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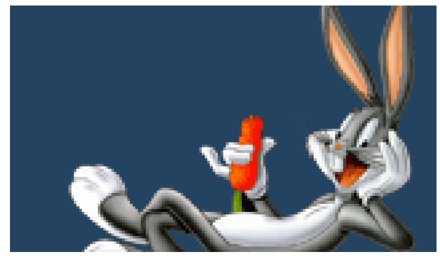

BMS-NCM NEWS: the Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

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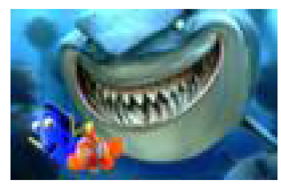

BMS-NCM NEWS

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Special Issue, September 03, 2007

Letter from the editor

Hello everybody!

Welcome to this special issue of our Newsletter. It is dedicated to the second

PhD-Day

organized by the Society to give the opportunity to young Belgian Mathematicians to advertise their work as researchers in mathematics.

Have a nice day!

Françoise Bastin

The Belgian young mathematicians

TELL US WHAT THEY ARE DOING

on the occasion of the second

PhD-Day

on **Monday, 10 September 2007**

at the **Université Libre de Bruxelles**

(Room **Forum E**, Campus Plaine, Boulevard du Triomphe, 1050 Bruxelles)

Organizer of this event: the Belgian Mathematical Society

On this day, opportunity is given to Belgian PhD students to present their research and to get to know their colleagues from all over the country.

PROGRAMME OF THE DAY

10h00 Welcome from the president of the Society
10h15 Godeaux Lecture:

"*Simon Stevin (1548-1620), Mathematician, physicist, ..., Uomo universale*",
by G. Vanden Berghe

11h15 Coffee
11h45 Poster presentations (Session 1)
12h45 Lunch (free for BMS members)
14h15 Oral presentations
16h35 Poster presentations (Session 2)
17h15 Reception (drink and award for best poster)

1 The list of participants

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2 Exhibitions and sponsors

During the whole event, it is possible to visit the stands of

ULB Press,
Unité de Recherche sur l'Enseignement des Mathématiques,
Belgian Mathematical Society

Prizes are sponsored by

ULB Press, Springer, European Mathematical Society

The BMS also thanks the

Ecole Doctorale Thématique en mathématique du FNRS

for its support; this day contributes to the "formation doctorale" offered by the EDT.

3 About the "Godeaux Lecture" and about the speaker

On that day the first ***Godeaux Lecture*** will be delivered by Prof. Guido Vanden Berghe (Universiteit Gent).

The Godeaux Lecture will be organised at least once every two years during a BMS event. These lectures honoring the memory of Lucien Godeaux are organised with the assets of the Belgian Center for Mathematical Studies which were transferred to the BMS after the dissolution of this Center. Lucien Godeaux (1887-1975) was one of the world's most prolific mathematicians (with more than 1200 papers published, 644 of which you 'll find in MathReviews) and took many initiatives to encourage young mathematicians to communicate their research. He was the founder of the Belgian Center for Mathematical Studies in 1949.

Simon Stevin (1548-1620) Mathematician, physicist, . . . , Uomo universale

Guido Vanden Berghe

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In this talk we shall give in first instance attention to the family and the life of Simon Stevin. Born in Bruges his grandparents were original wealthy inhabitants of Ypres and Veurne. In the second place we shall present a comprehensive picture of the activities and the creative heritage of Simon Stevin, who made outstanding contributions to various fields of science in particular, physics and mathematics and many more. Among the striking spectrum of his ingenious achievements, it is worth emphasizing, that Simon Stevin is rightly considered as the father of the system of decimal fractions as it is in use today. Stevin also urged the universal use of decimal fractions along with standardization in coinage, measures and weights. This was a most visionary proposal. Stevin was the first since Archimedes to make a significant new contribution to statics and hydrostatics. His activities as an engineer will be discussed; in particular the construction of fortifications,windmills and the famous sailing chariot will be illustrated. He truly was ìuomo universalisî.

References
(1) J.T. Devreese en Guido Vanden Berghe, Wonder en is gheen wonder , De geniale wereld van Simon Stevin. 1548-1620, Davidsfonds, Leuven, 2003.
(2) J.T. Devreese and Guido Vanden Berghe, Magic is No Magic: The Wonderful World of Simon Stevin , WITpress Southampton (UK), 2007 (in preparation)

About the Speaker: G. Vanden Berghe

Guido Vanden Berghe is the head of the group “Applied Mathematics and Informatics” at the University of Ghent. He got a master in Physics at that University in 1968 and his PhD thesis in 1971 was devoted to modelling and describing atomic nuclii. In the same year he also got a diploma in theoretical nuclear physics. He has been teaching since then in the general areas of numerical analysis and history of sciences.

He spent several periods abroad. He is the author of about 220 publications in theoretical physics, applied mathematics, numerical analysis, and history of mathematics. He is on the editorial board of several jounals and was awarded in 2005 the Honorary Fellowship of the European Society of Computational Methods in Sciences and Engineering and in 2006 he received the Sarton medal of Ghent University for his research in the History of Science.

This last subject has made him a much wanted speaker for several socio-cultural organizations. He has special interest in the development of sciences in the Low Countries during Renaissance. Simon Stevin is his favorite research theme.

In 1998, 450 years after the birth of Simon Stevin, he co-organized an exposition about the life and work of Simon Stevin and also an international conference about the influence of Stevin’s work on current science. He was invited to publish in 2002 “The Low Countries”, an English edition of “Ons Erfdeel” entitled “Simon Stevin, Flemish tutor to a Dutch Prince”. Together with J. Devreese the book “Wonder en is gheen wonder, de geniale wereld van Simon Stevin” (Het Davidsfonds) in 2003. An English translation “Magic is No Magic: The Wonderful World of Simon Stevin” appeared in 2007 (WITPress). In 2004 he contributed to the prestegious exposition “Simon Stevin, De geboorte van een nieuwe wetenschap”(The birth of a new science) in the Royal Library in Brussels and published the book “Exponential fitting” (Kluwer).

4 Award

As announced in the program, a prize will be offered for the best poster; the winner will be congratulated on the occasion of the drink (around 17:30, Monday September 10, 2007).

The award will be issued to the participant who has best advertised his/her PhD work and raised most interest from the audience with the poster.

5 Miscellaneous and practical information

Becoming a member of the BMS on this occasion?? On September 10 in the morning, it will be possible to contact directly a member of the BMS Committee and to register for membership and then ... get a free lunch.

Where and how to go? Information concerning the ULB (and the place where the event will take place): see

http://www.ulb.ac.be/docs/campus/plaine.html

About parking If you come by car, you can use **Parking 2, ULB (main parking)** and also parking 4 (if not possible in parking 2). Nevertheless, **you have to specify that you are a welcome visitor using the poster at the end of this newsletter.** Just put the poster in your car so that it can be read easily from outside.

About certificate for participation Participants who delivered a talk or presented a poster can ask for a “*official certificate*” : just ask Paul GODIN, on Monday September 10.

6 Detailed program

Morning poster session

1. **Khoshsiar Ghaziani** : *Bufurcation software* CL_MATCONTM *and applications*
2. **Gerlo** : *Metrically Generated Theories*
3. **Janssen** : *Hopf-Galois theory and generalizations*
4. **Deschamps** : *Affine actions on nilpotent Lie groups*
5. **Tibboel** : *The asymptotic behaviour of recurrence coefficients for orthogonal polynomials with varying exponential weights*
6. **Goemans** : *Special Cases of Weingarten Translation Surfaces in Euclidean and Minkowski 3-space*
7. **Carette** : *Automorphisms of groups acting on trees*
8. **Engelbeen** : *Realization of intensity profiles by static and dynamic multileaf collimators in radiotherapy*
9. **Falguières** : *Outer automorphism groups of type II_1 factors*

Afternoon poster session

1. **Vankerschaver** : *Geometric methods for classical field theories*
2. **Michel** : *Domino tilings of rectangles with fixed width*
3. **Vercruysse** : *Galois Theory for Corings and Comodules*
4. **Jahanara** : *Symmetries in Riemannian Geometry*
5. **Vernaeve** : *Extending a first order predicate calculus with partially defined iota terms*
6. **Verelst** : *Zeno's Paradoxes: A Cardinal Problem*
7. **Bogaerts** : *Permutation Arrays and Isometries of Sym(n)*
8. **Voglaire** : *Strict quantization of Hermitian symmetric spaces*

SCHEDULE OF TALKS

Time	SESSION A	SESSION B
14h15 – 14h35	**Moonens** : *Linear functionals for which $\omega + d\zeta = F$ has a continuous solution*	**Peña Peña** : *Extensions theorems for holomorphic and two-sided biregular functions*
14h35 – 14h55	**Kheibarshekan** : *A Dynamic Model for the onset of the Endocycle in the plant Arabidopsis*	**Verstringe** : *Reduction of diffeomorphisms using analytic methods*
14h55 – 15h15	**Hautphenne** : *On a particular class of branching processes*	**Schillewaert** : *Generalised dual arcs and Veronesean surfaces, with applications to cryptography*
15h15 – 15h35	**Grumiau** : *Lane-Emden problem : algorithms and symmetries*	**Heistercamp** : *Weinstein Conjecture with multiplicities*
15h35 – 15h55	**Bouchez** : *Symmetry results for the problem $-\Delta u = \lambda_2 \lvert u\rvert^{p-2}u$: a drop in the ocean of the implicit function theorem's applications*	**Goffa** : *Classes of prime Noetherian maximal orders*
15h55 – 16h15	**Smet** : *Irrationality proof of certain Lambert series using little q-Jacobi polynomials*	**van Roosmalen** : *Classification of Hereditary Categories*
16h15 – 16h35	**Ramos Quoirin** : *Multiplicity of solutions for p-laplacian problems*	**Vander Vennet** : *Poisson boundaries of discrete groups and discrete quantum groups*

7 The abstracts

As announced, a talk is a 15 minutes presentation; then there are 5 minutes for questions and to switch rooms.

Permutation Arrays and Isometries of $Sym(n)$

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Let $Sym(n)$ be the group of all permutations of n elements. If ϕ, ψ are two permutations such that ϕ and ψ coincide in λ positions, the Hamming distance between ϕ and ψ is the integer $d_n(\phi, \psi) = n - \lambda$.
A permutation array (PA) $\Gamma_{(n,d)}$ of size s and minimum distance d is a set of s permutations of n elements such that the distance between any two permutations is at least d.
Some data-transmission codes use PA's of maximum size s with respect to n and d. We use the group $Iso(Sym(n))$ of isometries of $Sym(n)$ to study and construct PA's.
Classification of PA's up to isometry requires the use of invariants to distinguish nonisometric PA's. For some classes of PA's, we give properties of the stabiliser of the PA's in $Iso(Sym(n))$.

Symmetry results for the problem $-\Delta u = \lambda_2|u|^{p-2}u$: a drop in the ocean of the implicit function theorem's applications

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(joint work with D. Bonheure, C. Grumiau, J. Van Schaftingen)

The implicit function theorem (shortly the *i.f.t.*) is a very simple result taught in first class of mathematics at university, which has an ocean of applications. We can for example mention the very beautiful Theorem ensuring that problem

$$u' = \epsilon f(t, u), \quad u(0) = u(1),$$

where $f : [0, 1] \times \mathbb{R}^N \to \mathbb{R}^N$ is a continuously differentiable function, has a solution u_ϵ for $\epsilon \in \mathbb{R}$ close enough to 0, if we assume that equation $F(a) = 0$ has a solution a_* such that $\det F'(a_*) \neq 0$, with $F(a) := \int_0^1 f(s, a)ds$.

Let us consider the Dirichlet boundary value problem

$$-\Delta u = \lambda_2|u|^{p-2}u \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where $p \geq 2$, λ_2 is the second eigenvalue of $-\Delta$, and Ω a bounded open set of $\mathbb{R}^N$. The *Energy functional*

$$J : H_0^1(\Omega) \to \mathbb{R} : u \mapsto \frac{1}{2}\int_\Omega |\nabla u|^2 - \frac{\lambda_2}{p}\int_\Omega | u |^p$$

is an application defined on the Sobolev space $H_0^1(\Omega)$ which allows to formulate this problem geometrically like the research of functions u such that $J'(u) = 0$. A *least energy nodal solution* u is an unsigned solution such that $J(u) \leq J(v)$ for all other unsigned solution v.

If Ω admits some symmetry hyperplane(s), we prove that for p close enough to 2, the least energy nodal solutions of this problem has the same symmetry property-(ies) as their orthogonal projection in E_2, the second

eigenspace of $-\Delta$. Because, if Ω is a rectangle, a square or a disc, the functions of E_2 have some symmetry properties, our result seems have some pertinent applications.

After recalling the *i.f.t.*, we will give an idea of the proof of the two previous results while noting the similarities (the presence of degenerate -$u' = 0$- and undegenerate -$F(a) = 0$- equations) and the differences (the use of the existence part or the uniqueness part of the *i.f.t.*).

Automorphisms of groups acting on trees

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(joint work with Richard Weidmann)

Our work relies on Bass-Serre theory, or the theory of groups acting on trees. This theory provides a combinatorial tool that yields insight on the structure of group constructions coming from algebraic topology, such as free products $G_1 * G_2$ or free products with amalgamation $G_1 *_H G_2$. Our focus is to study the automorphism group of a group G, given an action of G on a tree. We use the theory of foldings set up by Kapovich, Myasnikov and Weidmann [1] to extract information on $\mathrm{Aut}(G)$.

The groups G we are interested in lie in various classes. We consider the class of free groups, which are in fact free products of infinite cyclic groups $F_n \cong \mathbf{Z} * \cdots * \mathbf{Z}$. We also consider a free product decomposition of a group $G = *_i G_i$ such that each G_i cannot be further split as a non trivial free product. In this case we are able to write an explicit presentation of $\mathrm{Aut}(G)$ in terms of presentations for each G_i and $\mathrm{Aut}(G_i)$.

References

[1] I. Kapovich, A. Myasnikov and R. Weidmann, *Foldings, graphs of groups and the membership problem*, Internat. J. Alg. Comput. **15** (2005) 95?-128.

Affine actions on nilpotent Lie groups

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(joint work with Dietrich Burde and Karel Dekimpe)

As a natural generalization of the usual affine group $\mathrm{Aff}(\mathbb{R}^n) = \mathbb{R}^n \rtimes \mathrm{GL}(n, \mathbb{R})$, we consider the affine group of a connected and simply connected nilpotent Lie group N, which is defined as $\mathrm{Aff}(N) = N \rtimes \mathrm{Aut}(N)$ and which acts on N via ${}^{(m,\alpha)}n = m \cdot \alpha(n)$, for all $m, n \in N$, $\alpha \in \mathrm{Aut}(N)$.

On one hand we study subgroups Γ in $\mathrm{Aff}(N)$ acting properly discontinuously and cocompactly on N. This situation is a natural generalization of the so-called affine crystallographic groups. We have proved that for all dimensions $1 \le n \le 5$ the generalized Auslander conjecture holds, i.e., that such subgroups are virtually polycyclic.

On the other hand we focus on simply transitive actions of one simply connected nilpotent Lie group G on another one, say N, via a map $\rho : G \to \mathrm{Aff}(N)$ and refer to such actions as NIL-affine actions.

In the usual affine case (i.e. $N = \mathbb{R}^n$) the notion of a simply transitive affine action has been translated completely towards the Lie algebra level. We can show that an analogous translation is available for the much more general simply transitive NIL-affine actions.

When we focus on abelian simply transitive NIL-affine actions, i.e. $\rho : \mathbb{R}^n \to \mathrm{Aff}(N)$, as a nice setting next to the affine case, we discovered that the existence of such an abelian NIL-affine action is equivalent to the existence of a special Lie-compatible algebra structure on the Lie algebra of N, which we refer to as a LR-structure.

We discovered that a Lie algebra admitting such a LR-structure, has to be two-step solvable. We can prove the existence of an LR-structure on several classes of two-step solvable Lie algebras. Conversely we also present an example of a three-step nilpotent Lie algebra on four generators without such a structure. We also study LR-structures in more detail, e.g. concerning ideals.

Realization of intensity profiles by static and dynamic multileaf collimators in radiotherapy

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(joint work with Samuel Fiorini)

When a cancer is diagnosed, a physician can prescribe radiotherapy sessions. These sessions consist in destructing tumor(s) by radiation without damaging the organs located in the radiation field. Thus one has to modulate the radiation. Mathematically, finding this modulation is equivalent to the following problem: given an integer matrix I of dimension $M \times N$ find a decomposition of I as weighted sum of binary $M \times N$ matrices which have the consecutive ones property, that is, the one on each row are grouped in one block. A first objective considered in the literature is to minimize the sum of the coefficients of the decomposition, which have to be integer, because they represent the radiation time we have to minimize for medical reasons. This problem is known to be polynomial. We are studying variants of this problem where the matrices of the decomposition have to satisfy extra constraints, like the cart's constraint which stems from a mecanic constraint. We prove that this problem is also polynomial. A second objective is to minimize the number of matrices in the decomposition. This second problem is NP-hard, even when the matrix I has only one unimodal row. The second objective we consider allows to take into account the comfort of the patient. This motivates the following approach : find, among the decompositions wich minimize the radiation time, a decomposition which minimizes the number of matrices in the decomposition.

References

[1] D. Baater, M. Ehrgott, H.W. Hamacher, G.J. Woeginger, *Decomposition of Integer Matrices and Multileaf Collimator Sequencing,* Discrete Applied Mathematics. 152: 6-34, 2005.

Outer automorphism groups of type II_1 factors

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A von Neumann algebra is a non-commutative generalization of a measure space, the algebras $L^\infty(X, \mu)$ being the abelian examples. By definition, a von Neumann algebra is a weakly closed unital $*$-subalgebra of $B(H)$ for some Hilbert space H. A factor is a von Neumann algebra with trivial center. We are particularly interested in infinite dimensional factors endowed with a tracial state. These are called type II_1 factors.

The outer automorphism group $\mathrm{Out}(M)$ of a II_1 factor M provides, in principle, a useful invariant to distinguish between families of II_1 factors. But this group $\mathrm{Out}(M)$ is extremely hard to compute.

Breakthrough rigidity results in the theory of II_1 factors were obtained recently by Popa (see [3, 4, 5]) and are based on Popa's deformation/rigidity technique.

- In [2], it is shown that there exists, for every *compact abelian* group K, a type II_1 factor M with $\mathrm{Out}(M) \cong K$. The result in [2] is an existence theorem and answered in particular the long standing open problem on the existence of II_1 factors without outer automorphisms.
- In [6], type II_1 factors M with $\mathrm{Out}(M)$ any *discrete group of finite presentation* are explicitly constructed. This in particular gave the first explicit examples of II_1 factors without outer automorphisms.
- More examples of explicit computations of the outer automorphism group of II_1 factors were recently obtained by Vaes in [7]. Actually, a lot more is done in [7]: Vaes gives the first examples of explicit computations of all *finite index bimodules for some* II_1 *factors.*

In [1], in a joint work with Stefaan Vaes we prove the existence of II_1 factors M such that $\mathrm{Out}(M)$ is any, possibly non-abelian, compact group. We use the methods of [2].

References

[1] Falguières Sébastien and Stefaan Vaes, *Every compact group arises as the outer automorphism group of a* II_1 *factor*, `math.OA/0705.1420v1`

[2] A. Ioana, J. Peterson & S. Popa, *Amalgamated free products of* w*-rigid factors and calculation of their symmetry group.* To appear in *Acta Math.* `math.OA/0505589`

[3] S. Popa, *Strong rigidity of* II_1 *factors arising from malleable actions of w-rigid groups, Part I.* Invent. Math. **165** (2006), 369–408.

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Metrically Generated Theories

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Our interest goes out to constructs $\mathbf{X}$ for which natural functors describe the transition from (generalized) metric spaces to metrizable objects in the given construct $\mathbf{X}$. With a (generalized) metric d one can associate for example a topology $\mathcal{T}_d$, a uniformity $\mathcal{U}_d$, an approach structure δ_d or a bornological structure $\mathcal{B}_d$. In each of these examples, a natural functor $K : \mathcal{C} \to \mathbf{X}$ from a category of (generalized) metric spaces $\mathcal{C}$ to the category $\mathbf{X}$ is given. In [1], necessary and sufficient conditions were formulated on such a functor $K : \mathcal{C} \to \mathbf{X}$ in order to ensure that $\mathbf{X}$ is generated by "$\mathcal{C}$-metrizable" spaces, or equivalently, that $\mathbf{X}$ can be isomorphically described as a full concretely coreflective subconstruct of a model category $\mathbf{M}^{\mathcal{C}}$ with objects sets structured by collections of $\mathcal{C}$-metrics. Topological constructs $\mathbf{X}$ for which there exists such a functor $K : \mathcal{C} \to \mathbf{X}$ are called $\mathcal{C}$-metrically generated or simply metrically generated. Many examples of topological constructs are captured in this way. This unified setting of metrically generated constructs admits a general treatment of many interesting aspects of categorical topology, such as the solution of the epimorphism problem for separated objects or the study of completeness and compactness [2, 3].

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Special Cases of Weingarten Translation Surfaces in Euclidean and Minkowski 3-space

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Some results on Weingarten translation surfaces and more specific translation surfaces with vanishing second Gaussian curvature, in 3-dimensional Euclidean and Minkowski space, are presented.

For a surface M with non-degenerate second fundamental form II, the *second Gaussian* and *second mean curvature* are, respectively, defined as the Gaussian and mean curvature of (M, II). They are denoted with K_{II} and H_{II} respectively.

We define a surface to be a *Weingarten surface* if it is an (A, B)-W-surface for $A \neq B \in \{K, H, K_{II}, H_{II}\}$. An (A, B)-*W-surface* is a surface on which there exists a non-trivial functional relation $\Phi(A, B) = 0$ between two curvatures A and B of the surface.

Special examples of Weingarten translation surfaces will be investigated, namely those surfaces for which K_{II} is zero. The minimal translation surfaces of Scherk are among these examples. Other non-trivial examples of surfaces that satisfy the curvature condition examined, can be expressed in terms of the Lambert W function. Most attention will be paid to this last kind of special cases.

Classes of prime Noetherian maximal orders

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(joint work with Eric Jespers and Jan Okniński)

In general, it remains an unsolved problem to characterize when an arbitrary semigroup algebra is a prime Noetherian maximal order. Even for group algebras this is unknown. The only class of groups for which a positive answer has been given is that of the polycyclic-by-finite groups (K.A.Brown). Hence it is natural to consider the problem first for semigroup algebras of submonoids of polycyclic-by-finite groups. In [4], the authors gave a structural characterization of such algebras $K[S]$ that are Noetherian and in [2] they described when a semigroup algebra of a submonoid of a finitely generated abelian-by-finite group G is a prime Noetherian maximal order. The characterization obtained generalizes the one given in [3] in case G is torsion free and it shows that the action of G on minimal primes of some abelian submonoid of S is very important (as was also discovered for the special case where S is a finitely generated monoid of IG-type in [1]). Knowledge of height one prime ideals of $K[S]$ turns out to be fundamental and the result reduces the problem to the structure of the monoid S, in particular the monoid S itself needs to be a maximal order. If S is a submonoid of a finitely generated abelian-by-finite group G (with A the abelian subgroup of finite index in G), the semigroup S is a maximal order in G if and only if the semigroup S is maximal among all subsemigroups T of G such that $T \cap A = S \cap A$ is a maximal order ([2, 3.1]). So, abelian maximal orders are interesting to understand the non-abelian ones. That is why we will end by illustrating a characterization of finitely presented abelian monoids that are cancellative maximal orders, in terms of a defining presentation up to two relations. Since we are interested in abelian monoids that satisfy the ascending chain condition on ideals, finitely presented monoids are natural to consider. Descriptions of abelian Noetherian maximal order semigroup algebras, that are not necessarily finitely generated, have been obtained earlier by Anderson, Gilmer and Chouinard.

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Lane-Emden problem : algorithms and symmetries.

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(joint work with D. Bonheure, V. Bouchez, C. Troestler, J. Van Schaftingen)

We will be interested in the problem

$$\begin{cases} -\Delta u = |u|^{p-2}u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where $2 < p < \frac{2N}{N-2}$ and Ω is a bounded open set in $\mathbb{R}^N$, for $N \geq 2$. More precisely, we will study the ground state and the least energy nodal solutions. They are respectively one-signed solutions and sign changing solutions whith minimal energy, where the energy is given by

$$\mathcal{E} : H_0^1(\Omega) \to \mathbb{R} : u \mapsto \frac{1}{2}\int_\Omega |\nabla u|^2 - \frac{1}{p}\int_\Omega |u|^p.$$

In the first part of this talk, after citing some important historical results, we will use the Mountain-Pass algorithm of Y. S. Choi and P. J. McKenna to approch ground states and to illustrate a celebrated theorem of B. Gidas, W. M. Ni and L. Nirenberg which asserts that ground states respect the symmetries of Ω, when Ω is convex. In the second part, we will use the modified Mountain-Pass algorithm of D. G. Costa, Z. Ding and J. M. Neuberger to approch least energy nodal solutions and to illustrate "old" and recently obtained symmetry results. In particular, we will illustrate in three cases the result which asserts that, for p close to 2 and under some additional assumptions, any least energy nodal solution respects symmetries of its orthogonal projection in the second eigenspace of $-\Delta$ in $H_0^1(\Omega)$: second eigenvalue of $-\Delta$ is simple (by example : the rectangle), ball and square.

Time permitting, we will speak about numerical experiments which show the existence of a symmetry breaking.

On a particular class of branching processes

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A large number of questions in fields like biology and telecommunications may be modeled as continuous-time multi-type branching processes. We focus our attention here on a particular class of such branching processes, named the Markovian binary trees (MBT). The MBT was introduced for the first time in 2005 by N. Kontoleon in his thesis [1] where he show that most of the macroevolutionary processes are embedded in the MBT. He focused on the investigation of the MBT extinction probability.

With the help of the branching processes theory and the matrix analytic methods, we are dealing here with new methods to compute the extinction probability of the MBT, and we study for instance the distribution of the time until its extinction, the distribution of the number of particles alive at some time t, the distribution of the total progeny of the process until extinction, provided this extinction occurs. Finally, we investigate the behavior of the process obtained if an immigration or a catastrophe process is added to our initial MBT.

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Weinstein Conjecture with multiplicities

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(joint work with Felix Schlenk)

An old question in Hamiltonian dynamics originating in celestial mechanics is: Does a compact energy surface carry a periodic orbit? Let Q be the configuration space of a dynamical system. The corresponding phase space T^*Q of the manifold Q is equipped with a canonical symplectic structure $\omega = d\lambda$, where the 1-form $\lambda = \sum_i p_i dq_i$ is the Liouville form. If $H : T^*Q \to \mathbf{R}$ is the Hamiltonian function of the mechanical system, its evolution $\gamma(t) = (q(t), p(t))$ in T^*Q is given by the Hamiltonian equation

$$\dot{\gamma} = X_H(\gamma(t)).$$

In this equation, X_H is the Hamiltonian vector field associated to the function $H(q,p)$, and is characterised by $\iota(X_H)\omega = dH$. Consider now an energy surface $\Sigma_c = \{(q,p)|H(q,p) = c\}$. Suppose that Q is a compact manifold without boundary, and that c is such that Σ_c is a smooth hypersurface. Suppose also that Σ_c is compact and surrounding Q. In this situation, the Weinstein conjecture tells us that Σ_c carries a closed orbit of X_H. This conjecture is proved by Hofer and Viterbo in [1]. The Hamiltonian $H(q,p) = \frac{1}{2}|p^2|$ describes the geodesic flow. In this case, we know that for many manifolds Q each hypersurface Σ_c carries infinitely many closed geodesics. In analogy, we can expect a quantitative version of the Weinstein conjecture, i.e., there always exist many or even infinitely many closed orbits of X_H on Σ_c.

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Symmetries in Riemannian Geometry

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(joint work with Stefan Haesen, Mira Petrovic, Zerin Senturk and Leopold Verstraelen)

Abstract. Presentation of characterizations (in terms of the work of Schouten parallel transport) and on of metrical interpretations (in terms of the squaroids of Levi-Civita) of the main natural curvature symmetries occuring in Riemannian Geometry.

Hopf-Galois theory and generalizations

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(joint work with S. Caenepeel and S.H. Wang)

Let H be a Hopf algebra. We say that an H-comodule algebra A, with H-coaction $\rho : A \to A \otimes H$, is a (faithfully flat) H-Galois extension of the subalgebra $B = A^{coH} = \{a \in A|\rho(a) = a \otimes 1_H\}$ of coinvariants of A if the canonical map $\kappa : A \otimes_B A \to A \otimes H, \kappa(a \otimes_B a') = a\rho(a')$ is bijective (and ${}_BA$ is faithfully flat). This notion unifies amongst others classical Galois extensions of fields and rings and strongly graded algebras.

An important Structure Theorem proved by Schneider states that the category $\mathcal{M}_A^H$ of relative Hopf modules over a faithfully flat H-Galois extension A is equivalent to the category $\mathcal{M}_B$ of right B-modules.

This result can be formulated more generally in the context of corings. An A-coring is a comonoid in the monoidal category of A-bimodules. Over the years the theory of corings has proven to be an appropriate framework to unify elegantly all kinds of results stemming from ring and Hopf algebra theory.

We generalized Schneider's Theorem in two different directions. Inspired by the notion of a partial group action we introduced in [1] *partial* H-(co)actions and stated a Structure Theorem for relative Hopf modules over a faithfully flat *partial* H-Galois extension. In fact the latter is an application of the coring approach we mentioned earlier. In [2] we extended Galois theory for corings to so-called *group corings* and obtained as an application a Hopf-Galois-type theory for so-called *Hopf group coalgebras*.

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A Dynamic Model for the onset of the Endocycle in the plant Arabidopsis

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(joint work with Willy Govaerts, Bart Sautois from Ghent University and Martin Kuiper, Steven Vercruysse, Lieven De Veylder from VIB)

We build a mathematical ODE model using computer simulations and numerical continuation techniques to understand the dynamics of the cell cycle process in the plant Arabidopisis. In the first phase we concentrate on some interactions which prevent the cell to divide when going through the cell cycle, so that the cell transits to the endocycle. Corresponding to the pathways of the interactions we build an ODE model that mimics the experimental observations. To predict the qualitatively different behaviors of the system with respect to parameters changes, we use the numerical Matlab continuation package Matcont which is a powerful software for the study of bifurcations.In the second phase we try to find out which types of proteins have to be included in the pathways of the endocycle in order to improve the model. In this case we extract the information from the database MineMap which is based on the most recent papers in this area.

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Bufurcation software Cl_MatcontM and applications

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We consider the map $x_{n+1} = F(x_n, \alpha)$ where $x \in R^n$ and $\alpha \in R^k$ are vectors of state variables and parameters, respectively. CL_ matcontM [1] is a Matlab software for bifurcation analysis of iterations of maps. It supports:

- Continuation of fixed points with respect to a control parameter, detection and location of all codim-1 bifurcation points and their continuation in two control parameters, as well as detection of all codim-2 points.
- Continuation of the secondary branch of fixed point in a branch point and curve of fixed point of double period in a flip point.

- Switching to the branches of codim-1 bifurcations that emanate at codim-2 bifurcation points.
- For all codim-1 and codim-2 bifurcation points the normal form coefficients are computed using directional finite differences, symbolic derivatives and automatic differentiation.

As applications, we consider examples in mathematical biology and economics. In the first example we study the dynamical behavior of a Leslie-Gower competition model in two control parameters, where we discover two species coexistence under strong competition. In the second example we consider a two-dimensional nonlinear duopoly model. We compute stability boundaries of cycles in which nearby orbits converge to fixed points. In particular we compute regions of the parameter space in which there is multistability of different cycles of the map.

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Domino tilings of rectangles with fixed width

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Let $t(m,n)$ be the number of ways to tile a $m \times n$ rectangle with 1×2 rectangles ("dominos"). For $m = 2$, $t(m,n)$ is the $(n+1)$-th Fibonacci number. Therefore,

$$t(2, n+2) = t(2, n+1) + t(2, n) \qquad \text{for any } n \in \mathbf{N}_0.$$

More generally, for each fixed m, the sequence $(t(m,n))_{n\in\mathbf{N}_0}$ verifies a homogeneous linear recurrence relation with constant coefficients. We prove that the minimum order of such a recurrence relation governing the sequence $(t(m,n))_{n\in\mathbf{N}_0}$ is $2^{\lceil \frac{m}{2} \rceil}$. The proof is based on the following formula found by Kasteleyn [1] and Temperley and Fisher [2]:

$$t(m,n) = 2^{\frac{mn}{2}} \prod_{k=1}^{m} \prod_{l=1}^{n} \sqrt[4]{\cos^2 \frac{k\pi}{m+1} + \cos^2 \frac{l\pi}{n+1}}.$$

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Linear functionals for which $\omega + d\zeta = F$ has a continuous solution

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(joint work with Th. De Pauw)

This work relies on a previous paper by Th. De Pauw and W.F. Pfeffer.

On the space $\mathscr{D}^m = \mathscr{D}(\mathbb{R}^n, \Lambda^m \mathbb{R}^n)$ of all infinitely differentiable differential forms having a compact support in $\mathbb{R}^n$ one puts a locally convex topology τ such that $\omega_k \to \omega, k \to \infty$ holds for τ if and only if there exists a compact set $K \subseteq \mathbb{R}^n$ such that $\operatorname{supp} \omega_k \subseteq K$ holds for each $k \in \mathbb{N}$ and that for each $i_1, \ldots, i_n \in \mathbb{N}$ one has

$$\frac{\partial^{i_1}}{\partial x_1^{i_1}} \cdots \frac{\partial^{i_n}}{\partial x_n^{i_n}} \omega_k \quad \to \quad \frac{\partial^{i_1}}{\partial x_1^{i_1}} \cdots \frac{\partial^{i_n}}{\partial x_n^{i_n}} \omega,$$

uniformly in $\mathbb{R}^n$ when k goes to infinity. An element of the topological dual space $\mathscr{D}_m = (\mathscr{D}^m)'$ is called a *m-current* in $\mathbb{R}^n$.

Let T be a m-current in $\mathbb{R}^n$. Suppose one can find Radon measures μ, ν in $\mathbb{R}^n$ and two bounded multivectorfields v and w measurable w.r.t. μ and ν respectively for which the equalities

$$\langle T, \omega \rangle = \langle \mu \llcorner v, \omega \rangle \quad \text{and} \quad \langle \partial T, \zeta \rangle := \langle T, d\zeta \rangle = \langle \nu \llcorner w, \zeta \rangle$$

hold for each $\omega \in \mathscr{D}^m$ and each $\zeta \in \mathscr{D}^{m-1}$. Then one will say that T is a m-normal current in $\mathbb{R}^n$. Obviously each m-normal current can be extended to a linear functional on the space $C^m = C^m(\mathbb{R}^n, \Lambda^m \mathbb{R}^n)$ consisting of all continuous differential forms in $\mathbb{R}^n$.

Given a linear functional on the space $\mathbf{N}_m$ of all m-normal currents in $\mathbb{R}^n$, one will say that $(\omega, \zeta) \in C^m \times C^{m-1}$ is a solution of the equation $\omega + d\zeta = F$ if for each $T \in \mathbf{N}_m$ the equality $\langle T, \omega \rangle + \langle \partial T, \zeta \rangle = F(T)$ holds.

One will show that such a continuous solution exists if and only if F satisfies an interesting continuity condition.

Extensions theorems for holomorphic and two-sided biregular functions

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(joint work with R. Abreu Blaya, J. Bory Reyes and F. Sommen)

Let $\mathbb{C}_m$ be the complex Clifford algebra constructed over $\mathbb{R}^m$. Then we consider $\mathbb{C}_m$-valued functions f, defined in open subsets of $\mathbb{R}^{2m}$, which satisfy the so-called *isotonic system*

$$\partial_{\underline{x}_1} f + i\tilde{f}\partial_{\underline{x}_2} = 0$$

where $\partial_{\underline{x}_1}$, $\partial_{\underline{x}_2}$ are Dirac type operators and where $\tilde{\cdot}$ denotes the main involution in $\mathbb{C}_m$.

In this communication, we discuss an integral representation formula for the solutions of the above system (see [3]). We also define the isotonic Cauchy transform and prove the Sokhotski-Plemelj formulae for this integral operator (see [1]).

Finally, as the holomorphic and two-sided biregular functions satisfy the isotonic system we use our results to obtain some extension theorems for these types of functions (see [2]).

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Multiplicity of solutions for p-laplacian problems

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(joint work with F.O. de Paiva)

The p-Laplace operator $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$ stands for a model operator in nonlinear analysis and appears in several physical problems such as reaction-diffusion and nonlinear diffusion equations. We deal with the equation

$$-\Delta_p u = f(x,u), \quad u \in W_0^{1,p}(\Omega) \tag{1}$$

by considering the eigenvalue problem

$$-\Delta_p u = \lambda m(x)|u|^{p-2}u, \quad u \in W_0^{1,p}(\Omega). \tag{2}$$

By relating $m(x)$ to the asymptotic behavior of f, we are able to obtain multiplicity of solutions of (1). Depending on $\lambda_1(m)$, the first eigenvalue of (2), we prove the existence of 2 or 3 nontrivial solutions of (1).

Generalised dual arcs and Veronesean surfaces, with applications to cryptography

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Abstract

We start by defining generalised dual arcs, the motivation for defining them comes from cryptography, since they can serve as a tool to construct authentication codes and secret sharing schemes. We extend the characterisation of the tangent planes of the Veronesean surface V_2^4 in $PG(5,q)$, q odd, described in [1], as a set of q^2+q+1 planes in $PG(5,q)$, such that every two intersect in a point and every three are skew. We show that a set of q^2+q planes generating $PG(5,q)$ and satisfying the above properties can be extended to a set of q^2+q+1 planes still satisfying all conditions. This result is a natural generalisation of the fact that a q-arc in $PG(2,q)$, q odd, can always be extended to a $(q+1)$-arc. This extension result is then used to study a regular generalised dual arc with parameters $(9,5,2,0)$ in $PG(9,q)$, q odd, where we obtain an algebraic characterization of such object as being the image of a *cubic* Veronesean.

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Irrationality proof of certain Lambert series using little q-Jacobi polynomials

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We apply the Padé technique to find rational approximations to

$$h^{\pm}(q_1, q_2) = \sum_{k=1}^{\infty} \frac{q_1^k}{1 \pm q_2^k}, \qquad 0 < q_1, q_2 < 1, \quad q_1 \in \mathbb{Q}, \quad q_2 = 1/p_2, \quad p_2 \in \mathbb{N} \setminus \{1\}.$$

In this construction we make use of little q-Jacobi polynomials. Our rational approximations are good enough to prove the irrationality of $h^{\pm}(q_1, q_2)$ and give an upper bound for the irrationality measure. Moreover, if the two parameters q_1, q_2 are related in a certain way, we obtain better results. This is the case when $q_i = q^{r_i}, r_i \in \mathbb{N}, q = 1/p, p \in \mathbb{N} \setminus \{1\}$.

The asymptotic behaviour of recurrence coefficients for orthogonal polynomials with varying exponential weights

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We consider orthogonal polynomials $\{p_{n,N}(x)\}_{n=0}^{\infty}$ on the real line with respect to a weight $w(x) = e^{-NV(x)}$ and in particular the asymptotic behaviour of the coefficients $a_{n,N}$ and $b_{n,N}$ in the three term recurrence $x\pi_{n,N}(x) = \pi_{n+1,N}(x) + b_{n,N}\pi_{n,N}(x) + a_{n,N}\pi_{n-1,N}(x)$. For one-cut regular V we show, using the Deift-Zhou method of steepest descent for Riemann-Hilbert problems, that the diagonal recurrence coefficients $a_{n,n}$ and $b_{n,n}$ have asymptotic expansions as $n \to \infty$ in powers of $1/n^2$ and powers of $1/n$, respectively.

Poisson boundaries of discrete groups and discrete quantum groups

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To a random walk on a discrete group, one can associate a Markov operator. Harmonic functions with respect to the Markov operator are represented by the Poisson integral, using the action of the group on the Poisson boundary and the harmonic measure. The identification of this Poisson boundary for given groups and random walks, is a major problem. The theory of random walks can be extended to discrete quantum groups. We compute the associated Poisson boundary in several concrete examples, like the duals of the universal orthogonal quantum groups and the quantum automorphism groups. Moreover, we describe the behavior of Poisson boundaries under monoidal equivalence of quantum groups.

Classification of Hereditary Categories

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(joint work with Michel Van den Bergh)

Abelian categories are an abstract framework for homology theories; they arise naturally in representation theory and commutative algebraic geometry. In such a category $\mathcal{A}$ we may consider kernels, images and hence also exact sequences. We will only consider k-linear categories where k is a field, thus $\mathrm{Hom}(A, B)$ is a k-vectorspace and composition of morphisms is compatible with the action of k.

It is well-known that the functor $\mathrm{Hom}(A, -) : \mathcal{A} \to \mathrm{Vect}\, k$ is only left exact, thus turning an exact sequence $0 \to X \to Y \to Z \to 0$ in an exact sequence

$$0 \to \mathrm{Hom}(A, X) \to \mathrm{Hom}(A, Y) \to \mathrm{Hom}(A, Z).$$

In homology theory, one introduces the functors $\mathrm{Ext}^n(A, -)$ for $n \geq 1$ such that previous exact sequence may be completed to the long exact sequence

$$\begin{aligned} 0 \to \mathrm{Hom}(A, X) \to \mathrm{Hom}(A, Y) \to \mathrm{Hom}(A, Z) &\to \mathrm{Ext}^1(A, X) \to \mathrm{Ext}^1(A, Y) \to \mathrm{Ext}^1(A, Z) \\ &\to \mathrm{Ext}^2(A, X) \to \mathrm{Ext}^2(A, Y) \to \mathrm{Ext}^2(A, Z) \\ &\to \cdots \end{aligned}$$

We will say $\mathcal{A}$ is hereditary if $\mathrm{Ext}^1(A, -)$ is right exact for every $A \in \mathrm{Ob}\, \mathcal{A}$. Examples of such abelian categories are given by the category of coherent sheaves on a smooth projective curve and the category of representations of a quiver without relations.

Our aim is to study and classify hereditary categories under additional, mostly geometrically inspired, conditions such as Serre duality.

Galois Theory for Corings and Comodules

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(joint work with G. Böhm, S. Caenepeel, E. De Groot, L. El Kaoutit, J. Gómez-Torrecillas, M. Iovanov, S. Wang)

Corings and comodules were defined by Sweedler in 1975 and are defined as coalgebras (or co-monoids) in the monoidal category of bimodules over a (non-commutative) ring. In this way they are, from a categorical point of view, the dual notion of a ring (extension). The most interesting examples of corings were found only in 1999 by Takeuchi, who observed that corings and comodules can be constructed out of entwining structures and entwined modules, and therefore as well out of all kinds of structures that were intensively studied previously in Hopf algebra theory. In fact, by passing from entwining structures and entwined modules to corings and comodules, it turned out that most structural theorems could be preserved, moreover calculations became easier and more transparent and many results could be clarified as they were presented in a broader perspective.

Galois theory for corings and comodules orriginates from the classical work on Galois extensions of commutative fields. A group action can be generalized to a Hopf algebra (co)action, which has lead to the development of Hopf-Galois theory. Hopf-Galois theory as well has a formulation in terms of corings and is one of the major research subjects in present coring theory. It allows one to characterize in general equivalences between categories of comodules over a coring and modules over a (possibly non-unital firm) ring.

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Zeno's Paradoxes: A Cardinal Problem

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Although *no one has ever touched Zeno without refuting him (Whitehead)*, it will be the aim of this contribution to show that, whatever it was that was refuted, it was certainly not Zeno. We demonstrate that upon direct analysis of the Greek sources [3], an underlying structure common to both the Paradoxes of Plurality and the Paradoxes of Motion exists. This structure bears on a correct — Zenonian — interpretation of a simultaneous "division through and through" [1]. Essentially, Zeno's divisional procedure implies the construction of a well-ordered continuum. A mathematical representation will be set up that catches this common structure, based on the notion of a divisional tree [4], and expressed in the language of domain theory [2]. This representation will prove to be aequivalent to Cantor's Continuum Hypothesis [6, 5].

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Extending a first order predicate calculus with partially defined iota terms

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Partial functions and "undefinedness" have been around in mathematics for a long time, without causing any trouble. It was only when mathematics and computer science met in projects on "automatization" of formal reasoning that some problems came up [4]. Where humans are able to avoid the application of a partial function outside "the domain" of the function, formalising the rules for this activity seems to be less trivial. In [1] Farmer states that there does not exist a consensus on how partial functions should be mechanized. We want to add one more possibility by extending a two-valued first order sequent calculus for predicate logic with identity [2] to handle partial functions, essentially by generalising Hilbert and Bernays's approach [3] to partially defined iota terms of the form $\iota x_\psi(\varphi)$ which represent the unique x satisfying φ whenever the condition ψ is fulfilled. In our approach, a sequent is sound only if all terms appearing in it are defined. In this sense the sequent $\Gamma \vdash \iota x_\psi(\varphi) = z$ is sound only if $\Gamma \vdash \psi$.

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Reduction of diffeomorphisms using analytic methods

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It has been studied since the time of H. Poincaré whether a diffeomorphism of Euclidean space can be locally linearized near a fixed point using coordinate transforms. A first reason for non-linearizability is the existence of resonant terms. These resonances are an obstruction to find a formal solution of the problem $\phi^{-1} \circ f \circ \phi(x) = Ax$. In this formula ϕ is identity in first order, and $f(x)$ is in first order Ax. Leaving the path of linearization one could ask whether it is possible to remove all but the resonant terms. There is no formal obstruction to do so, but the formal solution does not converge in general due to the existence of small denominators. The condition of convergence depends essentially on some intricate diophantine relations between the eigenvalues of A. This condition is not always stable when adding a small parameter to our system. However, when all eigenvalues lie on the same side of the unit circle in the complex plane, it was shown by H. Poincaré that an analytic solution exists; for a nice proof see [1]. In an upcoming article our study will include the so-called saddle case, i.e. when the eigenvalues do not necessarily lie on the same side of the unit circle. An important point is that we allow dependence on a small parameter. More in particular: we show that it is possible to linearize all but a cone of terms around the resonant ones. In the case of vector fields, similar results have already been obtained by P. Bonckaert and P. De Maesschalck in [2].

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Strict quantization of Hermitian symmetric spaces

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The general problem of quantizing symplectic or Poisson manifolds allows intrinsically for many answers. The notion of quantization we use is the *WKB quantization* in the particular form developped by P. Bieliavsky [1] upon initial ideas of Karasev, Zakrzewski and Weinstein (see [2], [3]). It consists roughly in defining a new associative, non commutative product $\star$ on the functions on the manifold M, in the form

$$(u \star v)(x) = \int_{M\times M} A(x,y,z) e^{\frac{i}{\hbar}S(x,y,z)} u(y)v(z)dydz,$$

with $A \in C^\infty(M)[[\hbar]]$ and $S \in C^\infty(M)$ such that the product is invariant under the largest possible subgroup of the automorphism group of the manifold.

In the case of Hermitian symmetric spaces of the non compact type, there is hope that the highly symmetric geometry allows to write explicit formulas for the amplitude and phase functions A and S, leading to products with a great invariance group.

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