

QCD tests from $e^+e^- \rightarrow I = 1$ hadrons data and implication on the value of α_s from τ -decays

S. Narison

Laboratoire de Physique Mathématique
Université de Montpellier II
Place Eugène Bataillon
34095 - Montpellier Cedex 05, France

Abstract

We re-examine the estimate of α_s and of the QCD condensates from $e^+e^- \rightarrow I = 1$ hadrons data. We conclude that e^+e^- at low energies gives a value of Λ compatible with the one from LEP and from tau inclusive decay. Using a τ -like inclusive process and QCD spectral sum rules, we estimate the size of the $D=4$ to 9 condensates by a fitting procedure *without invoking stability criteria*. We find $\langle \alpha_s G^2 \rangle = (7.1 \pm 0.7)10^{-2} \text{ GeV}^4$, $\rho \alpha_s \langle \bar{u}u \rangle^2 = (5.8 \pm 0.9)10^{-4} \text{ GeV}^6$, which confirm previous sum rules estimate based on stability criteria. The corrections due to the $D = 8$ condensates and to instantons on the vector component of τ -decay are respectively $\delta_1^{(8)} = -(1.5 \pm 0.6)10^{-2}(1.78/M_\tau)^8$ and $\delta_1^{(9)} = -(7.0 \pm 26.5)10^{-4}(1.78/M_\tau)^9$, which indicate that the $\delta_1^{(8)}$ is one order magnitude higher than the vacuum saturation value, while the $D \geq 9$ instanton-like contribution to the the vector component of the τ -decay width is a negligible correction. We also show that, due to the correlation between the $D = 4$ and $1/M_\tau^2$ contributions in the ratio of the Laplace sum rules, the present value of the gluon condensate already excludes the recent estimate of the $1/M_\tau^2$ -term from FESR in the axial-vector channel. Combining our non-perturbative results with the resummed perturbative corrections to the τ -width R_τ , we deduce from the present data $\alpha_s(M_\tau) = 0.33 \pm 0.03$.

1 Introduction

Measurements of the QCD scale Λ and of the q^2 -evolution of the QCD coupling are one of the most important test of perturbative QCD. At present LEP and τ -decay data [1]-[7] indicate that the value of α_s is systematically higher than the one extracted from deep-inelastic low-energy data. The existing estimate of α_s from QCD spectral sum rules [8] à la SVZ [9] in e^+e^- data [10, 11] also favours the low value of α_s from deep-inelastic scattering¹, which is, however, in contradiction with the recent CVC-test performed by [13] using e^+e^- data. It is therefore essential to test the reliability of the low-energy predictions before speculating on the phenomenological consequences implied by the previous discrepancy. Deep-inelastic scattering processes need a better control of the parton distributions, the higher-twist and instanton-like contributions in order to be competitive with LEP and tau-decay measurements. In addition, perturbative corrections in these processes should be pushed so far such that the remaining uncertainties will only be due to the re-summation of the perturbative series at large order. Indeed, the τ -decay rate has been calculated including the α_s^3 -term [3], while an estimate [14] and a measurement [15] of the α_s^4 coefficient is done. Moreover, a resummation of the $(\beta_1\alpha_s)^n$ of the perturbative series is now available [16]. The QCD spectral sum rule (QSSR) [8] à la SVZ [9] applied to the $I = 1$ part of the $e^+e^- \rightarrow$ hadrons total cross-section has a QCD expression very similar to the τ -decay inclusive width, such that on a theoretical basis, one can have a good control of it. In a previous paper [17], we have derived in a model-independent way the running mass of the strange quark from the difference between the $I = 1$ and $I = 0$ parts of the $e^+e^- \rightarrow$ hadrons total cross-section. In this paper, we pursue this analysis by re-examining the estimate of α_s and of the condensates including the instanton-like and the *marginal* $D=2$ -like operators obtained from the $I = 1$ channel of the e^+e^- data. In so doing, we re-examine the exponential Laplace sum rule used by [10] in e^+e^- , which is a generalization of the ρ -meson sum rule studied originally by SVZ [9]. We also expect that the Laplace sum rule gives a more reliable result than the FESR due to the presence of the exponential weight factor which suppresses the effects of higher meson masses in the sum rule. This is important in the particular channel studied here as the data are very inaccurate above 1.4–1.8 GeV, where the optimal result from FESR satisfies the so-called heat evolution test [11, 18, 19]. That makes the FESR prediction strongly dependent on the way the data in this region are parametrized, a feature which we have examined [13, 20] for criticizing the work of [21]. We also test the existing and controversial results [18, 19] of the $D = 2$ -type operator obtained from QSSR. Combining our different non-perturbative results with the recent resummed perturbative series [16], we re-estimate the value of α_s from τ -decays.

2 α_s from $e^+e^- \rightarrow I = 1$ hadrons data

Existing estimates of α_s or Λ from different aspects of QSSR sum rules for $e^+e^- \rightarrow I = 1$ hadrons data [10, 11] lead to values much smaller than the present LEP and τ -decay measurements [3]-[7]. However, such results contradict the stability-test on the extraction of α_s from τ -like inclusive decay [13] obtained using CVC in e^+e^- [22] for different values of the τ -mass. In the following, we shall re-examine the reliability of these sum rule results. We shall not re-consider the result from FESR [11] due to the drawbacks of this method mentioned previously,

¹However, new results of jet studies in deep-inelastic ep -scattering at HERA for photon momentum transfer $10 \leq Q^2 [\text{GeV}^2] \leq 4000$ give a value of α_s [12] compatible with the LEP-average.

and also, because the FESR-analysis has been re-used recently [18, 19] for a determination of the $D = 2$ -type operator, which we shall come back later on. Λ_3 and the condensates have been extracted in [10] from the Laplace sum rule:

$$\mathcal{L}_1 \equiv \frac{2}{3}\tau \int_{4m_\pi^2}^{\infty} ds e^{-s\tau} R^{I=1}(s) \quad (1)$$

and from its $\tau \equiv 1/M^2$ derivative:

$$\mathcal{L}_2 \equiv \frac{2}{3}\tau^2 \int_{4m_\pi^2}^{\infty} ds s e^{-s\tau} R^{I=1}(s), \quad (2)$$

where:

$$R^I \equiv \frac{\sigma(e^+e^- \rightarrow I \text{ hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}. \quad (3)$$

In the chiral limit $m_u = m_d = 0$, the QCD expressions of the sum rule can be written as:

$$\mathcal{L}_i = 1 + \sum_{D=0,2,4,\dots} \Delta_i^{(D)}. \quad (4)$$

The perturbative corrections can be deduced from the ones of $R^{I=1}$ obtained to order α_s^3 :

$$R^{I=1}(s) = \frac{3}{2} \left\{ 1 + a_s + F_3 a_s^2 + F_4 a_s^3 + \mathcal{O}(a_s^4) \right\}, \quad (5)$$

where, for 3 flavours: $F_3 = 1.623$ [23], $F_4 = 6.370$ [24]; the expression of the running coupling to three-loop accuracy is:

$$\begin{aligned} a_s(\nu) &= a_s^{(0)} \left\{ 1 - a_s^{(0)} \frac{\beta_2}{\beta_1} \log \log \frac{\nu^2}{\Lambda^2} \right. \\ &+ \left. \left(a_s^{(0)} \right)^2 \left[\frac{\beta_2^2}{\beta_1^2} \log^2 \log \frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} \log \log \frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} + \frac{\beta_3}{\beta_1} \right] + \mathcal{O}(a_s^3) \right\}, \end{aligned} \quad (6)$$

with:

$$a_s^{(0)} \equiv \frac{1}{-\beta_1 \log(\nu/\Lambda)} \quad (7)$$

and β_i are the $\mathcal{O}(a_s^i)$ coefficients of the β -function in the $\overline{MS}$ -scheme for n_f flavours:

$$\begin{aligned} \beta_1 &= -\frac{11}{2} + \frac{1}{3}n_f \\ \beta_2 &= -\frac{51}{4} + \frac{19}{12}n_f \\ \beta_3 &= \frac{1}{64} \left[-2857 + \frac{5033}{9}n_f - \frac{325}{27}n_f^2 \right]. \end{aligned} \quad (8)$$

For three flavours, we have:

$$\beta_1 = -9/2, \quad \beta_2 = -8, \quad \beta_3 = -20.1198. \quad (9)$$

In the chiral limit, the $D = 2$ -contribution vanishes. It has also been proved recently [16] that renormalon-type contributions induced by the resummation of the QCD series at large order

cannot induce such a term. In the chiral limit, the $D = 4$ non-perturbative corrections read [9, 3]:

$$\begin{aligned}\Delta_1^{(4)} &= \frac{\pi}{3} \tau^2 \langle \alpha_s G^2 \rangle \left(1 - \frac{11}{18} \frac{\alpha_s}{\pi} \right) \\ \Delta_2^{(4)} &= -\Delta_1^{(4)}.\end{aligned}\tag{10}$$

The $D = 6$ non-perturbative corrections read [9]:

$$\begin{aligned}\Delta_1^{(6)} &= -\frac{448\pi^3}{81} \tau^3 \rho \langle \bar{u}u \rangle^2 \\ \Delta_2^{(6)} &= -2\Delta_1^{(6)}.\end{aligned}\tag{11}$$

We shall use the conservative values of the condensates [8, 3]:

$$\langle \alpha_s G^2 \rangle = (0.06 \pm 0.03) \text{ GeV}^4 \quad \rho \langle \bar{u}u \rangle^2 = (3.8 \pm 2.0) 10^{-4} \text{ GeV}^6,\tag{12}$$

and *high* values of Λ from LEP and tau-decay data [1]-[4] for 3 flavours:

$$\Lambda_3 = 375_{-85}^{+105} \text{ MeV},\tag{13}$$

corresponding to $\alpha_s(M_Z) = 0.118 \pm 0.06$. The phenomenological side of the sum rule has been parametrized using analogous data as [10] and updated using the data used in [13]. The confrontation of the QCD and the phenomenological sides of the sum rules is done in Fig.1a and in Fig. 2a for a giving value of $\Lambda_3 = 375$ MeV and varying the condensates in the range given previously. One can conclude that one has a good agreement between the two sides of $\mathcal{L}_1$ for $M \geq 0.8$ GeV and of $\mathcal{L}_2$ for $M \geq 1.0 \sim 1.2$ GeV. The effects of the condensates are important below 1 GeV for $\mathcal{L}_1$ and below 1.3 GeV for $\mathcal{L}_2$. In Fig. 1b and Fig. 2b, we fix the condensates at their central values and we vary Λ_3 in the range given above. One can notice that a value of Λ_3 as high as 525 MeV is still allowed by the data, while the shape of the QCD curve for $\mathcal{L}_2$ changes drastically for a high value of Λ_3 . This phenomena is not informative as, below 1 GeV, higher dimension condensates can already show up and may break the Operator Product Expansion (OPE). By comparing these results with the ones of [10], one can notice that our QCD prediction for $\mathcal{L}_1$ corresponding to the previous set of parameters is as good as the fit of [10], while for that of $\mathcal{L}_2$, the agreement between the two sides of the sum rule is obtained here at a slightly larger value of M for high-values of Λ_3 . However, what is clear from our analysis is that the exponential Laplace sum rules *do not exclude* values of Λ_3 obtained from LEP and τ -decay data, though they cannot give a *more precise* information on the real value of Λ_3 if the condensates are left as free-parameters in the analysis. It is also informative and reassuring, that our analysis supports the value of Λ_3 obtained from τ -decay and used via CVC [22] for e^+e^- , in order to test the stability of the prediction for different values of the τ -mass [13] from the expression which we shall discuss below.

3 The condensates from τ -like decays

In so doing, we shall work with the vector component of the τ decay-like quantity deduced from CVC [22]:

$$R_{\tau,1} \equiv \frac{3 \cos^2 \theta_c}{2\pi\alpha^2} S_{EW} \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2} \right)^2 \left(1 + \frac{2s}{M_\tau^2} \right) \frac{s}{M_\tau^2} \sigma_{e^+e^- \rightarrow I=1},\tag{14}$$

where $S_{EW} = 1.0194$ is the electroweak correction from the summation of the leading-log contributions [25]. This quantity has been used in [13] in order to test the stability of the α_s -

Table 1: Phenomenological estimate of $R_{\tau,1}$

$M_\tau[\text{GeV}]$	$R_{\tau,1}$
1.0	1.608 ± 0.064
1.2	1.900 ± 0.075
1.4	1.853 ± 0.072
1.6	1.793 ± 0.070
1.8	1.790 ± 0.081
2.0	1.818 ± 0.097

prediction obtained at the τ -mass of 1.78 GeV. It has also been used to test CVC for different exclusive channels [13, 26]. Here, we shall again exploit this quantity in order to deduce *model-independent* informations on the values of the QCD condensates. The QCD expression of $R_{\tau,1}$ reads:

$$R_{\tau,1} = \frac{3}{2} \cos^2 \theta_c S_{eW} \left(1 + \delta_{EW} + \delta^{(0)} + \sum_{D=2,4,\dots} \delta_1^{(D)} \right), \quad (15)$$

where $\delta_{EW} = 0.0010$ is the electroweak correction coming from the constant term [27]; the perturbative corrections read [3]:

$$\delta^{(0)} = \left(a_s \equiv \frac{\alpha_s(M_\tau)}{\pi} \right) + 5.2023 a_s^2 + 26.366 a_s^3 + \dots, \quad (16)$$

The a_s^4 coefficient has also been estimated to be about 103 [14, 15], though we shall use $(78 \pm 25)a_s^4$ where the error reflects the uncalculated higher order terms of the D -function, while the first term is induced by the lower order coefficients after the use of the Cauchy integration. In the chiral limit $m_i = 0$, the quadratic mass-corrections contributing to $\delta_1^{(2)}$ vanish. Moreover, it has been proved [16] that the summation of the perturbative series cannot induce such a term, while the one induced eventually by the freezing mechanism is safely negligible [28, 18]. Therefore, we shall neglect this term in the first step of our analysis. We shall test, later on, the internal consistency of the approach if a such term is included into the OPE. In the chiral limit $m_i = 0$, the $D = 4$ contributions read [3]:

$$\delta_1^{(4)} = \frac{11}{4} \pi a_s^2 \frac{\langle \alpha_s G^2 \rangle}{M_\tau^4}, \quad (17)$$

which, due to the Cauchy integral and to the particular s -structure of the inclusive rate, the gluon condensate starts at $\mathcal{O}(a_s^2)$. This is a great advantage compared with the ordinary sum rule discussed previously. The $D = 6$ contributions read [3]:

$$\delta_1^{(6)} \simeq 7 \frac{256 \pi^3}{27} \frac{\rho \alpha_s \langle \bar{\psi}_i \psi_i \rangle^2}{M_\tau^6}, \quad (18)$$

The contribution of the $D = 8$ operators in the chiral limit reads [3]:

$$\delta_1^{(8)} = -\frac{39\pi^2}{162} \frac{\langle \alpha_s G^2 \rangle^2}{M_\tau^8}. \quad (19)$$

The phenomenological parametrization of $R_{\tau,1}$ has been done using the same data input as in [18, 13]. We give in Table 1 its value for different values of the tau mass. Neglecting the $D = 4$ -contribution which is of the order α_s^2 , we perform a two-parameter fit of the data for each value of Λ_3 corresponding to the world average value of $\alpha_s(M_Z) = 0.118 \pm 0.006$ [1, 2] and by letting the $D = 6$ and $D = 8$ condensates as free-parameters. We show the results of the fitting procedure in Table 2 for different values of Λ_3 . The errors take into account the effects

Table 2: Estimates of d_6 and d_8 from $R_{\tau,1}$ for different values of Λ_3

Λ_3 [MeV]	d_6 [GeV ⁶]	$-d_8$ [GeV ⁸]
480	$-.07 \pm 0.43$	1.15 ± 0.40
375	0.27 ± 0.34	0.69 ± 0.31
290	0.58 ± 0.29	0.83 ± 0.27

of the τ -mass moved from 1.6 to 2.0 GeV, which is a negligible effect, and the one due to the data. One can notice that the estimate of the $D = 8$ condensates is quite accurate, while the one of the $D = 6$ is not very conclusive for $\Lambda_3 \leq 350$ MeV. Indeed, only above this value, one sees that the $D = 6$ contribution is clearly positive as expected from the vacuum saturation estimate. This fact also explains the anomalous low value of $-d_8$ around this transition region. Using the average value of Λ_3 in Eq. (13), we can deduce the result:

$$d_8 \equiv M_\tau^8 \delta_1^{(8)} = -(0.85 \pm 0.18) \text{GeV}^8 \quad d_6 \equiv M_\tau^6 \delta_1^{(6)} = (0.34 \pm 0.20) \text{GeV}^6, \quad (20)$$

which we shall improve again later on once we succeed to fix the value of d_6 .

4 The condensates from the ratio of the Laplace sum rules

Let us now improve the estimate of the $D = 6$ condensates. In so doing, one can remark that, though there are large discrepancies in the estimate of the absolute values of the condensates from different approaches, there is a consensus in the estimate of the ratio of the $D = 4$ over the $D = 6$ condensates ²:

$$r_{46} [\text{GeV}^{-2}] \equiv \frac{\langle \alpha_s G^2 \rangle}{\rho \alpha_s \langle \bar{u}u \rangle^2} = 94.80 \pm 23 \quad [29]$$

²We have multiplied the original error given by [30] by a factor 10. The constraint obtained in [31] is not very conclusive as it leads to $r_{46} \leq 110 \text{ GeV}^{-2}$ and does not exclude ≤ 0 value of the condensates.

$$\begin{aligned}
& 96.20 \pm 35 & [11] \\
& 114.6 \pm 16 & [30] \\
& 92.50 \pm 50 & [32].
\end{aligned} \tag{21}$$

from which we deduce the *average*:

$$r_{46} = (105.9 \pm 11.9) \text{ GeV}^{-2}. \tag{22}$$

We use the previous informations on d_8 and r_{46} for fitting the value of the $D = 4$ condensates from the ratio of the Laplace sum rules:

$$\mathcal{R}(\tau) \equiv \tau^{-2} \frac{\mathcal{L}_2}{\mathcal{L}_1}, \tag{23}$$

used previously by [29] for a simultaneous estimate of the $D = 4$ and $D = 6$ condensates. We recall that the advantage of this quantity is its less sensitivity to the leading order perturbative corrections. The phenomenological value of $\mathcal{R}(\tau)$ is given in Fig. 2. Using a one-parameter fit, we deduce:

$$\langle \alpha_s G^2 \rangle = (6.1 \pm 0.7) 10^{-2} \text{ GeV}^4. \tag{24}$$

Then, we re-inject this value of the gluon condensate into the tau-like width in Eq. (14), from which we re-deduce the value of the $D = 8$ condensate. After a re-iteration of this procedure, we deduce our *final* results:

$$\langle \alpha_s G^2 \rangle = (7.1 \pm 0.7) 10^{-2} \text{ GeV}^4 \quad d_8 = -(1.5 \pm 0.6) \text{ GeV}^8. \tag{25}$$

Using the mean value of r_{46} , we also obtain:

$$\rho \alpha_s \langle \bar{u}u \rangle^2 = (5.8 \pm 0.9) 10^{-4} \text{ GeV}^6. \tag{26}$$

We consider these results as an improvement and a confirmation of the previous result in Eq. (12). It is also informative to compare these results with the ALEPH and CLEO II measurements of these condensates from the moments distributions of the τ -decay width. The most accurate measurement leads to [5]:

$$\langle \alpha_s G^2 \rangle = (7.8 \pm 3.1) 10^{-2} \text{ GeV}^4, \tag{27}$$

while the one of d_6 has the same absolute value as previously but comes with the wrong sign. Our value of d_8 is in good agreement with the one $d_8 \simeq -0.95 \text{ GeV}^8$ in [13, 6] obtained from the same quantity, but it is about one order of magnitude higher than the vacuum saturation estimate proposed by [33] and about a factor 5 higher than the CLEO II measurement. However, it is lower by a factor 2~3 than the FESR result from the vector channel [32]³. The discrepancy with the vacuum saturation indicates that this approximation is very crude, while the one with the FESR is not very surprising. Indeed, the FESR approach done in the vector and axial-vector channels [11, 32] tends *always to overestimate* the values of the QCD condensates. The discrepancy with the CLEO II measurement can be understood from the wrong sign of the $D = 6$ condensate obtained there and to its correlation with the $D = 8$ one.

³In the normalization of [32], our value of d_8 translates into $C_8 \langle O_8 \rangle = (0.18 \pm 0.04) \text{ GeV}^8$.

5 Instanton contribution

Let us now extract the size of the instanton-like contribution by assuming that it acts like a $D \geq 9$ operator. A good place for doing it is $R_{\tau,1}$ as, in the Laplace sum rules, this contribution is suppressed by a $8!$ factor. Using the previous values of the $D = 6$ and $D = 8$ condensates, we deduce:

$$\delta_1^{(9)} = -(7.0 \pm 26.5)10^{-4}(1.78/M_\tau)^9, \quad (28)$$

which, though inaccurate indicates that the instanton contribution is negligible for the vector current and has been overestimated in [34] ($\delta_V^{inst} \approx 0.03 \sim 0.05$). Our result supports the negligible effects found from an alternative phenomenological [35] ($\delta_V^{inst} \approx 3 \times 10^{-3}$) and theoretical [36] ($\delta_V^{inst} \approx 2 \times 10^{-5}$) analysis. Further cancellations in the sum of the vector and axial-vector components of the tau widths are however expected [34, 35] ($\delta^{inst} \approx \frac{1}{20}\delta_V^{inst}$).

6 Test of the size of the $1/M_\tau^2$ -term

Table 3: Estimates of $\langle \alpha_s G^2 \rangle$ from $R(\tau)$ for different values of d_2

d_2 [GeV] ² [18]	$\langle \alpha_s G^2 \rangle 10^2$ [GeV ⁴]	$-d_2$ [GeV] ² [19]	$\langle \alpha_s G^2 \rangle 10^2$ [GeV ⁴]
0.03	7.8 ± 0.5		
0.05	8.1 ± 0.5	0.2	3.2 ± 0.29
0.07	8.6 ± 0.5	0.3	1.2 ± 0.29
0.09	9.1 ± 0.5	0.4	-0.7 ± 0.6

Let us now study the size of the $1/M_\tau^2$ -term. From the QCD point of view, its possible existence from the resummation of the PTS due to renormalon contributions [28] has not been confirmed [16], while some other arguments [28, 37] advocating its existence are not convincing. Postulating its existence (whatever its origin!), [18] has estimated the strength of this term by using FESR and the ratio of moments $\mathcal{R}(\tau)$. As already mentioned earlier, the advantage in working with the ratio is that the leading order perturbative corrections disappear such that in a *compromise* region where the high-dimension condensates are still negligible, there is a *possibility* to pick up the $1/M_\tau^2$ -contribution. Indeed, using usual stability criteria and *allowing a large range of values around the optimal result*, [18] has obtained the *conservative* value:

$$d_2 \equiv C_2 \equiv \delta_1^{(2)} M_\tau^2 \simeq (0.03 \sim 0.08) \text{ GeV}^2, \quad (29)$$

while the estimate of [18] from FESR applied to the vector current has not been very conclusive, as it leads to the inaccurate value:

$$d_2 \simeq (0.02 \pm 0.12) \text{ GeV}^2. \quad (30)$$

However, the recent FESR analysis from the axial-vector current obtained at about the same value of the continuum threshold t_c satisfying the so-called evolution test [11], is *surprisingly*

very precise [19] and disagrees in sign and magnitude with our previous estimate from the ratio of moments. Assuming a quadratic dependence in Λ_3 , the result of [19] reads:

$$d_2 \simeq -(0.3 \pm 0.1) \text{ GeV}^2, \quad (31)$$

which is *surprisingly* very precise taking into account the fact that the spectral function of the axial-vector current is not better measured than that of the vector current. We test the reliability of this result, by remarking that d_2 (if it exists!) is strongly correlated to d_4 in the analysis of the ratio of Laplace sum rules $R(\tau)$, while it is not the case between d_2 or d_4 with d_6 and d_8 . Using our previous values of d_6 and d_8 , one can study the variation of d_4 given the value of d_2 . The results given in Table 3 indicate that the present value of the gluon condensate excludes the value of d_2 in Eq. (31) and can only permit a negligible fluctuation around zero of this contribution, which should not exceed the value $0.03 \sim 0.05$. This result rules out the possibility to have a sizeable $1/M_\tau^2$ -term [28, 37] and justifies its neglect in the analysis of the τ -width. More precise measurement of the gluon condensate or more statistics in the τ -decay data will improve this constraint.

7 Sum of the non-perturbative corrections to R_τ

Using our previous estimates, it is also informative to deduce the sum of the non-perturbative contributions to the decay widths of the observed heavy lepton of mass 1.78 GeV. In so doing, we add the contributions of operators of dimensions $D = 4$ to $D = 9$ and we neglect the expected small $\delta^{(2)}$ -contribution. For the vector component of the tau hadronic width, we obtain ⁴:

$$\delta_V^{NP} \equiv \sum_{D=4}^9 \delta_1^{(D)} = (2.38 \pm 0.89)10^{-2}, \quad (32)$$

while using the expression of the corrections for the axial-vector component given in [3], we deduce:

$$\delta_A^{NP} = -(7.95 \pm 1.12)10^{-2}, \quad (33)$$

and then:

$$\delta^{NP} \equiv \frac{1}{2}(\delta_V^{NP} + \delta_A^{NP}) = -(2.79 \pm 0.62)10^{-2}, \quad (34)$$

Our result confirms the smallness of the non-perturbative corrections measured by the ALEPH and CLEO II groups [5]:

$$\delta^{NP} = (0.3 \pm 0.5)10^{-2}, \quad (35)$$

though the exact size of the experimental number is not yet very conclusive.

8 Implication on the value of α_s from R_τ

Before combining the previous non-perturbative results with the perturbative correction to R_τ , let us test the accuracy of the resummed $(\alpha_s\beta_1)^n$ perturbative result of [16]. In so doing, we fix $\alpha_s(M_\tau)$ to be equal to 0.32 and we compare the resummed value of $\delta^{(0)}$ including the α_s^3 -corrections with the one where the coefficients have been calculated in the $\overline{MS}$ -scheme [23].

⁴We have used, for $M_\tau = 1.78$ GeV, the conservative values: $\delta_V^{(9)} \approx -\delta_A^{(9)} \simeq -(0.7 \pm 2.7)10^{-3}$ and $\delta^{(9)} \approx 1/20\delta_V^{(9)}$ [34].

Table 4: QCD predictions for R_τ using the *contour coupling*-expansion

$\alpha_s(M_\tau)$	a_s^3	a_s^4	a_s^6	a_s^8
0.26	3.364 ± 0.022	3.370	3.380 ± 0.019	3.381
0.28	3.402 ± 0.024	3.411	3.426 ± 0.019	3.426
0.30	3.442 ± 0.026	3.453	3.474 ± 0.021	3.472
0.32	3.484 ± 0.030	3.498	3.526 ± 0.023	3.520
0.34	3.526 ± 0.033	3.546	3.582 ± 0.031	3.568
0.36	3.571 ± 0.040	3.594	3.640 ± 0.045	3.613
0.38	3.616 ± 0.040	3.645	3.706 ± 0.069	3.655
0.40	3.664 ± 0.040	3.700	3.775 ± 0.108	3.685

We consider the two cases where R_τ is expanded in terms of the usual coupling α_s or in terms of the *contour coupling* [4]. In both cases, one can notice that the approximation used in the resummation technique tends to overestimate the perturbative correction by about 10%. Therefore, we shall reduce systematically by 10%, the prediction from this method from the α_s^5 to α_s^9 contributions. We shall use the coefficient 27.46 of α_s^4 estimated in [14, 15]. Noting that, to the order where the perturbative series (PTS) is estimated, one has alternate signs in the PTS, which is an indication for reaching the asymptotic regime. Therefore, we can consider, as the best estimate of the resummed PTS, its value at the minimum. That is reached, either for truncating the PTS by including the α_s^6 or the α_s^8 contributions. The corresponding value of R_τ including our non-perturbative contributions in Eq. (34) is given in Table 4. We show for comparison the value of R_τ including the α_s^3 -term, where we have used the perturbative estimate in [6] (the small difference with the previous papers [4, 13, 6, 7, 20] comes from the different non-perturbative term used here), while the error quoted there comes from the naïve estimate $\pm 50a_s^4$. However, one can see that the estimate of this perturbative error has taken properly the inclusion of the higher order terms, while the truncation of the series at α_s^3 already gives a quite good evaluation of the PTS. One can also notice that there is negligible difference between the PTS to order α_s^6 and α_s^8 for small values of α_s , while the difference increases for larger values. We consider as a final perturbative estimate of R_τ the one given by the PTS including the α_s^6 -term at which we encounter the first minimum. The error given in this column is the sum of the non-perturbative one from Eq. (34) with the perturbative conservative uncertainty, which we have estimated like the effect due to the last term i.e $\pm 34.53(-\beta_1 a_s/2)^6$ at which the minimum is reached, which is a legitimate procedure for asymptotic series [38]. We have also added to the latter the one due to the small fluctuation of the minimum of the PTS from the inclusion of the α_s^6 or α_s^8 -terms. One can notice that for $\alpha_s \leq 0.32$, the error in R_τ is dominated by the non-perturbative one, while for larger value of α_s , it is mainly due to the one from the PTS. Using the value of R_τ in Table 4, we deduce:

$$\alpha_s(M_\tau) = 0.33 \pm 0.030, \quad (36)$$

where we have used the experimental average [2]:

$$R_\tau = 3.56 \pm 0.03. \quad (37)$$

Our result from the optimized resummed PTS is in good agreement with the most recent estimate obtained to order α_s^3 [6, 5, 7]:

$$\alpha_s(M_\tau) = 0.33 \pm 0.030. \quad (38)$$

9 Conclusion

Our analysis of the isovector component of the $e^+e^- \rightarrow \text{hadrons}$ data has shown that there is a consistent picture on the extraction of α_s from high-energy LEP and low-energy τ and e^+e^- data. It has also been shown that the values of the condensates obtained from QCD spectral sum rules based on *stability criteria* are reproduced and improved by fitting the τ -like decay widths and the ratio of the Laplace sum rules. Our estimates are in good agreement with the determination of the condensates from the the τ -hadronic width moment-distributions [5], which needs to be improved from accurate measurements of the e^+e^- data or/and for more data sample of the τ -decay widths which can be reached at the τ -charm factory machine. Finally, our consistency test of the effect of the $1/M_\tau^2$ -term, whatever its origin, does not support the recent estimate of this quantity from FESR axial-vector channel [19] and only permits a small fluctuation around zero due to its strong correlation with the $D = 4$ condensate effects in the ratio of Laplace sum rules analysis, indicating that it cannot affect in a sensible way the accuracy of the determination of α_s from tau decays. As a by-product, we have reconsidered the estimate of $\alpha_s(M_\tau)$ from the τ -widths taking into account the recent resummed result of the perturbative series. Our result in Eq. (36) is a further support of the existing estimates.

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Figure captions

Fig. 1a: The Laplace sum rule $\mathcal{L}_1$ versus the sum rule parameter M . The dashed curves correspond to the experimental data. The full curves correspond to the QCD prediction for $\Lambda_3=375$ MeV, $\langle\alpha_s G^2\rangle = 0.06 \pm 0.03$ GeV⁴ and $\rho\langle\bar{u}u\rangle^2 = (3.8 \pm 2.0)10^{-4}$ GeV⁶. **Fig. 1b:** The

same as Fig. 1a but for different values of Λ_3 and for $\langle\alpha_s G^2\rangle = 0.06$ GeV⁴ and $\rho\langle\bar{u}u\rangle^2 = 3.8 \cdot 10^{-4}$ GeV⁶. **Fig. 2a:** The same as Fig. 1a but for $\mathcal{L}_2$. **Fig. 2b:** The same as Fig. 1b but for

$\mathcal{L}_2$. **Fig. 3:** Experimental value of the ratio of Laplace sum rules $\mathcal{R}(\tau)$ versus the sum rule variable $\tau \equiv 1/M^2$.

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