Comparative Study of Different Techniques of Electrical Machine Design

M. N. Bandyopadhyay *

Abstract

A comparative study of the various methods for design of an induction motor is presented. This deals solutions of a large number of non-linear equations and consequently some modifications of the methods have been suggested for use in practice. Modifications of the gradient theory, dynamic programming and statistical search is also made for optimal design. Besides, dual criteria optimization problem of induction motor has been solved by theory of games.

Keywords

Gradient, Optimum, Constraint, Penalty, Monte-Carlo, induction Motor.

Notations

V_1 = Ampere conductor /meter

V_2 = length/ pole pitch

V_3 = core depth

V_4 = stator tooth depth/width

V_5 = maximum flux density in the air gap, Wb/m^2

V_6 = rotor slot depth /slot depth

C = cost of machine

W = summation of cost and penalty function

η = Efficiency

T = temperature rise in stator winding

K_1 = Starting current /Full load current

K_M = Maximum torque /Full load torque

K_{ST} = Starting torque/ Full load torque

K = Constant of proportionality

S_{TK} = Full load torque /Starting torque.

*Dr. M. N. Bandyopadhyay is with the Indian expatriate professor at the Arbaminch Water Technology Institute, Arbaminch, Ethiopia.

1 Introduction

Design problem of an Induction Motor is essentially a non-linear programming. Mathematically, the problem can be formulated by determining the independent variables V_1, V_2, V_3, ..., V_6 and other performance. Characteristics such that certain criteria of optimization, which is a function of the independent variables,

$$C = C(V_1, V_2, V_3, \ldots\ldots, V_6) \quad (1)$$

Should be minimum. The constraints are the functions of the independent variables.

T (V_1, V_2, V_3, ..., V_6) 65^o C

(1/K_M) = M_K (V_1, V_2, V_3, ..., V_6 ≤ 0.5

(1/K_{ST}) = S_{TK} (V_1, V_2, V_3, ..., V_6) ≤ 0.7

K_1 (V_1, V_2, V_3, ..., V_6) ≤ 7

The independent variables are all positive, i.e.

$$vi > 0 (I = 1, 2, ...6) \quad (2)$$

The objective function C has been taken to be the total cost of energy loss for one year. Production cost includes cost of active materials, fabrication and labor. Labor is taken as 25% of cost of the materials as well as fabrication, which is about 5% of the cost of active materials. Energy loss has been calculated when the machine is assumed to run at full load for 8 hours every day for one year. The problem has been studied with a 7.5 KW, 400 V, 4 Pole, 50 Hz, three phase squirrel cage induction motor. The following optimization techniques have been applied with appropriate modifications.

(a) Gradient Technique: Use of Jordan elimination technique for optimization. Use of penalty functions in the gradient method Use of the gradient method with zigzag motion along the boundary.

(b) Dynamic programming.

(c) Monte Carlo method.

A machine having a minimum cost of production and maximum efficiency will, in many cases, satisfy the optimum requirements of both the manufacturer and the consumer. Here two contradictory criteria of optimization have to be simultaneously satisfied. If the cost of the machine is decreased, then efficiency will also decrease, but a machine, which has minimum production cost and maximum efficiency is desired. The problem is successfully dealt by the Theory of Games.

2 Principle of Gradient Theory

Let C be a function of n independent variables X_1, X_2, X_3, ..., X_n, If C is to be optimized, then the movement should be in the direction of the steepest path. To have an idea of the steepest path of a function of n independent variables, the equation takes the form

$\Delta X_n = K \delta C / \delta X_n$ where
n = 1, 2, 3....n) (4)

If C is minimized, then the path of the movement is in the direction of steepest descent. Hence the vector must have a negative sign. Therefore the final equation for minimization becomes $\Delta X_n = -K \delta C / \delta X_n$ where n = 1, 2, 3....n) (5)

Schinizinger[2] has cautioned that proper care has to be taken of the fact that each of the variables is dimensioned and scaled differently. On account of the wide variations in magnitude of the gradient principle. If proper scaling is done, or in other words, the value of K is taken different for variables, there will be quicker convergence to the optimum value.

2.1 Use Of Different Techniques

Different techniques are used for optimization of the proposed machine. Jordan Elimination Technique A matrix of the form

	Δv_1	Δv_2	Δv_3	Δv_4	Δv_5	Δv_6
Δ C	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
Δ T	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}

	Δ T	Δv_2	Δv_3	Δv_4	Δv_5	Δv_6
Δ C	a_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}
Δv_1	1	$-a_{22}$	$-a_{23}$	$-a_{24}$	$-a_{25}$	$-a_{26}$

Where bij = aij * a_{21}-ail *a2j (Ï # 2, j # 1) According to elimination technique, these matrices are identical.

2.2 Theory Of Gradient Principle Using Penalty

The gradient principle using penalty function is utilized in a more generalized form. The weighted objective function when the constraints are out of limit, is given by $W = C + K_1 T + K_2 M_k + K_3 S_{TK} + K_4 K_I$ If T is out of limit, then the equation becomes $W = C + K_1 T$. Again, if both T and M_k are out of limit, it will take the form $W = C + K_1 T + K_2 M_K$. Now, the steepest descent method can be applied on these objective functions. When the constraints are out of limit, then they will have a tendency to go inward of the optimal zone during optimization. If it is found that one of the constraints is not satisfied during optimization, then the value of penalty is to be increased for that particular constant. During optimization selection of the value of penalty. If the value of C is equal to that of the previous iteration or the difference between them is negligible when all the constraints are satisfied, then it can be said that optimum point has been attained.

2.3 Gradient Method With Zigzag Motion Along The Boundary

In this method C is optimized by the steepest descent method, but when the constraints are out of limit, then C can no longer be optimized, but the constraints will be optimized to insert them into feasible range. It is quite obvious that there is possibility of obtaining different minimum points. But if the step lengths are made very small by keeping the constant of proportionality both for objective function and for constraints, then the difference between the values of the lowered. Figure 1 and Figure 2 explain the principal with the help of block diagrams.

3 Principle Of Dynamic Programming [1,5,6]

Dynamic programming is termed the principle of optimality. An optimal sequence of decision in a multistage decisions must constitute an optimal sequence of decisions for the remaining problem, with the stage and state resulting from the first decision considered as initial conditions. Suppose a sequence of decisions $Z_1, Z_2, Z_3, \ldots Z_N$ is to be made to minimize some function C (X^N)where there is a known input-output relation at each stage, $X^n = F^n (X^{n-1}, Z_n)$, n = 1, 2, . . . $_N$ say. According to Bellman, n optimal policy has the property that whatever the initial state and initial dimensions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions. Now according to definitions, X_1 is determined choosing to Z_1, the remaining decisions must be chosen so that C (X^N) is minimized for the particular value of X^1. Similarly having chosen $Z_2 \ldots Z^{N-1}$and thus determining X^{N-1} the remaining decision N must be chosen so that C (X^N) is a minimum for that X^{N-1}. The proof of this statement of optimality is clear by the contradiction; if

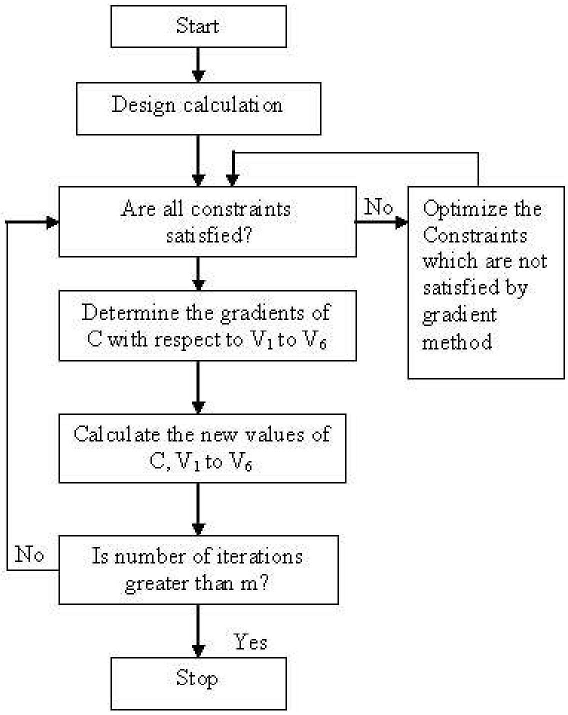

Figure 1: Block diagrams for preliminary stage of gradient method by zigzag motion along the boundary.

the choice Z_1 becomes the first one and the remaining decisions are not, optimal with reference to X^1, C (X^N)be made smaller by choosing another set of remaining decisions.

3.1 Application Of Dynamic Programming In Electrical Machine Design.

In electrical machine design problems, it is not possible to make any input-output relationship at each stage. Hence, for cost optimization, dynamic programming can be utilized taking the essence of the principle. Here, the problem is to choose the independent variables V_1, V_2, V_3, V_4, V_5 and V_6 and such that the cost, which is a nonlinear function of the independent variables is minimum. The minimum value of C will mean constraints. The combinations of all the independent variables from V_1 to V_6 are briefly explained (Figure 3). At block1, V_1 is changed and the minimum value of cost satisfying all the constraints is obtained. Then it goes over to block2. If, by changing the value of V_2, it is found that the same minimum value of cost is obtained, then it goes to block3. If the value of minimum cost changes with the value of V_2, having all the constraints satisfied, then it goes to block1 again. If the result of block1 is same as in block2, from the minimum cost point of view, then it will go back to block 3, but, if the value of V_1 changes with minimum cost, then it will go to block2. The same procedure is repeated.

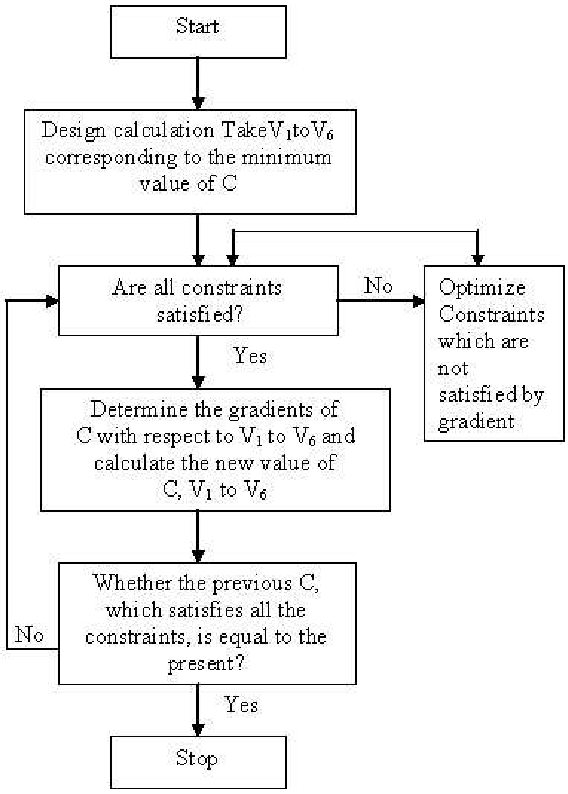

Figure 2: Block diagram for the final stage of gradient method along the boundary.

Figure 3.

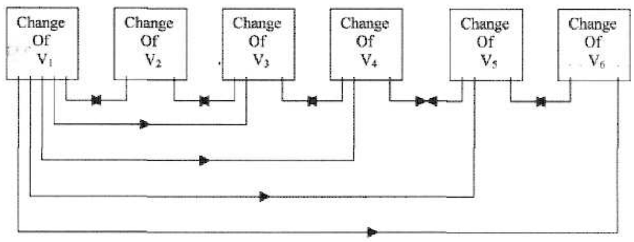

Figure 3: Stages of variation of six independent variables in dynamic programming.

4 Principle Of Monte Carol Method.
9,10,11

This method is based on the fact that analytical representation of a problem can be replaced by a statistical mode in which the probability of some quantity gives the solution of the problem. Six sets of uniformly distributed random numbers for the six independent variables were obtained from the computer by power residue method. The six differently chosen independent variable, if V_1 to V_6 should be scaled unity. For example, if V_1 is allowed to very between a to b, then the value of V_1 will vary as V_1 = X (b-a) + a. Where X is the random number having magnitude less than unity. Similarly other independent variables can be formulated. The statistical search is made in several stages. In the first stage, wide ranges of variations of the independent variables are taken. A small number of searches are made. From the values of the cost in the flexible range, the minimum one can be taken out. The second search is made in a smaller cube having minimum cost at the center.

5 Application Of Game Theory To Induction Motor Design [12]

The principle of Game Theory can be used in induction motor design. The efficiency and cost of the machine are taken as the two objective functions. Whenever efficiency is increased, the cost of the machine will increase and whenever the cost is allowed to decrease, the efficiency will decrease. The designer's intention should be to minimize the cost of the machine and maximize the efficiency. Any three of these variables from V_1 to V_6 may be varied such that the machine cost will be minimized and the other three will be varied such that the efficiency will be maximized, Gradient method is used in deciding the moves in the games. According to gradient method, if cost is to be minimized and efficiency is to be maximized, then the independent variables should be changed in the following manner.

$$\Delta V_n = K(\delta\eta/\delta V_n) \qquad (Where\ n = 1, 2, 3)$$
$$\Delta V_n = -K(\delta c/\delta V_n) \qquad (Where\ n = 4, 5, 6)$$

6 Discussion

In Tabe11, computer results of Jordan elimination technique are given. Since it is found that δ T / δ V_3 has the greatest absolute value, according to elimination principle, V_3 will not be an independent variable after T has crossed 65^oC. Analyzing the results by Jordan's elimination technique, the following observations are noted. In order to increase the effectiveness of this method two different steps can be taken. Viz.

(a) To take a very small value of K after the limiting value of the constraint is crossed or

(b) to progressively decrease the value of K for each iteration.

In case of penalty method, initially the values of penalty for all the constraints are taken as 1and few results are tabulated in Table 2. Table 3 shows the results considering the

S_I. No.	C	T	K_M	K_{ST}	K_I
1	17105	61.235	2.6298	1.4200	6.1802
- -	- -	- -	- -	- -	- -
16	1620.7	63.983	2.88	1.5876	6.8061
17	1610.9	65.145	2.8821	1.5907	6.8033
18	1588.2	65.205	2.9412	1.6105	6.9029
19	1572.8	65.820	2.9767	1.6253	6.9603
20	1559.4	66.477	3.0025	1.6333	6.9954

Table 1: Results of Jordan's elimination technique

value of penalties for T, M_K, S_{TK}, and K_I taken as 10, 200, 100 and 100 respectively. If the mode of change of iterations are noted, then it can be easily understood from table 2, when the value of penalty for T is only 1, then the system fails to decrease the value of T, but when gradually the penalty for T is increased to 200, then T decreases and the cost is found lower at R_s. 1617.4 compared to 1628.2. Another point to be noted is that during optimization it may so happen that the cost function is not minimized but the weighted function W is.

S_I.No	C	T	K_M	K_{ST}	K_I
1	2062.0	54.733	2.3013	0.85572	4.5196
2	2007.8	55.489	2.3546	0.88906	4.6409
3	1977.1	55.755	2.4251	0.93361	4.7878
4	1966.5	57.181	2.4600	0.96252	4.8740
–	–	–	–	–	–
10	1730.3	63.522	2.6695	1.4303	6.1107
11	1709.3	64.944	2.6872	1.4367	6.1331
12	1689.3	66.350	2.6830	1.4415	6.1526
13	1671.5	67.476	2.6833	1.4410	6.1650
14	1654.3	68.577	2.6921	1.4393	6.1753
15	1641.6	70.268	2.6878	1.4408	6.1765

Table 2: Results for Gradient method with single penalty.

Iteration	C	T	K_M	K_{ST}	K_I
1	1684.1	61.336	2.7222	1.441	6.3895
2	1663.2	62.722	2.7281	1.4469	6.4016
3	1628.2	64.133	2.8504	1.5907	6.7415
4	1613.9	65.776	2.8498	1.5875	6.7300
5	1614.2	65.341	2.8205	1.5492	6.6615
6	1615.0	65.050	2.7957	1.5118	6.5879
7	1617.4	64.877	2.6691	1.4747	6.5114

Table 3: Results of gradient method with different values of penalty.

Iteration	C	T	K_M	K_{ST}	K_I	V_1	V_2	V_3	V_4	V_5	V_6
17	1640.8	64.46	2.73	1.48	6.49	25379	0.99	0.0353	1.79	0.82	2.13
60	1632.7	64.59	2.88	1.61	6.79	24749	1.05	0.0321	1.81	0.83	2.09
39	1633.7	64.93	2.86	1.61	6.80	24643	1.09	0.0337	1.75	0.82	2.21
45	1637.4	64.26	2.84	1.62	6.81	24775	1.06	0.0323	1.82	0.83	2.16

Table 5: Results of Monte Carlo method.

Iteration	C	T	K_M	K_{ST}	K_I	V_1	V_2	V_3	V_4	V_5	V_6
4	1623.1	64.869	2.8437	1.5786	6.7442	24946	1.0378	0.034	1.772	0.83	2.14
13	1622.9	64.888	2.8565	1.5792	6.7363	24987	1.0214	0.0328	1.808	0.836	2.1196
42	1622.9	64.206	2.8677	1.5852	6.777	24820	1.0184	0.0338	1.778	0.8406	2.1719
44	1621.9	64.50	2.8716	1.5779	6.7506	24854	0.999	0.0324	1.8087	0.8399	2.1419

Table 6: Results of Monte Carlo method near optimum value.

Iter-ation	C	T	K_M	K_{ST}	K_I
1	1620.7	63.983	2.8800	1.5867	6.8061
2	1617.3	64.370	2.8807	1.5886	6.8052
3	1614.1	64.757	2.8814	1.589	6.8043
4		65.145			
5	1620.7	65.969	2.8801	1.5876	6.8062
6	1617.4	64.859	2.8808	1.5886	6.8054
7	1614.1	64.749	2.8814	1.5897	6.8044

Table 4: Results of Gradient method by zigzag motion along boundary.

	V_1	V_2	V_3
V_4	89.436	89.249	89.058
V_5	89.349	89.139	89.107
V_6	89.287	89.124	89.181

Table 7: Results of Game theory for efficiency.

	V_1	V_2	V_3
V_4	1216.8	1190.0	1166.7
V_5	1200.6	1180.3	1169.9
V_6	1200.6	1180.3	1170.7

Table 8: Results of Game theory for cost.

	V_2	V_3	V_4
V_5	C=1226	1226	1226.2
V_6	1226	1226	1226.2
V_1	1226	1226	1226.2

Table 9:

	V_2	V_3	V_4
V_5	η = 89.465	η = 89.465	η = 89.467
V_6	η = 89.465	η = 89.465	η = 89.467
V	η = 89.465	η = 89.465	η = 89.467

Table 10:

	V_2	V_3	V_4
V_5	C = 1216.9 η = 89.368	C = 1229.3 η = 89.495	C = 1261.1 η = 89.699
V_6	C = 1226 η = 89.461	C = 1206.3 η = 89.959	C = 1331.9 η = 89.689
V_1	C = 1223.5 η = 89.475	C = 1226.9 η = 89.463	C = 1235.2 η = 89.507

Table 11:

In Table 3, the iteration numbers 4 and 7 indicate C increases from 1613.9 to1617.4 while W is decreasing. Hence it may be concluded that it is better to minimize the constraints to their limiting values during constrained optimization without considering the main cost function C. The penalty method is now explained on gradient method by zigzag motion along the boundary. This method is indicated by keeping the values of K different for different independent variables and some minimum points of Care determined. The minimum most cost chosen by the above method is repeated making the value of K very small and the same for all independent variables. This results in quicker convergence and increases accuracy. The results are tabulated in table 4. The results substantiate the validity of the generalize method of search by zigzag motion along the boundary. The result of dynamic programming is found

	V_3	V_4	V_5
V_6	C = 1206.8 η = 88.959	C = 1331.9 η = 89.689	C = 1226.0 η = 89.433
V_1	C = 1226.9 η = 89.463	C = 1232.7 η = 88.507	C = 1221.9 η = 89.481
V_2	C = 1227.3 η = 89.481	C = 1233.7 η = 89.626	C = 1224.8 η = 89.449

Table 12:

	V_4	V_5	V_6
V_1	C = 1235.2 η = 89.507	C = 1221.9 η = 89.481	C = 1224.0 η = 89.473
V_2	C = 1232.7 η = 89.626	C = 1224.8 η = 89.446	C = 1225.8 η = 89.418
V_3	C = 1195.6 η = 89.335	C = 1224.2 η = 89.459	C = 1219.6 η = 89.442

Table 13:

	V_5	V_6	V_1
V_2	C = 1224.8 η = 89.446	C = 1225.8 η = 89.448	C = 1224.8 η = 89.446
V_3	C = 1224.2 η = 89.459	C = 1219.6 η = 89.442	C = 1224.2 η = 89.460
V_4	C = 1216.8 η = 89.436	C = 1214.6 η = 89.428	C = 1216.8 η = 89.436

Table 14:

	V_6	V_1	V_2
V_3	C = 1219.6 η = 89.442	C = 1224.2 η = 89.460	C = 1220.7 η = 89.446
V_4	C = 1214.6 η = 89.428	C = 1216.8 η = 89.436	C = 1215.1 η = 89.430
V_5	C = 1217.8 η = 89.372	C = 1209.6 η = 89.379	C = 1215.9 η = 89.369

Table 15:

when C = 1628.0. In other words, the minimum cost point which satisfies all the constraints is found to be 1628.0. In Monte Carlo method, the problem under investigation is solved taking the independent variables V_1 = 24600 to 25600, V_2 = 0.92 to 1.15, V_3 = 0.32 to 0.036, V_4 = 1.73 to 1.91, V_5 = 0.79 to 0.83, V_6 = 1.9 to 2.5. From 60 iterations, it is found that iterations 17, 39, 45, 60 had lesser values of cost as shown in table 5. After studying these results, the ranges of the independent variables were made much smaller for the next stage of calculations. A further 60 iterations were made(Table 6) which shows that iterations 4, 13, 42, 44 give costs which are less than those given in Table 5. Thus by making the ranges smaller and smaller in the feasible region, the optimum value is obtained. The values of efficiencies and cost of machine are tabulated in Table 7 and 8. Table 7 gives the maximum of the column minimal equal to 89.287 and the minimum of row maximal as 89.287. Hence the saddle point is 89.287. Similarly in Table 8, the saddle point is 1200.6. hence, the best result is the machine which costs 1200.6 and efficiency 89.287. Again, the games are replaced by interchanging the places of independent variables. In the next step V_2, V_3, V_4 are taken for efficiency maximization and V_5, V_6, V_1 for machine cost optimization. In this case, it is found, when the game is played between V_3 and V_1, the value of constraint goes out of feasible region and it is difficult to take it bake the region. Hence the constant of proportionality, K, is reduced for constraint optimization. The position of independent variables are interchanged and the results given in tables 11 to 15. In Table 13, the maximum of column minimal shows η = 89.446 and c = 1221.9 and the minimum of the row maxim shows η = 89.459 and c = 1224.2. Hence in table 13, three points are obtained (1221.99, 89.481), (1224.8, 89.446), (1224.2, 89.459). Amongst these (1221.9, 89.481) is the best result. Again in table 15, among the three points, maximum efficiency is represented by point (1216.8, 89.436) and minimum cost by the point (1209.6, 89.379). The other point (1215.1, 89.430) which is the best result as it gives machine cost little lower than 1216.8 and efficiency near to 89.436.

7 Conclusion

The greatest advantage of the zigzag method is, wherever the initial point may be, either in the feasible region or in the infeasible region, the search will ultimately direct towards the optimum points. The repeated task of locating the initial points in the feasible zone, as is required in all the earlier methods, is avoided-a positive relief to the practical designer. Secondly, in zigzag method the possibility of obtaining local minimum is least, as a large number of the final stage steps of minimum points are studied along the boundary and at the final stage the steps of changes are also taken small. The dynamic programming is essentially an iterative method and for large number of parameters it is time consuming. This type of programming is applicable in small regions where the independent variable will not change to a greater extent. In that case, more accurate results can be obtained by taking the intervals small. The Monte Carlo method is very helpful where a function is not known well. It locates the feasible and infeasible regions of the function. Programming is simple by this method as there is not much complicated logic in the algorithm except a random number counter. The main body of computation is a series of design calculations with the aid of usual algebraic formulae. Only after the whole sequence of calculations is done, the results are compared. Thus it is convenient to change any design calculation any time during investigation. Game theory has proved to be quite useful in optimizing the dimensions of an induction motor. In order to facilitate computation, a proper starting should be chosen. This may be obtained by optimizing the machine for only one criterion, cost, say, by any method, viz, gradient method. From this starting point by use of game theory a machine may be obtained having good efficiency without much increase in cost. The saddle point obtained by the first matrix game may be taken as initial point for another set of games, in this way, a condition will come when there will be no change of cost or efficiency at saddle points for subsequent matrix games.

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