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Efficient short-term scheduling of refinery operations based on a continuous time formulation

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Abstract

The problem addressed in this work is to develop a comprehensive mathematical programming model for the efficient scheduling of oil-refinery operations. Our approach is first to decompose the overall problem spatially into three domains: the crude-oil unloading and blending, the production unit operations and the product blending and delivery. In particular, the first problem involves the crude-oil unloading from vessels, its transfer to storage tanks and the charging schedule for each crude-oil mixture to the distillation units. The second problem consists of the production unit scheduling which includes both fractionation and reaction processes and the third problem describes the finished product blending and shipping end of the refinery. Each of those sub-problems is modeled and solved in a most efficient way using continuous time representation to take advantage of the relatively smaller number of variables and constraints compared to discrete time formulation. The proposed methodology is applied to realistic case studies and significant computational savings can be achieved compared with existing approaches.

Keywords: Refinery operation; Spatial decomposition; Continuous time formulation; Scheduling

1. Introduction

In the literature, mathematical programming technologies have been extensively concerned and developed in the area of long-term refinery planning (Bodington, 1995; Ravi & Reddy, 1998), while short-term scheduling has received less attention. Refinery planning optimization is mainly addressed through successive linear programming approach, such as GRTMPS (Haverly Systems), PIMS (Aspen Technology) and RPMS (Honeywell Hi-Spec Solutions), while nonlinear programming models are also increasingly becoming a common methodology for the refinery optimization (Moro, Zanin, & Pinto, 1998; Pinto, Joly, & Moro, 2000).

Scheduling has been mainly addressed for batch plants. Extensive reviews can be found in Ierapetritou and Floudas (1998a), Pinto and Grossmann (1998), Reklaitis (1992). Continuous plants, however have received less attention in the open literature concerning the scheduling optimization problem. Sahinidis and Grossmann (1991) considered the problem of cyclic scheduling of multiproduct continuous plants for the single stage case, and by Pinto and Grossmann (1994) for the multistage case (typical of lube-oil production). Ierapetritou and Floudas (1998b) extended their batch scheduling model to consider continuous and mixed production facilities.

Fewer publications have been appeared to address the short-term scheduling of refinery operations. Zhang and Zhu (2000) proposed a decomposition approach, which decomposes the overall refinery model into a site level and a process level. The site level determines operating rules for each process and the process level returns updated performance to the site level for further optimization. This procedure continues until a specified tolerance is met. The problem of crude-oil unloading with inventory control is addressed by Lee, Pinto, Grossmann, and Park (1996) based on time discretization. Shah (1996) applied mathematical programming techniques to crude-oil scheduling. However the models are prohibitively expensive due to the nature of discrete time representation.

Gasoline blending is a crucial step in refinery operation as gasoline can yield 60–70% of a refinery's profit. The process involves mixing various stocks, which are the intermediate products from the refinery, along with some additives, such as antioxidants and corrosion inhibitors, to produce blends with certain qualities (DeWitt, Lasdon, Waren, Brenner, & Meihem, 1989). A variety of support systems have been

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Nomenclature

Indices

n	event points

Problem 1

i	storage tanks
j	charging tanks
k	key components
l	CDUs
v	vessels

Problem 2

$s1$	raw materials
$s2$	intermediates
$s3$	intermediates
$s4$	final products
t	storage tanks

Problem 3

c	component tanks
o	orders
p	productstock tanks
s	products
w	components

Sets

N	event points within the time horizon

Problem 1

I_j	storage tanks which can transfer crude-oil to charging tank j
I_v	storage tanks which can be fed by vessel v
J_i	charging tanks which can be fed by tank i
J_l	charging tanks which can charge CDU l
L_j	CDUs which can be charged by charging tank 1
V_i	vessels which can feed crude-oil to tank i

Problem 2

$S1_{s2}$	raw materials which can produce material $s2$
$S2_{s1}$	materials which can be produced from raw material $s1$
$S2_{s3}$	materials which can produce material $s3$
$S3_{s2}$	materials which can be produced from material $s2$
$S3_{s4}$	materials which can produce final product $s4$
$S4_{s3}$	final products which can be produced from material $s3$
T_{s1}	storage tanks which can store raw material $s1$
T_{s2}	storage tanks which can store intermediate $s2$
T_{s3}	storage tanks which can store intermediate $s3$
T_{s4}	storage tanks which can store final product $s4$

Problem 3

C_w	componentstock tanks which can store component w
O_p	orders which can be performed in productstock tank p
O_s	orders which order product s
P_o	productstock tanks which are suitable for performing order o
P_s	productstock tanks which can store product s

S_p	products which can be stored in productstock tank p
W_c	components which can be stored in componentstock tank c

Parameters

H	time horizon
NE	total number of event points

Problem 1

c_{inv}	inventory cost of storage tanks per volume per day
c_{sea}	sea waiting cost per day
c_{set}	changeover cost per time
c_{un}	unloading cost per day
$D_0(i, k)$	initial concentration of component k in the crude-oil of storage tank i
$D_0(j, k)$	initial concentration of component k in the crude mix of charging tank i
$D_{\max}(i, k)$	maximum concentration of component k in the crude-oil of storage tank i
$D_{\max}(j, k)$	maximum concentration of component k in the crude mix of charging tank j
$D_{\min}(i, k)$	minimum concentration of component k in the crude-oil of storage tank i
$D_{\min}(j, k)$	minimum concentration of component k in the crude mix of charging tank j
$D_v(v, k)$	concentration of component k in the crude-oil of vessel v
DM(j)	demand of crude mix from charging tank j
$f_{\max}$	maximum volume flow rate
$f_{\min}$	minimum volume flow rate
f_{rate}	volume flow rate of crude-oil or mix transfer
NST	total number of storage tanks
c_{inc}	inventory cost of storage tanks per volume per day
$T\text{arr}(v)$	arrival time of vessel v
$V_{\max}$	upper bound of volume of oil being transferred
$V_{\min}$	lower bound of volume of oil being transferred

Problem 2

demand($s4$)	demand of final product $s4$
$h$vol(t)	maximum capacity of storage tank t
initial 1($s1$, t)	amount of raw material $s1$ stored in tank t initially
initial 2($s2$, t)	amount of material $s2$ stored in tank t initially
initial 3($s3$, t)	amount of material $s3$ stored in tank t initially
initial 4($s4$, t)	amount of final product $s4$ stored in tank t initially
$l$vol(t)	minimum amount of material stored in tank t if tank t is utilized
low	lower bound on the amount of material being processed
max rate 1	maximum flow rate of extraction unit
max rate 2	maximum flow rate of dewaxingunit
max rate 3	maximum flow rate of hydrofinishing unit
min rate 1	minimum flow rate of extraction unit
min rate 2	minimum flow rate of dewaxing unit
min rate 3	minimum flow rate of hydrofinishing unit
price($s4$)	unit price of final product $s4$
switch($s1$, $s1'$)	1 if switch-over of raw materials $s1$ and $s1'$ is allowed
yield 1($s1$, $s2$)	yield of feed $s1$ and product $s2$
yield 2($s2$, $s3$)	yield of feed $s2$ and product $s3$
yield 3($s3$, $s4$)	yield of feed $s3$ and product $s4$

Problem 3

B_{flow}	flow rate of product being produced and transferred to productstock tanks
$l(o)$	lifting rate of order o
$P_{\text{rod\_end}}(o)$	time by which order o is due

$P_{\text{rod\_srt}}(o)$	time by which order o can start
recipe(s, w)	the proportion of component w to in product s
$V_{\text{initial}}(p, s)$	amount of product s stored in tank p initially
$V_{\max}(p)$	maximum capacity of productstock tank p
$V_{\min}(p)$	minimum amount of product stored in tank p if tank p is utilized
Variables of problem 1	
$B(i, j, n)$	volume of crude-oil that storage tank i transfers to charging tank j at event point n
$B(i, k, n)$	volume of component k that storage tank i transfer to charging tank j at event point n
cost	operating cost
$T\text{f}(i, j, n)$	end time of storage tank i transferring crude-oil to charging tank j at event point n
$T\text{ft}(v, i, n)$	time that vessel v finishes unloading crude-oil into storage tank i
$T\text{s}(i, j, n)$	starting time of storage tank i transferring crude-oil to charging tank j at event point n
$T\text{st}(v, i, n)$	time that vessel v starts unloading crude-oil into storage tank i
$V(i, k, n)$	volume of component k in storage tank i at event point n
$V(i, n)$	volume of crude-oil in storage tank i at event point n
$x1(v, i, n)$	binary variables that assign the beginning of v unloading crude-oil to i at event point n
$x2(i, j, n)$	binary variables that assign the beginning of i transferring crude-oil to j at event point n
$x3(j, l, n)$	binary variables that assign the beginning of j charging crude-oil mix to l at event point n
Variables of problem 2	
$b2(s2, t, n)$	amount of material $s2$ in tank t at event point n
in $2(s2, t, n)$	amount of material $s2$ being produced to tank t at event point n
out $2(s2, t, n)$	amount of material $s2$ being consumed from tank t at event point n
$Te2(s2, s3, n)$	end time of producing $s3$ from $s2$ while it starts at event point n
$Ts2(s2, s3, n)$	starting time of producing $s3$ from $s2$ at event point n
uv $2(s2, s3, n)$	binary variables that assign the beginning of producing $s3$ from $s2$ at event point n
$y2(s2, t, n)$	binary variables that assign material $s2$ being stored in tank t at event point n
Variables of problem 3	
$b(s, p, n)$	amount of product s in tank p at event point n before new product being transferred from blender
$bc(w, c, n)$	amount of component w in comp. tank c at event point n
blnd(s, p, n)	amount of product s being delivered from blender to tank p at event point n
comp(w, c, n)	amount of component w being transferred to the blender at event point n
cracking(w, c, n)	amount of component w being transferred from separation units to comp. tank c at event point n
lift(i, j, n)	amount of product being lifted for order i from tank j at event point n
$T\text{f}(o, p, n)$	ending time of order o in tank p while it starts at event point n
$T\text{f}(s, p, n)$	ending time of product s being produced and transferred to productstock tank p at event point n
$T\text{s}(o, p, n)$	starting time of order o in tank p at event point n
$T\text{s}(s, p, n)$	starting time of product s being produced and transferred to productstock tank p at event point n
$z1(o, p, n)$	binary variables that assign the beginning of order o in tank p at event point n
$z2(s, p, n)$	binary variables that assign product 5 being stored in tank p at event point n
$z3(s, n)$	0–1 continuous variables that assign product s being produced at event point n

developed to address planning and scheduling of blending operations. StarBlend (Rigby, Lasdon, & Waren, 1995), for example, which is developed by Texaco, uses a multi-period blending model written in GAMS that facilitates the incorporation of future requirements into current blending decisions. Glismann and Gruhn (2001a, 2001b) proposed a mixed-integer linear model (MILP), which is based on a resource-task network representation, to solve the task of short-term scheduling of blending processes. The recipe optimization problem is then formulated as a non-linear program and the results are returned to the scheduling problem, so that an overall optimization can be achieved. A fuzzy linear formulation is applied to the blending facilities by Dujkanovic, Babic, Milosevic, Sobajic, and Pao (1996), in order to address the problem of uncertainty of input information within the fuel scheduling optimization. Singh, Forbes, Vermeer, and Woo (2000) addressed the problem of blending optimization for in-line blending for the case of stochastic disturbances in feedstock qualities. They presented a real-time optimization method that can provide significantly improved profitability. Deterministic global optimization methods, such as branch-and-bound, cutting plane algorithms were studied by Horst and Tuy (1993). Ryoo and Sahinidis (1995) proposed a novel branch-and-bound-based method for discrete/continuous global optimization, using the idea of range reduction tests of variables.

The objective of this paper is to propose a new mathematical model that addresses the simultaneous optimization of short-term scheduling problem of refinery operations as stated in Section 2. In Section 3, the mathematical formulations of three sub-problems are presented and then applied to case studies in the following section. The state-task network (STN) representation introduced by Kondili, Pantelides, and Sargent (1993) is used throughout this paper.

2. Problem definition

The overall oil-refinery system is decomposed into three parts as depicted in Fig. 1. The first part (problem 1, Fig. 1) involves the crude-oil unloading, mixing and inventory control, the second part (problem 2, Fig. 1) consists of the production unit scheduling which includes both fractionation and reaction processes, and the third part (problem 3, Fig. 1), depicts the finished product blending and shipping end of the refinery. The key information available from external sources are:

(a) key component concentration ranges;
(b) yields between feed grades and product grades;
(c) recipe of each product which is assumed fixed to maintain model's linearity;
(d) amount of product required for each order;
(e) minimum and maximum flow rates;
(f) capacity limitations of all tanks;
(g) types of materials that can be stored in each tank; and
(h) time horizon under consideration.

The objective is to determine the following variables:

(a) starting and end times of tasks taking place at each stage;
(b) amount and type of material being produced or consumed at each time; and
(c) amount and type of material being stored at each time in each tank, so as to minimize the operation cost, maximize the total profit and process all the orders in specific time periods. The overall problem is first decomposed into three sub-problems as illustrated in Fig. 1, and each of those is modeled and solved in an efficient way based on a continuous time formulation, as described in the following sections.

3. Mathematical formulation

The following assumptions are made regarding the refinery operations:

(a) the times required for unit mode change are neglected;
(b) perfect mixing is assumed in the tanks; and
(c) the property state of each crude-oil or mixture is decided only by specific key components. Specific assumptions for each of the problems are stated at the corresponding sections.

The mathematical model involves mainly material balance constraints, allocation constraints, sequence constraints, and demand constraints. Material balance constraints connect the amounts of material in one unit at one event point to that at the next event point. Al location constraints set the delivery

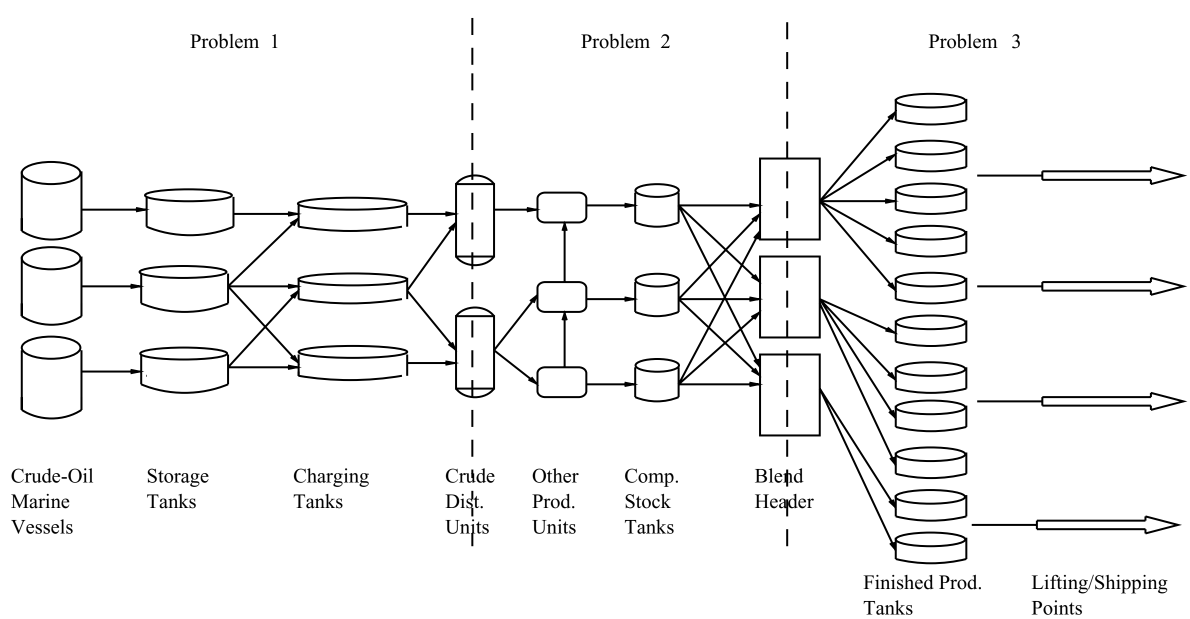

Fig. 1. Graphic overview of refinery system.

assignments between two consecutive stages, and the beginning and finishing times of each operation are determined by the sequence constraints. Demand constraints ensure that all the demands will be satisfied during the time horizon. In the following sections the detailed models for each one of the problems are presented.

3.1. Crude-oil unloading and blending (problem 1)

A typical crude-oil unloading system considered here consists of crude-oil marine vessels—used for crude-oil transportation, storage tanks—serving as the off-loading tanks for incoming crude-oils, charging tanks—used for crude-oil blending, and crude-oil distillation units—where the main hydrocarbon separation into downstream feedstocks takes place. Crude-oil vessels unload crude-oil into storage tanks after arrival at the refinery docking station. Then the crud-oil is transferred from storage tanks to charging tanks, in which a crude-oil mix is produced. The crude-oil mix in each charging tank may then be charged into one or more crude-oil distillation units.

3.1.1. Material balance constraints for vessel (v), storage tank (i) and charging tank (j)

Each vessel (v) initially has ($V_0(v)$) crude-oil and unloads all its crude-oil to storage tanks before leaving the docking station:

$$V_0(v) = \sum_{i \in I_v} \sum_{n} B(v, i, n), \quad \forall v \in V \tag{1a}$$

Constraints (1b)–(1g) express that the volumes of crude-oil or oil mix in vessels or tanks at event point (n) is equal to that at event point ($n - 1$) adjusted by any amounts fed from previous stage or transferred to next stage.

$$V(v, n) = V_0(v) - \sum_{i \in I_v} B(v, i, n), \quad \forall v \in V,\ n = 1 \tag{1b}$$

$$V(v, n) = V(v, n-1) - \sum_{i \in I_v} B(v, i, n), \quad \forall v \in V,\ n \in N,\ n \neq 1 \tag{1c}$$

$$V(i, n) = V_0(i) + \sum_{v \in V_i} B(v, i, n) - \sum_{j \in J_i} B(i, j, n), \quad \forall i \in I,\ n = 1 \tag{1d}$$

$$V(i, n) = V(i, n-1) + \sum_{v \in V_i} B(v, i, n) - \sum_{j \in J_i} B(i, j, n), \quad \forall i \in I,\ n \in N,\ n \neq 1 \tag{1e}$$

$$V(j, n) = V_0(j) + \sum_{i \in I_j} B(i, j, n) - \sum_{l \in L_j} B(j, l, n), \quad \forall j \in J,\ n = 1 \tag{1f}$$

$$V(j, n) = V(j, n-1) + \sum_{i \in I_j} B(i, j, n) - \sum_{l \in L_j} B(j, l, n), \quad \forall j \in J,\ n \in 1,\ n \neq 1 \tag{1g}$$

3.1.2. Material balance constraints for the component (k) in storage tank (i) and charging tank (j)

Based on the assumption that the property of each crude-oil is decided by key components, such as sulfur or metals, in the crude-oil, the appropriate material balance constraints for the components are formulated to monitor the quantities and restrain the concentrations from exceeding pre-defined specifications.

Constraints (2a) express that the initial amount of component (k) in storage tank (i) is determined by [the initial amount of crude-oil in tank (i) × [the initial concentration of component (k) in tank (i)]. The amount of component (k) in storage tank (i) at event point (n) is equal to that at event point ($n - 1$) adjusted by any amounts unloaded from vessels or transferred to the charging tanks as determined by constraints (2b).

$$V(i, k, n) = V_0(i) \times D_0(i, k) + \sum_{v \in V_i} B(v, i, n) \times D(v, k) - \sum_{j \in J_i} B(i, j, k, n), \quad \forall i \in I,\ k \in K,\ n = 1 \tag{2a}$$

$$V(i, k, n) = V(i, k, n-1) + \sum_{v \in V_i} B(v, i, n) \times D(v, k) - \sum_{j \in J_i} B(i, j, k, n), \quad \forall i \in I,\ k \in K,\ n = N,\ n \neq 1 \tag{2b}$$

Constraints (2c) and (2d) express the requirement that the concentration of component (k) in storage tank (i) should be in the required concentration range.

$$V(i, n) \times D_{\min}(i, k) \leq V(i, k, n) \leq V(i, n) \times D_{\max}(i, k), \quad \forall i \in I,\ k \in K,\ n \in N \tag{2c}$$

$$B(i, j, n) \times D_{\min}(i, k) \leq B(i, j, k, n) \leq B(i, j, n) \times D_{\max}(i, k), \quad \forall i \in I,\ j \in J_i,\ k \in K,\ n \in N \tag{2d}$$

Similar to constraints (2a)–(2d), the following constraints represent the material balance for the components in charging tanks. Note that in constraint (2f), $\sum_{i \in I_j} B(i, j, k, n)$ can be written as $\sum_{i \in I_j} B(i, j, n) \times D_0(i, k)$ in the case that the concentration of components in the storage tanks is constant.

$$V(j,k,n) = V_0(j) \times D_0(j,k) + \sum_{i \in I_j} B(i,j,k,n) - \sum_{l \in L_j} B(j,l,k,n), \quad \forall j \in J,\ k \in K,\ n = 1 \quad (2e)$$

$$V(j,k,n) = V(j,k,n-1) + \sum_{i \in I_j} B(i,j,k,n) - \sum_{l \in L_j} B(j,l,k,n), \quad \forall j \in J,\ k \in K,\ n \in N,\ n \neq 1 \quad (2f)$$

$$V(j,n) \times D_{\min}(j,k) \le V(j,k,n) \le V(j,n) \times D_{\max}(j,k), \quad \forall j \in J,\ k \in K,\ n \in N \quad (2g)$$

$$B(j,l,n) \times D_{\min}(j,k) \le B(j,l,k,n) \le B(j,l,n) \times D_{\max}(j,k), \quad \forall j \in J,\ l \in L_j,\ k \in K,\ n \in N \quad (2h)$$

3.1.3. Flow rate constraints

The following constraints (3a) and (3b) express the requirement that the volume of crude-oil or mix being transferred should be: [duration time × flow rate], while constraints (3c) express the limitations of minimum and maximum flow rate when an oil mix is being charged into CDU.

$$(T_{\mathrm{f}}(v,i,n) - T_{\mathrm{s}}(v,i,n)) \times f_{\mathrm{rate}} = B(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N \quad (3a)$$

$$(T_{\mathrm{f}}(i,j,n) - T_{\mathrm{s}}(i,j,n)) \times f_{\mathrm{rate}} = B(i,j,n), \quad \forall i \in I,\ j \in J_i,\ n \in N \quad (3b)$$

$$(T_{\mathrm{f}}(j,l,n) - T_{\mathrm{s}}(j,l,n)) \times f_{\min} \le B(j,l,n) \le (T_{\mathrm{f}}(j,l,n) - T_{\mathrm{s}}(j,l,n)) \times f_{\max}, \quad \forall j \in J,\ l \in L_j,\ n \in N \quad (3c)$$

Note that equal flow rate is assumed for the tasks of transferring crude-oil and mix since those tasks can be performed simultaneously for the same tank.

3.1.4. Demand constraints

The demand of crude-oil mix (j) should be met by the total amount of crude-oil mix being lifted from a charging tank (j).

$$\sum_{l \in L_j} \sum_{n \in N} B(j,l,n) = \mathrm{DM}(j), \quad \forall j \in J \quad (4)$$

3.1.5. Sequence constraints: vessel (v) → storage tank (i)

The requirements that all vessels can start unloading crude-oil only after their arrival and must empty their cargo before the end of the time horizon are expressed through constraints (5a) and (5b), respectively.

$$T_{\mathrm{s}}(v,i,n) \ge T_{\mathrm{arr}}(v) \times x1(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N \quad (5a)$$

$$T_{\mathrm{f}}(v,i,n) \le H, \quad \forall v \in V,\ i \in I_v,\ n \in N \quad (5b)$$

Each vessel (v) starting to unload to tank (i) at event point ($n+1$) should be after the end of any unloading activity at event point (n), note that (v and v') in constraint (5c) could be the same or different vessels.

$$T_{\mathrm{s}}(v,i,n+1) \ge T_{\mathrm{f}}(v',i,n) - H \times (1 - x1(v',i,n)), \quad \forall v,\ v' \in V,\ i \in I_v,\ n \in N,\ n \neq \mathrm{NE} \quad (5c)$$

$$T_{\mathrm{s}}(v,i,n+1) \ge T_{\mathrm{s}}(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N,\ n \neq \mathrm{NE} \quad (5d)$$

$$T_{\mathrm{f}}(v,i,n+1) \ge T_{\mathrm{f}}(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N,\ n \neq \mathrm{NE} \quad (5e)$$

If vessel (v') arrives at the docking station later than vessel (v), then it cannot start unloading until vessel (v) finishes and leaves as described by constraints (5f).

$$\sum_n T_{\mathrm{st}}(v',i,n) \ge \sum_n T_{\mathrm{ft}}(v,i,n), \quad \forall v \in V_i,\ v' \in V_i,\ i \in 1,\ T_{\mathrm{arr}}(v') > T_{\mathrm{arr}}(v) \quad (5f)$$

Similar sequence constraints are imposed for the crude-oil transfer between storage tank (i) and charging tank (j) (constraints (6a)–(6d)) and charging tank (j) and CDU (l) (constraints (7a)–(7d)).

3.1.6. Sequence constraints: storage tank (i) → charging tank (j)

$$T_{\mathrm{s}}(i,j,n+1) \ge T_{\mathrm{f}}(i,j',n) - H \times (i - x2(i,j',n)), \quad \forall i \in I,\ j,\ j' \in J_i,\ n \in N,\ n \neq \mathrm{NE} \quad (6a)$$

$$T_{\mathrm{s}}(i,j,n+1) \ge T_{\mathrm{s}}(i,j,n), \quad \forall i \in I,\ j \in J_i,\ n \in N,\ n \neq \mathrm{NE} \quad (6b)$$

$$T_{\mathrm{f}}(i,j,n+1) \ge T_{\mathrm{f}}(i,j,n), \quad \forall i \in I,\ j \in J_i,\ n \in N,\ n \neq \mathrm{NE} \quad (6c)$$

$$T_{\mathrm{f}}(i,j,n) \le H, \quad \forall i \in I,\ j \in J_i,\ n \in N \quad (6d)$$

Constraints (6e) and (6f), together with constraints (5c) and (6a), state that any activity that takes place at event point ($n+1$) can only start after the end time of activity at event point (n). If storage tank (i) receives crude-oil and delivers mix at event point (n) simultaneously, the delivering task can only start after receiving task avoiding any infeasibility at the storage tanks.

$$T_s(v,i,n+1) \geq T_f(i,j,n) - H \times (1 - x2(i,j,n)), \quad \forall v \in V_i,\ i \in I,\ n \in N,\ n \neq \text{NE} \tag{6e}$$

$$T_s(i,j,n+1) \geq T_f(v,i,n) - H \times (1 - x1(i,j,n)), \quad \forall v \in V_i,\ i \in I,\ j \in J_i,\ n \in N,\ n \neq \text{NE} \tag{6f}$$

$$T_s(i,j,n) + H \times (1 - x2(i,j,n)) \geq T_s(v,i,n) - H \times (1 - x1(v,i,n)), \quad \forall v \in V_i,\ i \in I,\ j \in J_i,\ n \in N \tag{6g}$$

3.1.7. Sequence constraints: charging tank (j) → CDU (1)

$$T_s(j,l,n+1) \geq T_f(j,l,n) - H \times (1 - x3(j,l,n)), \quad \forall j \in J,\ l \in L_j,\ n \in N,\ n \neq \text{NE} \tag{7a}$$

$$T_s(j,l,n+1) \geq T_s(j,l,n), \quad \forall j \in J,\ l \in L_j,\ n \in N,\ n \neq \text{NE} \tag{7b}$$

$$T_f(j,l,n+1) \geq T_f(j,l,n), \quad \forall j \in J,\ l \in L_j,\ n \in N,\ n \neq \text{NE} \tag{7c}$$

$$T_f(j,l,n) \leq H, \quad \forall j \in J,\ l \in L_j,\ n \in N \tag{7d}$$

Each charging tank (j) can either be fed from storage tank (i) or charge CDU (l) at any event point (n).

$$T_s(i,j,n+1) \geq T_f(j,l,n) - H \times (1 - x3(j,l,n)), \quad \forall i \in I,\ j \in J_i,\ l \in L_j,\ n \in N,\ n \neq \text{NE} \tag{7e}$$

$$T_s(j,l,n+1) \geq T_f(i,j,n) - H \times (1 - x2(i,j,n)), \quad \forall i \in I,\ j \in J_i,\ l \in L_j,\ n \in N,\ n \neq \text{NE} \tag{7f}$$

Charging tank (j) should start charging CDU (l) after completing the charging of CDU (l') in previous event points.

$$T_s(j,l,n+1) \geq T_f(j,l',n) - H \times (1 - x3(j,l'n)), \quad \forall j \in J,\ l \in L_j,\ l' \in L_j,\ n \in N,\ n \neq \text{NE} \tag{7g}$$

Each CDU (l) must operate continuously over the entire time horizon. Thus, the total operation time of each CDU (l) should be equal to the time horizon (H). Constraints (7i) and (7j) express that if CDU (l) is charged at event point (n), then the next charge should start exactly at the ending time of this event point (n).

$$\sum_n \sum_{j \in J_l} (T_f(j,l,n) - T_s(j,l,n)) = H, \quad \forall l \in L,\ n \in N \tag{7h}$$

$$T_s(j,l,n+1) \geq T_f(j',l,n) - H \times (1 - x3(j',l,n)), \quad \forall j \in J_l,\ j' \in J_l,\ l \in L,\ n \in N,\ n \neq \text{NE} \tag{7i}$$

$$T_s(j,l,n+1) \leq T_f(j',l,n) + H \times (1 - x3(j',l,n)), \quad \forall j \in J_l,\ j' \in J_l,\ l \in L,\ n \in N,\ n \neq \text{NE} \tag{7j}$$

3.1.8. Beginning–ending time consideration

The starting and end times of unloading of vessel (v) are described as $T_{st}(v,i,n) = T_s(v,i,n) \times x_1(v,i,n)$ and $T_{ft}(v,i,n) = T_f(v,i,n) \times x_1(v,i,n)$ that involve bilinear terms (continuous × binary). By applying Glover's transformation (Floudas, 1995) to constraints (8a)–(8d), linearity can be preserved.

$$T_s(v,i,n) - H \times (1 - x1(v,i,n)) \leq T_{st}(v,i,n) \leq T_s(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N \tag{8a}$$

$$T_{st}(v,i,n) \leq H \times x1(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N \tag{8b}$$

$$T_f(v,i,n) - H \times (1 - x1(v,i,n)) \leq T_{ft}(v,i,n) \leq T_f(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N \tag{8c}$$

$$T_{ft}(v,i,n) \leq H \times x1(v,i,n), \quad \forall v \in V,\ i \in I_v,\ n \in N \tag{8d}$$

3.1.9. Objective function

The objective of this problem is to minimize the total operating cost.

$$\begin{aligned}\text{cost} = {} & c_{sea} \times \sum_v \sum_{i \in I_v} \sum_n (T_{st}(v,i,n) - T_{arr}(v)) \\ & + c_{un} \times \sum_v \sum_{i \in I_v} \sum_n (T_{ft}(v,i,n) - T_{st}(v,i,n)) \\ & + c_{inv} \times H \times \sum_i \left(\frac{\sum_n V(i,n) + V_0(i)}{\text{NE}+1} \right) \\ & + c_{inc} \times H \times \sum_j \left(\frac{\sum_n V(j,n) + V_0(j)}{\text{NE}+1} \right) \\ & + c_{set} \times \left(\sum_i \sum_{l \in L_j} \sum_n x3(j,l,n) - \text{NST} \right)\end{aligned}$$

The first term in the objective function is the sea waiting cost, where $\sum_v \sum_{i \in I_v} \sum_n (T_{st}(v,i,n) - T_{arr}(v))$ is the total waiting time at sea for all the vessels. The second term represents the unloading cost, where $\sum_v \sum_{i \in I_v} \sum_n (T_{ft}(v,i,n) - T_{st}(v,i,n))$ is the total unloading duration of all the vessels. The total inventory levels of storage tanks and charging tanks are approximated by $\sum_i (\sum_n V(i,n) + V_0(i))/(\text{NE}+1)$ and $\sum_j (\sum_n V(j,n) + V_0(j))/(\text{NE}+1)$, respectively, which corresponds to the average value of the inventory level of the tanks over the time horizon under consideration. This approximation is selected in order to maintain the linearity of

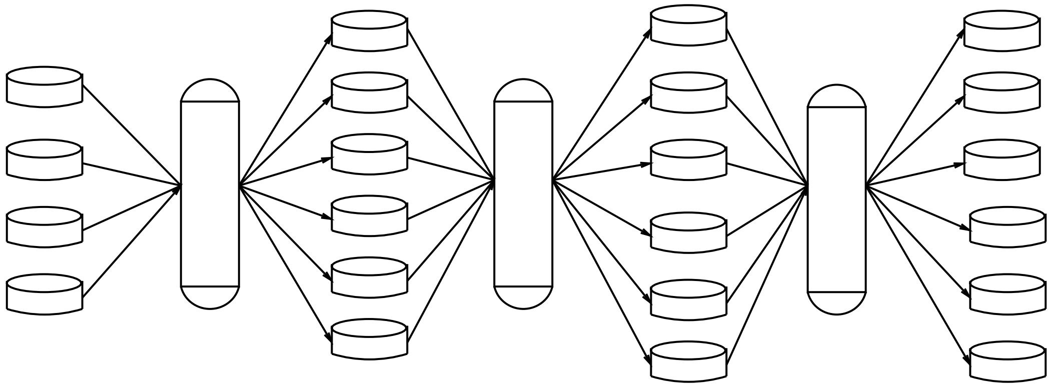

Fig. 2. Flowchart of production units for lube-oil refinery.

the model. As it will be shown in the examples in the next section, this simplification corresponds to a close approximation of the actual inventory cost and results in reasonable schedules.

3.2. Production unit scheduling (problem 2)

A representative example for production scheduling problem, that corresponds to a lube-oil refinery, is considered here and includes three processing stages: extraction, dewaxing and hydrofinishing (Fig. 2).

3.2.1. Material balance constraints for materials in tank (t)

Constraints (9a)–(9h) state that the amount of materials in tank (t) at event point (n) is equal to that at event point ($n-1$) adjusted by any amounts produced or consumed between the event points ($n-1$) and (n). For raw material ($s1$), there is no amount produced from the previous stage while for final product ($s4$), there is no amount transferred to the next stage.

$$b1(s1,t,n) = b1(s1,t,n-1) - \text{out}\,1(s1,t,n), \quad \forall s1 \in S1,\ t \in T_{s1},\ n \in N,\ n \neq 1 \quad (9a)$$

$$b1(s1,t,n) = \text{initial}\,1(s1,t) - \text{out}\,1(s1,t,n), \quad \forall s1 \in S1,\ t \in T_{s1},\ n = 1 \quad (9b)$$

$$b2(s2,t,n) = b2(s2,t,n-1) - \text{out}\,2(s2,t,n) + \text{in}\,2(s2,t,n-1) \quad \forall s2 \in S2,\ t \in T_{s2},\ n \in N,\ n \neq 1 \quad (9c)$$

$$b3(s3,t,n) = \text{initial}\,2(s2,t) - \text{out}\,2(s2,t,n), \quad \forall s2 \in S2,\ t \in T_{s2},\ n = 1 \quad (9d)$$

$$b3(s3,t,n) = b3(s3,t,n-1) - \text{out}\,3(s3,t,n) + \text{in}\,3(s3,t,n-1) \quad \forall s3 \in S3,\ t \in T_{s3},\ n \in N,\ n \neq 1 \quad (9e)$$

$$b3(s3,t,n) = \text{initial}\,3(s3,t) - \text{out}\,3(s3,t,n), \quad \forall s3 \in S3,\ t \in T_{s3},\ n = 1 \quad (9f)$$

$$b4(s4,t,n) = b4(s4,t,n-1) + \text{in}\,4(s4,t,n-1), \quad \forall s4 \in S4,\ t \in T_{s4},\ n \in N,\ n \neq 1 \quad (9g)$$

$$b4(s4,t,n) = \text{initial}\,4(s4,t), \quad \forall s4 \in S4,\ t \in T_{s4},\ n = 1 \quad (9h)$$

3.2.2. Material balance constraints for production units

Constraints (10a)–(10d) express that for each production unit, the amount of product grade at event point (n) is equal to the that of feed grade at the same event point multiplied by the yield. Since there is no one-to-one assignment between ($s1$) and ($s2$) at extraction stage, constraints (10a) and (10b) are active only if ($s1$) is consumed to produce ($s2$) at event point (n) (i.e., $\text{uv}\,1(s1,s2,n) = 1$).

$$\sum_{t \in T_{s1}} \text{out}\,1(s1,t,n) \times \text{yield}\,1(s1,s2) \leq \sum_{t \in T_{s2}} \text{in}\,2(s2,t,n) + U2 \times (1 - \text{uv}\,1(s1,s2,n)), \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N \quad (10a)$$

$$\sum_{t \in T_{s1}} \text{out}\,1(s1,t,n) \times \text{yield}\,1(s1,s2) \geq \sum_{t \in T_{s2}} \text{in}\,2(s2,t,n) - U2 \times (1 - \text{uv}\,1(s1,s2,n)), \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N \quad (10b)$$

$$\sum_{t\in T_{s2}} \mathrm{out\,2}(s2,t,n)\times \mathrm{yield\,2}(s2,s3) = \sum_{t\in T_{s3}} \mathrm{in\,3}(s3,t,n),\quad \forall s2\in S2,\ s3\in S3_{s2},\ n\in N \quad (10c)$$

$$\sum_{t\in T_{s3}} \mathrm{out\,3}(s3,t,n)\times \mathrm{yield\,3}(s3,s4) = \sum_{t\in T_{s4}} \mathrm{in\,4}(s4,t,n),\quad \forall s3\in S3,\ s4\in S4_{s3},\ n\in N \quad (10d)$$

3.2.3. Capacity constraints

If there is material in tank (t) at event point (n), then constraints (11a)–(11d) correspond to the upper and lower bounds on the capacity ($b(s,t,n)$).

$$l_{\mathrm{vol}}(t)\times y1(s1,t,n)\le b1(s1,t,n)\le h_{\mathrm{vol}}(t)\times y1(s1,t,n),\quad \forall s1\in S1,\ t\in T_{s1},\ n\in N \quad (11a)$$

$$l_{\mathrm{vol}}(t)\times y2(s2,t,n)\le b2(s2,t,n)\le h_{\mathrm{vol}}(t)\times y2(s2,t,n),\quad \forall s2\in S2,\ t\in T_{s2},\ n\in N \quad (11b)$$

$$l_{\mathrm{vol}}(t)\times y3(s3,t,n)\le b3(s3,t,n)\le h_{\mathrm{vol}}(t)\times y3(s3,t,n),\quad \forall s3\in S3,\ t\in T_{s3},\ n\in N \quad (11c)$$

$$l_{\mathrm{vol}}(t)\times y4(s4,t,n)\le b4(s4,t,n)\le h_{\mathrm{vol}}(t)\times y4(s4,t,n),\quad \forall s4\in S4,\ t\in T_{s4},\ n\in N \quad (11d)$$

Similarly, constraints (11e)–(11g) impose the upper and lower bounds on the amount of material being processed in the production units at event point (n). $U1$, $U2$ and $U3$ are obtained by summing up the maximum capacities of suitable tanks, while *low* is chosen as approximately two to three times of normal flow rate.

$$\mathrm{low}\times \mathrm{uv\,1}(s1,s2,n)\le \sum_{t\in T_{s1}} \mathrm{out\,1}(s1,t,n) \le U1\times \mathrm{uv\,1}(s1,s2,n),\quad \forall s1\in S1,\ n\in N \quad (11e)$$

$$\mathrm{low}\times \mathrm{uv\,2}(s2,s3,n)\le \sum_{t\in T_{s2}} \mathrm{out\,2}(s2,t,n) \le U2\times \mathrm{uv\,2}(s2,s3,n),\quad \forall s2\in S2,\ n\in N \quad (11f)$$

$$\mathrm{low}\times \mathrm{uv\,3}(s3,s4,n)\le \sum_{t\in T_{s3}} \mathrm{out\,3}(s3,t,n) \le U3\times \mathrm{uv\,3}(s3,s4,n),\quad \forall s3\in S3,\ n\in N \quad (11g)$$

3.2.4. Allocation constraints

Constraints (12a) represent that at most one material can be stored in tank (t) at event point (n).

$$\sum_{s1} y1(s1,t,n)+\sum_{s2} y2(s2,t,n)+\sum_{s3} y3(s3,t,n)+\sum_{s4} y4(s4,t,n)\le 1,\quad \forall t\in T,\ s1\in S1_t,\ s2\in S2_t,\ s3\in S3_t,\ s4\in S4_t,\ n\in N \quad (12a)$$

According to constraints (12b)–(12d), at most one reaction can take place in one production unit at even point (n).

$$\sum_{s1}\sum_{s2} \mathrm{uv\,1}(s1,s2,n)\le 1,\quad \forall s1\in S1,\ s2\in S2_{s1},\ n\in N \quad (12b)$$

$$\sum_{s2}\sum_{s3} \mathrm{uv\,2}(s2,s3,n)\le 1,\quad \forall s2\in S2,\ s3\in S3_{s2},\ n\in N \quad (12c)$$

$$\sum_{s3}\sum_{s4} \mathrm{uv\,3}(s3,s4,n)\le 1,\quad \forall s3\in S3,\ s4\in S4_{s3},\ n\in N \quad (12d)$$

3.2.5. Mode switch-over constraints

If different raw materials ($s1$) and ($s1'$) are stored in the same tank consecutively, only certain switch-overs ($\mathrm{switch}(s1, s1') = 1$) are allowed. Constraint (13) prevents all other disallowed switch-overs.

$$y1(s1,t,n)+y1(s1',t,n+1)\le 1,\quad \forall t\in T,\ s1\in S1_t,\ s1'\in S1_t,\ \mathrm{switch}(s1,s1')\ne 1,\ n\in N,\ n\ne \mathrm{NE} \quad (13)$$

3.2.6. Demand constraints

The initial amount of final product ($s4$) combined with the amount produced during the time horizon should meet the requirement (demand($s4$)).

$$\sum_{t\in T_{s4}}\sum_{n} (\mathrm{in\,4}(s4,t,n)+\mathrm{initial\,4}(s4,t))\ge \mathrm{demand}(s4),\quad \forall s4\in S4,\ n\in N \quad (14)$$

3.2.7. Sequence constrains for same reaction in the same unit

The following three sets of constraints state that one reaction starting at event point ($n+1$) should start after the end time of the same reaction in the same production unit which has started at event point (n). Every reaction should start and end within the time horzion (H).

$$T_{s1}(s1,s2,n+1) \ge T_{e1}(s1,s2,n)-H\times(1-\mathrm{uv\,1}(s1,s2,n)),\quad \forall s1\in S1,\ s2\in S2_{s1},\ n\in N,\ n\ne \mathrm{NE} \quad (15a)$$

$$T_{s1}(s1, s2, n+1) \geq T_{s1}(s1, s2, n), \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N,\ n \neq \text{NE} \quad (15b)$$

$$T_{e1}(s1, s2, n+1) \geq T_{e1}(s1, s2, n), \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N,\ n \neq \text{NE} \quad (15c)$$

$$T_{s1}(s1, s2, n) \leq H, \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N \quad (15d)$$

$$T_{e1}(s1, s2, n) \leq H, \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N \quad (15e)$$

$$T_{s2}(s2, s3, n+1) \geq T_{e2}(s2, s3, n) - H \times (1 - \text{uv}\,2(s2, s3, n)), \quad \forall s2 \in S2,\ s3 \in S3_{s2},\ n \in N,\ n \neq \text{NE} \quad (16a)$$

$$T_{s2}(s2, s3, n+1) \geq T_{s2}(s2, s3, n), \quad \forall s2 \in S2,\ s3 \in S3_{s2},\ n \in N,\ n \neq \text{NE} \quad (16b)$$

$$T_{e2}(s2, s3, n+1) \geq T_{e2}(s2, s3, n), \quad \forall s2 \in S2,\ s3 \in S3_{s2},\ n \in N,\ n \neq \text{NE} \quad (16c)$$

$$T_{s2}(s2, s3, n) \leq H, \quad \forall s2 \in S2,\ s3 \in S3_{s2},\ n \in N \quad (16d)$$

$$T_{e2}(s2, s3, n) \leq H, \quad \forall s2 \in S3,\ s3 \in S3_{s2},\ n \in N \quad (16e)$$

$$T_{s3}(s3, s4, n+1) \geq T_{e3}(s3, s4, n) - H \times (1 - \text{uv}\,3(s3, s4, n)), \quad \forall s3 \in S4,\ s4 \in S4_{s3},\ n \in N,\ n \neq \text{NE} \quad (17a)$$

$$T_{s3}(s3, s4, n+1) \geq T_{s3}(s3, s4, n), \quad \forall s3 \in S4,\ s4 \in S4_{s3},\ n \in N,\ n \neq \text{NE} \quad (17b)$$

$$T_{e3}(s3, s4, n+1) \geq T_{e3}(s3, s4, n), \quad \forall s3 \in S4,\ s4 \in S4_{s3},\ n \in N,\ n \neq \text{NE} \quad (17c)$$

$$T_{s3}(s3, s4, n) \leq H, \quad \forall s3 \in S4,\ s4 \in S4_{s3},\ n \in N \quad (17d)$$

$$T_{e3}(s3, s4, n) \leq H, \quad \forall s3 \in S4,\ s4 \in S4_{s3},\ n \in N \quad (17e)$$

3.2.8. Sequence constrains for different reactions in the same unit

If two reactions take place in the same production unit, then they should be performed at most consecutively.

$$T_{s1}(s1, s2, n+1) \geq T_{e1}(s1', s2', n) - H \times (1 - \text{uv}\,1(s1', s2', n)), \quad \forall s1, s1' \in S1,\ s2 \in S2_{s1},\ s2' \in S2'_{s1},\ s2 \neq s2',\ n \in N,\ n \in N,\ n \neq \text{NE} \quad (18a)$$

$$T_{s2}(s2, s3, n+1) \geq T_{e2}(s2', s3', n) - H \times (1 - \text{uv}\,2(s2', s3', n)), \quad \forall s2, s2' \in S2,\ s3 \in S3_{s2},\ s3' \in S3'_{s2},\ s3 \neq s3'\ n \in N,\ n \in N,\ n \neq \text{NE} \quad (18b)$$

$$T_{s3}(s3, s4, n+1) \geq T_{e3}(s3', s4', n) - H \times (1 - \text{uv}\,3(s3', s4', n)), \quad \forall s3, s3' \in S3,\ s4 \in S4_{s3},\ s4' \in S4'_{s3},\ s4 \neq s4'\ n \in N,\ n \neq \text{NE} \quad (18c)$$

3.2.9. Sequence constrains for different reactions in different units

Constraints (19a) and (19b) are written for two consecutive stages and express the requirements that the product of the first stage is the feed of the second one. For example, in constraint (19a), if ($s2$) is produced at event point (n) (i.e., $\text{uv}\,1(s1, s2, n) = 1$), then the starting time of dewaxing stage where ($s2$) is consumed is later than finishing time of extraction stage.

$$T_{s2}(s2, s3, n+1) \geq T_{e1}(s1, s2, n) - H \times (1 - \text{uv}\,1(s1, s2, n)), \quad \forall s1 \in S1_{s2},\ s2 \in S2,\ s3 \in S3_{s2},\ n \in N,\ n \neq \text{NE} \quad (19a)$$

$$T_{s3}(s3, s4, n+1) \geq T_{e2}(s2, s3, n) - H \times (1 - \text{uv}\,2(s2, s3, n)), \quad \forall s2 \in S2_{s3},\ s3 \in S3,\ s4 \in S4_{s3},\ n \in N,\ n \neq \text{NE} \quad (19b)$$

3.2.10. Duration constraints

Constraints (20a)–(20c) express the duration of each reaction should be between the amount of material being processed at that event point divided by the minimum and maximum flow rates.

$$\frac{\sum_{t \in T_{s1}} \text{in}\,2(s2, t, n)/\text{yield}\,1(s1, s2)}{\text{max rate}\,1} \leq T_{e1}(s1, s2, n) - T_{s1}(s1, s2, n) \leq \frac{\sum_{t \in T_{s1}} \text{in}\,2(s2, t, n)/\text{yield}\,1(s1, s2)}{\text{min rate}\,1}, \quad \forall s1 \in S1,\ s2 \in S2_{s1},\ n \in N \quad (20a)$$

$$\frac{\sum_{t \in T_{s2}} \text{out}\,2(s2, t, n)}{\text{max rate}\,2} \leq T_{e2}(s2, s3, n) - T_{s2}(s2, s3, n) \leq \frac{\sum_{t \in T_{s2}} \text{out}\,2(s2, t, n)}{\text{min rate}\,2}, \quad \forall s2 \in S2,\ s3 \in S3_{s2},\ n \in N \quad (20b)$$

$$\frac{\sum_{t\in T_{s3}} \text{out}\,3(s3,t,n)}{\text{max rate}\,3} \leq T_{e3}(s3,s4,n) - T_{s3}(s3,s4,n) \leq \frac{\sum_{t\in T_{s3}} \text{out}\,3(s3,t,n)}{\text{min rate}\,3}, \quad \forall s3\in S3,\ s4\in S4_{s3},\ n\in N \quad (20c)$$

3.2.11. Objective function

The objective of this problem is to maximize the total profit over the time horizon, expressed as:

$$\text{profit} = \sum_{s4}\left(\sum_{t\in T_{s4}}\sum_{n}(\text{in}\,4(s4,t,n) - \text{initial}\,4(s4,t)) \times \text{price}(s4)\right), \quad \forall s4\in S4, n\in N$$

3.3. Gasoline blending and distribution (problem 3)

The gasoline blending system consists of four stages all linked together through various piping segments, flowmeters and valves. They are in order: componentstock tanks, blend header, productstock tanks and lifting ports. Components from the componentstock tanks are fed to the blend header according to the recipes. Thus, different products can be produced and then stored in their suitable productstock tanks. The final step is to lift those products during the specified time periods in order to satisfy all the orders.

3.3.1. Material balance constraints for productstock tank (p)

Constraint (21a) expresses that the amount of product (s) in tank (p) at event point ($n+1$) ($b(s,p,n+1)$) is equal to that at event point (n) adjusted by any amounts transferred from blender ($\text{blnd}(s,p,n)$) or lifted at event point (n) ($\sum_{o\in O_s}\text{lift}(o,p,n)$). Constraint (21b) states that the amount of product (s) being lifted from tank (p) at the last event point (NE) should not exceed the amount of product (s) stored in tank (p).

$$b(s,p,n+1) = b(s,p,n) + \text{blnd}(s,p,n) - \sum_{o\in O_s}\text{lift}(o,p,n), \quad \forall s\in S,\ p\in P_s,\ n\in N,\ n\neq \text{NE} \quad (21a)$$

$$b(s,p,n) + \text{blnd}(s,p,n) \geq \sum_{o\in O_s}\text{lift}(o,p,n), \quad \forall s\in S,\ p\in P_s,\ n\in N \quad (21b)$$

3.3.2. Capacity constraints

Constraint (22) imposes a volume capacity limitation of product (s) in tank (p) at event point (n).

$$V_{\min}(p)\times z2(s,p,n) \leq b(s,p,n) + \text{blnd}(s,p,n) \leq V_{\max}(p)\times x2(s,p,n), \quad \forall s\in S,\ p\in P_s,\ n\in N \quad (22)$$

3.3.3. Allocation constraints

According to constraint (23a), $z1(o,p,n)$ is equal to 1 if the amount of product being lifted from tank (p) for order (o) is not zero at event point (n), that is, $\text{lift}(o,p,n)\neq 0$; $z1(o,p,n)$ equals 0, otherwise. $U1$ and $U2$ correspond to lower and upper bound on the amount of product lifted, respectively, and are chosen according to the smallest order and the maximum capacities of tanks.

$$U1\times z1(o,p,n) \leq \text{lift}(o,p,n) \leq U2\times z1(o,p,n), \quad \forall o\in O,\ p\in P_o,\ n\in N \quad (23a)$$

To avoid task splitting, constraints (23b)–(23d) state that order (o) should be processed only once if it is a small-size order and at most twice if it is a medium-size order. Otherwise, it can be processed at most three times. For different problems, $U3$ and $U4$ are chosen accordingly to define small- and medium-size orders. Constraint (23e) expresses that for large-size orders which are defined as greater than or equal to $U5$, the minimum order splitting is 25 Mbbl.

$$\sum_{n}\sum_{p\in P_o} z1(o,p,n) = 1, \quad \forall \sum_{s} P_{\text{rod\_ord}}(o,s) \leq U3, \quad o\in O,\ n\in N \quad (23b)$$

$$\sum_{n}\sum_{p\in P_o} z1(o,p,n) \leq 1, \quad \forall \sum_{s} P_{\text{rod\_ord}}(o,s) \leq U4, \quad o\in O,\ n\in N \quad (23c)$$

$$\sum_{n}\sum_{p\in P_o} z1(o,p,n) \leq 3, \quad \forall o\in O,\ n\in N \quad (23d)$$

$$25\times z1(o,p,n) \leq \text{lift}(o,p,n), \quad \forall \sum_{s} P_{\text{rod\_ord}}(o,s) \leq U5, \quad o\in O,\ p\in P_o,\ n\in N \quad (23e)$$

Constraint (23f) forces $z2(s,p,n)$ to be equal to 1 when $\text{blnd}(s,p,n)$ is not zero, otherwise, $z2(s,p,n)$ equals 0.

$$V_{\min}(p)\times z2(s,p,n) \leq \text{blnd}(s,p,n) \leq V_{\max}(p)\times z2(s,p,n), \quad \forall s\in S,\ p\in P_s,\ n\in N \quad (23f)$$

3.3.4. Demand constraints

Constraints (24a) and (24b) state that order (o) can be processed at most once in one tank during the time horizon under consideration and that the amount of product being lifted from all the productstock tanks should be equal to the amount ordered ($\sum_s P_{\text{rod\_ord}}(o,s)$).

$$\sum_{n} z1(o,p,n) \leq 1, \quad \forall o \in O,\ p \in P_o,\ n \in N \tag{24a}$$

$$\sum_{n}\sum_{p\in P_o} \text{lift}(o,p,n) = \sum_{s} P_{\text{rod\_ord}}(o,s), \quad \forall s \in S,\ o \in O,\ n \in N \tag{24b}$$

3.3.5. Sequence constraints

Constraints (25a)–(25c) state that order (o) starting in tank (p) at event point ($n+1$) should start after the ending time of the same order processed in the same tank which has started at event point (n). Constraints (25d) and (25e) express that order (o) should start and finish during the specific time period based on the order requirement These constraints are relaxed if $z1(o,p,n)$ is zero, which means the order (o) is not lifted from tank (p) at event point (n).

$$T_{\text{s}}(o,p,n+1) \geq T_{\text{f}}(o,p,n) - H \times (1 - z1(o,p,n)), \quad \forall o \in O_p,\ p \in P,\ n \in N,\ n \neq \text{NE} \tag{25a}$$

$$T_{\text{s}}(o,p,n+1) \geq T_{\text{s}}(o,p,n), \quad \forall o \in O_p,\ p \in P,\ n \in N,\ n \neq \text{NE} \tag{25b}$$

$$T_{\text{f}}(o,p,n+1) \geq T_{\text{f}}(o,p,n), \quad \forall o \in O_p,\ p \in P,\ n \in N,\ n \neq \text{NE} \tag{25c}$$

$$T_{\text{s}}(o,p,n) \geq P_{\text{rod\_srt}}(o) \times z1(o,p,n), \quad \forall o \in O_p,\ p \in P,\ n \in N \tag{25d}$$

$$T_{\text{f}}(o,p,n) \leq P_{\text{rod\_end}}(o) + H \times (1 - z1(o,p,n)), \quad \forall o \in O_p,\ p \in P,\ n \in N \tag{25e}$$

3.3.6. Duration constraints

If order (o) is processed in tank (p) at event point (n), that is, $z1(o,p,n) = 1$, then both sides of constraint (26a) are equal so that the task duration is given by $\text{lift}(o,p,n)/l(o)$, where $l(o)$ is the lifting rate of order (o). If $z1(o,p,n) = 0$, then the duration is zero according to constraint (26b).

$$\frac{\text{lift}(o,p,n) - \sum_s P_{\text{rod\_ord}}(o,s) \times (1 - z1(o,p,n))}{l(o)} \leq T_{\text{f}}(o,p,n) - T_{\text{s}}(o,p,n) \leq \frac{\text{lift}(o,p,n)}{l(o)}, \quad \forall o \in O_p,\ p \in P,\ n \in N \tag{26a}$$

$$T_{\text{f}}(o,p,n) - T_{\text{s}}(o,p,n) \leq \frac{\sum_{s\in S_p} P_{\text{rod\_ord}}(o,s) \times z1(o,p,n)}{l(o)}, \quad \forall o \in O_p,\ p \in P,\ n \in N \tag{26b}$$

3.3.7. Blending stage consideration

The consideration of the blending stage requires the incorporation of the following constraints.

3.3.8. Material balance constraints for the blender

Constraint (27) is introduced to express that the required amount of component (w) in order to produce product (s) at event point (n) ($\sum_s (\text{recipe}(s,w) \times \sum_{p\in P_s} \text{blnd}(s,p,n)))$ should be equal to the total amount of component (w) being transferred from all the component tanks at that event point ($\sum_{c\in C_w} \text{comp}(w,c,n)$).

$$\left(\sum_s (\text{recipe}(s,w) \times \sum_{p\in P_s} \text{blnd}(s,p,n)\right) = \sum_{c\in C_w} \text{comp}(w,c,n), \quad \forall s \in S,\ w \in W,\ n \in N \tag{27}$$

Note that in order to avoid the introduction of bilinear terms in the mass balance equations and keep the model linear, the assumption of constant production recipe is used. Work is underway where nonlinear blending rules and variable recipe are incorporated.

3.3.9. Material balance constraints for component tank (c)

The amount of component (w) in tank (c) at event point ($n+1$) ($bc(w,c,n+1)$) is equal to that at event point (n) ($bc(w,c,n)$) adjusted by any amounts transferred from separation units ($\text{cracking}(w,c,n)$) or delivered to the blender at event point (n) ($\text{comp}(w,c,n)$). This relation is expressed by constraint (28a). Constraint (28b) imposes the upper and the lower bounds on the flow rates of component (w) transferred from tank (c) to the blender.

$$bc(w,c,n+1) = bc(w,c,n) + \text{cracking}(w,c,n) - \text{comp}(w,c,n), \quad \forall w \in W_c,\ n \in N \tag{28a}$$

$$\text{flow}_{\text{min}} \times z4(w,c,n) \leq \text{comp}(w,c,n) \leq \text{flow}_{\text{max}} \times z4(w,c,n), \quad \forall w \in W,\ c \in C_w,\ n \in N \tag{28b}$$

3.3.10. Allocation constraints for productstock tank (p)

Constraint (29) state that product (s) cannot be transferred to productstock tank (p) and distributed at the same event point (n)

$$\sum_{s\in S_p} z2(s,p,n) + z1(o,p,n) \leq 1, \quad \forall o \in O_p,\ p \in P,\ n \in N \tag{29}$$

3.3.11. Allocation constraints for blender

According to constraints (30a), $z3(s,n)$ equals 1 if product (s) is produced and transferred to at least one tank at event point (n), whereas $z3(s,n)$ equals 0 if product (s) is not transferred to any of the tanks at event point (n). Constraint

(30b) expresses that only one product can be produced in the blender at the same event point (n).

$$z2(s,p,n) \le z3(s,n) \le \sum_{p\in P_s} z2(s,p,n), \quad \forall s\in S,\ n\in N \quad (30a)$$

$$\sum_s z3(s,n) \le 1, \quad \forall s\in S,\ n\in N \quad (30b)$$

3.3.12. Sequence constraints

Similar to constraints (25a)–(25c), constraints (31a)–(31c) state that product (s) should start being transferred to tank (p) at event point ($n+1$) after the ending time of the same product transferred to the same tank which has started at event point (n), whereas constraints (31d) and (31e) represent the requirement of all the transfers to happen within the time horizon (H).

$$T_s(s,p,n+1) \ge T_f(s,p,n) - H\times(1-z2(s,p,n)), \quad \forall s\in S_p,\ p\in P,\ n\in N,\ n\ne \mathrm{NE} \quad (31a)$$

$$T_s(s,p,n+1) \ge T_s(s,p,n), \quad \forall s\in S_p,\ p\in P,\ n\in N,\ n\ne \mathrm{NE} \quad (31b)$$

$$T_f(s,p,n+1) \ge T_f(s,p,n), \quad \forall s\in S_p,\ p\in P,\ n\in N,\ n\ne \mathrm{NE} \quad (31c)$$

$$T_s(s,p,n) \le H, \quad \forall s\in S_p,\ p\in P,\ n\in N \quad (31d)$$

$$T_f(s,p,n) \le H, \quad \forall s\in S_p,\ p\in P,\ n\in N \quad (31e)$$

If the blender provides product (s) for more than one productstock tanks at event point (n), then the starting and finishing times for all the tanks should be the same.

$$T_s(s,p,n) + H\times(1-z2(s,p,n)) \ge T_s(s,p',n) - H\times(1-z2(s,p',n)), \quad \forall s\in S_p,\ p\in P_s,\ p'\in P_s,\ p\ne p',\ n\in N \quad (32a)$$

$$T_s(s,p,n) - H\times(1-z2(s,p,n)) \le T_s(s,p',n) + H\times(1-z2(s,p',n)), \quad \forall s\in S_p,\ p\in P_s,\ p'\in P_s,\ p\ne p',\ n\in N \quad (32b)$$

$$T_f(s,p,n) + H\times(1-z2(s,p,n)) \ge T_f(s,p',n) - H\times(1-z2(s,p',n)), \quad \forall s\in S_p,\ p\in P_s,\ p'\in P_s,\ p\ne p',\ n\in N \quad (32c)$$

$$T_f(s,p,n) - H\times(1-z2(s,p,n)) \le T_f(s,p',n) + H\times(1-z2(s,p',n)), \quad \forall s\in S_p,\ p\in P_s,\ p'\in P_s,\ p\ne p',\ n\in N \quad (32d)$$

Constraints (33a) and (33b) express that product transfer and distribution should be performed consecutively in the same productstock tank (p).

$$T_s(o,p,n+1) \ge T_f(s,p,n) - H\times(1-z2(s,p,n)), \quad \forall o\in O_p,\ s\in S_p,\ p\in P,\ n\in N,\ n\ne \mathrm{NE} \quad (33a)$$

$$T_s(s,p,n+1) \ge T_f(o,p,n) - H\times(1-z1(o,p,n)), \quad \forall o\in O_p,\ s\in S_p,\ p\in P,\ n\in N,\ n\ne \mathrm{NE} \quad (33b)$$

According to constraints (34), two different products (s) and (s') being transferred to the same or different productstock tanks have to be transferred consecutively according to the allocation constraint for the blender.

$$T_s(s,p,n+1) \ge T_f(s',p',n) - H \times(1-z2(s',p',n)), \quad \forall s\in S_p,\ s'\in S_p,\ s\ne s',\ p\in P,\ p'\in P,\ n\in N,\ n\ne \mathrm{NE} \quad (34)$$

3.3.13. Duration constraints

The minimum run length of 6 h is imposed to the blender by constraint (35a)

$$\sum_{p\in P_s} \mathrm{blnd}(s,p,n) \ge 6\times B_{\mathrm{flow}}, \quad \forall s\in S,\ n\in N \quad (35a)$$

Constraints (35b) define the duration of product (s) being transferred to the tanks at event point (n) as the difference between the ending time ($T_f(s,p,n)$) and starting time ($T_s(s,p,n)$), if it takes place in tank (p). Constraints (35c) express that the duration of transferring product (s) from the blender to tank (p) corresponds to the amount of product (s) being transferred divided by the flow rate. The purpose of introducing artificial variables ($\mathrm{arti}(s,n)$) is to find a feasible solution in case a larger flow rate is required.

$$T_f(s,p,n) - T_s(s,p,n) - H\times(1-z2(s,p,n)) + H \le \mathrm{duration}(s,n) \le T_f(s,p,n) - T_s(s,p,n) \times(1-z2(s,p,n)), \quad \forall s\in S_p,\ p\in P,\ n\in N \quad (35b)$$

$$\mathrm{duration}(s,n) = \frac{\sum_{s\in S_p}\mathrm{blnd}(s,p,n)}{B_{\mathrm{flow}}} - \mathrm{arti}(s,n), \quad \forall s\in S_p,\ p\in P,\ n\in N \quad (35c)$$

3.3.14. Objective function

The objective of the scheduling problem is to minimize the sum of artificial variables in the duration constraints of

Table 1
Computational results and comparisons for example 1

Example	Variables	0–1 Variables	Constraints	Objective	Nodes	Iterations	CPU time
1	139	24	382	247.0[a]	12	635	0.28[b]
1 (Lee et al., 1996)	192	36	331	217.667	208	1695	17.1[c]
2	341	56	990	413.48	509	14430	4.89
2 (Lee et al., 1996)	4566	70	825	352.55	10525 (904)[d]	331493 (21148)	4158.8 (287.9)
3	301	48	888	343.10	110	3284	1.46
3 (Lee et al., 1996)	581	84	1222	296.56	>8993 (2519)	>515541 (60663)	>7744 (1089.4)
4	485	76	1382	387.15	718	18270	7.87
4 (Lee et al., 1996)	N/A (991)	N/A (105)	N/A (2154)	N/A (420.99)	N/A (5011)	N/A (157883)	N/A (4372.8)

[a] No direct comparison can be made since the objective obtained by the proposed methodology corresponds to an approximation of the reported value due to the continuous nature of the formulation.
[b] Sun Ultra 60 Workstation.
[c] IBM RS-600.
[d] Results in parenthesis are obtained with the use of SOS1 and priority.

blender so as to determine a feasible solution with a flow rate as close to (B_{flow}) as possible. The formulation however is general to accommodate different objective functions targeting the optimization of production.

$$\text{objective} = \sum_{s}\sum_{n} \text{arti}(s, n), \quad \forall s \in S,\ n \in N$$

4. Case studies: results and comparisons

For crude-oil unloading and blending problem (problem 1), four examples are studied with the data obtained from Lee et al. (1996). Example 1 deals with a small-size problem, while Example 4 presents a problem with three vessels, six storage tanks and four charging tanks that constitute an industrial size problem. As shown in Table 1, the proposed MILP formulation results in much smaller model in terms of constraints, continuous and binary variables. Consequently, the solutions of those examples are much easier and require less CPU time. To improve problem solving efficiency, a cheater parameter is used in CPLEX options. The gap for all the solutions are found to be zero. Note that for the industrial size problem (example 4), the proposed formulation can be solved efficiently using 718 nodes and 7.87 CPU seconds using GAMS/CPLEX 7.0. The resulting Gantt charts of crude-oil transferring between vessels, storage tanks, charging tanks and CDUs for this example are provided in Fig. 3. As illustrated in the Gantt charts, the CDUs are operated continuously throughout the entire time horizon, and charging tanks cannot be fed by storage tanks and charge CDUs at the same time. No comparison with previous results can be made since Lee et al. (1996) required the use of priority and SOS1 constraints to solve example 4. However, example 3 can be used for comparison which is smaller in terms of the number of vessels (3), storage and charging tanks (3). Using the proposed formulation, this example requires 110 nodes and 1.46 CPUs, as compared with >8993 nodes and >7744 CPUs required by Lee et al. (1996). The comparison of real and approximate values of inventory cost is provided in Table 2, which illustrates that the approximation of inventory levels in objective function is reasonable.

Table 2
Comparison of real and approximate inventory cost

Example	Real value of inventory cost	Approximated inventory cost
1	94.65	107.00
2	154.76	153.48
3	135.75	105.60
4	243.99	235.65

The case study considered for production unit scheduling stage (problem 2) is based on realistic data provided by Honeywell Hi-Spec Solutions. It consists of 5 raw materials, 3 processing stages and 10 products. Evaluation of the proposed formulation is performed by testing smaller scale instances of the problem involving the consideration of four, six, and eight products. The detailed data are presented in Table 3, while the computational characteristics of the models are tabulated in Table 4. For the case with 10 products, the proposed formulation can be solved within 1747 nodes

Table 3
System information for problem 2

Scheduling horizon (h)			168		
Feed grade	Product grade	Minimum flow rate	Maximum flow rate	Yield	Reactor
SP	SPLR	25	35	0.74	Extraction unit
SP	SPHR	27	35	0.72	
LN	LNLR	24	38	0.71	
LN	LNHR	27	36	0.70	
SPLR	SPLD	30	72	0.72	Dewaxing unit
SPHR	SPHD	28	70	0.74	
LNLR	LNLD	26	72	0.71	
LNHR	LNHD	28	70	0.72	
SPLD	SPLF	23	57	0.91	Hydro-finishing unit
SPHD	SPHF	20	55	0.92	
LNLD	LNLF	23	58	0.93	
LNHD	LNHF	20	55	0.92	

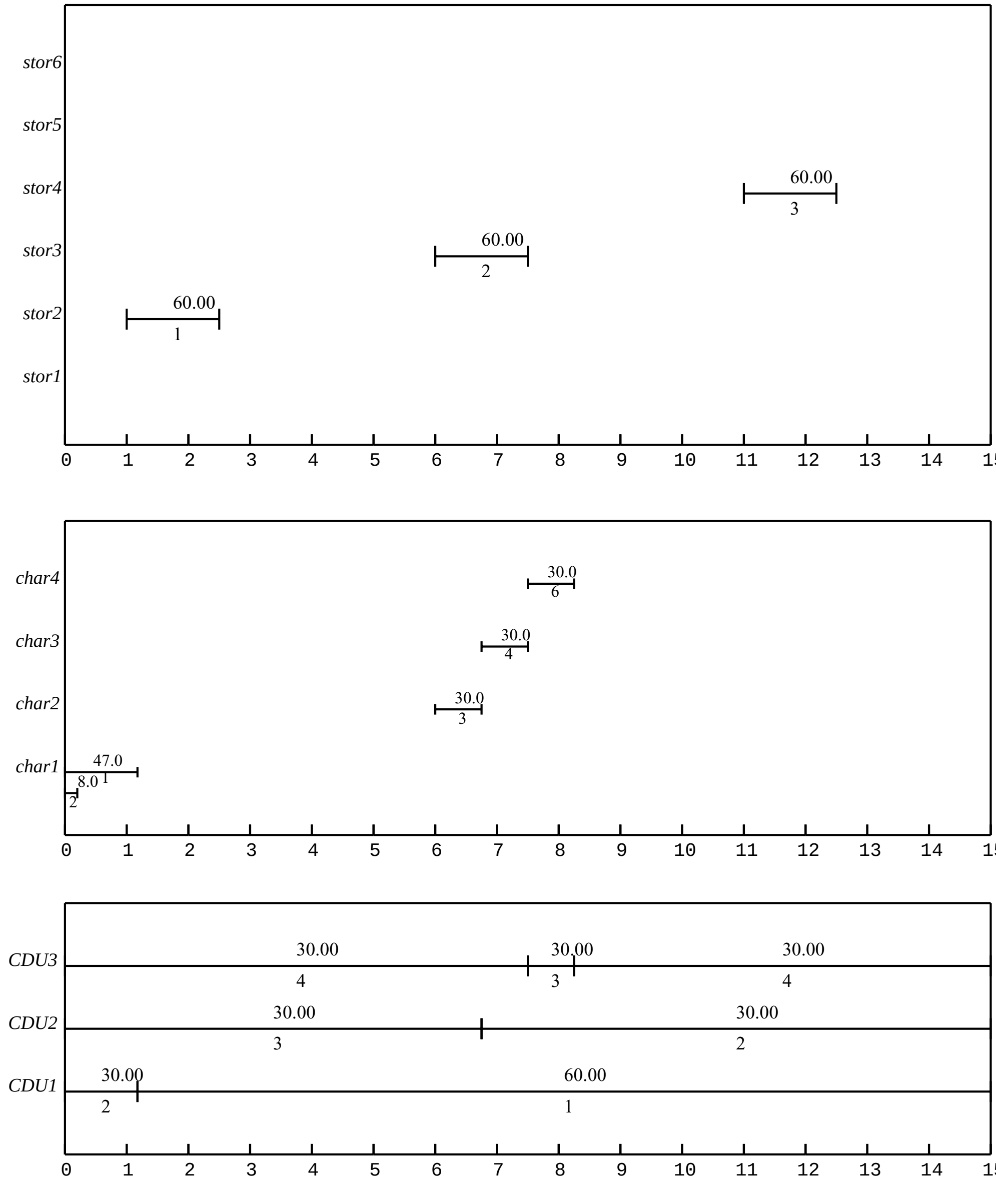

Fig. 3. Gantt charts for example 4 of problem 1.

and 6431 CPU seconds using GAMS/CPLEX 7.0. The resulting Gantt chart for this example is shown in Fig. 4. The blending and distribution problem (problem 3) considered here consists of 45 orders of 4 different products that are stored in 11 productstock tanks. The incorporation of the blending stage adds the consideration of 9 components and 20 component tanks. Smaller-scale instances of the problem are constructed to test the proposed formulation involving the consideration of 10, 16, 23, 30, and 37 orders. The detailed data for the case of 10 orders are presented in Tables 5 and 6. GAMS/CPLEX 7.0 is used for the solution of the resulting MILP formulation. For the case study examined

Table 4
Computational results for problem 2

No. of products	Continuous variables	0–1 Variables	Constraints	Objective value	Nodes	Iterations	CPU time (s)
4	6269	728	4388	2493.91	32	2417	2.58
6	18217	1668	11097	3855.86	134	27301	113.77
8	24307	2250	15095	4727.49	824	180086	1024.03
10	47255	4320	29220	5321.38	1747	734209	6431.16

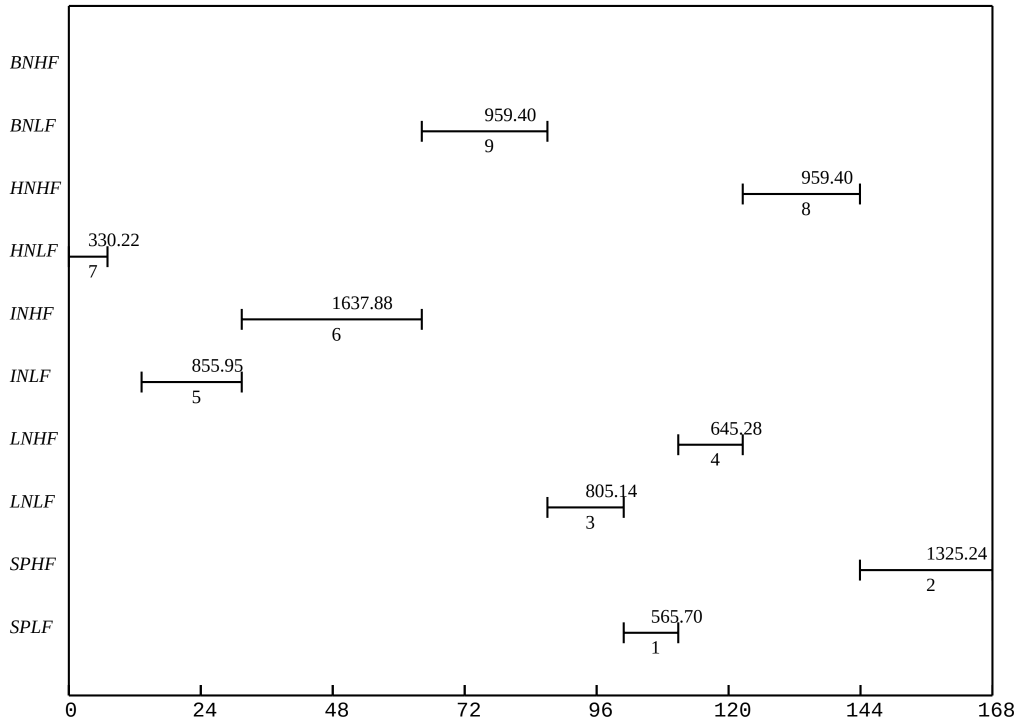

Fig. 4. Gantt chart for the example with 10 products of problem 2.

Table 5
Distribution data for example with 10 orders

Order	*o*1	*o*2	*o*3	*o*4	*o*5	*o*6	*o*7	*o*8	*o*9	*o*10	
Product	N4	W4	W4	N4	W4	N4	W4	N4	W4	N5	
Amount	11	3	3	11	3	11	3	132	3	175	
Time that can start	0	0	24	24	48	48	96	118	144	150.5	
Due date	24	24	48	48	72	72	120	190	168	185.5	
Lifting rate	50	50	50	50	50	50	50	8	50	5	
Time horizon	192										
Productstock tank	pt1	pt2	pt3	pt4	pt5	pt6	pt7	pt8	pt9	pt10	pt11
Products that can be stored	E4, W4	E4, W4	E4, W4	E4, N5, W4	E4, W4	E4, W4	N4, N5	N4, N5	N4, N5	N4, N5	N4, N5
Initial product	E4	–	W4	N5	W4	W4	N4	N4	N5	N4	N4
Amount	90.20	–	14.08	87.51	28.49	57.59	13.79	12.36	23.96	85.11	12.36
Maximum capacity	92	92	94	91	92	84	94	92	92	91	82
Minimum capacity	0.92	0.92	0.94	0.91	0.92	0.84	0.94	0.92	0.92	0.91	0.82

Table 6
Blending data for example with 10 orders

Component		A	C7	C6	M	C4	C5	CR	AR	CG
Tanks that can be stored in		ct10	ct9	ct8	ct15, 52, cts53, 54	ct51	ct57, 58, ct60	ct4, ct13	ct11, cts5	ct7, 12, 17, cts56, 59
Recipe of products	N4	0	0.0767	0	0	0.14	0.2742	0.4018	0	0.1073
	N5	0	0	0.0419	0	0.0121	0.5178	0	0.0443	0.384
	E4	0	0	0	0	0.2729	0	0.3897	0	0.3078
	W4	0.6527	0	0	0	0.1591	0	0.1882	0	0
Amount of component and tanks that it is initially stored in		26.46, ct10	67.90, ct9	59.44, ct8	7.30, ct15	0.59, ct51	0.29, ct57	27.38, ct4	25.63, ct11	34.58, ct7
					5.75, ct52		8.90, ct58	19.35, ct13	13.84, ct55	49.34, ct51
					3.10, ct53		1.64, ct60			53.41, ct56
					28.29, ct54					4.25, ct59s
Blending rate	50									

Table 7
Computational results for the blending and distribution system

Orders	Continuous variables	0–1 Variables	Constraints	First integer solution				Second integer solution			Optimal solution		
				Nodes	Iterations	CPU time (s)	Objective value	Nodes	Iterations	Objective value	Nodes	Iterations	CPU time (s)
10	1706	420	7130	21	1495	6.15	0	N/A	N/A	N/A	21	1495	6.15
16	4205	1032	18737	20	3614	29.03	0	N/A	N/A	N/A	20	3614	29.03
23	5974	1470	26746	40	13474	210.24	0	N/A	N/A	N/A	40	13474	210.24
30	9056	2232	40793	80	24906	627.13	0	N/A	N/A	N/A	80	24906	627.13
37	13955	3444	63308	828	177746	4081.49	4.934	1338	244258	0.793	1353	246176	5016.51
45	25454	6289	116452	361	194954	7406.48	6.645	4138	838195	5.094	4280	874051	20351.18

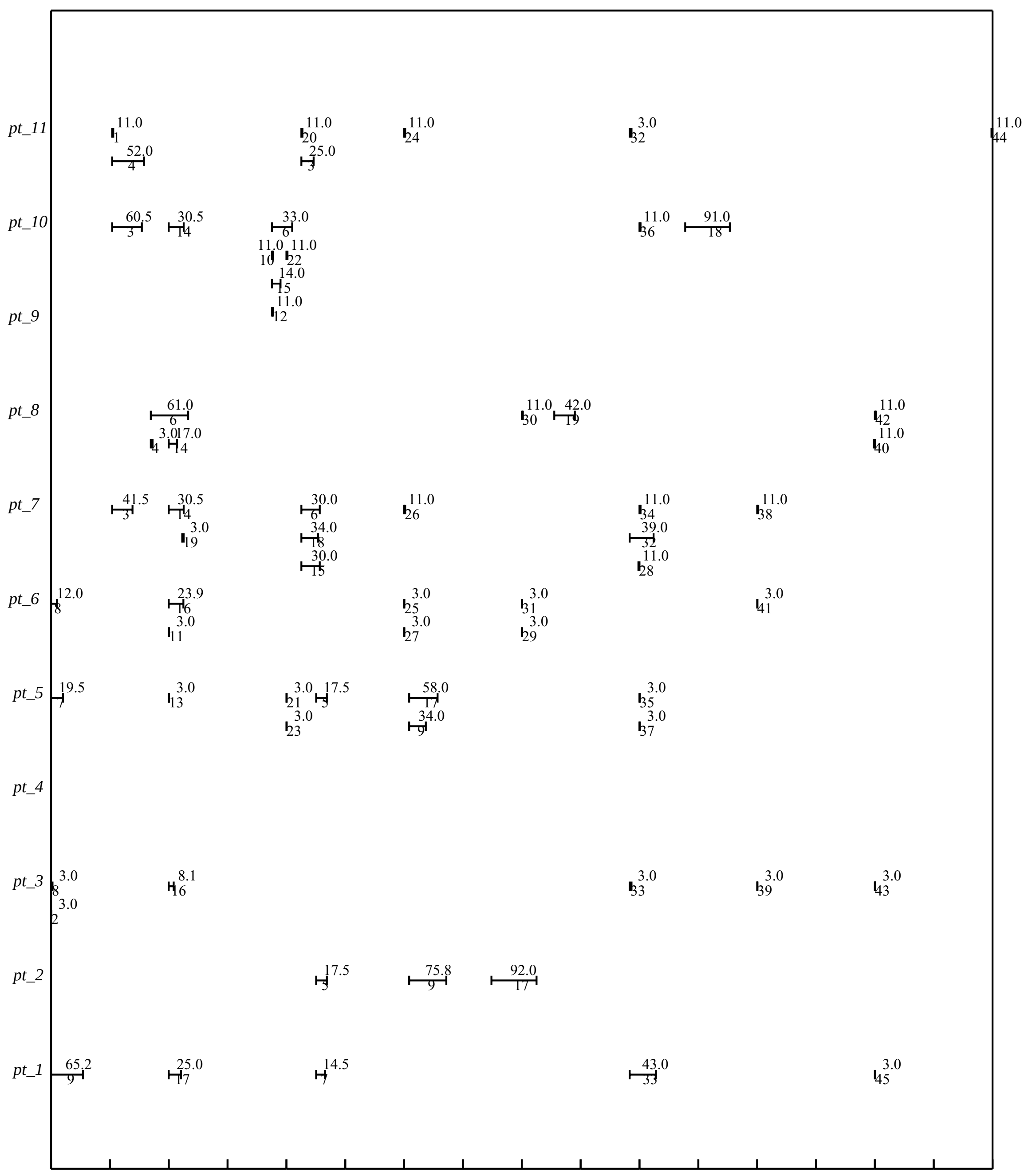

Fig. 5. Gantt chart for the example with 45 orders of problem 3.

however as shown in Table 7, even the full scale problem involving 45 orders converged to a feasible solution requiring 4280 nodes in approximately 5 h CPU time which is a reasonable time for the solution of the integrated scheduling of blending and distribution problem with a time horizon of 8 days. The optimal solution with zero integrality gap as well as the first and second integer solutions are shown. The resulting Gantt chart of the largest case involving 45 orders is shown in Fig. 5. Compared to the commonly used Gantt chart for scheduling purpose the difference here is that the number below the line corresponds to the order number, whereas the number above to the amount of product lifted from this particular tank.

5. Summary and future directions

In this paper, a continuous time formulation was presented for the short-term scheduling of refinery operations. It is shown that the resulting model can be solved efficiently even for realistic large-scale problems. The main advantage of the proposed approach is the full utilization of the time continuity. This results in smaller models in terms of variables and constraints since only the real events have to be modeled. In contrary, discrete time formulations which are commonly used for refinery operations result in excessive number of variables and constraints due to unnecessary time discretization.

For the cases that there are demands for oil intermediates, the demand constraints and lifting sequence constriants used in problem 3 can be applied to other sub-problems accordingly. Our final aim is to solve the short-term scheduling for overall refinery operations, and it can be solved either forward (from crude-oil unloading) or backward (from the production distribution). Each of the sub-problems has been sovled sucessfully, work is currently performed that involves the integration of all three different problems (Fig. 1) by applying heuristic-based lagrangean decomposition methodology (Wu & Ierapetritou, 2003) and will be the subject of future publication.

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