Preliminary design optimization of profile losses in multi-stage axial compressors based on complex method

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Abstract: This paper illustrates a numerical technique undertaken for preliminary design optimization of profile losses in multi-stage axial compressors. Design process has been carried out based on one-dimensional row by row calculations along compressor mean line. The main objective of the optimization process was to find the best distribution of pressure ratios along compressor stages in order to maximize the overall efficiency, which is itself a non-linear function of governing variables. In this respect, only profile losses, identified as the most dominant ones, are focused on and minimized during the optimization process. Pressure ratios of stages have been taken as design variables. Diffusion factor and De Haller number of each blade row were considered as the main constraints during the optimization process. Numerical optimization was based on a sequential search technique, referred to as complex method. Design process in parallel to the numerical optimization is examined on a ten-stage axial compressor of known general performance data. Final results showed an increase of about 2.7 per cent in the total efficiency relative to its initial value calculated during the preliminary design process.

Keywords: axial compressor, numerical optimization, complex method, diffusion factor, De Haller number

1 INTRODUCTION

Components design of a gas turbine engine, specially its compressor of possible several stages, is a formidable challenge for designers [**1**, **2**]. Because, it usually involves a large number of design parameters which must be optimized for its safe and efficient operation [**3**, **4**]. It is almost impossible for an experienced engineer to find an optimum configuration through the trial and error technique only.

In today's scientific and industrial world, numerical optimization techniques occupy an important position, and enjoy a large spectrum of applications. In general, engineering problems are normally formulated in order to satisfy a certain technically defined requirements with the least expense or the most benefits. When an engineering problem of design is formulated in a mathematical model, it has an objective function to represent the merits, and a number of constraints to be observed and not to be violated during the design process. It is the objective function that is to be maximized or minimized whichever the case maybe, during the iterative optimization process, in a manner not to violate the constraints.

Axial multi-stage compressors are used extensively in the civil and military gas turbine engines [**5**]. In particular, in the aerospace industry utilizing the gas turbine engines as their propulsion system, compressors play the vital rule. Therefore, it is of paramount importance that the efficiency of such compressors to be as high as possible, enhancing the whole performance of the engines.

In the process of designing a multi-stage compressor, a number of limitations and constraints have to be checked, continuously [**6**]. Nonetheless, the designer must constantly be aware of the efficiency parameter in the design process. In this regard, the optimization process can offer invaluable assistance and provide guidelines for the designer as to which way the preliminary design must proceed [**7**].

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In general, design calculations must be performed in two stages; one for the base design, then, followed by the optimization process to provide the necessary guidelines for design modifications. A large amount of information can be passed from one stage to the other in each design cycle. The iterative design procedure continues in this manner until the convergence within an acceptable tolerance is achieved.

Of the effective ways for optimization purposes are those which are somehow linked to the computational fluid dynamics (CFD) operations. In this case, more details of flow properties are taken into account which can be led to ultimate more accurate results. Ghisu *et al.* [**8**] have developed and tested an automatic framework for design optimization of s-shaped ducts and its initial application to the optimization of an axial-symmetric intercompressor duct. They have applied the use of CFD for performance evaluation together with optimization techniques for seeking the best design. Their optimization methods (either deterministic of stochastic) are linked to CFD solvers to provide a more thorough exploration of the design space, trying to produce a better design than the datum one, subject to a number of constraints.

There can also be found other optimization techniques in the literature. Some of these methods are based on the second derivation, which are generally used in the non-linear and non-gradient constraints problems. As a general rule, in solving unconstrained non-linear problems, gradient and second-derivative methods converge faster than direct search methods [**9**]. However, in practice, the derivative-type methods have two main barriers to their implementation. First, in problems with a modestly large number of variables, it is laborious or impossible to provide analytical functions for the derivatives needed in gradient or second-derivative algorithms. Although evaluation of the derivatives by different schemes can be substituted for evaluation of the analytical derivatives, the numerical error introduced particularly in the vicinity of the extremes can impair the use of such substitutions. In any case, search methods do not require regularity and continuity of the objective function and the existence of derivatives. Second, optimization techniques based on the evaluation of the first and possibly second derivatives require a relatively large amount of problems preparation by the user, before he introduces the problems into the algorithm, as compared with search techniques.

Because of the difficulties described above, direct search optimization algorithms have been devised that, although slower to execute for simple problems, in practice, may prove more satisfactory from the user's point than gradient or second-derivative methods.

There are many search methods used in optimization process [**10**], such as the direct search (complex) method [**11–13**] and the flexible polyhedron search (simplex) method [**14–18**]. The complex method is based on a sequential search technique, which has proven to be effective in solving problems with non-linear objective function subjected to non-linear inequality constraints. No derivatives are required and the procedure should tend to find the global maximum due to the fact that the initial set of points are randomly scattered throughout the feasible region. More details about the complex algorithm and its advantages, relative to the other methods, are explained in subsequent sections of this paper.

The simplex method maximizes (or minimizes) a function of n independent variables using $(n + 1)$ vertices of a flexible polyhedron in E^n. Each vertex can be defined by a vector $\boldsymbol{x}$. The vertex (point) in E^n which yields the lowest value of objective function $f(\boldsymbol{x})$ is projected through the centre of gravity (centroid) of the remaining vertices. Improved values of the objective function are found by successively replacing the point with the lowest value of $f(\boldsymbol{x})$ by better points until the maximum of $f(\boldsymbol{x})$ is found. In the search process for a maximum of the objective function $f(\boldsymbol{x})$, trial $\boldsymbol{X}$ vectors can be selected at points in E^n located at the vertices of the simplex, as originally suggested by Spendley *et al.* [**15**]. The objective function can be evaluated at each of the vertices of the simplex, and a projection made from the point yielding the lowest value of the objective function through the centroid of the simplex. This point is deleted and a new simplex, termed a reflection, is formed composed of the remaining old points and the one new point located along the projected line at the proper distance from the centroid. Continuation of this procedure, always deleting the vertex that yields the lowest value of the objective function, plus rules for reducing the size of the simplex and for preventing cycling in the vicinity of the extremity, permit a derivation-free search in which the step size on any stage K is fixed but the direction of search is permitted to change.

Certain practical difficulties in the regular simplex procedure, namely, that it does not provide for acceleration of the search and encounters difficulty in carrying on the search in curving valleys or on curving ridges, have led to several improvements [**9**].

In this research work, each of the simplex and complex optimization methods has been applied to the compressor of interest. The compressor was a ten-stage axial compressor, which has been initially redesigned based on the well-known row by row analysis along the compressor mean line. Analytical processes, based on the above optimization methods, together with the consequent results showed that the complex method could be more effective than the simplex one. On the other hand, with the augmentation of the compressor stages to more than five stages the simplex method showed no effects on the compressor overall efficiency and produced unacceptable results, such as negative efficiencies or efficiencies more than 100 per cent. These inconsistencies are mainly due to the nature of the simplex method, which produce new points based on the last produced points. For example, consider the situation in which the function becomes indefinite. In this case, the algorithm returns to the previous stage for finding the new point. However, while the new point is produced as a function of previous existing points, these points do not change. Thus, the algorithm will produce the same unsuitable resulting point. Since, this algorithm is an unconstrained method, these kinds of failures are inevitable. In this method, it is possible to obtain the out of range values such as negative efficiencies. This may basically be produced, because, this method is a constraint-free algorithm. Thus, when the algorithm finds a direction through which there is a tendency for maximizing the function, it would not check if this maximization will result in values within the acceptable logical domain of the problem or not. This is one of the main differences between the simplex and complex algorithms.

All the above limitations encouraged the authors to use the complex method for their optimization purposes. As has already been mentioned, this technique is a sequential search technique which has proven to be effective in solving problems with non-linear objective functions subjected to non-linear inequality constraints, without any requirement on derivatives.

2 DESIGN PROCEDURE AND LOSS ESTIMATION

In the early stages, aerodynamic design of an axial compressor is performed based on the basic design parameters which would be specified by technical requirements. In general, these would consist of overall pressure ratio, speed of rotation, mass flowrate, input flow Mach number and total temperature and pressure at the inlet [**19**, **20**].

Having the above data, one can find the number of stages needed, which depends mainly on the maximum allowable pressure and temperature ratio for any stage. Usually, the temperature changes in the first and final stages would be less than that of the intermediate stages. When the inlet guide vanes are not used, the flow input to the first stage would be axial and as a result, will come up with less compression input work [**19**]. Flow emerging from the final stage would nearly be axial, and the work required to compress the air flow would be less. Once the number of required stages is decided on, design of every stage can be started. Design process has been carried out based on one-dimensional row by row calculations along the compressor mean line.

Typically a new design starts based on knowledge about the existing compressors of the same class and category. This will provide rough estimations of necessary input data for the initial stages of design process. Input data, in addition to the basic design parameters, which are preassigned, include more detailed ones such as initial load factors for each stage, outlet angle from the stator vanes, solidity, aspect ratio, thickness ratio, and position of its maximum value for each of the stator and rotor blades of that stage. These initial values can be modified in the design cycles, due to considerations of limitations related to the manufacturing technology and gas dynamics characteristics. The output from each design cycle would be the geometry of the air passage, velocity triangles at each of the inlet and outlet cross-sections of every row of blades, pressure ratio of each blade row, diffusion factor (DF), reaction coefficient and efficiency.

At the end of the design cycle, following the row by row analysis along the mean radius, using the radial equilibrium law based on for example the free vortex assumption, provides to calculate all the necessary gas dynamic properties together with the aerofoil geometries along the other radial positions [**1**]. This will yield in an estimation of the three-dimensional geometry of the blades.

During the design process some constraints such as the well-known De Haller number (DH) and DF must be checked to verify that they are in the acceptable ranges. If design goals are not fulfilled, the values of these essential control parameters may help to distinguish the problems, which are basically produced from unsuitable values of the design variables considered at the beginning of the design process.

It should be emphasized that the precise design procedure must take into account the three-dimensionality nature of the flow and also the effects of the fluid viscosity and existence of shock waves. However, as has already been mentioned, this paper is more focused on utilizing one of the effective methods in preliminary aerodynamics design optimization. As a result, effects of tip clearance, end walls and secondary flows especially on the total losses are not included here. It is worth mentioning that in the absence of shock waves, the most dominant loss in the axial compressors is the profile loss. This kind of loss considers the viscous effects including the possible flow separation due to the flow turn angle and the subsequent adverse pressure gradient along the blade surface.

The profile losses and effects of flow compressibility are encountered through the design and optimization steps, utilizing the method described Magdy and Schobeiri [**20**]. They have presented a set of curves as a function of the total loss parameter ($\eta \sin(\alpha_{\text{in}}, \beta_{\text{in}})/2\sigma$) versus modified DF ($D_{\text{m}}$) at different immersion factors (defined as $H_{\text{r}} = [r_{\text{tip}} - r]/[r_{\text{tip}} - r_{\text{hub}}]$). These curves are obtained through extensive experimental works on different kinds of blade rows of various cross-sectional geometries and solidities. The functional form of the above graphs can be stated by equation (1)

$$\frac{\eta \sin(\alpha_{\text{in}}, \beta_{\text{in}})}{2\sigma} = f(D_{\text{m}}, H_{\text{r}}) \tag{1}$$

In the above equation, α_{in}, β_{in} are the absolute and relative entry flow angles to the stator or rotor blades. σ is the blade row solidity and η is the loss coefficient.

The modified DF expression by Magdy and Schobeiri [**20**] includes compressibility effects and, therefore, is capable of handling highly compressible flow such as in transonic compressors. This expression is

$$D_{\text{m}} = 1 - \frac{W_{\text{ex}}}{W_{\text{in}}} + \frac{r_{\text{ex}} W_{\theta,\text{ex}} - r_{\text{in}} W_{\theta,\text{in}}}{\sigma(r_{\text{in}} + r_{\text{ex}}) W_{\text{in}}} \left(\frac{\rho_\infty}{\rho_{\text{in}}}\right) \tag{2a}$$

where the subscripts 'in' and 'ex' refer to the inlet and exit of the blade, respectively.

Using the relations between the velocity components and performing some algebraic operations, equation (2a) can be extended to the stator and rotor blade rows with the final forms of equations (2b) and (2c), respectively

$$(D_{\text{m}})_{\text{stator}} = 1 - \frac{\sin\alpha_1}{\mu \sin\alpha_2} + \frac{\sin^2\alpha_1}{2\sigma}\left(\cot\alpha_1 - \frac{\cot\alpha_2}{\mu\upsilon}\right) \times \left[1 - M_1^2 \left(\frac{\sin\alpha_1}{\mu\sin\alpha_2}\right)\left(\frac{\sin\alpha_1}{\mu\sin\alpha_2} - 1\right)\right] \tag{2b}$$

$$(D_{\text{m}})_{\text{rotor}} = 1 - \frac{\sin\beta_2}{\mu \sin\beta_3} + \frac{\sin^2\beta_2}{2\sigma} \times \left(\frac{1-\upsilon^2}{\mu\upsilon\phi} + \cot\beta_2 - \frac{\cot\beta_3}{\mu\upsilon}\right) \times \left[1 - M_1^2 \left(\frac{\sin\beta_2}{\mu\sin\beta_3}\right)\left(\frac{\sin\beta_2}{\mu\sin\beta_3} - 1\right)\right] \tag{2c}$$

Dimensionless quantities appeared in equations (2b) and (2c) are introduced as

$$\phi = \frac{V_{\text{ax.3}}}{U_3}, \mu = \frac{V_{\text{ax.in}}}{V_{\text{ax.ex}}} \quad \text{and} \quad \upsilon = \frac{U_2}{U_3} \tag{2d}$$

In equations (2b) and (2c), M_1 is the relative flow Mach number at the blade entry section.

3 OBJECTIVE FUNCTION

One of the most important quantities of an axial compressor, like any other device, is its overall efficiency. This parameter is taken herein as the objective function in the optimization process. The design variables, which are taken as the pressure ratios of the stages, must be figured out such that the overall efficiency of the compressor would be a maximum, of course with respecting the constraints. The efficiency for each stage of the compressor is expressed by equation (3), in which, π_{ck} is the pressure ratio of the kth stage and τ_{ck} is its total temperature ratio.

$$\eta_{ck} = \frac{\pi_{ck}^{(\gamma-1)/\gamma} - 1}{\tau_{ck} - 1} = \frac{\pi_{ck}^{(\gamma-1)/\gamma} - 1}{\pi_{ck}^{(\gamma-1)/\gamma\times\text{ec}} - 1} \tag{3}$$

In the above equation, γ is the ratio of the specific heats of the gas and 'ec' is the stage polytropic efficiency.

In the case of a multi-stage compressor, the overall efficiency (η_{c}) can be written similar to equation (3) as follows

$$\eta_{\text{c}} = \frac{\pi_{\text{c}}^{(\gamma-1)/\gamma} - 1}{\tau_{\text{c}} - 1} \tag{4}$$

where π_{c} and τ_{c} are the overall pressure and temperature ratios, respectively. The latter is the product of the temperature ratios of all the stages as shown by equation (5)

$$\tau_{\text{c}} = \prod_{k=1}^{N_{\text{St}}} \tau_{ck} \tag{5}$$

where N_{St} represents the number of the compressor stages. Using equation (3), equation (5) can be rewritten as

$$\tau_{\text{c}} = \prod_{k=1}^{N_{\text{St}}} \left\{ \frac{1}{\eta_{ck}} \left[\pi_{ck}^{(\gamma-1)/\gamma} - 1\right] + 1 \right\} \tag{6}$$

Substituting equation (6) in equation (4), compressor efficiency can be obtained as follows

$$\eta_{\text{c}} = \frac{\pi_{ck}^{(\gamma-1)/\gamma} - 1}{\prod_{k=1}^{N_{\text{St}}} \left\{ \frac{1}{\eta_{ck}} \left[\pi_{ck}^{(\gamma-1)/\gamma} - 1\right] + 1 \right\} - 1} \tag{7}$$

Replacing the stage efficiency from equation (3) into equation (7) one can obtain

$$\eta_{\text{c}} = \frac{\pi_{ck}^{(\gamma-1)/\gamma} - 1}{\prod_{k=1}^{N_{\text{St}}} \left\{ \frac{\pi_{ck}^{(\gamma-1)/\gamma\times\text{ec}} - 1}{\pi_{ck}^{(\gamma-1)/\gamma} - 1} \left[\pi_{ck}^{(\gamma-1)/\gamma} - 1\right] + 1 \right\} - 1} \tag{8}$$

Equation (8) represents the objective function, in which π_{ck} appear as variables. As is seen from this algebraic expression, the function is non-linear in terms of the variables and would require the use of any of the suitable non-linear optimization algorithms in the literature.

4 DESIGN CONSTRAINTS

In engineering optimization problems, constraint functions are often expressed analytically in explicit algebraic forms, which may be linear or non-linear. However, there are situations when such relationships do not have an explicit analytical form or their algebraic expressions are too involved and intractable (implicit constraints). In such cases numerical algorithms must be employed in order to get computational means of evaluation of their values given in certain magnitudes of design variables.

In the optimization problem studied in this article there are two explicit and three implicit constraints. The explicit constraints are minimum and maximum pressure ratios for each stage. According to the experimental observations the pressure ratios greater than about 1.6 may lead to flow separation and considerable losses in common rotor blade rows. It may also lead to some mechanical failure due to the strength of materials used in constructing compressor blades. Consequently, in the present study, the stage pressure ratio should not exceed 1.6. In addition, it should always stay higher than about 1.1. These two latter limits outline the explicit constraints in the present study.

The first implicit constraint is the modified DF of each stage, which is itself a function of velocity triangles and consequently of the stage pressure ratio. The equations used in this study for calculating the modified DF are already introduced in equations (2a) to (2c).

In the early state of design, modified DF can serve as a criterion for the flow failure. Experiments performed on a large variety of cascades, have shown that those cases having modified DFs greater than 0.5 are associated with considerable separated flows [**6**]. According to the above considerations, in the present study, $D_{\mathrm{m}} < 0.48$ is considered as the first implicit constraint. The usual DF, in parallel to its modified version, is also calculated and used as an important constraint through the optimization process. The DF is defined in equation (9) as follows

$$\mathrm{DF} = 1 - \frac{W_{\mathrm{ex}}}{W_{\mathrm{in}}} + \frac{W_{\theta,\mathrm{in}} - W_{\theta,\mathrm{ex}}}{2\sigma W_{\mathrm{in}}} \quad (9)$$

The second implicit constraint is the compressor total pressure ratio, which must remain unchanged before and after the optimization process.

The last implicit constraint is the well-known DH, which is defined in equation (10)

$$\mathrm{DH} = \frac{W_{\mathrm{ex}}}{W_{\mathrm{in}}} \quad (10)$$

According to the De Haller criterion, the velocity ratio $W_{\mathrm{ex}}/W_{\mathrm{in}}$ should be kept always greater than 0.72 in order to avoid flow separation.

5 OUTLINES OF THE COMPLEX METHOD

The optimization algorithm adopted here is based on the complex method after Box [**9**], which has been proved to be a useful procedure in many engineering optimization problems. This method is based on a sequential search technique which has proven to be effective in solving problems with non-linear objective function subjected to non-linear inequality constraints. No derivatives are required and the procedure should tend to find the global maximum due to the fact that the initial set of points are randomly scattered throughout the feasible region. The complex algorithm proceeds as follows.

1. An original complex of $K \geqslant N + 1$ points are generated consisting of a feasible starting point and $K - 1$ additional points generated from random numbers and constraints for each of the independent variables. It can be written

$$X_{i,j} = G_i + R_{i,j}(H_i - G_i), \quad i = 1, 2, \ldots, N \quad \text{and} \quad j = 1, 2, \ldots, K - 1 \quad (11)$$

   where $X_{i,j}$ are optimization variables, $R_{i,j}$ are random numbers between 0 and 1, and H and G are the upper and lower constraints, respectively.
2. The selected points must satisfy both the explicit and implicit constraints. If at any time the explicit constraints are violated, the point is moved a small distance δ inside the violated limit. If an implicit constraint is violated, the point is moved one half of the distance to the centroid of the remaining points. This can be expressed mathematically as follows

$$X_{i,j}(\mathrm{new}) = \frac{X_{i,j}(\mathrm{old}) + \bar{X}_{i,\mathrm{cn}}}{2}, \quad i = 1, 2, \ldots, N \quad (12)$$

   where the coordinates of the centroid of the remaining points, i.e. $\bar{X}_{i,\mathrm{cn}}$ are defined by

$$\bar{X}_{i,\mathrm{cn}} = \frac{1}{K-1}\left[\sum_{j=1}^{K} X_{i,j} - X_{i,j}(\mathrm{old})\right], \quad i = 1, 2, \ldots, N \quad (13)$$

   This process is repeated as necessary until all the implicit constraints are satisfied.
3. The objective function is evaluated at each point. The point having the lowest function value is

replaced by a point which is located at a distance λ times as far from the centroid of the remaining points as the distance of the rejected point on the line joining the rejected point and the centroid. This process can be expressed mathematically as follows

$$X_{i,j}(\text{new}) = \lambda(\bar{X}_{i,\text{cn}} - X_{i,j}(\text{old})) + \bar{X}_{i,\text{cn}}, \quad i = 1, 2, \ldots, N \tag{14}$$

4. If a point repeats in giving the lowest function value on consecutive trials, it is moved one half the distances to the centroid of the remaining points.
5. The new point is checked against the constraints and is adjusted as before if the constraints are violated.
6. Convergence is assumed when the objective function values at each set of point are within ε units for previously determined number of consecutive iterations. Iteration is defined as the calculation required selecting a new point which satisfies the constraints and does not repeat in yielding the lowest function value.

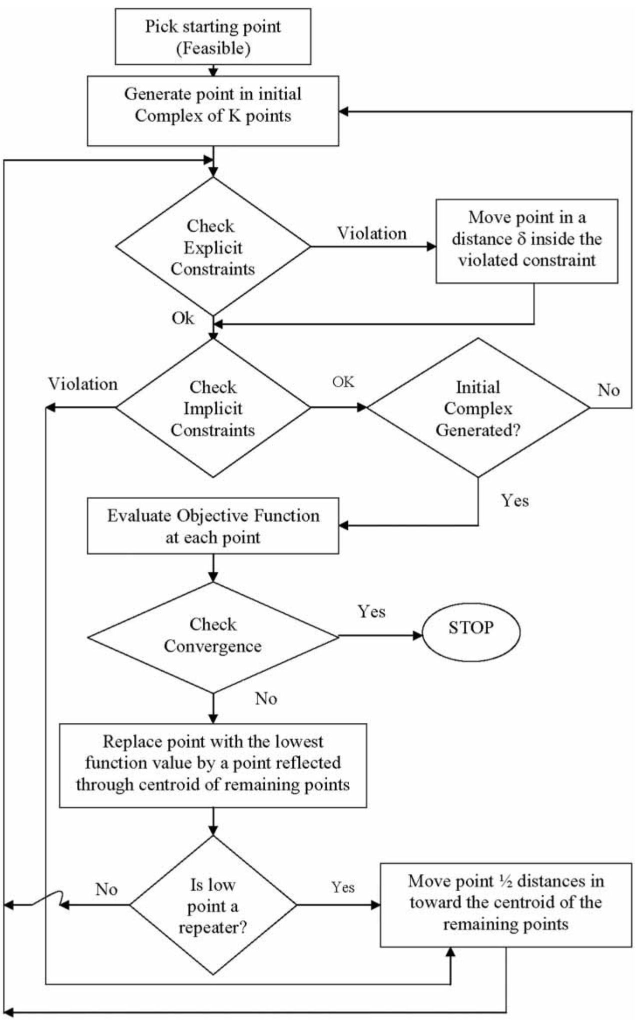

Fig. 1 Algorithm of numerical optimization based on complex method

The optimization algorithm is summarized and shown in Fig. 1.

6 OPTIMIZATION PROCESS

1. The zero stage of optimization process commences with exporting the preliminary design data to the optimization module of computerized code and calculation of the objective function, i.e. the overall efficiency of the compressor which is already introduced by equation (8).
2. The initial stage of the optimization starts with generating of random points in the feasible region of the design domain. Number of these points is two times the variables of the proposed objective function. As has already been mentioned, the pressure ratio of each stage is considered as the optimization variable.
3. These points are checked to be within the logical ranges that are put as the pressure limits (explicit constraints) for each compressor stage.
4. After finishing with the validation of explicit constraints, the pressure distribution along the proposed compressor can be established. Then, the velocity triangle of each stage of the compressor is calculated at the blade entry and exit regions along the mean-line. Absolute and relative flow angles (shown by symbols α and β, respectively) are also calculated at this stage. Thus, having the new values of α, β, and velocity components one can calculate the new D_m, DF, and DH for each blade row using equations (2), (9), and (10), respectively.
5. Calculating the D_m, DF, and DH for each blade row, the optimization algorithm check to see whether they violate their limits, considered by designer in the optimization part of software as implicit constraints, or not. If this check was OK the algorithm goes on and checks the next stage for the values of constraint parameters, and if, it was a violation the optimization algorithm moves the pressure ratio of that stage 0.5 distance in towards the centroid of the remaining points, and then calculates the efficiency of that stage and then goes to the next stage.
6. After verifying implicit constraints for all stages and calculating the total loss parameter, the objective function (compressor overall efficiency) is evaluated to see whether it becomes better or worse. If it is better, the convergence criteria will be checked. If convergence criteria had achieved, the algorithm is finished. Otherwise, after moving points (pressure ratios) towards the centre of the points of which the overall efficiency has its highest value, all above procedures will be repeated.

7 ADVANTAGES OF THE COMPLEX METHOD

Advantages of the complex method in optimization problems can be categorized as follows:

(a) optimization of non-linear functions;
(b) considering linear and non-linear constrains;
(c) considering implicit and explicit constrains;
(d) point generation of two times the variables in feasible region;
(e) ability of preventing the repetition of inconvenient points;
(f) ability of optimizing function with no defined number of variables (which, here means the capability to optimize compressors from one to infinite number of stages);
(g) ability to define constraints for each stage, separately;
(h) no requirement on derivatives.

As can be easily understood the forth advantage of this method is very important and useful because, in optimization process there is always the risk of losing data during the optimization process. In most of the optimization algorithms there are produced only that number of points or some times one or two points more than what is needed to cover the feasible region of interest. Therefore, if in the optimization process one or two of these points get lost, which usually occurs, one or even more dimensions of the searching space will be lost and this will lead the process to be unsuccessful. However, in the present method, two times the required points are produced. This means that if some points are lost algorithm can go on. Since the complex method check each of the new generated points to prevent the repetition, one will never face with an obstacle during the optimization process and can easily escape the sharp points of the function that is intended to optimize it.

8 RESULTS

The example chosen, to demonstrate the capability of the complex method in the present optimization process, is a ten-stage existing compressor introduced in references [**21**] and [**22**]. General performance data for this compressor were used to redesign the compressor and obtain more data needed for the next step. Then, the computerized code, developed for the optimization purpose based on the complex method, was executed.

In Fig. 2 distribution of pressure ratio before and after optimization process is plotted for each stage. As can be seen in this figure, the pressure ratio for the beginning stages have been more than 1.6 before optimization. This may cause many problems in compressor performance and the strength of the blades. After optimization process, the highest pressure ratio of 1.51 is obtained, which is acceptable and appropriate for practical purposes.

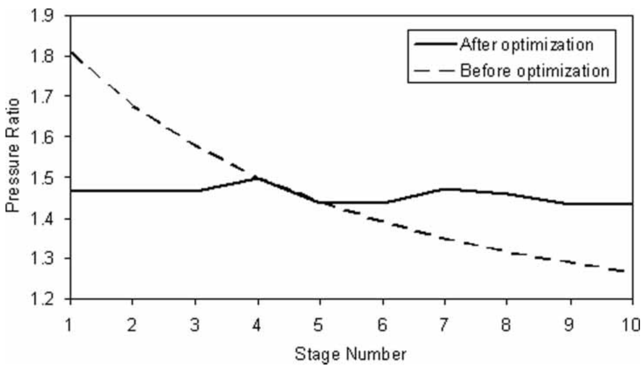

Fig. 2 Pressure ratios of the compressor stages

Distribution of modified DFs of the rotor blade rows in the base design is as shown in Fig. 3(a). In the same figure, the results after optimization are also plotted. As can be seen in this figure the rotor D_m values of eight stages of compressor before optimization is more than 0.5 which means great losses due to the flow separation. After optimization, the maximum value of D_m is less than 0.48 which verifies the power of the complex method in taking into the account the effect of implicit constraint during the optimization process. Ordinary DF results for the rotor blade rows are also shown in Fig. 3(b). Initial DF is increasing by moving from the compressor head towards its end. After stage three, DF has exceeded its upper limit, indicating unacceptable final results obtained through initial design process. As can be detected from Fig. 3(b), optimization technique, undertaken in this research work, has produced acceptable results. Although DF is increasing along the compressor axis, but it is always kept lower than the upper limit of 0.5.

Figure 4 shows the rotor DH distribution before and after optimization along the stages. This figure shows for the other time the capability of the complex method in taking into account the implicit constraints. As it is obvious in this figure, the DH number after optimization is above 0.72 for each row, which is satisfactory.

Figure 5 shows the stage efficiency before and after optimization in which one can clearly observe the augmentation of efficiency for each stage after the optimization process (pay attention to the different scales of the y-axis in this figure).

Figure 6 demonstrates the variation of compressor overall efficiency as the optimization procedure is going on. It can be observed in this figure that there are some points with the overall efficiency higher than the final state. However, it should be noted that there are some constraints which should be considered throughout the optimization process, such as D_m

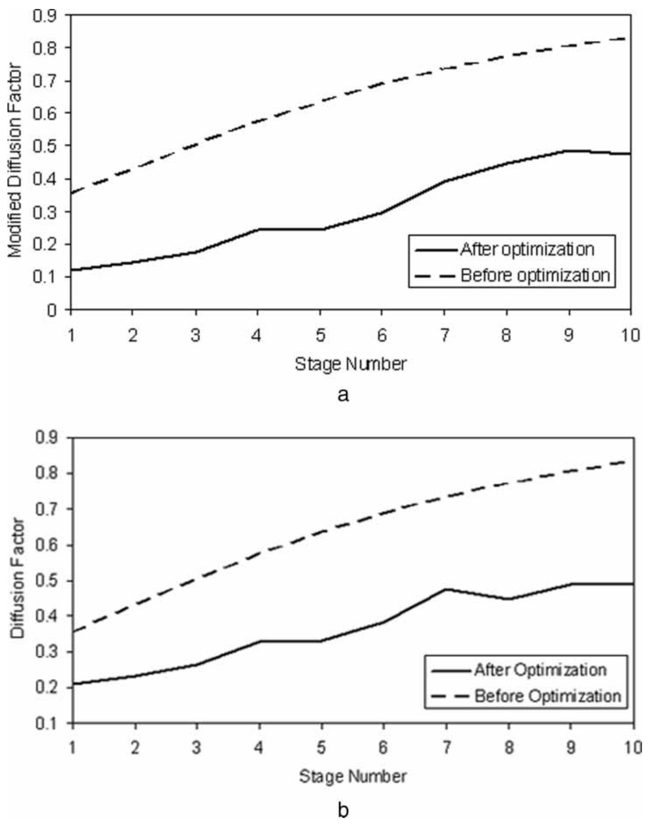

Fig. 3 (a) Modified DFs of the rotor blade rows of the compressor stages and (b) DFs of the rotor blade rows of the compressor stages

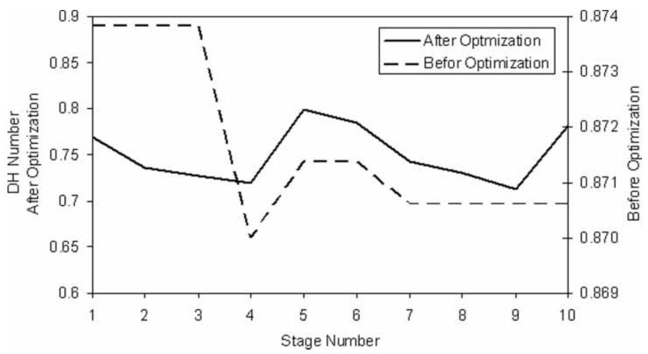

Fig. 4 DH of the rotor blade rows of the compressor stages

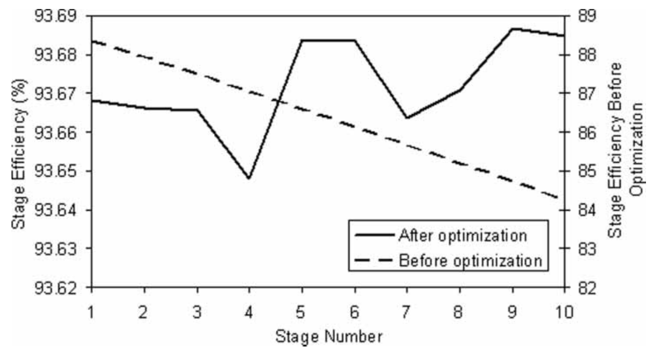

Fig. 5 Efficiencies of the compressor stages

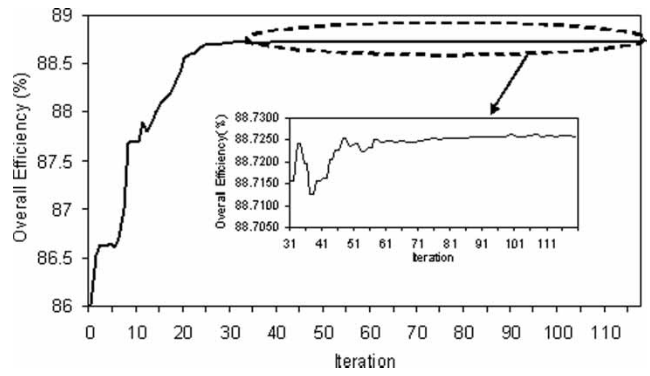

Fig. 6 Overall efficiency convergence

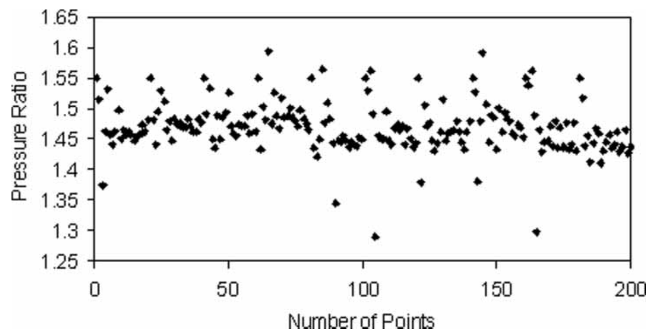

Fig. 7 Distribution of pressure ratios produced by the complex method at the initial stage of optimization process

or maximum pressure ratio that is acceptable for each stage. Thus, even if the efficiency is higher than the final value in some cases, but, because of violation of optimization constraints these values are not acceptable. Finally, as the changes in the compressor overall efficiency becomes lower than a given value it can be understood that the optimization process is arriving at the best point and the process is finished.

As is shown in Fig. 6, the total efficiency in the base design has been computed to be equal to 86 per cent, which after modification based on the present method, it is raised to 88.7 per cent. Therefore, using the present optimization method, the overall efficiency of the base design has been improved by 2.7 per cent. It should be noted that constant polytropic efficiency of 92 per cent has been assumed for all the stages throughout the calculations.

In Fig. 7, the distribution of the points (pressure ratio for each stage) produced in the feasible region by the complex method is demonstrated. These points are the random points which are produced at the second stage of optimization process. As can be seen in this figure, these points have not any organization and are distributed in the limits of 1.1 and 1.6 which are the explicit constraints of the present optimization algorithm.

After termination of the optimization process, these points have converged into different groups, as shown in Fig. 8. These groups are representative of the pressure ratio of each stage and clearly demonstrate and verify the convergence of the complex method. As it can be understood from these two figures, the optimization process has produced 200 points, which are equal to two times the total optimization variables for each stage (20 points for each stage, or totally 200 points for ten stages of the studied compressor). The points in Fig. 8, which are fallen apart from these ten distinguished groups, are the points which are wasted during the optimization process. This is one of the greatest advantages of complex method which produces two times the necessary points at the first stage of optimization. This characteristic gives the ability to the complex method to cover all the dimensions of space even after wasting many points as the optimization process goes on.

Figures 9 to 14 show the variations of different parameters as the optimization process is in progress. Because of the large number of stages only the results of odd stages are presented in the above figures. The convergence procedure for each parameter is obvious, showing the success of the complex method through the optimization process.

Of the important parameters during the design process can be referred to the absolute and relative Mach numbers values at the blade entry section, especially for the beginning stages of the compressor. Entry Mach numbers to the blade rows are usually kept below about 0.7 by designers. Otherwise, associated shock

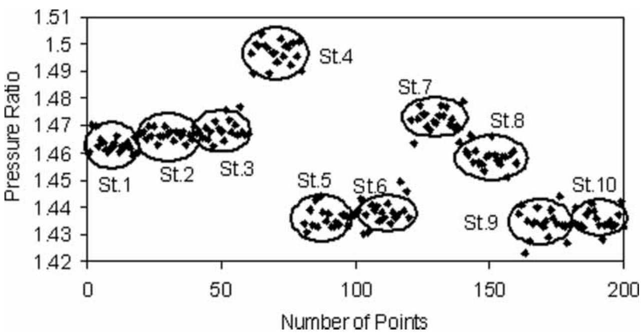

Fig. 8 Distribution of pressure ratios produced by the complex method at the last stage of optimization process

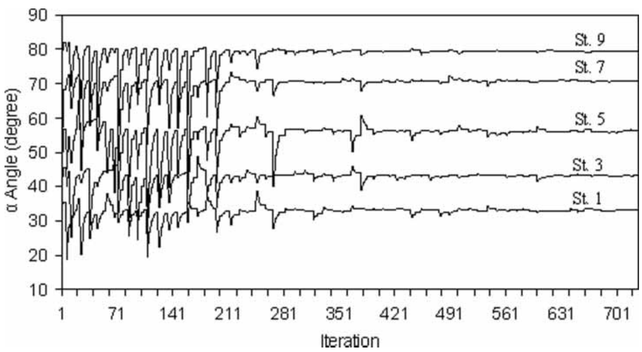

Fig. 9 Variations of stage absolute entry flow angle as the optimization is in progress

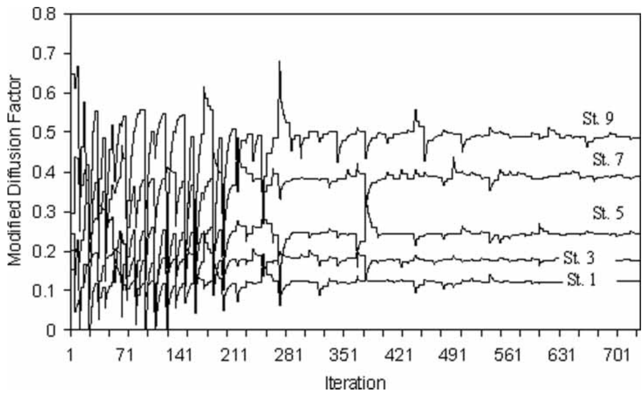

Fig. 10 Variations of modified DF as the optimization is in progress

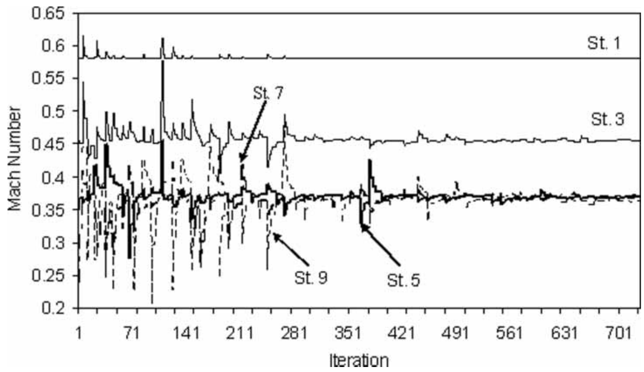

Fig. 11 Variations of stage entry Mach number as the optimization is in progress

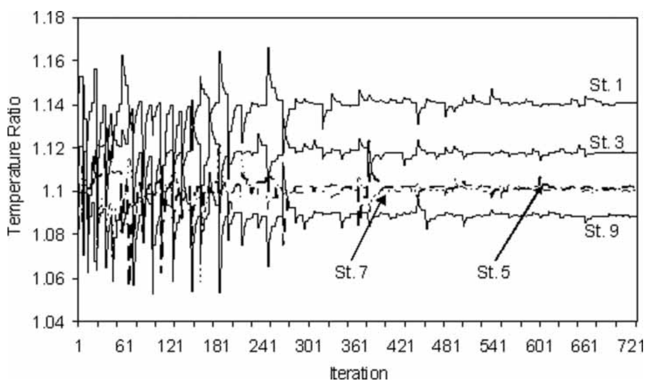

Fig. 12 Variations of stage temperature ratio as the optimization is in progress

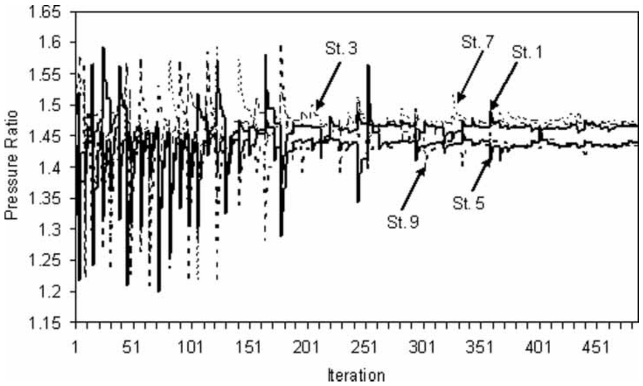

Fig. 13 Variations of stage pressure ratio as the optimization is in progress

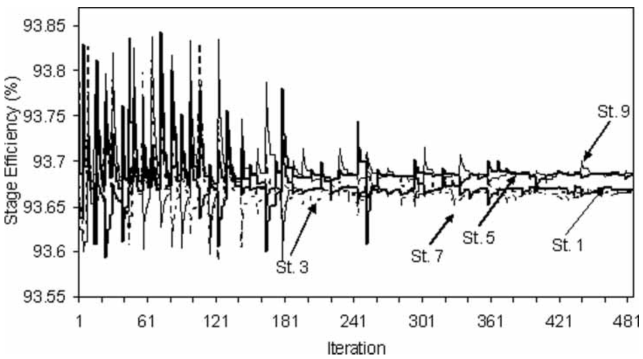

Fig. 14 Variations of stage efficiency as optimization algorithm is in progress

waves at the entry and within the blades passages could cause strong losses and reduce the compressor overall efficiency. Nevertheless, it should be noted that the above limitation does not force all designers to keep away from considering entry Mach numbers slightly greater than 0.7. However, as can be detected from Fig. 11, the Mach number has always been kept below 0.6 for all stages as the optimization has been on progress.

9 CONCLUSIONS

Considering what discussed in this article and the results obtained, one can conclude that the complex method is a suitable method for optimizing engineering problems with non-linear objective functions and implicit and explicit non-linear constraints. In this method, existence of non-continuities or sharp edges in the objective function is not a barrier on the optimization process. It also covers all the dimensions of the space even after wasting many points during the optimization process. Final results clearly demonstrate the convergence speed of the complex method. After a relatively short number of iterations the complex algorithm finds out the best ways of optimizing the variables and the oscillation amplitude of these variables decrease rapidly.

After finishing with preliminary design of a ten-stage compressor, applying the proposed numerical optimization technique provided to calculate the optimum distribution of pressure ratios of the stages in order to maximize the compressor overall efficiency. Final results showed an increase of about 2.7 per cent in the total efficiency relative to its initial value, calculated during the preliminary design process.

REFERENCES

1 **Mattingly, J. D.** *Elements of gas turbine propulsion*, 1996 (McGraw-Hill, New York).

2 **Cohen, H., Rogers, G. F. C.,** and **Saravanamuttoo, H. I. H.** *Gas turbine theory*, 3rd edition, 1987 (Longman, London).

3 **Beknev, V. S., Egorov, I. N.,** and **Talyzina, V. S.** Multicriterial design optimization of the multistage axial flow compressor. In the 5th ASME, 'COGEN-TURBO-V', Budapest, 1991.

4 **Egorov, I. N.** Optimization of a multistage axial compressor in a gas-turbine engine system. ASME 92-GT-424, 1992.

5 **Hill, P. D.** and **Peterson, C. R.** *Mechanics & thermodynamics of propulsion*, 2nd edition, 1992 (Addison Wesley, Reading, Massachusetts).

6 **Gresh, T. M.** *Compressor performance: aerodynamics for the user*, 2001 (Butterworth Heinemann, Boston).

7 **Keskin, A.** and **Bestle, D.** Application of multi-objective optimization to axial compressor preliminary design. *Aerosp. Sci. Technol.*, 2006, **10**(7), 581–589.

8 **Ghisu, T., Molinari, M., Parks, G. T., Dawes, W. N., Jarrett, J.,** and **Clarkson, P. J.** Axial compressor intermediate duct design & optimization. In the 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Sheraton Waikiki, Honolulu, Hawaii, 23–26 April 2007, AIAA paper 2007-1868.

9 **Box, M. J.** A new method for constrained optimization and a comparison with other methods. *Comput. J.*, 1965, **8**(1), 42–52.

10 **Powell, M. J. D.** An efficient method for finding the minimum of a function of several variables without calculating derivatives. *Comput. J.*, 1964, **7**(2), 155–162.

11 **Wood, C. F.** Application of direct search to the solution of engineering problems. Westinghouse Research Laboratory, Science paper 6-41210-1-P1, 1960.

12 **Hooke, R.** and **Jeeves, T. A.** Direct search solution of numerical and statistical problems. *J. Assoc. Comput. Math*, 1961, **8**(2), 212–229.

13 **Kuster, J. L.** and **Mize, J. H.** *Optimization technique with FORTRAN*, 1972 (McGraw Hill, New York).

14 **Nedler, J. A.** and **Mead, R. A.** Simplex method for function minimization. *Comput. J.*, 1964, **7**(4), 308–313.

15 **Spendley, W., Hext, G. R.,** and **Himsworth, F. R.** Sequential application of simplex designs in optimization and evolutionary operations. *Technometrics*, 1962, **4**(4), 441–461.

16 **Campey, I. G.** and **Nickols, D. G.** *Simplex minimization*, 1961 (Imperial Chemical Industries, Ltd, UK).
17 **McKinnon, K. I. M.** Convergence of the Nelder–Mead simplex method to a non-stationary point. *SIAM J. Optim.*, 1999, **9**(1), 148–158.
18 **Rykov, A.** Simplex algorithms for unconstrained optimization. *Probl. Control Inf. Theory*, 1983, **12**(4), 195–208.
19 **Cumpsty, N. A.** *Compressor aerodynamics*, 1st edition, 1989 (Longman, USA).
20 **Magdy, S. A.** and **Schobeiri, M. T.** A new method for the prediction of compressor performance maps using one-dimensional row-by-row analysis. In the International Gas Turbine and Aero-Engine Congress and Exposition, Houston, Texas, 1995, ASME paper 95-GT-434.
21 **Saravanamuttoo, H., Rogers, G.,** and **Cohen, H.** *Gas-turbine theory*, 5th edition, 2001 (Prentice Hall, UK).
22 **Downing, M. R.** and **Finger, H. B.** *Performance of the 10-stage axial-flow compressor*, 1946 (Bureau of Aeronautics, Aircraft Research Laboratory, Cleveland, Ohio).

APPENDIX

Notation

D_m	modified DF
ec	stage polytropic efficiency
G	lower constraints in numerical optimization
H	upper constraints in numerical optimization
M	Mach number at blade entry
r	radial position
$R_{i,j}$	random numbers between 0 and 1
U	rotor blade rotational velocity
V	absolute velocity
W	relative velocity
$X_{i,j}$	optimization variables
α	absolute flow angle
β	relative flow angle
η	overall efficiency
λ	reflection factor defined in complex method
π	total pressure ratio
σ	cascade solidity
τ	total temperature ratio
ϕ	flow coefficient

Subscripts

ax	axial
c	compressor
cn	centroid
k	number of stage
θ	circumferential direction
∞	free stream inside the blade channel
1	stator inlet station number
2	rotor inlet (stator outlet) station number
3	rotor outlet station number