SOME FUNDAMENTAL PROPERTIES OF MMSE FILTER BANKS

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ABSTRACT

We design filter banks that are best matched to input signal statistics in M-channel subband coders, using a broad class of rate–distortion criteria. We present fundamental properties and analytical expressions for minimum mean-squared error (MMSE) filter banks, without constraints on filter length, under optimal bit allocation requirements. We also investigate a constrained–length version of this problem, which is applicable to practical coding scenarios. While the optimal filter banks are nearly perfect-reconstruction at high rates, we show that MMSE FIR filter banks enjoy a significant advantage (in the MSE sense) over optimal perfect–reconstruction FIR filter banks at all rates.

1. INTRODUCTION

We consider M–channel subband coders with analysis filters $\{H_i(f), 0 \leq i < M\}$ and synthesis filters $\{\tilde{H}_i(f), 0 \leq i < M\}$. Fig. 1 shows an equivalent representation of the codec in terms of the $M \times M$ analysis and synthesis polyphase matrices $\mathcal{H}(f)$ and $\tilde{\mathcal{H}}(f)$. The problem of interest here is to design the filters so as to optimize the rate–distortion performance of subband coders that use uniform scalar quantizers in each channel. The distortion measure is mean–squared reconstruction error. We explore fundamental properties of MMSE filter banks (for which the perfect–reconstruction (PR) is not imposed a priori [1]). We also present analytical expressions for the resulting optimal filter banks in terms of bit rate and second-order input signal statistics.

1.1. Basic Model for Signal and Quantization Noise

The input $x(n)$ to the subband coder is assumed to be real–valued and wide–sense stationary with zero mean and spectral density $S(f)$. Throughout, we assume that $S(f)$ is bounded away from zero. The total bit budget is R bits per sample, to be allocated to the quantizers in each channel. Quantizer Q_i in channel i operates on a signal $y_i(n)$ with variance σ_i^2, is scalar and uniform, and is allocated R_i bits, where $\frac{1}{M}\sum_{i=0}^{M-1} R_i = R$. We make the assumption that the quantization noise is additive, white and independent of the signal; and that the quantization noise sources in different channels are mutually independent. This is a standard model which is valid at high bit rates, but not at low bit rates. Each quantizer is assumed to have a distortion–rate function $\sigma_i^2 \overline{D}(R_i)$, where in this context distortion is quantization noise variance. The following properties are required : $\overline{D}(.)$ is *strictly positive, strictly monotonic decreasing*, and *strictly convex*. For Theorems 2.1 and 2.2 to hold, we also require $\ln\overline{D}(.)$ to be *concave*. The standard exponential model $\overline{D}(R_i) = \gamma\sigma_i^2 2^{-2R_i}$ for the rate–distortion function satisfies all of the assumptions above. We first state the design criterion for PR filter banks in Sec. 1.2 and extend it to MMSE filter banks in Sec. 1.3.

Work supported by NSF grant MIP-97-07633 and DOE contract W-31-109-Eng-38.

1.2. Design Criterion for PR Systems

Under the assumptions above, the reconstruction error $\hat{x}(n)-x(n)$ is a cyclostationary process with period M. For PR systems, the goal is to minimize the expected mean–squared error (MSE) [2]

$$\frac{1}{M}\sum_{n=0}^{M-1} E|\hat{x}(n) - x(n)|^2 = \frac{1}{M}\sum_{i=0}^{M-1} \overline{D}(R_i)\sigma_i^2 ||\tilde{h}_i||^2 \qquad (1)$$

where $||\tilde{h}_i||^2$ represents the amplification factor for white noise passed through synthesis filter $\tilde{h}_i$. In order to find the optimal bit allocation given the filter banks, the optimization problem (1) with bit rate constraints is transformed into the Lagrange optimization problem

$$\text{Minimize } \tfrac{1}{M}\textstyle\sum_{i=0}^{M-1} \overline{D}(R_i)\sigma_i^2 ||\tilde{h}_i||^2 - \mu\tfrac{1}{M}\textstyle\sum_{i=0}^{M-1} R_i$$

where $-\mu$ is the Lagrange multiplier. The solution satisfies the condition,

$$\sigma_i^2 ||\tilde{h}_i||^2 \left.\frac{d\overline{D}(R)}{dR}\right|_{R_i} = \mu, \quad 0 \leq i < M. \qquad (2)$$

Hence, for optimal bit allocation, the slope of the distortion–rate function $\frac{1}{M}\overline{D}(R_i)\sigma_i^2||\tilde{h}_i||^2$ at the encoder's operating point must be the same for all i. It has been assumed here that R is large enough so that the positivity constraints $R_i \geq 0$ are all inactive. Due to the strict convexity of $\overline{D}(.)$, the optimal bit allocation problem has a unique solution. Let $\mathcal{S}(f)$ be the $M \times M$ spectral density matrix for the polyphase vector $\underline{x}(n)$, input to $\mathcal{H}(f)$ in Fig. 1. We have

$$\sigma_i^2 = \int_{-0.5}^{0.5} (\mathcal{H}\mathcal{S}\mathcal{H}^\dagger)_{ii}\, df, \quad ||\tilde{h}_i||^2 = \int_{-0.5}^{0.5} (\tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}})_{ii}\, df.$$

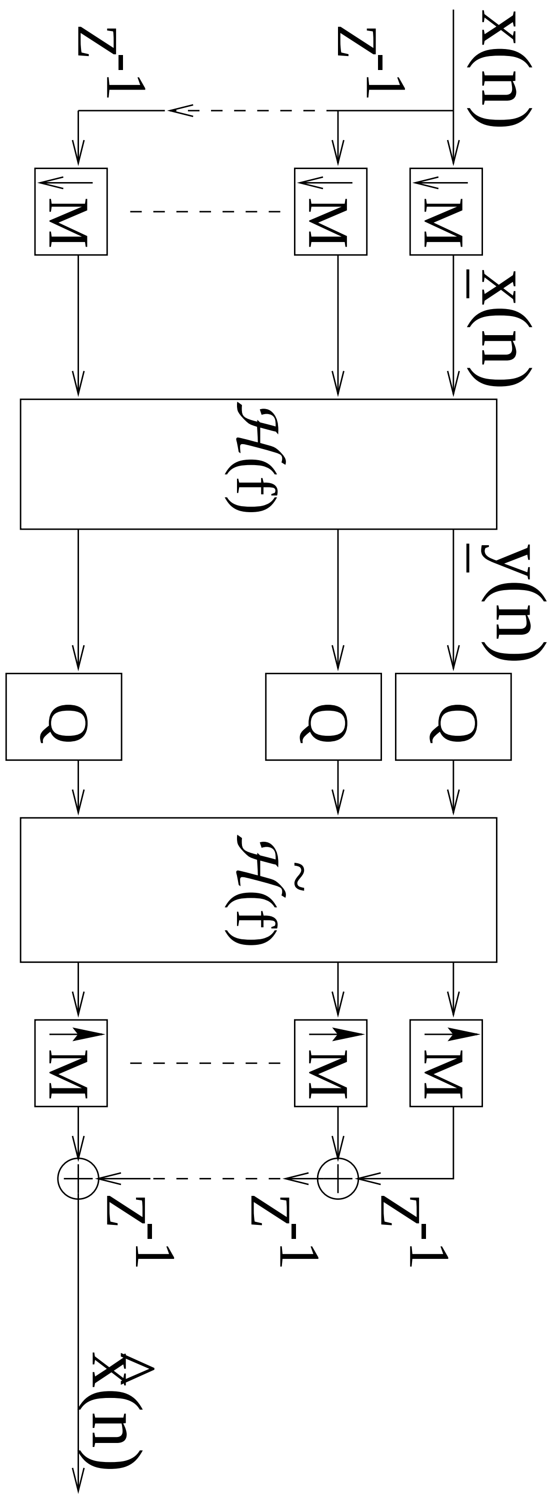

Figure 1: Polyphase representation of M–channel subband coder and decoder using analysis filters $H_i(f)$ and synthesis filters $\tilde{H}_i(f)$. The polyphase components of $H_i(f)$ (resp. $\tilde{H}_i(f)$) are contained in row (resp. column) i of the polyphase matrix $\mathcal{H}(f)$ (resp. $\tilde{\mathcal{H}}(f)$.)

1.3. Design Criterion for MMSE Filter Banks

In the MMSE filter bank approach, the PR constraints are relaxed so as to trade off systematic reconstruction errors (due to lack of PR) against quantization noise. The solution is *nearly* identical to the PR solution at high bit rates (low quantization noise), but notable improvements over PR designs have been obtained using numerical simulations at lower bit rates [1]. Here, we seek analytical expressions for the filters and bit allocation $\{R_i\}$ that jointly minimize the MSE. Using the model in Sec. 1.1, signal and quantization noise are assumed to be independent, so the MSE is the sum of the noise term (1) and a signal term. The MSE is

$$\begin{aligned}\mathcal{E} = \; & \frac{1}{M}\sum_{i=0}^{M-1}\overline{D}(R_i)\int(\mathcal{H}\mathcal{S}\mathcal{H}^\dagger)_{ii}\int(\tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}})_{ii} \\ & + \frac{1}{M}Tr\int(\tilde{\mathcal{H}}\mathcal{H}-I_M)\,S\,(\tilde{\mathcal{H}}\mathcal{H}-I_M)^\dagger, \qquad (3)\end{aligned}$$

to be minimized over $\mathcal{H}, \tilde{\mathcal{H}}$, and $\{R_i\}$. The optimal bit allocation condition is still given by (2).

2. FUNDAMENTAL PROPERTIES OF MMSE FILTER BANKS

We have recently proven that optimal (in the sense (1)) PR filter banks enjoy two fundamental properties: total decorrelation of subband channels, and spectral majorization [2, 3]. These properties were previously known to apply only to paraunitary filter banks, in which case the solution is a principal–component filter bank (PCFB) [4, 5, 6]. We now show that these fundamental properties hold even if the PR conditions are relaxed, and the cost function (3) is used. The proof of the two theorems below uses variational techniques and parallels the proof of Theorem 2.3 and Lemma 2.6 in [2].

Theorem 2.1 *(***Total Decorrelation is Necessary for Optimality.***) The system $\mathcal{H}, \tilde{\mathcal{H}}$, $\{R_i\}$ minimizes (3) only if the matrices $\mathcal{H}\mathcal{S}\mathcal{H}^\dagger$ and $\tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}}$ are diagonal, and $\{R_i\}$ satisfies (2).*

Theorem 2.2 *(***Spectral Majorization is Necessary for Optimality.***) Let $\mathcal{H}, \tilde{\mathcal{H}}$, $\{R_i\}$ be a minimizer of (3), and $\mathcal{M} = \tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}}$. Without loss of generality, assume that $R_0 \geq R_1 \geq \cdots \geq R_{M-1}$. The normalized spectral densities $\frac{1}{w_i\sigma_i^2}\mathcal{S}_{y,ii}(f)$ for the subband signals satisfy the spectral majorization property:*

$$\frac{1}{w_0\sigma_0^2}\mathcal{S}_{y,00}(f) \geq \cdots \geq \frac{1}{w_{M-1}\sigma_{M-1}^2}\mathcal{S}_{y,M-1,M-1}(f), \quad \forall f,$$

where $w_i \triangleq \frac{1}{M}\overline{D}(R_i)$. Likewise, the normalized quantities $\frac{1}{w_i||\tilde{h}_i||^2}\mathcal{M}_{ii}(f)$ satisfy the spectral majorization property

$$\frac{1}{w_0||\tilde{h}_0||^2}\mathcal{M}_{00}(f) \geq \cdots \frac{1}{w_{M-1}||\tilde{h}_{M-1}||^2}\mathcal{M}_{M-1,M-1}(f), \quad \forall f.$$

The particular filter bank structure shown in Fig. 2 was shown to be optimal in the PR class [2, 3], in which case $\tilde{G}_i(f) = 1/G_i(f)$. The first block in this structure, $\mathcal{U}(f)$, is a PCFB. While at this point we are not able to ascertain

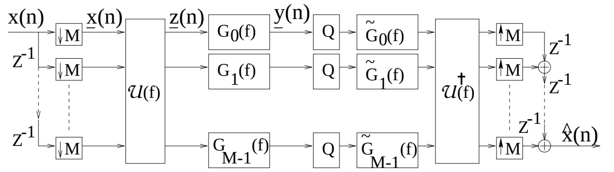

Figure 2: Cascade of a principal–component filter bank $\mathcal{U}(f)$ and a set of prefilters $G_i(f)$ and postfilters $\tilde{G}_i(f)$ around each quantizer. This system satisfies the total decorrelation and spectral majorization properties.

whether the structure in Fig. 2 is also optimal for MMSE filter banks, we are able to compute the best filters $G_i(f)$ and $\tilde{G}_i(f)$. We then prove some important properties of the resulting filter bank [1].

We can write the cost function (3) in terms of the pre– and postfilters $G_i(f)$ and $\tilde{G}_i(f)$ as

$$\mathcal{E} = \frac{1}{M}\sum_{i=0}^{M-1} \overline{D}(R_i) \int |G_i\sqrt{S_i}|^2 \int |\tilde{G}_i|^2 + \frac{1}{M}\sum_{i=0}^{M-1} \int |\tilde{G}_i G_i - 1|^2 S_i, \quad (4)$$

where $S_i(f)$ is the spectral density for the signal $z_i(n)$ in Fig. 2. Analytical expressions for the optimal filters $G_i(f)$, $\tilde{G}_i(f)$ depend on the bit rates $\{R_i\}$.

Theorem 2.3 *For any $\{R_i\}$, the filters that minimize the MSE (3) for the system in Fig. 2 are*

$$|G_i(f)| = c_i\sqrt{P_i(f)}S_i^{-1/4}(f), \quad |\tilde{G}_i(f)| = c_i^{-1}\sqrt{P_i(f)}S_i^{1/4}(f) \quad (5)$$

where c_i are arbitrary positive scaling factors, and the product filters $P_i(f) = G_i(f)\tilde{G}_i(f)$, $0 \le i < M$, are given by

$$P_i(f) = \max\left(0, 1 - \frac{\overline{D}(R_i)}{1+\overline{D}(R_i)} \frac{\int_+ \sqrt{S_i(f)}\,df}{\sqrt{S_i(f)}}\right). \quad (6)$$

The MSE for these filters is given by

$$\mathcal{E}(\mathcal{H},\tilde{\mathcal{H}}) = \frac{1}{M}\sum_{i=0}^{M-1}\left[\int_+ \sqrt{S_i(f)}\,df\right]^2 \frac{\overline{D}(R_i)}{1+\overline{D}(R_i)} + \frac{1}{M}\sum_{i=0}^{M-1}\int_- S_i(f)\,df, \quad (7)$$

where $\int_+$ means integration over the (unique) set $\mathcal{F}_i^+$ that satisfies $\mathcal{F}_i^+ = \left\{f \mid \sqrt{S_i(f)} > \frac{\overline{D}(R_i)}{1+\overline{D}(R_i)}\int_+\sqrt{S_i}\right\}$, and $\int_-$ means integration over the complementary set [2].

Outline of the Proof : By the Cauchy-Schwartz inequality, (4) is lower bounded by $\mathcal{E}_{LB} = \frac{1}{M}\sum_{i=0}^{M-1}\overline{D}(R_i)\left|\int G_i\tilde{G}_i\sqrt{S_i}\right|^2 + \frac{1}{M}\sum_{i=0}^{M-1}\int\left|G_i\tilde{G}_i - 1\right|^2 S_i$, with equality iff $G_i\sqrt{S_i} = c_i\tilde{G}_i^*$. Additionally, $c_i > 0$ and $P_i(f) \ge 0$ (see [2]). The functional $\mathcal{E}_{LB}$ is strictly convex in P_i, and its unique minimizer is given by (6). Substituting into $\mathcal{E}_{LB}$, we obtain (7).

The optimal MSE (7) is *upper–bounded* by the optimal value $\mathcal{E} = \frac{1}{M}\sum_{i=0}^{M-1}\overline{D}(R_i)\left(\int_{-0.5}^{0.5}\sqrt{S_i(f)}\,df\right)^2$ for the IIR biorthogonal case [2], and tends to this limit as $R \to \infty$. The performance of FIR MMSE filter banks converges to (7) as the filter length tends to infinity. As discussed in [2, 3], PR filter banks do not enjoy a similar property: FIR filter banks of arbitrary length must satisfy the constraint $\det\mathcal{H}(f) \equiv 1$, and the performance of these filters does not converge to that of IIR filter banks (for which the constraint $\det\mathcal{H}(f) \equiv 1$ is not applicable). Even at high bit rates, FIR MMSE filter banks of sufficient length can vastly outperform FIR biorthogonal filter banks of arbitrary length. While conditions (5),(6) for optimality of the filter bank apply for arbitrary $\{R_i\}$, the optimal $\{R_i\}$ do satisfy (2).

3. IMPORTANT SPECIAL CASES

Consider the classical model $\overline{D}(R_i) = \gamma 2^{-2R_i}$ for the rate–distortion function with optimal bit allocation. Then (2) yields the closed–form solution $2^{2R_i} \propto \sigma_i^2\|\tilde{h}_i\|^2$. (Just before going to press, we discovered the paper [7] which derives the optimal set of pre– and postfilters for a single–channel problem. However, their framework apparently does not lend itself to joint optimization of filter banks and bit allocation [7, p. 1024].) The resulting distortions $\sigma_i^2\|\tilde{h}_i\|^2\overline{D}(R_i)$ are identical for all channels. The expected MSE (3) is then

$$\mathcal{E}(\mathcal{H},\tilde{\mathcal{H}}) = \gamma 2^{-2R}\left[\prod_{i=0}^{M-1}\int(\mathcal{H}\mathcal{S}\mathcal{H}^\dagger)_{ii}\int(\tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}})_{ii}\right]^{1/M} + \frac{1}{M}Tr\int(\tilde{\mathcal{H}}\mathcal{H} - I_M)\,S\,(\tilde{\mathcal{H}}\mathcal{H} - I_M)^\dagger. \quad (8)$$

For the exponential $\overline{D}(R_i)$ model and optimal bit allocation, the general expressions (6),(7) are specialized in Theorem 3.1 below. Analytical expressions for the optimal filters are given in terms of constants $\{\sigma_i^2\}$ which are solutions to a nonlinear system. If it is known that $\mathcal{F}_i^+ = \left[-\frac{1}{2}\ \frac{1}{2}\right]$, then the only constant to be solved for is $\mathcal{E}_q$.

Theorem 3.1 *The filters that minimize the MSE (8) for the system in Fig. 2 are given by (5) with product filters*

$$P_i(f) = \max\left(0, 1 - \frac{\mathcal{E}_q/\sigma_i^2}{\sqrt{S_i(f)}}\right), \quad 0 \le i < M. \quad (9)$$

Here

$$\sigma_i^2 = \frac{1}{2}\int_+\sqrt{S_i(f)}\,df \pm \sqrt{\left(\frac{1}{2}\int_+\sqrt{S_i(f)}\,df\right)^2 - \mathcal{E}_q|\mathcal{F}_i^+|}, \quad (10)$$

are the normalized variances of the subband signals (using $c_i = 1$),

$$\mathcal{E}_q = \gamma 2^{-2R}\left(\prod_{i=0}^{M-1}\sigma_i^2\right)^{2/M}, \quad (11)$$

and $\mathcal{F}_i^+ = \left\{f \mid \sqrt{S_i(f)} > \frac{\mathcal{E}_q}{\sigma_i^2}\right\}$. The MSE for these filters

[1] Since the class of filters considered in Fig. 2 contains the optimal PR filter bank, the solution is clearly guaranteed to be at least as good as the optimal PR filter bank.

[2] At high bit rates, $\mathcal{F}_i^+ = \left[-\frac{1}{2}\ \frac{1}{2}\right]$. As our numerical results have shown, $\mathcal{F}_i^+ = \left[-\frac{1}{2}\ \frac{1}{2}\right]$ at *all* bit rates for AR(1) processes.

is given by

$$\mathcal{E}(\mathcal{H},\tilde{\mathcal{H}}) = \mathcal{E}_q + \frac{1}{M}\sum_{i=0}^{M-1}\left(\frac{\mathcal{E}_q}{\sigma_i^2}\right)^2 |\mathcal{F}_i^+| + \frac{1}{M}\sum_{i=0}^{M-1}\int_{-} S_i(f)\,df. \quad (12)$$

Note that according to (10), there exist two possible candidates for the solution σ_i^2 in each channel. Each of these solutions is a local extremum for (8). The optimal solution is the one that minimizes (8). For bit rates that are high enough, each sign in (10) must be positive.

2–Channel Case

Substantial simplifications arise if the number of channels is $M = 2$. The product filters are given by

$$P_0(f) = \max\left(0, 1 - \frac{\gamma 2^{-2R}\sigma_1^2}{\sqrt{S_0(f)}}\right),$$
$$P_1(f) = \max\left(0, 1 - \frac{\gamma 2^{-2R}\sigma_0^2}{\sqrt{S_1(f)}}\right). \quad (13)$$

The optimal filters are again given by (5). Here the normalized variances of the subband signals are (using $c_i = 1$)

$$\sigma_0^2 = \frac{\int_+ \sqrt{S_0(f)}\,df - |\mathcal{F}_0^+|\gamma 2^{-2R}\int_+\sqrt{S_1(f)}\,df}{1-(\gamma 2^{-2R})^2|\mathcal{F}_0^+||\mathcal{F}_1^+|},$$
$$\sigma_1^2 = \frac{\int_+ \sqrt{S_1(f)}\,df - |\mathcal{F}_1^+|\gamma 2^{-2R}\int_+\sqrt{S_0(f)}\,df}{1-(\gamma 2^{-2R})^2|\mathcal{F}_0^+||\mathcal{F}_1^+|}. \quad (14)$$

The MSE corresponding to these filters is given by

$$\mathcal{E} = \gamma 2^{-2R}\sigma_0^2\sigma_1^2 + \frac{1}{2}(\gamma 2^{-2R})^2(|\mathcal{F}_0^+|\sigma_0^4 + |\mathcal{F}_1^+|\sigma_1^4) + \frac{1}{2}\sum_{i=0}^{1}\int_{-} S_i(f)\,df. \quad (15)$$

(14) is a nonlinear system in σ_0^2, σ_1^2. But if it is known that $\mathcal{F}_0^+ = \mathcal{F}_1^+ = \left[-\frac{1}{2}\ \frac{1}{2}\right]$, (14) (15) no longer contains unknowns.

4. SIMULATION RESULTS

An AR(1) process with correlation coefficient $r = 0.8$ in the two–band case has been considered to illustrate the analysis above. In this case, the PCFB is the traditional filter bank with ideal low and high pass filters. The optimization problem (8) for the pre– and postfilters has been solved for various rates R and the results have been compared with optimal IIR biorthogonal filter banks and optimal unconstrained–length FIR biorthogonal filter banks [2, 3]. At all bit rates, $\mathcal{F}_i^+ = \left[-\frac{1}{2}\ \frac{1}{2}\right]$. For $R = 1.76$, the bit rate in the high–pass channel becomes zero, in which case the criterion (8) becomes clearly invalid. The performance of the optimal unconstrained–length MMSE filter banks is very close to optimal IIR biorthogonal filter banks at very high rates, but improvements become quite significant as R decreases (Fig. 3). At very high rates, the optimal filter banks are close to the (PR) IIR biorthogonal solution. At lower bit rates, the filters differ significantly from those optimal PR filters. Frequency responses are shown in Fig. 4 for an AR(1) process with correlation coefficient $r = 0.8$, and rate $R = 2.91$. The scaling factors c_0 and c_1 for all three filter banks have been chosen so that the frequency responses are the same at $f = 0$ and at $f = 0.5$.

The remarks at the end of Sec. 2 motivated us to investigate the constrained–length version of this design. A simple rectangular windowing technique was used to design constrained–length FIR–MMSE filterbanks from the optimum unconstrained–length solution. As shown in Fig. 5, the results are excellent at medium bit rates. At $R = 2.91$, the length–63 FIR–MMSE filter bank outperforms optimal FIR biorthogonal filter banks of arbitrary length, and the length–103 FIR–MMSE filter bank outperforms optimal IIR biorthogonal filter banks. Similar advantages hold at arbitrarily high bit rates, but longer FIR filters are needed to break the performance bounds for FIR and IIR biorthogonal filters. Refinements in the FIR–MMSE filter design method are likely to yield further improvements.

5. REFERENCES

[1] K. Gosse and P. Duhamel, "Perfect–Reconstruction versus MMSE Filter Banks in Source Coding," *IEEE Trans. SP*, Vol. 45, No. 9, pp. 2188—2202, Sep. 1997.

[2] P. Moulin, M. Anitescu and K. Ramchandran, "Theory of Rate–Distortion–Optimal, Constrained Filter Banks–Application to IIR and FIR Biorthogonal Designs," submitted to *IEEE Trans. Sig. Proc.*, 1998.

[3] P. Moulin, K. Ramchandran and M. Anitescu, "Asymptotic Performance of Subband Coders Using Constrained, Signal–Adapted FIR Filter Banks," *Proc. 32nd CISS Conference*, Princeton, NJ, March 1998.

[4] M. Unser, "An Extension of the Karhunen-Loève Transform for Wavelets and Perfect–Reconstruction Filterbanks," *SPIE Vol. 2034*, pp. 45—56, 1993.

[5] M. K. Tsatsanis and G. B. Giannakis, "Principal Component Filter Banks for Optimal Multiresolution Analysis," *IEEE SP*, Vol. 43, pp. 1766—1777, Aug. 1995.

[6] P.P. Vaidyanathan, "Theory of Optimal Orthonormal Subband Coders," *IEEE Trans. Sig. Proc.*, Vol. 46, No. 6, pp. 1528—1543, June 1998.

[7] J. Tuqan and P.P. Vaidyanathan, "Statistically Optimum Pre– and Postfiltering in Quantization," *IEEE CAS II*, Vol. 44, No. 1, pp. 1015—1031, Dec. 1997.

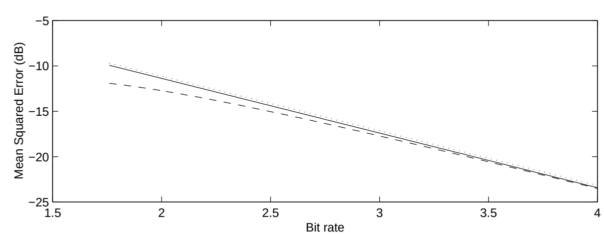

Figure 3: AR(1) process with correlation coefficient $r = 0.8$: Optimum values of MSE as a function of overall bit rate R for different filter design methods. Solid line: IIR biorthogonal (half-whitening) filters [2],[3]; Dotted line: FIR biorthogonal filters [2],[3]; Dashed line: MMSE filters.

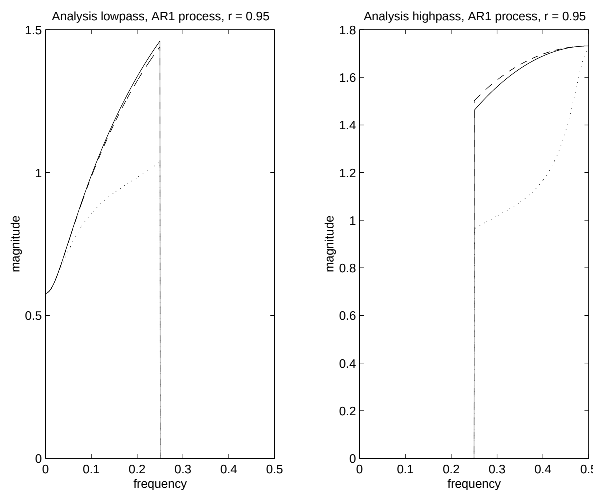

Figure 4: Frequency responses of optimal, unconstrained length analysis filters in two-band case, for AR(1) process at bit rate $R = 2.91$. Solid line: IIR biorthogonal (half-whitening) filters [2],[3]; Dotted line: FIR biorthogonal filters [2],[3]; Dashed line: MMSE filters.

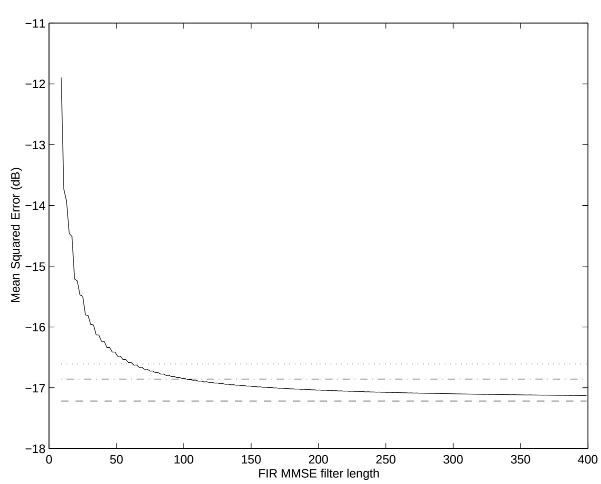

Figure 5: AR(1) process, $R = 2.91$: Convergence of MSE (solid curve) for FIR–MMSE filterbanks to -17.22 dB limit (dashed line) for unconstrained–length MMSE filter banks. Compare with MSEs for IIR biorthogonal filter banks (dash–dotted line, -16.86 dB), and unconstrained–length FIR biorthogonal filter banks (dotted line, -16.61 dB).