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A New Grey Bass Equation for Modelling New Product Diffusion

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Abstract. A novel grey Bass model with a power exponent (Grey Bass Power Model) was proposed, and the related parameter-packets of the model were deduced. A nonlinear programming method was employed to optimize the grey parameters and improve the accuracy of the grey Bass model. The results show that GM(1,1) model, GPM (1,1) model, grey Verhulst model and grey Bass model are the special examples of grey Bass power model. The actual example of product diffusion shows the modelling accuracy has been remarkably improved by using the new model.

Introduction

Economic growth is driven largely by technological innovation, in addition to capital and labor and other factors. This means that technological innovation is also an important productivity. In the process of technological innovation, the new product diffusion has gotten so much attention that it has become a hot research issue. Bass model [1] is one of the most outstanding models for revealing the mechanism of new technology or product diffusion by establishing a differential dynamic speed equation. Now, Bass model has been widely applied in agriculture, education, medicine, information technology, and service industries etc [2-4].

As the parameters are the main structural factors affecting the prediction accuracy, the method of parameters estimation is an important theoretical issue. Ordinary least squares method [5], maximum likelihood estimation method [6], nonlinear least of squares method [7] has been widely employed to estimate the parameters of Bass model in the case of an adequate amount of data. The greyed Bass model [8] was been proposed to solve the actual problem of lacking adequate valid data via the method of greying the differential method [9].

The power exponent is introduced to grey Bass model as a grey parameter. The corresponding nonlinear optimization model is employed to whiten the grey parameters in the new grey Bass model. Then we believe that the new model can be helpful to improve the modeling precision of grey Bass model.

The Grey Bass Model with a Power Exponent

We first introduce the power exponent (a grey parameter) to the grey Bass model proposed by reference [8] and analyze the relationships between the new model and the classical ones.

Definition 1. Let $x^{(0)}(k)$ be the market diffusion sequence of a new product,

$$x^{(1)}(k) = AGOx^{(0)}(k),$$

$$z^{(1)}(k) = MEANx^{(1)}(k) = 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right).$$

The grey Bass power model of $x^{(0)}(k)$ is:

$$x^{(0)}(k) = a\left(M - z^{(1)}(k)\right) + bz^{(1)}(k)\left(1 - \frac{z^{(1)}(k)}{M}\right)^{\otimes_\alpha},$$

where the power exponent $\otimes_\alpha$ is a grey parameter.

From the expression of the above model, we can get the following conclusions:

1° When the internal influence coefficient $b=0$, grey Bass power model equals GM(1,1);

2° When the external influence coefficient $a=0$, grey Bass power model equals grey power model GPM (1, 1);

3° When the external influence coefficient $a=0$, $\otimes_\alpha=1$, grey Bass power model equals grey Verhulst model.

3° When $a\neq 0$, $b\neq 0$ and $\otimes_\alpha=1$, grey Bass power model equals grey Bass model in [8].

Thus, grey Bass power model has close contacts to the existing grey models. GM(1,1) model, GPM (1, 1) model, grey Verhulst model and grey Bass model are the special examples of the new model.

Parameters Identification of Grey Bass Power Model

In practices, we need to use experts predict or analogy study to estimate the maximum capacity M. Therefore, this paper assumes the maximum market capacity M is known, and on this basis to discuss the parameters identification of the model.

Theorem 1. The primacy parameter package $P_{\mathrm{I\,B}}$ of grey Bass power model:

$$P_{\mathrm{I\,B}}=(a,b)=\left(\frac{GE-CH}{FG-C^2},\frac{FH-CE}{FG-C^2}\right).$$

The second parameter package $P_{\mathrm{II\,B}}$ of grey Bass power model is:

$$P_{\mathrm{II\,B}}=(C,E,F,G,H)$$

where

$$C=\sum_{k=2}^{n}z^{(1)}(k)\left(M-z^{(1)}(k)\right)\left(1-\frac{z^{(1)}(k)}{M}\right)^{\otimes_\alpha},E=\sum_{k=2}^{n}x^{(0)}(k)\left(M-z^{(1)}(k)\right),$$

$$F=\sum_{k=2}^{n}\left(M-z^{(1)}(k)\right)^2,G=\sum_{k=2}^{n}\left(z^{(1)}(k)\left(1-\frac{z^{(1)}(k)}{M}\right)^{\otimes_\alpha}\right)^2,H=\sum_{k=2}^{n}x^{(0)}(k)z^{(1)}(k)\left(1-\frac{z^{(1)}(k)}{M}\right)^{\otimes_\alpha}.$$

Proof. Let $k=2,3,\cdots,n$ in the grey Bass power model, we have

$$y_N=BP_{\mathrm{I\,B}}$$

where

$$B=\begin{bmatrix} M-z^{(1)}(2) & z^{(1)}(2)\left(1-\frac{z^{(1)}(2)}{M}\right)^{\otimes_\alpha} \\ M-z^{(1)}(3) & z^{(1)}(3)\left(1-\frac{z^{(1)}(3)}{M}\right)^{\otimes_\alpha} \\ \vdots & \vdots \\ M-z^{(1)}(n) & z^{(1)}(n)\left(1-\frac{z^{(1)}(n)}{M}\right)^{\otimes_\alpha} \end{bmatrix},\quad y_N=\begin{bmatrix} x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots \\ x^{(0)}(n)\end{bmatrix},\quad P_{\mathrm{I\,B}}=\begin{bmatrix}a\\ b\end{bmatrix}$$

Using the rule of OLS, we have

$$P_{\mathrm{I\,B}}=\left(B^TB\right)^{-1}B^Ty_N.$$

For $B^TB=\begin{bmatrix}F & C\\ C & G\end{bmatrix},B^Ty_N=\begin{bmatrix}E\\ H\end{bmatrix}$,thus,

$$P_{\mathrm{I\,B}}=\left(B^TB\right)^{-1}B^Ty_N=\frac{1}{FG-C^2}\begin{bmatrix}G & -C\\ -C & F\end{bmatrix}\begin{bmatrix}E\\ H\end{bmatrix}=\begin{bmatrix}\frac{GE-CH}{FG-C^2}\\ \frac{FH-CE}{FG-C^2}\end{bmatrix}=\begin{bmatrix}a\\ b\end{bmatrix}.$$

Optimization the Grey Parameter

It can be seen from Theorem 1, both of the structural parameters a and b are necessarily dependent on the grey parameter $\otimes_\alpha$. Supposing that the objective function is the minimization of simulated average relative error and the constraint is the relationships among these parameters, the optimization model can be established as following:

$$\underset{\otimes_\alpha}{\operatorname{Min}}\, error_{avg} = \frac{1}{n-1}\sum_{k=2}^{n}\left|\hat{x}^{(0)}(k)/x^{(0)}(k)-1\right|$$

$$s.t.\quad \begin{cases} \hat{x}^{(0)}(k) = a\left(M - z^{(1)}(k)\right) + bz^{(1)}(k)\left(1-\dfrac{z^{(1)}(k)}{M}\right)^{\tilde{\otimes}_\alpha} \\ a = \dfrac{GE-CH}{FG-C^2} \\ b = \dfrac{FH-CE}{FG-C^2} \\ k = 2,3,\cdots,n. \end{cases}$$

where $\tilde{\otimes}_\alpha$ is the whitened value of the grey parameter $\otimes_\alpha$.

The optimal parameters $\tilde{\otimes}_\alpha$, a and b can be obtained by solving the model with operations research software LINGO. The whitened parameter $\tilde{\otimes}_\alpha$ can make the average modeling error minimum.

Case Study

In this section, the advantage of the grey Bass power model over the traditional grey Bass model is demonstrated by a actual example of China's blog diffusion in [8]. The total number of China's valid blog space from 2002 to 2007 is in Table 1.

Table1 China's blog scale in the past year(ten thousand persons)

Year	2002	2003	2004	2005	2006	2007
Accumulative number	8	30	150	900	2080	4200

By using the optimization method proposed in this paper, we get the optimal whitening value $\tilde{\otimes}_\alpha = 17.7006$ by method proposed in this paper. According to the survey results from reference [4], China's blog market potential M is taken to be 702 million people.

According to Theorem 1, we have

$$P_{1B} = (a,b)^T = (-0.000067, 1.412519)^T.$$

The grey Bass power model is

$$\hat{x}^{(0)}(k) = -0.000067\left(70200 - z^{(1)}(k)\right) + 1.412519z^{(1)}(k)\left(1-z^{(1)}(k)/70200\right)^{17.7006}.$$

Take $e(k)\%$ as the relative absolute error, $e_{avg}\%$ as the average relative absolute error, $p_{avg}\%$ as the average precision. The modeling results of China's valid blog space are showed in the table 2.

According to Table 2, the grey Bass power model with $\tilde{\otimes}_\alpha = 17.7006$ shows a remarkably higher accuracy than that of the traditional grey Bass model with a fixed value $\tilde{\otimes}_\alpha = 1$. Comparatively, the precision of grey Bass power model is 91.37% which is much higher than that of grey Bass model 71.31%. Therefore, the optimization method for whiten the power exponent is effective.

As the initial sale of an innovative product in the market is unstable, the head data does not reflect the diffusion law. In the traditional grey Bass model, the simulated value for 2003 is too big. So, it is excluded in the calculation of e_{avg} and p_{avg}. In fact, this phenomenon is consistent with the Head Blind Axiom [10] proposed by Deng.

Table2 Comparison of the modeling results of the two grey Bass models

k	Actual value	grey Bass power model $\tilde{\otimes}_\alpha = 17.7006$		grey Bass model $\tilde{\otimes}_\alpha = 1$	
		$\hat{x}^{(0)}(k)$	$e(k)\%$	$\hat{x}^{(0)}(k)$	$e(k)\%$
2003	22	22.00	0.00	162.26	——
2004	120	119.57	0.36	210.26	75.22
2005	750	644.63	14.05	502.25	33.03
2006	1180	1435.05	21.61	1136.93	3.65
2007	2120	1968.60	7.14	2180.38	2.85
$e_{avg}\%$			8.63		28.69
$p_{avg}\%$			91.37		71.31

Conclusions

This paper expands the traditional grey Bass model to a novel form containing a power exponent. Comparatively, grey Bass power model prevails in the availability of acquiring $\tilde{\otimes}_\alpha$ that most fits the actual data by some techniques. Thus the model can well reflect the feature of raw data with high modeling accuracy.

As is widely known, factors influencing the discharge of new product diffusion are quite complex and changeable. Further development of the grey Bass model with a range of variations should be made that encompasses the influence of different factors.

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References

[1] Bass F M (1969). A New Product Growth Model for Consumer Durables [J]. *Management Science*, 15: 215-227.

[2] Sheng Y, Wu J Z and Zhang Z Q (1999). Study on Market proliferation model of new product [J]. *Foreign economy and manage*, 1:16-20.

[3] Zhang B, Yang G Y and Rong G H (2002). Applications of Diffusion Models for Internet Consumer Analysis [J]. *Chinese Journal of Management Science*, 10(2): 51-56 (in Chinese).

[4] Ling S W and Li N H (2008). Research on the diffusion mode of domestic blog based on BASS model [J]. *Hebei Journal of Industrial Science and Technology*, 25(4): 203-207 (in Chinese).

[5] Dragan J, Rudolf S (1996). The existence of optimal parameter of the generalized logistic function [J]. *Applied Mathematics and Computation*, 77(3): 281-294.

[6] Schmittlein D C and Mahajan V (1982). Maximum Likelihood Estimation for An Innovation Diffusion Models [J]. *Marketing Science*, 1: 57-78.

[7] Putsis W P and Srinivasan V (2000). *Estimation Techniques for Macro Diffusion Models* [M]. Kluwer Academid Publishers, Dordrecht, 264-291.

[8] Wang Z X, Dang Y G and Pei L L (2011). On Greying Bass Model and Its Application [J]. *The Journal of Grey System*, 23(1): 7-14.

[9] Deng J L (2007). *Introduction to Grey Mathematical Resource Science* [M]. HUST Press, Wuhan, China, 82-87 (in Chinese).

[10] Deng J L (2011). Grey Symmetry Balanced System in Impact [J]. *The Journal of Grey System*, 23(1): 1-6.