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Possible Nonvanishing Mass of Photon

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From phenomenological and field-theoretical considerations on photon mass, we first show that photon is not limited to being massless at the present stage. Next we illustrate a possibility of formulating a local field theory for massive photons coupled with nonconserved currents, while we cannot formulate it for massless photons.

§ 1. Introduction

Quantum electrodynamics is described by a massless gauge field, the photon field, interacting minimally with charged matter fields. The best upper bound on photon mass μ based on direct observations is given by the measurement of Jupiter's magnetic field. Goldhaber et al.[1] estimated μ as $\mu \lesssim 6\times 10^{-16}(\mathrm{eV}/c^2)$. In this paper, we present a merit of introducing nonvanishing photon mass.

In this section, we review the massive photon theory coupled with conserved current. At the first glance, one may afraid that the massive photon hypothesis, no matter how tiny μ may be, contradicts our observations. Especially, the fact that massive photon has one more degree of freedom, a longitudinal component, distinguishes the massive case from the massless one. Therefore, we investigate whether any observable effects of longitudinal photons (L-photons) appear in elementary processes and in the thermal equilibrium as far as photon is coupled with conserved current.

First we consider an elementary process in which a photon of the mass μ is emitted or absorbed. The transition amplitude for it is expressed as $\langle f|i\rangle = \varepsilon^{\mu}(k)M_{\mu}(k)$, where $\varepsilon^{\mu}(k)$ is a photon polarization vector satisfying $k\cdot\varepsilon=0$ and $\varepsilon^2=-1$ for k^{μ} given by $k^{\mu}=(\omega, \boldsymbol{k})$ with $\omega=(\boldsymbol{k}^2+\mu^2)^{1/2}$. When the massive photon is coupled with conserved currents as in QED, we have $k\cdot M(k)=0$. Note that ε for the L-photon going in the z-direction is written as

$$\varepsilon_L{}^{\mu}=\mu^{-1}(|\boldsymbol{k}|, 0, 0, \omega)=\mu^{-1}[(k^{\mu})+(\omega-|\boldsymbol{k}|)(-1, 0, 0, 1)]\,. \tag{1}$$

Then, we obtain the ratio of the production or absorption cross section of an L-photon to that of a transverse photon (T-photon) as

$$\sigma_L/\sigma_T \sim (\mu/\omega)^2 \tag{2}$$

if $\mu \ll \omega$, because the first term in the second expression of (1) makes no contribution to $\varepsilon\cdot M(k)$ while the second term gives a contribution of order of magnitude μ/ω. Equation (2) implies that both the production and detection of the L-photon are practically impos-

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sible within the precision of our experiment.

Second, we ask the reason why the Planck formula of black body radiation for massive photons can be valid without being multiplied by a factor 3/2, in spite of the existence of L-photons. Bass and Schrödinger investigated a system of massive photons in a closed box with perfectly conducting walls, making use of the classical Proca field theory.[2] Their important observation is that perfect conductor is almost transparent for L-photons while it reflects almost all T-photons; more precisely, that T-photons are reflected by the walls except for their small transparent component, only of the order of $(\mu/\omega)^2$, to be transformed into L-photons. It takes more than τ, $\tau \sim V^{1/3}(\omega/\mu)^2$ (V: volume of the box), to arrive at the true equilibrium of T- and L-photons from a temporary equilibrium of T-photons alone. The relaxation time is numerically given by $\tau \gtrsim 10^{20} V^{1/3}$ (sec) for $\omega=1$(eV), where $V^{1/3}$ is taken in units of cm. Notice that τ is much larger than the age of the universe, 10^{17}(sec).

The above argument is based on the unrealistic assumption of perfect conductor. This unsatisfactory point is improved by Kroll,[3] who calculated skin depths of a conducting matter with finite conductivity for three components of massive photons. He also described the photons by a classical Proca field coupled with a conserved current. His conclusion is that an inequality

$$\delta_L/\delta_T \geq (\omega/\mu)^2 \tag{3}$$

holds for an arbitrary value of ω, where δ_L is the skin depth for L-photons and δ_T for T-photons. From (3), we estimate the thickness D of the conducting wall which can confine L-photons as follows: $D \geq 10^{25}$(cm) for $\omega=1$(eV) and the typical value of $\delta_T=10^{-5}$(cm).

Thus we cannot exclude the hypothesis of possible tiny photon mass phenomenologically.*) Moreover, the massive photon theory has been established from field-theoretical points of view; that is, it is proved to be a unitary (ghost-confining), renormalizable gauge field theory in a covariant formulation with indefinite metric and to become QED in the massless limit.[5] Here it must be emphasized that the current conservation is essential for both the phenomenological and field theoretical aspects in the above; otherwise the conclusions are drastically changed.

Can we have a consistent local field theory for photon even in the presence of nonconserved currents ? Such an attempt is motivated by recent observations of the anomalous events $Z^0 \to e^+e^-\gamma$ at CERN,**) because it would suggest us a possible existence of an "excited" lepton e^* decaying into e and γ.[6] This possibility enforces us to use a nonconserved mass-changing current as long as e^* is regarded as a point-like particle. In this case we expect that the massive photon theory can be distinguished from the massless one and it gives us a merit of introducing photon mass. In fact, it is shown in the next section (§ 2) that a massive photon coupled with nonconserved currents can be described within the framework of a local field theory, while any massless photon coupled with them cannot. There, the unitarity of physical S-matrix is proved as a result of photon mass. The renormalizability of our theory is reported in a seperate paper.[7] Section 3 is devoted to discussion.

*) An attempt of introducing a photon mass was made by Georgi et al.,[4] who intended to explain the apparent discrepancy between a theoretical and an observed shape of the cosmic background radiation.

**) This phenomenon is not established by further experiments.

§ 2. Unitarity problem of photon theory coupled with nonconserved current

Two of the present authors (M. N. and K. Y.) discussed the massless photon theory coupled with nonconserved current and showed that a physical S-matrix can be unitary, as far as only virtual photon processes are concerned, by a suitable choice of parameters introduced.[8] Their ghost confinement is, however, still incomplete for the whole physical S-matrix, as will be illustrated below.

To clarify the above situation, let us consider the following Lagrangian density in the Landau gauge:

$$L=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-A_\mu\partial^\mu\zeta+\frac{1}{2}\partial_\mu\xi\partial^\mu\xi+fj_\mu'\left(A^\mu+\frac{1}{\mu}\partial^\mu\xi\right), \tag{4}$$

where ζ is a Lagrange multiplier field, and f and μ are constants. The current j_μ' consists of matter fields alone, and it is nonconserved; $\partial^\mu j_\mu'\neq 0$. An explicit form for the matter part of L is suppressed in (4). Another auxiliary field ξ is introduced to make L invariant under the c-number gauge transformation

$$\delta A_\mu=\partial_\mu\Lambda,\quad \delta\xi=-\mu\Lambda,\quad \delta\zeta=0,\quad \delta j_\mu'=0 \tag{5}$$

with Λ satisfying $\Box\Lambda=0$. Field equations resulting from (4) are

$$\left.\begin{aligned}&\partial^\nu F_{\mu\nu}=\partial_\mu\zeta-fj_\mu',\quad \partial^\mu A_\mu=0,\\&\Box\xi=-\mu^{-1}f\partial^\mu j_\mu',\end{aligned}\right\} \tag{6}$$

from which we can have a field η, $\eta\equiv\zeta+\mu\xi$, satisfying $\Box\eta=0$.

According to the usual prescription of canonical quantization, we can obtain 4-dimensional commutation relations

$$\left.\begin{aligned}&[\eta(x), A_\mu(0)]=i\partial_\mu D(x),\\&[\eta(x), \zeta(0)]=0,\\&[\eta(x), \xi(0)]=i\mu D(x)\end{aligned}\right\} \tag{7}$$

between η and the other fields. To eliminate ghost modes, we impose the supplementary condition

$$\eta^{(+)}(x)|\text{phys}\rangle=0 \tag{8}$$

on physical states by the positive frequency part of $\eta(x)$. Apparently, there is lacking one more constraint for sufficient conditions, for the field $\tilde{A}_\mu\equiv A_\mu+\mu^{-1}\partial_\mu\xi$ commutes with η. Since our physical state-vector subspace is generated by field operators commuting with η, $\tilde{A}_\mu$ becomes a physical operator. This shows that even the longitudinal component of $\tilde{A}_\mu$, which is a ghost mode, appears in physical sectors as well as the transverse one. We also note that such a situation does not alter even if we replace the present gauge symmetry with the Becchi-Rouet-Stora (BRS) symmetry[9] and $\eta^{(+)}(x)$ in (8) with the BRS charge by further introducing a pair of the Faddeev-Popov ghost fields.[10] Hence, we cannot construct a consistent field theory for massless photons in the presence of nonconserved currents no matter how the ghost fields are introduced.

However, the above fact brings us no difficulty for the massive photon case. We can

indeed develop a genuine massive $U(1)$ gauge theory[11] with nonconserved currents under a complete ghost confinement. Instead of (4), consider the following gauge invariant Lagrangian density:

$$L=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-A_\mu\partial^\mu\eta+\frac{\mu^2}{2}(A_\mu+\frac{1}{\mu}\partial_\mu\xi)^2+fj_\mu'(A^\mu+\frac{1}{\mu}\partial^\mu\xi) \tag{9}$$

with μ a bare mass of the photon described by A_μ, for which a c-number gauge transformation is defined by

$$\delta A_\mu=\partial_\mu\Lambda\,,\quad \delta\xi=-\mu\Lambda\,,\quad \delta\eta=0\,,\quad \delta j_\mu'=0\,. \tag{10}$$

The canonical theory given by (9) has very recently been studied in detail by the present authors.[7] Field equations resulting from (9),

$$\left.\begin{aligned}&\partial^\nu F_{\nu\mu}+\mu^2A_\mu=\partial_\mu\eta-\mu\partial_\mu\xi-fj_\mu'\,,\\&\partial^\mu A_\mu=0\,,\quad \Box\xi=-\mu^{-1}f\partial^\mu j_\mu'\,,\end{aligned}\right\} \tag{11}$$

yield $\Box\eta=0$, so that we can impose a supplementary condition on physical states in the same form as (8) by the present η.

Similarly to (7), we find

$$\left.\begin{aligned}&[\eta(x)\,,\ A_\mu(0)]=i\partial_\mu D(x)\,,\\&[\eta(x)\,,\ \eta(0)]=0\,,\\&[\eta(x)\,,\ \xi(0)]=i\mu D(x)\,.\end{aligned}\right\} \tag{12}$$

We also have a field $\tilde{A}_\mu$ such that

$$\tilde{A}_\mu\equiv A_\mu+\mu^{-1}\partial_\mu\xi\,,\qquad [\tilde{A}_\mu(x),\eta(0)]=0\,. \tag{13}$$

Hence, $\tilde{A}_\mu$ is a physical field as well as η. The present situation is, however, not the same as that we have encountered before, as will be seen below. The free part (gauge paro) of (9) yields

$$\left.\begin{aligned}&[A_\mu(x),A_\nu(0)]=-i[g_{\mu\nu}\Delta(x;\mu^2)+\mu^{-2}\partial_\mu\partial_\nu\{\Delta(x;\mu^2)-D(x)\}]\,,\\&[A_\mu(x),\xi(0)]=0\,,\\&[\xi(x),\xi(0)]=iD(x)\,,\end{aligned}\right\} \tag{14}$$

other than (12). We can regard these 4-dimensional commutators as those given for the corresponding asymptotic fields under some appropriate asymptotic limits as $x_0\to\pm\infty$.

In the system of the asymptotic fields, which generate our Fock space of state vectors, we have the following Proca field V_μ:

$$\left.\begin{aligned}&V_\mu\equiv A_\mu-\mu^{-2}(\partial_\mu\eta-\mu\partial_\mu\xi)\,,\\&(\Box+\mu^2)\,V_\mu=0\,,\quad \partial_\mu V^\mu=0\,,\\&[V_\mu(x),\eta(0)]=[V_\mu(x),\xi(0)]=0\,.\end{aligned}\right\} \tag{15}$$

Both of V_μ and η are physical fields. Then, V_μ generates a Hilbert space of positive norm, while η does a physical subspace of zero norm. We here notice that the asymptotic limit of $\tilde{A}_\mu$ in (13) is nothing but a linear combination of V_μ and $\partial_\mu\eta$, and that there is no unfavorable matter out of the physical unitarity.

§ 3. Discussion

In this paper we have constructed a gauge invariant theory for massive photon field coupled with nonconserved current and its unitarity of physical S-matrix. A few remarks may be appropriate here. We may use $\tilde{A}_\mu$ defined in (13) instead of A_μ from the beginning. In fact, $\tilde{A}_\mu$ is nothing but Proca field V_μ except for the zero norm field η. (See (15).) However if we used $\tilde{A}_\mu$ instead of A_μ originally, identifying $\tilde{A}_\mu$ with V_μ incorrectly, we would simply have Proca field theory which is neither gauge invariant nor renormalizable. In order to construct a gauge invariant neutral vector theory just as in the usual case with conserved current,[5] we must resort to an indefinite metric Hilbert space and proceed with A_μ itself. This is an essential point for the theory being unitary and renormalizable. We can have a renormalizable version of theory characterized by (9), as reported in Ref. 7), where the renormalizability of the theory is explicitly proved, consistently with the unitarity, in the sense of superpropagator approach for non-polynomial interactions.

Finally, we remark that in the presence of nonconserved currents the matrix element of a process emitting or absorbing a real L-photon becomes very large in proportion to the power of μ^{-1} due to the form of (1) without $k\cdot M=0$. (To be definite, we take $j_\mu'=\bar{e}^*\gamma_\mu e+\bar{e}\gamma_\mu e^*$.) We cannot follow the naive perturbation calculations for the transition matrix, but the reactance matrix method to preserve the unitarity. Then, we may parametrize the decay width of e^* of the mass m^* into e and an L-photon as $\Gamma(e^*\to e\gamma_L)=\alpha_f^{(\mathrm{eff})}m^*$ with a parameter $\alpha_f^{(\mathrm{eff})}$, while $\Gamma(e^*\to e\gamma_T)=\alpha_f m^*(\alpha_f=f^2/4\pi)$. The other elementary processes caused by virtual e^* emitting L-photons may give rise to observable effects. For example, consider the Compton scattering with an incoming T-photon and an outgoing L-photon. The ratio of its cross section σ_{TL} to the ordinary one σ_{QED} in the rest frame of the initial e is given by

$$\frac{\sigma_{TL}}{\sigma_{\mathrm{QED}}}\simeq\begin{cases}4\left(\dfrac{\Gamma}{m^*}\right)\dfrac{\alpha_f}{\alpha^2}\left(\dfrac{E}{m}\right)^3\left(\dfrac{E}{m^*}\right)^4, & (E\ll m)\\ 8\left(\dfrac{\Gamma}{m^*}\right)\dfrac{\alpha_f}{\alpha^2}\left(\dfrac{m}{m^*}\right)\left(\dfrac{E}{m^*}\right)^3\left(\dfrac{1}{2}+\ln\dfrac{E}{m}\right)^{-1}, & (E\gg m)\end{cases}\tag{16}$$

where m is the mass of e, E the incident photon energy and $\Gamma=\Gamma(e^*\to e\gamma_L)$. From this result, it is evident that the L-photon gives no substantial contribution to phenomena at our energy scale ($m^*\gg E$). When E approaches m^*, we expect the discrepancy from QED. Note that our theory of the massive photon characterized by (9) is applicable to phenomena of a possible point-like e^*.

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Note added in proof: The problem of nonconserved e. m. current was also discussed by Okun et al.[12] They claim the impossibility of nonvanishing photon mass because of catastrophically large "radiative" corrections. Nevertheless our conclusions are not altered, since our discussions are based on the renormalizable theory while theirs on nonrenormalizable one.