is smaller than $\underline{\pi}_i^s$, user i becomes so aggressive that it demands a large SNR increase that cannot be achieved even of all the resource is allocated to it. This is reflected by an infinite bid in (22). Now consider the case in which $\hat{\pi}_i^s<\underline{\pi}_i^s$. User i either cannot obtain a positive payoff or cannot achieve the desired SNR increase, and thus the best response is either 0 or ∞.

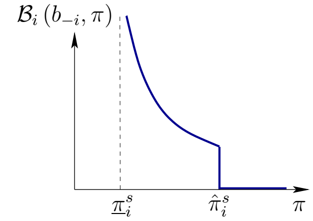

Fig. 3. User i's best response in SNR auction if $\underline{\pi}_i^s<\hat{\pi}_i^s$.

Combining (15) and (22), we know that if an NE exist, the relay power allocated for user i is

$$P_{r,d_i}(\pi)=\frac{f_i^s(\pi)}{f_i^s(\pi)+1}P, \qquad (23)$$

and $\sum_{i\in\mathcal{I}}\frac{f_i^s(\pi)}{f_i^s(\pi)+1}<1$. The strictly inequality is due to the positive reserve bid β.

Next we need to find the fixed point of all users' best responses, i.e., the NE. A trivial case would be $\hat{\pi}_i^s\leq\underline{\pi}_i^s$ for all user i, in which case there exists a unique all-zero NE $\boldsymbol{b}^*=\mathbf{0}$. The more interesting case would be the following.

Definition 1: A network is *SNR-regular* if there exists at least one user i such that $\hat{\pi}_i^s>\underline{\pi}_i^s$.

Theorem 2: Consider an SNR auction in an SNR-regular network. There exists a threshold price π_{th}^s such that a unique NE exists if $\pi>\pi_{th}^s$; otherwise no NE exists.

Remark 3: Unlike from the result in [13], the unique NE in Theorem 2 might not be a continuous function of π, due to the discontinuity of the best response function as shown in Fig. 3. This has been observed in the simulation results described in Section V. In particular, the unique NE could be all zero for any price $\pi>\pi_{th}^s$, even if the network is SNR-regular.

It can be seen that the "marginal utility equalization" property of a fair allocation (i.e., the constraint in Problem (14)) is satisfied at the NE of the SNR auction. However, there always exists some "resource waste" since some power will never be allocated to any user because of the positive reserve bid β. However, by choosing a price π larger than, but very close to, π_{th}^s, we could reduce the resource waste to a minimum and approximate the fair allocation. Formally, we define a reduced feasible set parameterized by δ as

$$\mathcal{P}_r^\delta \triangleq \left\{\boldsymbol{P}_r\left|\sum_i P_{r,d_i}\leq P(1-\delta), P_{r,d_i}\geq 0,\forall i\in\mathcal{I}\right.\right\}. \qquad (24)$$

Then we can show the following.

Theorem 3: Consider an SNR auction in an SNR-regular network, where $f_i^s(\pi)$ is continuous at π_{th}^s for each user i, and greater than zero for at least one user. For any sufficiently small δ, there exists a price $\pi^{s,\delta}$ under which the unique NE achieves the fair allocation $\boldsymbol{P}_r^{\text{fair}}$ with a reduced feasible set $\mathcal{P}_r^\delta$.

Remark 4: A sufficiently small δ makes sure that we are dealing with a regime in which $f_i^s(\pi)$ is continuous for any user i. This is also desirable in practice since we want to minimize the amount of resource wasted.

B. Power Auction

The best response function in the power auction is nonlinear and complicated in general. However, in the special case of low

SNR where Γ_{s_i,d_i} and $\triangle\mathtt{SNR}_i(b_i,b_{-i})$ are small for all i, i.e.,

$$W\log_2\left(1+\Gamma_{s_i,d_i}+\triangle\,\mathtt{SNR}_i(b_i,b_{-i})\right) \approx \frac{W}{\ln 2}\left(\Gamma_{s_i,d_i}+\triangle\,\mathtt{SNR}_i(b_i,b_{-i})\right), \quad (25)$$

$\mathcal{B}_i(b_{-i},\pi)$ has a linear form similar to that in (22). For each user, we can similarly define $f_i^p(\pi)$, $\underline{\pi}_i^p$, $\hat{\pi}_i^p$ and $g_i^p(\pi)$ as in the SNR auction case. One key difference here is that the value of $\hat{\pi}_i^p$ depends on the relative relationship between G_{s_i,d_i} and $G_{s_i,r}$. If $G_{s_i,d_i} > G_{s_i,r}$, then $\hat{\pi}_i^p = 0$ and user i never uses the relay. If $G_{s_i,d_i} < G_{s_i,r}$, $\hat{\pi}_i^p$ is the smallest positive root of $g_i^p(\pi)$. Details are omitted due to space limitations.

In terms of the existence, uniqueness and properties of the NE, we have the following.

Definition 2: A network is *power-regular* if $\hat{\pi}_i^p > \underline{\pi}_i^p$ for at least one user i.

Theorem 4: Consider a power auction in a power-regular network with low SNR. There exists a threshold price $\pi_{th}^p > 0$ such that a unique NE exists if $\pi > \pi_{th}^p$; otherwise no NE exists.

Theorem 5: Consider a power auction in a power-regular network with low SNR, where $f_i^p(\pi)$ is continuous at π_{th}^p for each user i, and greater than zero for at least one user. For any sufficiently small δ, there exists a price $\pi^{p,\delta}$ under which the unique NE achieves the efficient allocation $\boldsymbol{P}_r^{\text{efficient}}$ with a reduced feasible set $\mathcal{P}_r^{\delta}$.

C. Distributed Iterative Best Response Updates

The last question we want to answer is how the NE can be reached in a distributed fashion. Consider the SNR auction as an example. It is clear that the best response function in (22) can be calculated in a distributed fashion with limited information feedback from the relay. However, each user does not have enough information to calculate the best response of other users, which prevents it from directly calculating the NE. Nevertheless, the NE can be achieved in a distributed fasion if we allow the users to *iteratively* submit their bids based on best response functions.

Suppose users update their bids $\boldsymbol{b}(t)$ at time t according to the best response functions as in (21), based on other users' bids $\boldsymbol{b}(t-1)$ at time t, i.e.,

$$\boldsymbol{b}(t) = \boldsymbol{F}^s(\pi)\,\boldsymbol{b}(t-1) + \boldsymbol{f}^s(\pi)\,\beta, \quad (26)$$

where both $\boldsymbol{b}(t)$ and $\boldsymbol{b}(t-1)$ are column vectors, $\boldsymbol{F}^s(\pi)$ is an I-by-I matrix whose (i,j)th component equals $f_i^s(\pi)$, and $\boldsymbol{f}^s(\pi) = [f_1^s(\pi), ...f_I^s(\pi)]'$.

Theorem 6: If there exists a unique nonzero NE in the SNR auction, the best response updates in (26) converge globally to the NE from any positive $\boldsymbol{b}(0)$.

Similar convergence results can be proved for the power auction.

V. Simulation Results

We first simulate various auction mechanisms for a two-user network. As shown in Fig. 4, the locations of the two sources (s_1 and s_2) and two destinations (d_1 and d_2) are fixed at (200m,-25m), (0m,25m), (0m,-25m), (200m,25m). We fix

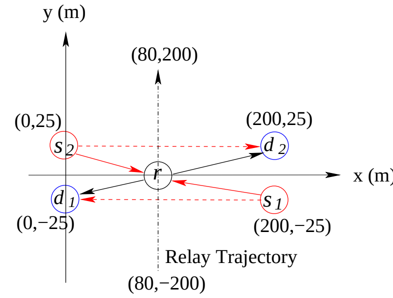

Fig. 4. A two-user cooperative network

the x coordinate of relay node r at 80m and its y coordinate varies within the range [-200m,200m]. In the simulation, the relay moves along a line. The propagation loss factor is set to 4, and the channel gains are distance based (i.e., time-varying fading is not considered here). The transmit power between a source and its destination is $P_{sd} = 0.01$W, the noise level is $\sigma^2 = 10^{-11}$W, and the bandwidth is $W = 1$MHz. The total power of the relay node is set to $P = 0.1$W.

In Fig. 5, we show the total rate increases achieved by two users in three auctions. The VCG auction achieves the efficient allocation by solving three non-convex optimization problems by the relay. For both the SNR auction and the power auction, the resource allocation depends on the choice of price π (but is independent of the reserve bid β). Every point on the curve represents an allocation in which the price is adjusted so that the total resource allocated to both users is more than $0.99P$ (unless this is not possible). The power auction achieves performance very close to that of the VCG auction. At those locations where VCG auction achieves a positive rate increase, power auction achieves a rate increase with an average of 95% of that achieved by the VCG auction. The SNR auction achieves less total rate increases but leads to fair resource allocations when both users use the relay (as can be seen in Fig. 6).

In Fig. 6, we show the individual rate increases of both users in the SNR auction and the power auction. The individual rate increases in the VCG auction are similar to that of power auction and thus not shown here. First consider the power auction. Since the relay movement trajectory is relatively closer to source s_2 than to source s_1, user 2 achieves an overall better performance compared with user 1. In particular, user 2 achieves a peak rate increase of 1.35 bits/Hz when the relay is at location 25m (y-axis), compared with the peak rate increase of 0.56 bits/Hz achieved by user 1 when relay is at location -25m. Things are very different in an SNR auction, where the resource allocation is fairer. Especially, since the distance between a source and its destination is the same for both users in our simulation, both users achieve the same positive rate increases when they both use the relay. This is the case when the relay is between locations -60m and 10m. At other locations, users just choose not to use the relay since they cannot both get equal rate increase while obtaining a positive payoff. This shows the tradeoff between efficiency and fairness.

Next, we consider the case in which there are multiple users in the network. To be specific, there are 20 users in the network, with their source nodes and destination nodes randomly and uniformly located within the square field that has the same

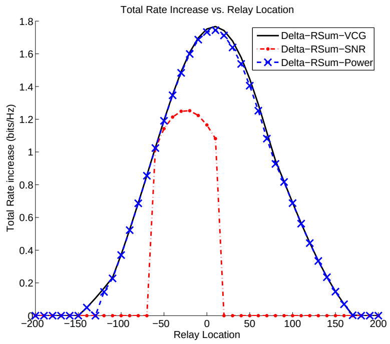

Fig. 5. Total rate increases vs. relay location (y-axis) for three auctions.

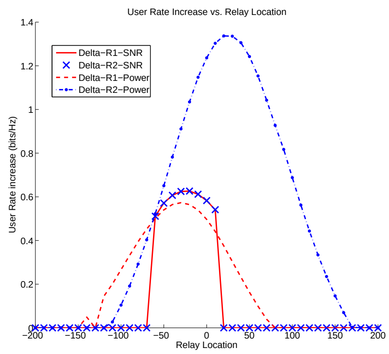

Fig. 6. Individual rate increase vs. relay location (y-axis) for SNR auction and power auction.

range of [-150m,150m] on both the x-axis and the y-axis. A single relay is fixed at the location (0m,0m). We change the total transmission power P of the relay from 0.04 W to 1 W. Figs. 7 and 8 show the corresponding simulation results. Each point in the figures represents the averaged result over 100 randomly generated network topologies. With an increasing amount of resource at the relay node, the total network rate increase improves in both auctions (as seen in Fig. 7), and the power auction achieves higher rate increase than the SNR auction. Fig. 8 shows the variance of the rate increase (among the users with positive rate increase), and it is clear that the SNR auction achieves a fairer resource allocation as indicated by the almost zero variance in all cases.

VI. Conclusions

Cooperative transmission can greatly improve communication system performance by taking advantage of the broadcast nature of wireless channels and cooperation among users. We proposed two share auction mechanisms, the SNR auction and the power auction, to distributively coordinate the relay power allocation among users. We have proven the existence and uniqueness of the Nash equilibrium in both auctions. We have shown that, under suitable conditions, the SNR auction achieves the fair allocation, while the power auction achieves the efficient allocation. Simulations results for both 2-user and multiple-user networks have been used to demonstrate the effectiveness of the auction mechanisms. In particular, the power auction achieves an average of 95% of the maximum rate in the 2 user case under a wide range of relay locations, and the SNR auction leads to performance improvement of users with little variations.

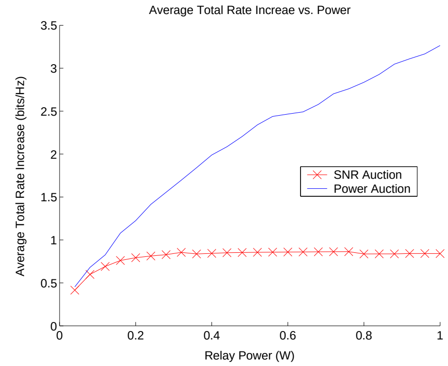

Fig. 7. Total network rate increase vs. Relay power.

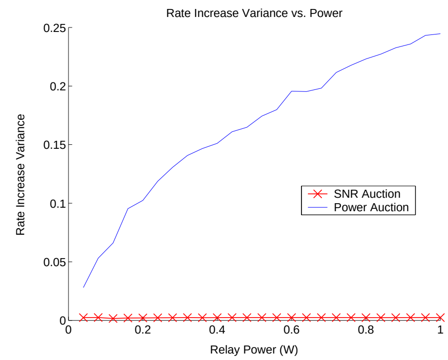

Fig. 8. Total positive rate increase variance vs. Relay Power.

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