South African Mathematical Society
45$^{\text{th}}$ Annual Congress

6–8 November 2002
University of Stellenbosch

Contents

Abstracts

Prepared by Jan Groenewald

THE FISCHER-CLIFFORD MATRICES OF A MAXIMAL SUBGROUP OF Fi'_{24}

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The Fischer group Fi'_{24} is the largest sporadic simple Fischer group of order

$$1255205709190661721292800 = 2^{21}.3^{16}.5^{2}.7^{3}.11.13.17.23.29 \ .$$

The group Fi'_{24} is the derived subgroup of the Fischer's 3- transposition group Fi_{24} discovered by Bernd Fischer. There are five classes of elements of order 3 in Fi'_{24} as represented in the ATLAS by $3A$, $3B$, $3C$, $3D$ and $3E$. A subgroup of Fi'_{24} of order 3 is called of type $3X$, where $X \in \{A, B, C, D, E\}$, if it is generated by an element in the class $3X$. There are six classes of maximal 3-local subgroups of Fi'_{24} as determined by Wilson. In this paper we determine the Fischer-Clifford matrices and conjugacy classes of one of these maximal 3-local subgroups $\bar{G} := N_{Fi'_{24}}(<N>) \cong 3^7{\cdot}O_7(3)$, where $N \cong 3^7$ is the natural orthogonal module for $\bar{G}/N \cong O_7(3)$ with 364 subgroups of type $3B$ corresponding to the totally isotropic points. The group $\bar{G}$ is a non- split extension of N by $G \cong O_7(3)$.

SAMS subject classification: 2

SOLVING NONLINEAR PDE'S IN HAUSDORFF CONTINUOUS FUNCTIONS

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The order completion method for solving nonlinear partial differential equations is based on the order completion of spaces of usual piecewise smooth functions defined on a set $\Omega \subset \mathbb{R}^n$ [2]. The application of Hausdorff continuous interval-valued functions is motivated by their exceptional property of being a Dedekind order complete set. In fact, the Dedekind order completion of $C(X)$, where X is a compact topological space, is exactly the set of all Hausdorff continuous functions defined on X [1]. We construct a space of extended interval-valued Hausdorff continuous functions defined on Ω which is order complete and contains the sets of piecewise smooth functions used in the order completion method. Thus, we consider the solutions by the order completion method of nonlinear PDEs in this space of Hausdorff continuous functions.

SAMS Subject Classification: 11

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ON THE NORMAL COMPLETION OF A BOOLEAN ALGEBRA

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An account of the properties of the familiar classical construction which assigns to each Boolean algebra A the complete Boolean algebra $\mathcal{N}A$ of its normal ideals, considered at the general level of an arbitrary topos, clarifying which aspects of $\mathcal{N}A$ are constructively valid. This is joint work with M. Mehdi Ebrahimi and Mojgan Mahmoudi of Shahid Beheshti University, Tehran, Iran.

SAMS Classification Numbers: 2, 5

MOLECULE FRAGMENTATION MODELS WITH MASS LOSS — REGULAR AND "SHATTERING" SOLUTIONS

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Fragmentation results from a variety of physical processes including erosion, polymer degradation, grinding, oxidation and dissolution. In some cases fragmentation occurs without mass-loss, that is, the mass of molecules resulting from the fragmentation equals the original mass of the ensemble. However, in many situation the process involves continuous mass loss due to dissolution of the porous surface of molecules leading to fragmentation due to breaking final bridges between different parts of the same particle. It can also involve discrete mass loss typified by chemical reactions of an explosive phase embedded in a much slower-reacting phase of the molecule, that breaks the molecule.

General fragmentation process is described by the integro-partial differential equation

$$\partial_t u(x,t) = -a(x)u(x,t) + \int_x^\infty a(y)b(x|y)u(y,t)dy + \partial_x[r(x)u(x,t)]. \tag{1}$$

This equation involves a particle mass distribution function u, a fragmentation rate a, a continuous mass loss rate r, and a non-negative measurable function b, where $b(x|y)$ describes the distribution of particle masses x spawned by the fragmentation of a particle of mass y. The continuous mass loss rate r is defined so that $r(m(t)) = -dm/dt$ for a particle of time-dependent mass $m(t)$. The balance of mass requires that b satisfies the equation

$$\int_0^y xb(x|y)dx = y - \lambda(y)y, \tag{2}$$

where $0 \leq \lambda \leq 1$ refers to the discrete mass loss. If there is no mass-loss in the model, both coefficients r and λ equal zero.

We analyse this problem by means of the semigroup theory in the space $L_1([0,\infty[, xdx)$ that is physically relevant for the model as the norm of u in this space gives the total mass of the ensemble.

Apart from standard solvability results, we identify regimes where the solution returns the conservation laws built into the model as well as regimes where there occurs an unexpected mass loss. In physical literature the latter is referred to as *shattering* and is attributed to the phase transition and creation of zero-size particles called a *fractal dust*.

SAMS subject classification: 11, 13, 17

GRAVITATIONAL COLLAPSE IN A 5-DIMESNIONAL SPACE-TIME

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We investigate the end state of gravitational collapse of an inhomogeneous dust cloud described by a self-similar 5-dimensional Tolman-Bondi spacetime. Both naked singularities and black holes are obtained. The degree of inhomogeneity of the collapsing matter necessary to form a naked singularity is given. Implications of the results in the context of the cosmic censorship conjecture are discussed.

SAMS subject classification: 13, 17

A NUMERICAL RELATIVITY CODE FOR EVOLVING A NEUTRON STAR NEAR A MASSIVE BLACK HOLE

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We present a numerical relativity code that evolves a particle (of mass m) near a black hole (of mass M). The particle's position is evolved by the geodesic equation; and the particle's mass acts as source for the gravitational field, which is evolved using the full Einstein equations by means of a characteristic code. This procedure is valid for $m \ll M$, which applies in astrophysically interesting cases such as a neutron star near a massive black hole.

SAMS subject classification: 13, 16

THE ROLE OF READING IN LEARNING MATHEMATICS

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Based on the results of research undertaken in 2000 and 2001 into the reading skills of foundation-phase mathematics students, a reading intervention programme was set up to determine whether explicit attention given to reading would develop student's reading skills, and enable them to adopt more effective reading strategies, so that they could become more adept at "reading to learn" their mathematics guides. This presentation reports on the specific reading problems identified during the first phase of the investigation, and on the intensive and extensive reading activities undertaken in the second phase (the intervention programme) and the implications of these results for studying mathematics.

SAMS subject classification: 18

ATOMS IN THE LATTICE OF MATRIC-EXTENSIBLE RADICALS

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Work done in conjunction with Prof H. France-Jackson.

SAMS subject classification: 2

ON CONNECTIONS BETWEEN SYSTEMS AND CONTROL THEORY AND INTEREST RATE MODLES

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A recent advance in financial mathematics has been the development of a geometric approach to interest rate theory ([1]). One of the advantages of this framework is that it facilitates the establishment of connections between the modelling of interest rates and systems and control theory. In this talk, we contribute to this growing debate by exposing relationships between system theoretic concepts such as controllability ([2] and [5]), observability ([2] and [6]) and realizability ([3] and [4]) and interest rate models of Heath-Jarrow-Morton-Musiela (HJMM) type ([7] and [8]), where the forward rates are driven by a multi-dimensional Wiener process. In this regard, we establish (necessary and sufficient) conditions that have to be satisfied in order for such models to be controllable and/or observable. Furthermore, we will demonstrate that the abstract state space associated with these HJMM interest rate models has meaning in an economic sense.

Key words: Controllability, Observability, Realizability; Interest Rate Models.
SAMS Subject Classification: 10 and 12.

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[8] Marek Musiela, Stochastic PDE's and Term Structure Models, Mathematical Finance **4** (1994), 563-576.

PROBLEM SOLVING SKILLS DEVELOPMENT IN THE EASTERN CAPE—A CASE STUDY

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In this talk the presenter will highlight the initiatives taken within the Mathematics and Applied Mathematics Department at UPE to (1) improve the mathematical problem solving performance of educators and (2) addressing the need for suitably qualified mathematics educators at FET level.

SAMS subject classification: 18

BAILEY'S LEMMA, PAIRS AND CHAINS

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In 1944, W. Bailey made a powerful remark known as Bailey's transform, followed by the famous Bailey lemma. In this talk, we will look at pairs of infinite sequences (α_n, β_n) called Bailey pairs. These pairs can, with Bailey's lemma, be used to produce identities of Rogers-Ramanujan type. I will then introduce the inversion formula which expresses β_n in terms of α_n. This formula will be used to produce implied identities. We will also "walk" along a Bailey chain, which is an iterative process to produce new Bailey pairs, this idea was originated by G. E. Andrews in 1984. Briefly, I will discuss Bailey lattices and Well-Poised Bailey pairs.

SAMS subject classification: 3

THE STRUCTURE OF RANK ONE ELEMENTS IN A BANACH ALGEBRA

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For a complex, unital Banach algebra A with invertible group A^{-1}, it is well known that the connected components of A^{-1} are exactly the cosets of the index group $A^{-1}/Exp(A)$, where $Exp(A)$ is the subgroup of A^{-1} consisting of generalised exponentials. With this in mind we investigate the component structure of the multiplicative semi-group, $\Im_1$, of rank one elements for semi-prime A. We then characterise the central rank one elements in terms of the components that contain them.

SAMS subject classification: 9

A THREE-VALUED SEMANTIC REPRESENTATION OF MENTAL MODEL THEORY

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Mental model theory explains the psychology of reasoning in terms of a semantic procedure of partial model construction. It presents an alternative to the natural deduction theory of rule-based reasoning systems in cognitive psychology. Its flaws from a formal logical perspective are well-documented, and it has remained to a large extent isolated from the influence of semantic reasoning systems developed in the context of artificial intelligence. Instead, it relies on classical logic, which is augmented with an informal, and varying, description of partial model construction.

We argue that mental model theory will benefit greatly from a more appropriate and rigorous semantic treatment that draws from experience gained in logics of artificial intelligence. We show that a three-valued logic of partial information provides a more suitable semantic framework for its analysis than classical logic.

SAMS subject classification: 1

ON THE ROLE OF MEASURABILITY IN LEBESQUE INTEGRATION

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In the classical approach to the Lebesgue integral one considers measurable functions over measurable sets. The talk will discuss the definition of a Riemann type generalisation of the Lebesgue integral for functions which are not necessarily measurable, over sets which need not be measurable. It will be proved that the two integrals coincide when the function to be integrated is measurable and the domain of integration is a measurable set.

SAMS subject classification: 8

NATURAL EXTENSIONS OF THE TOPOLOGICAL T_0 SPACES VIA FUNCTORIAL QUASI-UNIFORMITIES

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We impose compatible quasi-uniformities on all the T_0 topological spaces simultaneously in such a way that continuous maps become quasi-uniformly continuous. This amounts to taking a functor $F : \mathbf{Top_0} \to \mathbf{QU_0}$ such that TF is the identity functor on $\mathbf{Top_0}$, where $T : \mathbf{QU_0} \to \mathbf{Top_0}$ is the usual forgetful functor and $\mathbf{QU_0}$ is the category of T_0 quasi-uniform spaces and quasi-uniformly continuous maps.

In $\mathbf{QU_0}$ there is an analogue of the familiar completion of uniform spaces, called the bicompletion, denoted by $k_Y : Y \to KY$ for every Y in $\mathbf{QU_0}$. We consider any one of the above functors $F : \mathbf{Top_0} \to \mathbf{QU_0}$ and apply it to every X in $\mathbf{Top_0}$. Then we have $k_{FX} : FX \to KFX$. Applying the functor T we get $Tk_{FX} : X \to TKFX$, which is a dense topological embedding, natural in X.

Depending on the F that was used, such a natural extension of the T_0 spaces may be a prereflection, or a reflection, or something better, or part of a monad (TKF, TkF, μ) – or none of these. The constructions are analogous to the familiar ones that involved uniformities on the Tychonoff spaces, but the outcomes are rather different, reflecting the asymmetry of the T_0 spaces.

SAMS subject classification: 5

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PRESIDENT'S REPORT, PRESIDENT'S ADDRESS

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DETOUR GRAPHS AND A CONJECTURE OF ZELINKA

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A *detour graph* G is a simple graph in which every vertex is the end-vertex of a longest path in G. Zelinka [1] gave two constructions (type I and type II) for maximal non-traceable graphs, and conjectured that every such graph is either of type I or type II. Using a particular detour graph, we show how to construct an infinite family of maximal non-traceable graphs which are not of type I or II.

SAMS subject classification: 14

References

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REPRESENTATIONS OF SPACES OF MEASURABLE FUNCTIONS AND OPERATORS

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Spaces of measurable functions come equipped with a natural vector lattice structure. We consider the problem of characterizing those vector lattices which can be represented as a space of measurable functions. In the case of spaces of measurable operators associated with a von Neumann algebra the problem is complicated by the lack of lattice structure. We discuss possible alternative approaches in this context.

SAMS subject classification: 9

DISTANCES IN ORIENTATIONS OF GRAPHS

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We consider the following question: If all streets in a city are made one-way, by how much will the travel time between two randomly chosen points increase?
More formally: Let G be a connected graph (digraph). The average distance of G is defined to be the average of the distances between all pairs (ordered pairs) of vertices in G. A digraph D is an orientation of G if D is obtained from G by assigning an orientation to each edge of G. In our talk, we give bounds on the average distance of an orientation of G in terms of the average distance of G.
Joint work with Ortrud Oellermann, University of Winnipeg.

SAMS subject classification: 14

ON A CONJECTURE OF HARVEY FRIEDMAN

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Conjecture (Friedman): The set F of all n such that there exists $e < n$ such that n is the largest element of W_e is Turing complete.

Note that $W_e =$dom $\varphi_e = \{x : \varphi_e(x) \downarrow\}$

We prove the conjecture for the Gödel numberings used in Kolmogorov complexity. Let $F_{1:n}$ be the first n digits of the characteristic function of the set F. The strategy is to show that, given a time t we can diagonalize against the c.e. set of approximations ("n- patterns") to $F_{1:n}$ which have appeared by time t in the enumeration.

SAMS subject classification: 1

AUTOMORPHISMS OF AF ALGEBRAS

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A C* dynamical system is a triple (A, α, σ), where A is a C* algebra, α is an automorphism of A and σ is a state of A invariant under the automorphism $(\sigma \circ \alpha = \alpha)$. We consider dynamical systems where A is an AF algebra and study automorphisms of A in terms of recurrence theory. In particular we present results for finitely generated algebras and UHF algebras.

SAMS subject classification: 9

APPEARANCE OF THE CENTRAL SINGULARITY IN SPHERICAL COLLAPSE

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We analyze here the structure of non-radial nonspacelike geodesics terminating in the past at a naked singularity formed as the end state of inhomogeneous dust collapse. The spectrum of outgoing nonspacelike null geodesics is examined analytically. The local and global visibility of the singularity is also examined by integrating numerically the null geodesics equations. The possible implications of existence of such families towards the appearance of the star in late stages of gravitational collapse are considered. It is seen that the outgoing non-radial geodesics give an appearance to the naked central singularity as that of an expanding ball whose radius reaches a maximum before the star goes within its apparent horizon. The radiated energy (along the null geodesics) is shown to decay very sharply in the neighbourhood of the singularity. Thus the total energy escaping via non-radial null geodesics from the naked central singularity vanishes in the scenario considered here.

SAMS subject classification: 13

THE DISCRIMINANT OF THE NEWTON MAPPING

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Summary. Let N denote the Newton mapping and DN the Jacobian matrix of the Newton mapping. We discuss a combinatorial proof which relates the discriminant of DN to the Van der Monde determinant. This result plays a crucial role in Lojasiewicz's proof of the Malgrange preparation theorem.

SAMS subject classification: 8, 14

References

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MIXED VARIATIONAL FORMULATIONS OF PROBLEMS IN ELASTICITY II: FINITE ELEMENT APPROXIMATIONS

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A key challenge in the formulation of problems in elasticity, and in their approximation, is that of ensuring convergence in the incompressible limit. In the case of isotropic elasticity this is equivalent to establishing well-posedness and convergence of approximations in the limit as $\lambda \to \infty$, where λ is the Lamé parameter.

One means of circumventing problems of ill-posedness in the incompressible limit is to make use of mixed or saddle-point formulations. We consider one such formulation in which the unknown variables are displacement, stress and strain.

In the first part of a two-part presentation the continuous problem is shown to be well-posed, by recourse to an abstract result [1] on non-standard mixed problems. The second part of the presentation is devoted to finite element approximations. The conditions on the choice of spaces that will ensure convergence are established, and concrete examples of such choices are given. The presentation concludes with some numerical examples.

SAMS subject classification: 16, 17

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APPLICATIONS OF MULTISEQUENCES IN TOPOLOGY

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A *sequential cascade* is a tree with finite branches, which possesses a least element $\varnothing$, and such that the set t^+ of immediate successors of each non maximal element t is countably infinite. The natural topology of a sequential cascade is the coarsest with the property that the free sequence determined by t^+ converges to t for every non maximal t. A *multisequence on* A is a map from the set of maximal elements of a sequential cascade valued in A. A multisequence on subset A of a topological space X *converges to* x *in* X if it can be continuously extended to the whole cascade so that $\varnothing$ is mapped on x.

Applicability of multisequences to numerous aspects of sequentiality stems from the fundamental fact that a topological space X is *sequential* (that is, such that each sequentially closed subset of X is closed) if and only if x is in the closure of a subset A of X whenever there exists a multisequence on A which converges to x in X. The applications include

1. Estimates of sequential order of products and inverse limits;
2. Preservation of sequentiality and of Fréchetness by sundry operations;
3. Study of subsequential spaces;
4. Countable uniradiality;
5. Arens spaces of higher order.

SAMS subject classification: 5

MUST KNOW, SHOULD KNOW, COULD KNOW

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It's almost like searching for the Holy Grail, this quest to equip first year undergraduates with the basic tools to do problem solving. In first year Calculus one encounters quite a dazzling number of formulae, techniques and rules. In this maze there certainly is a core subset of knowledge that are essential for solving problems. This is also the essential bag of tools for entering the second year. Even if it is necessary to derive other formulae, these can be classified as “should knows” or even “could knows”. The question is: How do you motivate/ convince/ force first year students to indelibly master “must knows” and sensibly supplement it with “should knows” and “could knows”?

SAMS subject classification: 18

ERGODICITY IN QUANTUM MECHANICS?

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We consider discrete time dynamical systems in classical and quantum mechanics. Ergodicity is then discussed in a unified way to include both the classical and quantum case, using the language of $*$-algebras.

SAMS subject classification: 9, 13

A STABILITY RESULT FOR NONLINEAR NEUMANN PROBLEMS UNDER BOUNDARY VARIATIONS

Francois Ebobisse Bille (Joint work with G. Dal Maso & M. Ponsiglione, SISSA, Trieste Italy)
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In this talk I present a result on the stability of some nonlinear Neumann problems, involving strictly monotone operators in two-dimensional non-smooth domains, under perturbations of the domains in the Hausdorff complementary topology. More precisely, for every bounded open subset Ω of $\mathbb{R}^2$, we consider the following nonlinear Neumann problem:

$$\{ -\operatorname{div} a(x, \nabla u_\Omega) + b(x, u_\Omega) = 0\& \text{ in } \quad \Omega, a(x, \nabla u_\Omega) \cdot \nu = 0\& \text{ on } \quad \partial\Omega,$$

where $a : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ and $b : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$ are two Carathéodory functions that satisfy the standard monotonicity and growth assumptions of order p, with $1 < p \leq 2$.
Let Ω_n be a uniformly bounded sequence of open sets in $\mathbb{R}^2$ whose complements Ω_n^c have a uniformly bounded number of connected components. We prove that, if $\Omega_n^c \to \Omega^c$ in the Hausdorff metric and meas$(\Omega_n)\to$meas(Ω), then $u_{\Omega_n} \to u_\Omega$ and $\nabla u_{\Omega_n} \to \nabla u_\Omega$ strongly in L^p.
This kind of results represent a key point in some Shape Optimization problems and, more recently, also in some problems arising in the study of the quasi-static growth of fractures.
The strategy of the proof is to obtain the Mosco convergence of the corresponding Sobolev spaces $W^{1,p}(\Omega_n)$ to the Sobolev space $W^{1,p}(\Omega)$, for which the stability result follows.

Keywords: Boundary value problems for nonlinear elliptic PDE, Capacity, Hausdorff metric, Monotone operators, Mosco convergence
SAMS subject classification: 11, 17

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PRELIMINARY GROUP CLASSIFICATION OF THE NONLINEAR HEAT EQUATION

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SAMS subject classification: 11

THE IMPACT OF WEB-BASED UNDERGRADUATE MATHEMATICS TEACHING ON DEVELOPING ACADEMIC MATURITY

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After having presented web-based Calculus courses for a number of years, we investigate the impact that this medium of instruction has on the student's mathematical and general academic progress. Since the web-based model that we developed is much more flexible than the conventional contact lecture model, we suspected that this may contribute to the student becoming more of an independent learner than does the conventional programme. The conjecture was investigated using a qualitative as well as a quantitative instrument.

SAMS subject classification: 18

ACOUSTICALLY FILTERED NONHYDROSTATIC EQUATIONS AND COLD BUBBLES

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Weather forecasting models will soon reach horizontal resolutions that allow the representation of processes for which the hydrostatic approximation ceases to be valid. For this reason the implementation of nonhydrostatic numerical weather prediction models is considered a priority at many meteorological services and research institutions.

There are a few models that use the complete meteorological equations. They are called fully compressible fluid models, and they use the Euler equations. The main characteristic of these models is that they include high-frequency sound waves (Robert, 1993). These waves impose a severe restriction on the size of the time step that can be used to produce stable integrations. The problem with the fully compressible models is that they can be very time consuming, even on present supercomputers.

White (1989) formulated a flow model that is acoustically filtered but nonhydrostatic. The aim of this paper is to show that White=92s equations can be used to simulate nonhydrostatic effects satisfactorily, whilst the restriction imposed by sound waves on the time step is avoided. In order to make the model even more efficient a semi-implicit, semi-Lagrangian scheme is used to produce stable integrations with fewer calculations than the amount required with classical integration schemes. A novel feature of the present model is that it utilises a non-hydrostatic s- coordinate in the vertical. Recent nonhydrostatic models by Janjic (2001) and Laprise (1999) employ hydrostatic pressure as vertical coordinate. It is shown that the present approach allows the introduction of nonhydrostatic dynamics in close correspondence with the formulation of hydrostatic models.

To test the robustness of the numerical methods and the suitability of the set of equations to simulate non-hydrostatic effects, the integration scheme is subjected to the cold bubble test. In this test a density current is initiated as a cold blob of air that subsequently descends to the ground. As the density current spread out laterally at the lower boundary, Kelvin-Helmhotz shear instability rotors form along the top of the cold air boundary. Grid-converged solutions for this problem have been obtained with fully compressible equations by numerous authors. It is shown that with the present approach the same qualitative solutions can be obtained in a more efficient way.

SAMS subject classification: 11, 16

BANACH SPACE SEQUENCES AND PROJECTIVE TENSOR PRODUCTS

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Recently Q. Bu and J. Diestel (cf. [3]) considered a vector sequence space characterisation of the complete projective tensor product of the classical scalar sequences space ℓ^p with a Banach space. Their proof relies on deep vector measure theory. Following a different approach, we use basic results from the theory of tensor products of Banach spaces to establish the isometry of the space of $(1, p)$-summing sequences (also called strongly p-summable sequences) in a Banach space X with the space of nuclear X-valued operators on ℓ^p, as well as with the complete projective tensor product of ℓ^p with X. We'll discuss conditions for the space of (p, q)-summing sequences (recently introduced in [1]) to have the generalised AK-property, discuss some results about p-summing multipliers and show some connections with sequences in the ranges of vector measures. The results discussed in this talk, are from joint work with Ilse Röntgen (cf. [4]).

SAMS subject classification: 9

References

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ON AN ALMOST SUBIDEMPOTENT RADICAL PROPERTY

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In [3] G. Tzintzis introduced an interesting radical property Ξ, which is the upper radical property determined by the class ϵ of all rings without admissable ideals which are not ideals. He showed that Ξ contains the class α of all idempotent Brown-McCoy radical rings R such that every nonzero prime homomorphic image R' of R has a nonzero centre. He then asked the question wehter $\Xi\alpha$. In this talk we will answer thiw question in the negative by constructing a non-simple idempotent *-ring R with a zero centre.

SAMS subject classification: 2

References

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DIRECT CANCELLATION OF FINITE GROUPS

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Following Oger, a group A is cancellable if for any groups G and H with

$$A \times G \cong A \times H$$

implies that $G \cong H$. We show that finite groups are cancellable within the class of finitely generated groups with finite commutator subgroups.

SAMS subject classification: 2

References

[1] F. Oger, *The direct decompositions of a group G with G/G' finitely generated*, Trans. Amer. Math. Soc. **347**, No.6 (1995), 1997-2010.

UNICYCLIC DETOUR-SATURATED GRAPHS

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The *detour order*, $\tau(G)$, of a graph G is the number of vertices in a longest path of G. If $\tau(G+xy) > \tau(G)$ for every pair (x, y) of nonadjacent vertices in G, then G is *detour-saturated.* Kászonyi and Tuza [1] characterized the detour-saturated trees. A characterization of the unicyclic detour-saturated graphs will be presented.

SAMS subject classification: 14

References

1 L. Kászonyi and Z. Tuza, Saturated graphs with minimal number of edges, *J. Graph Theory* **10** (1986), 203–210.

REMARKS ON DIMENSION THEORY FOR FRAMES WITH STRONG INCLUSION

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Frames (or locales) are an important analogue of topological spaces. Frames with strong inclusion are the appropriate analogue of proximity spaces. We investigate some aspects of classical dimension theory for such objects.

SAMS subject classification: 5

ON FACTORIZATION OF INFINITE METACYCLIC GROUPS

Habtay Ghebrewold

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We study direct products of metacyclic groups as a first step towards understanding direct decompositions of groups. In this talk we discuss how some direct product of infinite metacyclic groups can be expressed as a different direct product using the greatest common factor and the least common multiple of the orders of the torsion subgroups.

SAMS subject classification: 2

References

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RINGS OF CONTINUOUS FUCNTIONS WITH COMPACT SUPPORT

Partha Pratim Ghosh* and S.K. Acharyya and K.C. Chattopadhyaya

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The main aim of this paper is to investigate a subring of the ring of continuous functions on a topological space X with values in a linearly ordered field F equipped with its order topology, namely the ring of continuous functions with compact support. Unless X is compact, these rings are commutative rings without unity. However, unlike many other commutative rings without unity, these rings turn out to have some nice properties, essentially in determining the property of X being locally compact non-compact or the property of X being nowhere locally compact. Also, one can associate with these rings a topological space resembling the structure space of a commutative ring with unity, such that the classical Banach Stone Theorem can be generalized to the case when the range field is that of the reals.

SAMS subject classification: 2, 5

σ-SPACES AND DOMAINS

Christopher Robert Anderson Gilmour

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σ-Spaces, in association with the semantic domain of programming languages, form a natural generalisation of topological spaces. In this talk I will introduce σ-spaces, and, with the help of a version of the Hofmann-Mislove theorem, investigate dualities between σ-spaces on the one hand and certain σ-frames on the other. I will then report on joint work with A. Pultr which investigates the connections with Scott information systems.

SAMS subject classification: 5, 15

ON CATEGORICAL CONSTRUCTIONS OF NON-COMMUTATIVE L^p-spaces

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SAMS subject classification: 9

ON THE ESSENTIAL LOWER BOUND OF ELEMENTS IN VON NEUMANN ALGEBRAS

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In this paper we obtain a number of interesting results on the connection that exists between the essential lower bound of an element in a von Neumann algebra and its Fredholm properties.These connections lead to an extension of a well-known perturbation theorem on operators.Finally we obtain for special cases necessary and sufficient conditions for elements in the closure of the group of invertible elements to be essentially topological divisor of zero.

SAMS subject classification: 9

ALGEBRAIC MODELS FOR TWO-PLAYER GAMES WITH ITERATIONS

Valentin Goranko (attendance unconfirmed at publication date)

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A recent trend in the mathematical theory of games develops a *structural approach* to games, whereby they are considered as algebraic objects built from *atomic games* by applying certain natural *game operations*. Furthermore, mathematical models for such games can be introduced, based on abstract '*game boards*' consisting of a set of *game states*, where a play of a game effects a transition from a state (*initial state*) to a state (*outcome state*), according to the choices of moves/strategies of each of the individual players. For every game, each player has certain *power* to determine the outcome of the game, and that power can be modelled by an *effectivity function* mapping states to sets of sets of states for which the player is *'effective'*. The effectivity functions are assigned to atomic games and then computed for the composite games according to rules corresponding to the game-buliding operations. This provides a set-theoretic semantics for the algebra of games and, in particular, a way for defining equivalence between game terms.

Here I study algebraic structures modeling perfect information games with two players, with game constructing operations: *choices of first and second player, composition, dualization, weak and strong iteration*. I axiomatize the valid identities in such structures with respect to game board models, thus establishing an algebraic characterization of equivalence between games.

This work is an extension and a continuation of [1].

SAMS subject classification: 1

References

[1] Goranko V., The Basic Algebra of Games, ILLC Prepublication report PP-2000-12, University of Amsterdam; available at http://general.rau.ac.za/maths/goranko/publ.htm; to appear in *Studia Logica*.

PRELIMINARY POTENTIAL GROUP CLASSIFICATION OF THE NONLINEAR HEAT CONDUCTION EQUATION

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SAMS subject classification: 11

LUBIN-TATE FORMAL GROUPS AND THE LIFTING PROBLEM FOR AUTOMORPHISMS OF FORMAL POWER SERIES RINGS

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Let R be a complete discrete valuation ring of characteristic 0, with residue field k of characteristic $p > 0$. The aim of this talk is to show how Lubin-Tate formal groups can be used to lift order p^n automorphisms of $k[\![X]\!]$, which occur as endomorphisms of a formal group of suitable height to automorphisms of $R[\![X]\!]$. This result has applications to the Oort-Sekiguchi conjecture on liftings of galois covers of smooth curves from characteristic p to characteristic 0.

SAMS subject classification: 2, 3

OPERATORS DEFINED BY RANDOM MEASURES AND CONDITIONAL EXPECTATION OPERATORS

J.J. Grobler* and D. Rambane

J.J. Grobler: Unit for Business Mathematics and Informatics, Potchefstroom University for CHE,. Email: srsjjg@puknet.puk.ac.za. D. Rambane: Department of Mathematics, University of Venda.

We study the lattice properties of operators defined by random measures and relate these operators to multiplication conditional expectation operators. We prove that every order bounded order continuous operator is a multiplication conditional expectation operator.

Keywords, Conditional expectation, kernel operator, Riesz space, Random measure, Order bounded operator.
SAMS subject classification: 9

PROTECTION OF A GRAPH

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For the vertex v of a simple n-vertex graph $G = (V, E)$ let $f(v)$ be the number of guards stationed at v. A guard at v can deal with a problem at any vertex in its closed neighbourhood.

We consider four strategies, i.e. properties of such functions, under which the entire graph may be deemed protected. The four properties are domination, Roman domination, weak Roman domination which have been previously studied and a new concept called secure domination.

The four parameters which give the minimum number of guards required to protect the graph under the various strategies, are discussed. Exact values or bounds are obtained for specific classes of graphs.

SAMS subject classification: 14

ON JACOBSON NEAR-RINGS

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In this paper we construct special radicals using class pairs of near-rings. We establish necessary conditions for a class pair to be a special radical class. We then define Jacobson-type near-rings and show that most cases the class of all near-rings of this type are special radical classes. Subsequently we investigate the relationship between each Jacobson-type of near-ring and the corresponding matrix near-ring.

SAMS subject classification: 2

J-SPECTRAL FACTORIZATION FOR ARBITRARY RATIONAL MATRIX FUNCTIONS

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The problem that we solve is to find (square and nonsquare) minimal J-spectral factors of an arbitrary rational matrix function with constant signature. Explicit formulas for these J-spectral factors are described in terms of a solution of a certain algebraic Riccati equation/inequality. All formulas are based on a special representation of a rational matrix function involving a quintet of matrices.
This talk is based on joint work with Mark Petersen.

SAMS subject classification: 9

CHARACTERISING LOTTERY STRUCTURES

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Suppose a lottery scheme consists of randomly selecting a winning t–set from a universal m–set, while a player participates in the scheme by purchasing a playing set of any number of n–sets from the universal set prior to the draw, and is awarded a prize if k or more numbers in the winning t–set match those of at least one of the player's n–sets in his playing set ($k \leq t, n \leq m$). It may well be that apparently different playing sets of equal cardinality yield equivalent results. In the

particular case where $t = n$, we investigate a method of characterising different lottery sets of a given order for small lotteries.

SAMS subject classification: 14

REDUCED PRODUCT OBJECTS IN MODEL CATEGORIES

Keith Allanson Hardie* and Peter J. Witbooi

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In his influential 1967 monograph *Homotopical Algebra*, D. Quillen described an abstract approach to homotopy theory enabling analogous theories to be defined in categories other than the category of spaces and continuous maps. Although the starting point in the classical homotopy theory of spaces and maps is the equivalence relation (between maps) of *homotopy*, in Quillen's approach it is that of a *model category*, that is to say, a category $\mathbf{C}$, together with three distinguished classes of morphisms, *we*, *cof*, *fib*, called weak equivalences, cofibrations, and fibrations, respectively. These are required to satisfy certain axioms which reflect typical properties of the classes of such maps in topology and they enable the construction of much of the basic machinery of homotopy theory in the category $\mathbf{C}$.

We define the reduced powers X_n of an object X in a suitable model category, or more generally, the objects $(X, A)_n$ and $(X, A)_\infty$ as in the work of Gray, associated with a cofibration $A \to X$. If, in particular, a certain *cube axiom* is satisfied, we prove the equivalence of the object X_∞ to $\Omega\Sigma X$ generalizing work of I. M. James and others.
SAMS subject classification: 5

ASSESSMENT POSSIBILITIES IN WEB MATHEMATICS

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Web-based courses in Mathematics offer a variety of possibilities as to the format in which questions are phrased. The best known of these is the multiple-choice format but it does not end here. A number of assessment techniques are discussed and compared.

SAMS subject classification: 18

THE WITT VECTORS AFTER HENDRIK LENSTRA

John Paul Harper

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SAMS subject classification: 3

DEVELOPMENT AND APPLICATION OF FLOTATION MODELS IN THE MINING INDUSTRY

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Flotation is the most widely used and economically viable method to separate valuable minerals from ore in the mining industry. The process itself is simple to implement but fundamentally describing the sub-processes occurring is extremely complicated. The flotation process itself is described and the equations derived to describe the mechanisms taking place are explained as well as how they can be put together in a simulation model. We show how a good simulation model could be used to increase a flotation circuit's efficiency and its potential application to the design and commissioning of new plants.

SAMS subject classification: 17

TOTAL DOMINATION IN GRAPHS

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A set S of vertices in a graph G without isolated vertices is a total dominating set of G if every vertex of G is adjacent to a vertex in S. The total domination number of G is the minimum cardinality of a total dominating set in G. In this talk, we discuss bounds on the total domination number in terms of the minimum degree of the graph, total domination in planar graphs with small diameter, and total domination in claw-free cubic graphs.

SAMS subject classification: 14

ON THE STABILITY OF NAVIER-STOKES FLOWS THROUGH PERMEABLE BOUNDARIES

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Experiments show that some fluids, when in motion, experience normal stresses which, in turn, are exerted on the solid boundaries. In a paper by R. Maritz and N. Sauer, of Pretoria University, the existence of these stresses was confirmed for viscous, incompressible second grade fluids, under certain conditions. However, for first grade fluids; otherwise known as Navier-Stokes fluids (air, water, smoke; etc.), the normal stresses on solid boundaries were calculated and found to be zerom by R Berker. Our research (an M.Sc. project), under the supervision of Prof. Niko Sauer, set out to investigate the existence or non-existence of these stresses on permeable boundaries for Navier-Stokes flows. A boundary permeation model that had been proposed by N. Sauer was used. The setting is a container filled with the Navier-Stokes fluid; and in turn, immersed in a similar fluid inside a larger container. The body fluid between the boundaries of the inner and outer containers is assumed to be a bounded domain $\Omega \subset R^3$, with a smooth boundary denoted by Γ. During the research, the Sobolev spaces $H^1(\Omega)$ and $H^2(\Omega)$ featured prominently; and hence, Sobolev's Imbedding theorem. The following mathematical model was derived, where the second last boundary condition is the boundary permeation model (proposed by N. Sauer):

$$\rho[\partial_t + \mathbf{v}.\nabla]\mathbf{v}(\mathbf{x}, t) = \nabla.\mathbf{T};\ \mathbf{v}(\mathbf{x},\mathbf{t}) \in H^2(\Omega);\ [\mathrm{Navier-Stokes\ equations}]$$

Where

$$\mathbf{T}(p,\mathbf{v}) = -p\mathbf{I} + 2\mu\mathbf{D}(\mathbf{v},\mathbf{v}) \text{ [Stress tensor]}$$

$$\mathbf{D} = \frac{1}{2}\left[\nabla\mathbf{v} + \nabla^T\mathbf{v}\right] \text{ [Rate of deformation or stretching tensor]}$$

Boundary conditions:

$$\nabla.\mathbf{v}(\mathbf{x},t) = 0 \text{ [Incrompressibilitycondition]}$$

$$\gamma_0\mathbf{v}(\mathbf{x},t) = -\eta_v(\mathbf{x},t)\mathbf{n}(\mathbf{x}) \text{ [Fluid velocity at } \Gamma\text{; permeation model]}$$

$$\partial_{\mathrm{t}}[\gamma_0\mathbf{v}] = -\mathbf{n}.\mathbf{T}\mathbf{n} \text{ [Acceleration at } \Gamma.\text{]}$$

The existence of normal stresses on permeable boundaries and the stability of Navier-Stokes flows through the same boundaries were confirmed and reported on in [1].

SAMS subject classification: 11

References

[1] Navier-Stokes equations: Theory and numerical methods (Vol. 223) Edited by: R. Salvi Published by: Marcel Dekker, Inc; (2001)(pp.33–43);New York; bassel.

CONVERGENCE OF RAY SEQUENCES OF PADÉ APPROXIMANTS FOR ${}_2F_1(a;1;c;z)$, FOR $c > a > 0$

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The Padé table of ${}_2F_1(a,1;c;z)$ is normal for $c > a > 0$. For $m \geq n-1$ and $c \notin \mathbb{Z}^-$, the denominator polynomial $Q_{mn}(z)$ in the $[m/n]$ Padé approximant $P_{mn}(z)/Q_{mn}(z)$ for ${}_2F_1(a,1;c;z)$ and the remainder term $Q_{mn}(z){}_2F_1(a,1;c;z) - P_{mn}(z)$ were explicitly evaluated by Padé. We show that for $c > a > 0$ and $m \geq n-1$, the poles of $P_{mn}(z)/Q_{mn}(z)$ lie on the cut $(1,\infty)$. We deduce that the sequence of approximants $P_{mn}(z)/Q_{mn}(z)$ converges to ${}_2F_1(a,1;c;z)$ as $m \to \infty$, $n/m \to \rho$ with $0 < \rho \leq 1$, uniformly on compact subsets of the unit disc $|z| < 1$ for $c > a > 0$.

Keywords: Padé approximation, zeros and poles of Padé approximants, convergence of ray sequences of Padé approximants, hypergeometric functions.

SAMS subject classification: 16

EXPLICIT TOWERS OF DRINFELD MODULAR CURVES I & II

A. Keet and Mohammed Rafiq Omar

Define

$$N_q(g) = \max(\#C(\mathbb{F}_q))$$

where C runs through all curves of genus g over $\mathbb{F}_q$. The number

$$A_q = \limsup_{g\to\infty} \frac{N_q(g)}{g}$$

is important in coding theory. If q is a square then

$$A_q = q^{\frac{1}{2}} - 1.$$

A sequence of curves $C_n/\mathbb{F}_q$ is *asymptotically optimal* if

$$\lim_{n\to\infty} \frac{\#C_n(\mathbb{F}_q)}{g(C_n)} = q^{\frac{1}{2}} - 1.$$

For the ring $\mathbb{F}_q[T]$, the Drinfeld modular curves $X_0(T^n)$ are asymptotically optimal. We discuss the work of Elkies on computing explicit equations for this tower of curves, and we consider the same problem for the tower $X_0((T^2+1)^n)$.

SAMS subject classification: 3

COCOMPACTNESS AND QUASI-UNIFORMIZABILITY OF COMPLETELY METRIZABLE SPACES

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Strengthening a result of Aarts, de Groot and McDowell, we show that a metrizable topological space X is completely metrizable if and only if it admits a quasi-uniformity $\mathcal{U}$ such that the topology induced by the conjugate quasi-uniformity $\mathcal{U}^{-1}$ on X is compact. Ciesielski, Flagg and Kopperman had asked whether that assertion holds, after establishing an analogous result for separable metrizable spaces in connection with some characterization of certain computational models.

SAMS subject classification: 5

LOGMODULARITY AND UNIQUENESS OF REPRESENTING MEASURES FOR OPERATOR ALGEBRAS

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We review the classical situation regarding the uniqueness of representing measures for logmodular uniform algebras before showing how these notions may be extended to the context of operator algebras. We then review the notion of a non-commutative H^∞-space and show how the previously discussed operator algebra notions may be applied to these spaces. In closing some significant challenges that remain are discussed, in particular the question regarding the validity of a non-commutative Szego theorem and Jensen inequality. All results discussed are the fruit of joint work with David Blecher.

SAMS subject classification: 8, 9

CROSS NORMS AND IDEALS OF OPERATORS

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Interactions between cross norms on tensor product of Banach spaces and the ideals of bounded operators that they generate will be considered.

SAMS subject classification: 9

LOAD-LOPPING: A PROBLEM IN ROBUST SCHEDULING UNDER UNCERTAINTY

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We briefly consider the difference between scheduling under perfect information and scheduling under uncertainty. We describe the problem of load-lopping, which occurs when a large electricity user attempts to save money on the peak demand tariff by generating their own electricity needed above some level, called the lopping level. In this case, an overly low level leads to large losses rather than savings, and so robustness is essential in the algorithm for estimating the lopping level at the start of a metering period. We propose and compare several algorithms.

SAMS subject classification: 17

COMPOSING PROBLEMS FOR A MATHEMATICS COMPETITION

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The purpose of this talk is to inspire the audience to become problem composers themselves. The main points are:

1. Your own teaching and research is your mainspring of inspiration.
2. Tailoring a problem to the required standard.
3. Case studies.

SAMS subject classification: 18

A CAREFUL LOOK AT THE NEW CURRICULUM FOR MATRIC MATHS: PANEL DISCUSSION

Henri Laurie
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The new FET NCS for maths (first matriculants due end 2006) has major implications for SA. Teachers will face steep new demands, universities and technikons will have to adjust to new FET exit quality, and professional mathematics will undoubtedly also be affected. In this 2-session panel discussion, Curriculum 2005 in general as well as the fine details of the FET NCS for Mathematics

will be presented and to some extent evaluated. This discussion will be relevant to an AGM motion; there will be ample time for discussion from the floor, and a limited number of copies of the maths NCS will be made available.

Part 1: The general structure of a C2005 FET qualification; the structure of mathematics in it; general policies on assessment. The Department of Education's assessment of teachers' ability to implement C2005, and its plans and budgets for teacher support, teacher training, overhaul of examinations process and teacher involvement in same, communication of C2005 to students, parents, further educationists, and potential employers.

Part 2: C2005 FET Maths. Details of topics, approach and timing in each of the four Learning Outcomes, with special emphasis on changes from the current matric HG maths syllabus. Overview of the level integration amongst these and evaluation of whether ambitious targets are likely to be achieved.

SAMS subject classification: 18

AN ANALYSIS OF FRW BRANE MOTION AND NULL GEODESIC EQUATIONS IN 5-D ADSS

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In this paper we consider a cosmological Friedmann Robertson Walker brane world imbedded in a 5-dimensional Anti de Sitter Schwarschild bulk. We obtain analytical solutions of the null geodesic equations in the bulk and in conjunction to this we write down the equation describing the motion of the FRW brane motion. We then show that in the flat and open cases $k = 0, -1$ null geodesics intersect the brane only once whereas in the closed case $k = +1$ the geodesics intersect the brane twice. Implication of the latter to the horizon problem are highlighted.

SAMS subject classification: 13, 17

SYMMETRY, SINGULARITY AND INTEGRABILITY IN DYNAMICAL SYSTEMS: THE FINAL QUESTION?

Peter Gavin Lawrence Leach

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Dynamical systems daily attain greater importance as the number of different fields of endeavour modelled by means of first order ordinary differential equations increases. Traditionally these dynamical systems are investigated in terms of the qualitative theory developed by Poincaré, but in recent years they have been increasingly subjected to examination in terms of their symmetries and singularity behaviours. These studies have revealed properties, mainly in relation to the connection between the possession of symmetry and integrability in the sense of Poincaré, which have led to the necessity for a reformulation of the theory of the symmetry basis of integrability.

SAMS subject classification: 7, 12, 13, 17

PARTIAL CONSTRAINT SATISFACTION ALGORITHMS

LOUISE LEENEN, KATARINA BRITZ, AND LUCAS VENTER
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A constraint satisfaction problem (CSP) involves finding values, selected from a finite domain, for all variables such that all constraints on the variables are satisfied. Partial constraint satisfaction involves finding values for all variables such that a subset of the constraints is satisfied. A satisfaction optimisation problem (CSOP) is a CSP that has some objective function defined in terms of some or all of the variables.

Most partial constraint satisfaction algorithms solve the maximal constraint satisfaction problem (Max-CSP), where the goal is to minimise the number of constraint violations. We investigate the viability of algorithms that satisfy subsets of constraints of increasing cardinality until a satisfactory solution is found. We focus on the partial solution of CSOPs.

SAMS subject classification: 16

MODULI OF INTEGRITY IN A BANACH ALGEBRA

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We use the basic definitions defined in Rickart C E for the left and right modulus of integrity:

$$\lambda(x) = \inf_{y \neq 0} \frac{\|xy\|}{\|y\|} \text{and } \rho(x) = \inf_{y \neq 0} \frac{\|yx\|}{\|y\|}$$

and obtain more properties of the moduli of integrity.

The invariance of the spectral radius under perturbation by radical elements is a direct consequence of a characterization of *Rad A* in terms of the spectrum. We show that a similar characterization of *Rad A* in terms of the singular spectrum holds, provided A is commutative.

SAMS subject classification: 9

References

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ASYMPTOTICALLY GOOD TOWERS OF CURVES OVER FINITE FIELDS

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For curves C defined over a finite field $\mathbb{F}_q$, we define

$$N_q(g) = \max\{\#C(\mathbb{F}_q)\}$$

where C runs through all curves of genus g over $\mathbb{F}_q$.

In this talk a survey of results on asymptotic bounds on $N_q(g)$ is given, in particular the Drinfeld-Vlăduţ upper bound. For square q this bound is attained, as shown by Ihara. It is however not known how good this bound is for non-square q.

Recently Van der Geer and Van der Vlugt have constructed an asymptotically good explicit tower of Artin-Schreier curves defined over $\mathbb{F}_8$, by building it up recursively by a simple equation and calculating the genus of each curve and number of points on each curve. We draw attention to this work as well as an open problem it suggests.

SAMS subject classification: 3, 6

APPROXIMATION THEORY: FROM CLASSICAL TO CONTEMPORARY DEVELOPMENT

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In this survey talk, we will give an overview of the systematic development of some of the main results of Approximation Theory. The unifying aspect of the functional analytic tools will be highlighted. Some generalizations and/or extensions will be discussed. Examples in concrete function spaces will also be considered.

SAMS subject classification: 9

THE RADON-NOKIDYM PROPERTY FOR TENSOR NORMS AND THE WEAKLY SEQUENTIAL COMPLETENESS OF TENSOR PRODUCTS

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We introduce the Radon-Nikodym property (RNP) and the Lewis-Radon-Nikodym property (Lewis-RNP) for tensor norms and show that the Hilbertian tensor norm h introduced in ([2], §3) has the Lewis-RNP but does not have the the RNP ([3]). Other examples of this ilk will be isolated. We shall also single out another of Grothedieck's natural tensor norms, the projective tensor norm $\wedge$, that does enjoy the RNP ([5], VIII §§3.8, 3.10 and 4.6), but hasten to point out why its *right injective hull* $\wedge\backslash$ does not enjoy the same property.

D.R. Lewis [7] (also see [5], page 257) has considered the conditions under which $X \overset{\vee}{\otimes} Y$ is weakly sequentially complete. Moreover, if X and Y are reflexive, then $X \overset{\vee}{\otimes} Y$ is weakly sequentially complete if and only if it is reflexive. Furthermore, the instances under which this is true for the classical reflexive spaces were considered by J.R. Holub [1]. Following suit, we close the discussion with a dash of application of the Radon-Nikodym property for tensor norms to this stability problem of weakly sequential completeness of tensor products *a la* [4], ([5], page 258, paragraph 2) and [6].

SAMS subject classification: 9

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ON KOOPMAN AND BANACH-LAMPERTI THEOREMS IN CLASSICAL AND QUANTUM DYNAMICAL SYSTEMS

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We extend the concept of implementability of semigroups of evolution operators associated with dynamical systems to quantum case. We show that such an extension can be properly formulated in terms of Jordan morphisms and isometries on non-commutative L^p spaces. We focus our attention on a non-commutative analog of the Banach-Lamperti theorem.

SAMS subject classification: 9, 13

SQUEEZING FLOW PROBLEM: A MATHEMATICAL MODEL

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In recent time, squeezing flow technique is commonly used for measuring rheological properties of liquid and semi-liquid foods substances like yogurt, melted cheese, peanut butter, cream cheese, tomato paste, butter, ketchup, mustard, and mayonnaise, (Suwonsichon and Peleg, 1999). This enables food rheologist to avoid or minimize problems encountered during rheological measurements of such products with convectional rotational viscometers (e.g., damage incurred during sample loading; possibility of slip during measurement). In this paper, a mathematical simulation of squeezing flow of an incompressible viscous fluid between two parallel plates is presented. The problem admits similarity solutions, thereby reducing the unsteady Navier-Stokes equations to a parameter dependent fourth order nonlinear ordinary differential equation. Analytical solutions are therefore constructed for the problem and important properties of the overall structure of the flow are presented graphically and discussed quantitatively.

Keywords: Unsteady flow, Parallel plates, Moving boundaries, Perturbation theory.
SAMS subject classification: 11, 12, 17

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INFLATION OF A HYPER ELASTIC INNER TUBE OR TYRE

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The inflation of a toroidal elastic tube by an internal pressure has important industrial applications, for instance in the inflation of a rubber inner tube or tyre.

The inflation of a thin-walled toroidal tube of hyperelastic material will be investigated. The change in the internal pressure as the torus is inflated will be investigated. The effect of different strain energy functions of the elastic material will be analysed.

SAMS subject classification: 11, 13

STUDENTS' PERCEPTION OF LOGIC IN MATHEMATICS

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Do first year students think mathematics is logical? From our experience, the answer is not necessarily yes. This talk will mainly be on work done by C Marais and A Simpson with first year students at the University of Warwick, UK. During this study, we used semi-structured interviews to assess the perception students have on logic in mathematics and their ability to use the tools of logic in mathematical discourse. We are also going to examine the place of logic in mathematics and what implications this has on mathematics education. Based on our findings, we will introduce a new hypothesis that will be the subject of a new study planned for next year by C Marais, J Engelbrecht, et al. This involves implementing an intervention course to assist students with the transition to advanced mathematical thinking. One thing to keep in mind is the relevance of the results obtained in the UK to the South African situation. Is our goal in undergraduate mathematics education in South Africa the same as in the UK? This is just one of the issues to consider when planning an intervention in our situation.

SAMS subject classification: 18

STRATIFICATION AND DOMINATION IN GRAPHS

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SAMS subject classification: 2

SOLUTIONS FOR SOME NON LINEAR SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS; NUMERICALLY AND THEORETICALLY

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There are several techniques for solving higher order differential equations. In this talk the object is to illustrate by means of the numerical solver *Mathematica* the graphics associated with the solution of an initial–value problem which is a non linear differential equation of grade two. This differential equation models the motion of a soft spring with no resistive force due to damping and is influenced by an external driving force $F_o cos\omega t$. The model we investigate

$$\begin{cases} \frac{d^2x}{dt^2} + kx - \varepsilon x^3 = 3DF_o \cos\omega t \\ x(0) = 3Da, \qquad \frac{dx(0)}{dt} = 3Db \end{cases}$$

Our aim is to predict the region where the solution is bounded. We consider a generalized *van der Pol* problem.

Shauder's Fixed Point Theorem is used to prove the existence of such solutions for a time T, and the effect of different initial values will be discussed.

SAMS subject classification: 12

THE MATHEMATICIAN SIR THOMAS MUIR

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This talk deals with the life and work of Thomas Muir, born on August 25, 1844 at Stonebyres, Lanarkshire, Scotland, died on March 21, 1934 at Rondebosch, Cape Town, South Africa. The first of Muir's 321 mathematical publications appeared in 1872 while he was Assistant to the Professor of Mathematics (Hugh Blackburn) at the University of Glasgow.

In 1892, Muir was offered the post of Superintendent General of Education (SGE) in the Cape Colony by Cecil John Rhodes, Prime Minister of the Cape Colony at the time. Muir will be remembered as one of the greatest organisers and reformers in the history of Cape education.

Through his whole adult life Muir had occupied himself with mathematical research. His *magnum opus* was a four-volume work with title "The Theory of Determinants in the Historical order of Development", published during the years 1906 – 1923, convering the development from Leibniz (1693) to 1900. A supplementary volume "Contributions to the History of Determinants" appeared in 1930 and it mainly covers the period 1900 – 1920. One finds more than 400 citations of Muir's work in the ISI-journals. Sir Thomas and lady Margaret were buried in the Maitland cemetery.

SAMS subject classification: 18

COMPARISON OF THE SPREADING OF A THIN LIQUID DROP AND A TWO-DIMENSIONAL THIN FLUID FILM ON A PERMEABLE HORIZONTAL BASE

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There are many industrial applications to the spreading of a thin fluid film on a porous base. For example, it has application in the spreading of water on a textile or over a powder and in ink-jet printing. In agriculture it has application in the analysis of runoff of rainwater over soils.

Two problems will be analysed and compared. The first is the axisymmetric spreading of a thin liquid drop on a horizontal base and the second is the two-dimensional spreading of a thin fluid film on a horizontal base. In both cases the base is permeable so that fluid can be sucked from the thin film or blown in the thin film. Group-invariant solutions for both problems will be presented. Each group-invariant solution depends on only one parameter. By giving the parameter different values a large range of solutions describing suction or blowing at the base is obtained. These solutions are discussed and compared.

SAMS subject classification: 11, 13

DIRECT AND INVERSE PROBLEM FOR AN INHOMOGENEOUS STRING WITH AN INTERVAL OF ZERO DENSITY

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Direct and Inverse spectral problems of an inhomogeneous string with a interval of zero density moving with damping are being investigated. By means of the Sturm Liouville transformation the former problem is reduced to the Sturm Liouville equation with boundary conditions depending on the eigenparameter λ. Finally we recover the density and other parameters from the knowledge of the spectrum and the length of the string.

SAMS subject classification: 11

FORCING LINEARITY : A SURVEY

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For a ring R and an R-module ${}_RV$, we investigate the relationship between the associated structures $\mathrm{End}_R(V)$ of R-endomorphisms (or R-linear maps) of V and $M_R(V) = \{f : V \to V \,|\, f(rv) = rf(v), \forall r \in R, v \in V\}$ of R-homogeneous functions of V.

How close homogeneous functions are from being linear is a problem that has been addressed within the context of various interpretations:

- When is $M_R(V)$ a ring?
- When is $M_R(V) = \mathrm{End}_R(V)$?
- Given that $f \in M_R(V)$ and f is linear on some prescribed set of proper submodules of ${}_RV$, when does it follow that $f \in \mathrm{End}_R(V)$?

For each of these questions, several (and by no means all) answers have been obtained. The purpose of this talk is to give an overview (enhanced by examples) on what has been done in this regard. The notion of a *Forcing Linearity Number* of a module will also be discussed.

SAMS subject classification: 2

SENSITIVITY ANALYSIS OF STAMPING PROCESSES TO VARIOUS FRICTION MODELS APPROPRIATE FOR NON-STATIONARY CONTACT PROBLEMS

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We use advanced simulation of manufacturing processes:

1. Deep-drawing of square cup

2. S-rail.

Advanced friction models that account for the dependence of friction on pressure and velocity are used. In each case the friction model is described and the simulation results are printed for comparison.

SAMS subject classification: 17

THE ESSENTIAL SPECTRUM OF A SYSTEM OF SINGULAR ORDINARY DIFFERENTIAL OPERATORS OF MIXED ORDER

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Kako [4] calculated the essential spectrum of an operator associated with a linearized magneto-hydrodynamic equation describing the oscillations of plasma in an equilibrium configuration in a cylindrical domain. This operator is given by a mixed-order 3×3-system of ordinary differential operators on $(0, r_0)$, where 0 is a singular endpoint. In different approaches, Hardt, Mennicken and Naboko [3] as well as Faierman, Mennicken, Möller [1, 2] generalized the results and gave detailed proofs. These results concern the so-called almost regular case and the much more involved quasi regular case. In the quasi regular case, an additional part of the essential spectrum may occur; in Kako's case, it is contained in the 'regular' part of the essential spectrum.

In this talk, I will review these result and show an extension to singular cases.

SAMS subject classification: 9, 12

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SHORT-TIME APPROXIMATE SOLUTIONS FOR GRAVITY-DRIVEN SPREADING OF A THIN FILM ON A ROTATING DISK

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The method of multiple-scales is used to introduce a "small-time" variable in the second-order nonlinear diffusion equation modelling gravity-driven spreading of a thin viscous liquid drop on a rotating disk.

Approximate group invariant solutions are determined and analysed.

Short-time solutions are important in the industrial application of spin- coating. The short term behaviour of a thin film may affect the spreading and flattening properties which are very important in the coating process.

SAMS subject classification: 11, 13

BINARY CODES FROM GRAPHS ON TRIPLES

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We examine the binary codes obtained from the adjacency matrix of each of the graphs on $\binom{n}{3}$ vertices, for $n \geq 7$, with adjacency defined by the vertices as 3-sets being adjacent if they have zero, one or two elements in common, respectively. This is a joint paper with J D Key (Clemson, USA) and B G Rodrigues (University of Natal, Pietermaritzburg).

SAMS subject classification: 2, 14

COMPLETENESS WITHOUT CAUCHY SEQUENCES

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In the talk we discuss the extension of distance spaces. It is well-known that a notion of H-closed spaces generalizes compactness for the Hausdorff spaces. We present a result that gives a strong compactness property for a quasi-pseudo-metric space X.

SAMS subject classification: 5

A SPECTRAL PROBLEM IN ORDERED BANACH ALGEBRAS

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An interesting problem in operator theory (originally formulated by Huijsmans and De Pagter) is to determine conditions under which it will be true that if T is a positive bounded linear operator on a complex Banach lattice with spectrum $\mathrm{Sp}\, T = \{1\}$, then $T \geq I$. In this talk we discuss the problem in an ordered Banach algebra context.

SAMS subject classification: 9

AN APPLICATION OF OPTIMAL SUBGROUPS OF PDEs

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We consider the solution of Partial Differential Equations by means of the Lie point symmetries. We are mainly concern with the reduction of the partial differential equations to an ordinary differential equation using any and all symmetries. The important question in this reduction process is "Can we find an optimal combination of symmetries to reduce the partial differential equation?". This is accomplished via the method of optimal subgroups. We apply the method to a particular partial differential equation that arises in general relativity which is given by $(u_t x)^2 - (u_t t + ku)(u_x x - ku) = 0$. We perform a comprehensive analysis of this equation so that we can find all the appropriate group invariant solutions to the partial differential equation.

SAMS subject classification: 7

SOLUTIONS OF SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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We establish Local and Global existence of solution to the constitutive equations for fibre suspension flows, for small data. The constitutive model investigated is that based on the use of orientation tensors, and these issues are examined in the context of a selection of closure approximations: the linear and quadratic closures [Advani SG and Tucker CL, Closure approximations for three-dimensional structure tensors. *Journal of Rheology* **34** (1990) 367–386.]. We implement a fixed point argument, using Schauder's Fixed Point Theorem, to show that a regular solution exists on small interval of time $(0, T^*)$. This solution satisfies an energy inequality, which implies its uniqueness in its class. For the global solution, some a priori bounds satisfied by the local solution are derived, which imply the existence of a unique solution defined for all time t. In the absence of body force that solution is exponentially stable.

SAMS subject classification: 11, 12, 15, 16

LOCAL CONNECTEDNESS OF FRAMES

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The notion of local connectedness of a frame is defined. Through the study of connected congruences of frames we obtain the Baboolal-Banaschewski local connectedness Theorem which says that the Stone-Cech compactification of a completely regular frame L is locally connected iff L is locally connected and pseudocompact. Furthermore, we show that an investigation of uniform local connectedness of uniform frames establishes the sufficiency part of this result.

SAMS subject classification: 5

A CONTROL PARAMETER ENHANCING TECHNIQUE APPROACH TO DISCRETE VALUED OPTIMAL CONTROL PROBLEMS

Colin Myburgh
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A computational method based on a control parametrization enhancing transform approach is developed for solving a class of discrete-valued control problems. These discrete-valued control problems usually have a lot of real-life applications, such as the optimal strategy of train driving and the on-off switch operation. Here, we solve a few real-life discrete-valued control problems to demonstrate the efficiency of our method.

SAMS subject classification: 12, 16

INDUSTRIAL MATHEMATICS SPECIAL SESSION: A (SMALL) POT-POURRI OF INDUSTRIAL MATHEMATICS PROBLEMS

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In this talk three problems from recent industrial mathematics study groups will be discussed. The first concerns the indentation of a porous media, as a test for fish freshness. The second is electrical heating and fluid flow through soil as a method of removing contaminants. The third is the flow and solidification of a thin fluid film, which has been applied as a model for ice accretion on aircraft and is currently employed in a commercial aircraft icing code.

In each case the industrial application will be described first. The mathematical model will then be discussed and finally it will be shown how the model may be used to assist in future industrial developments.

For details see the abstracts of D. Harris, D. Hatton, H. Laurie, O.D. Makinde, G.H. Maluleke, D.P. Mason, D. Mhlongo, and E. Momoniat.

SAMS subject classification: 11, 13, 17

THE FOUNDATION YEAR PROGRAMME IN MATHEMATICS AT THE UNIVERSITY OF PRETORIA

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The Foundation Year Programme in Mathematics and the Basic Sciences was started in 2001 in order to deal with the imbalance in student population at the University of Pretoria. The programme accepts Black students who show aptitude for Mathematics and Science but who did not perform well at school. We will discuss the broad aims of the Mathematics course in this programme and the curriculum chosen in accordance. Assessment techniques employed will also be reviewed. We will also discuss tracer studies conducted on the 2001 group.

SAMS subject classification: 18

THE BRIANCON-SKODA THEOREM AND THE COHEN-MACAULAYNESS OF REES MODULES

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The question of when the Rees ring or the associated graded ring of an ideal is Cohen-Macaulay, has been of central interest in commutative algebra. In S. Goto and Y. Shimoda (1979), the Cohen-Macaulay property of the Rees ring is related to the Cohen-Macaulay property of the associated grade ring. If I is an m-primary ideal, they state that if (R, m) is a Cohen-Macaulay local ring with infinite residue field and of dimension d, then the Rees algebra is Cohen-Macaulay if and only if $gr_I R$ is Cohen-Macaulay and the reduction number of I is at most $d-1$.Huneke(1993) has pointed out that the above result is related to the Briancon-Skoda Theorem and that one can delete the hypothesis about the reduction number in the statement above. In this paper, the above results are generalised to the Rees module $R(I, N)$ and the associated graded module $G(I, N)$, where N is a finitely generated R module and I an ideal of definition with respect to N. The tight closure of the ideal I with respect to N is defined. The definition of the integral closure of an ideal with respect to an ideal I is given and and a relationship beween the integral closure and the tight closure with respect to N is obtained.

SAMS subject classification: 2

SOLUTIONS BY SYMMETRY ANALYSIS OF THE PDE SATISFIED BY A DERIVATIVE DEPENDENT ON A FUTURES PRICE

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We determine some exact solutions of the second order PDE satisfied by a derivative dependent on futures prices. The equation is first reduced to an ODE, and then solved by specific techniques. Some of the symmetry properties of the solutions are translated into the properties of the derivative security as a financial instrument.

SAMS subject classification: 7, 11, 17

MALLIAVIN CALCULUS IN WHITE NOISE SPACE

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We use methods from nonstandard analysis to give a very simple construction of a space and show that its elements can be used to represent generalized Wiener functionals. This space is large enough to include the Kontratiev space and is flexible enough to handle generalized stochastic processes, stochastic differential equations and stochastic calculus. Moreover this space supports the Malliavin derivative, hence the Malliavin calculus can be formulated.

SAMS subject classification: 9, 10

SMOOTHLY PARACOMPACT SIKORSKI CW COMPLEXES

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The naive concept of a Sikorski CW complex is that of a Hausdorff Sikorski differential space that can be built up from cells and whose differential structure is defined in terms of differential structures on Euclidean unit closed balls. See [5] for differential spaces in the sense of Sikorski.

Our notion of Sikorski CW complex is related to the more familiar CW complex [4]concept in some important respects. In fact, the topology of a Sikorski CW complex defines a CW complex structure on the underlying space. The proof of this result can be found in [2]. The situation with regard to quotients is rather more complicated, because in general if (X, Y) is a Sikorski CW-pair, that is, X is a DW complex and Y is a subcomplex, X/Y may fail to be a Sikorski CW complex. However, if X is smoothly paracompact [1] and regular, and Y is compact, only then is X/Y a Sikorski CW complex.

Finally, as for ordinary CW complexes, there is an alternative description of Sikorski CW complexes as adjunctions of cells. But this way of describing Sikorski CW complexes is restrictive in the sense that it only works for finite Sikorski CW complexes. See [3].

SAMS subject classification: 5

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RADICALS AND CLASS PAIRS

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For near-ring ideal mappings ρ_1 and ρ_2, we investigate radical theoretical properties of and the relationships amongst the class pairs $(\rho_1 : \rho_2)$, $(\mathcal{S}_{\rho_2} : \mathcal{S}_{\rho_1})$ and $(\mathcal{R}_{\rho_2} : \mathcal{R}_{\rho_1})$. Conditions on ρ_1 and ρ_2 are given for a general class pair to form a radical class of various types. These types include the Plotkin and KA-radical varieties. A number of examples are shown to motivate the suitability of the theory of Hoehnke-radicals over KA-radicals when radical pairs of near-rings are studied. In particular, it is shown that $(\mathcal{P}_c : \mathcal{P}_3)$ forms a KA-radical class where $\mathcal{P}_c$ denotes the class of completely prime near-rings and $\mathcal{P}_3$ the class of 3-prime near-rings. This gives another near-ring generalization of the 2-primal ring concept. The theory of radical pairs are also used to show that, in general, the class of 3-semiprime near-rings is not the semisimple class of the 3-prime radical.

Keywords: preradical, radical, class pair, 3-prime, near-ring
SAMS subject classification: 2

NEW DEVELOPMENTS OF THE AMPLITUDE-SHAPE METHOD

Nabendra Parumasur and Mika, J.R.
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The Amplitude-Shape Method [1] transforms certain systems of stiff ordinary differential equations into subsystems consisting of a rapidly changing amplitude function and a slowly varying shape vector, respectively. In this paper we consider a new formulation of the amplitude and shape equations which is particularly well suited for certain classes of problems occuring in chemical kinetics. The resulting equations are treated with a mixed implicit-explicit scheme and displays a substantial saving in numerical effort when compared with classical solvers based on purely implicit schemes.

SAMS subject classification: 16

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A NONSTANDARD APPROACH TO THE HAUSDORFF DIMENSION

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We discuss a nonstandard approach to determining the Hausdorff dimension of compact subsets of [0,1] using a hyperfinite timeline. Some connections with the dimensions associated with Brownian paths are mentioned as well as an application to capacity.

SAMS subject classification: 10

TWO-COLOURABLE FINITE LINEAR SPACES

Lou Pretorius* (Joint work with Konrad Swanepoel)

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We determine the finite linear spaces with at most 15 points which admit two-colourings. We apply these results to a problem coming from the foundations of geometry:

(1) Does the Sylvester–Gallai property imply the Motzkin–Rabin property in linear spaces?

and to a problem coming from discrete geometry:

(2) Does the Motzkin–Rabin property hold in projective space over the complex numbers?

(In a linear space the Sylvester–Gallai property holds if for any finite noncollinear subset there is a line passing through exactly two points of this set, and the Motzkin–Rabin property holds if any finite noncollinear subset admits a two–colouring such that no line passing through at least two points of this set is monochromatic. Both properties hold in real projective space.)

Our results imply that if the answer to either question is negative, a counterexample would have at least 16 points.

SAMS subject classification: 4, 14

MAKING SENSE OF THE JOINT LAW OF A TWO-DIMENSIONAL BROWNIAN MOTION AND ITS LÉVY AREA PROCESS

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Paul Lévy first introduced the area integral of the planar Brownian motion, and calculated its distribution, 60 years ago. Unfortunately, the joint density of the area integral and the increment of the Brownian motion is in the form of an infinite integral with hyperbolic functions, and can only be calculated numerically. We will consider alternative (both analytical and numerical) approaches, with the goal of finding a more feasible formula for the joint law.

SAMS subject classification: 10, 16

CAI: AN ATTEMPT TO HELP DISTANCE EDUCATION STUDENTS "SEE" DIFFERENTIAL EQUATIONS

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The qualitative analysis of differential equations is a very tricky subject to teach in a distance education situation — the subject matter is dynamic, whereas even with the best pictures, the print-based tutorial matter can only be static! In an attempt to remedy this situation, UNISA has been developing a computer-aided instruction (CAI) package to help students understand and experiment with the concepts involved. In this talk, we will discuss the philosophy behind the lessons, the practical implementation, and future plans.

SAMS subject classification: 18

MIXED VARIATIONAL FORMULATIONS OF PROBLEMS IN ELASTICITY I: THE CONTINUOUS PROBLEM

J. Kamdem Djoko and B. Daya Reddy*

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A key challenge in the formulation of problems in elasticity, and in their approximation, is that of ensuring convergence in the incompressible limit. In the case of isotropic elasticity this is equivalent to establishing well-posedness and convergence of approximations in the limit as $\lambda \to \infty$, where λ is the Lamé parameter.

One means of circumventing problems of ill-posedness in the incompressible limit is to make use of mixed or saddle-point formulations. We consider one such formulation in which the unknown variables are displacement, stress and strain.

In the first part of a two-part presentation the continuous problem is shown to be well-posed, by recourse to an abstract result [1] on non-standard mixed problems. The second part of the presentation is devoted to finite element approximations. The conditions on the choice of spaces that will ensure convergence are established, and concrete examples of such choices are given. The presentation concludes with some numerical examples.

SAMS subject classification: 11, 16

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SOLUTIONS WITH TRANSITION LAYERS TO AN AUTONOMOUS SYSTEM OF ELLIPTIC EQUATIONS

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We consider the following system of semilinear elliptic equations

$$\begin{cases} -\varepsilon^2 \Delta u &= f(u) - v \quad \text{in} \quad \Omega; \\ -\Delta v &= \delta u - \gamma v \quad \text{in} \quad \Omega; \\ u &= v = 0 \quad \text{on} \quad \partial\Omega. \end{cases} \tag{3}$$

We assume Ω to be a smooth bounded domain in $\mathbb{R}^N$, with $N \geq 1$, and take $\varepsilon > 0$ and $\delta \geq 0$ as parameters. The nonlinearity is assumed to be of bistable type, the generic example being $f(u) = u(u-1)(a-u)$ with $0 < a < 1/2$. Motivation for studying this system comes, amongst other things, from its connection to morphogenesis. In particular we are interested in the existence of stable solutions with steep transition layers, both at the boundary and in the interior. In this presentation we shall discuss the existence and stability properties of such solutions.

SAMS subject classification: 9, 11, 12

ALGORITHMS FOR THE COMPUTATION OF EXACT MCD OF BIVARIATE SAMPLES

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Most inferential techniques in statistics assume that the data at hand are perfect, in the sense that no errors occurred during the acquisition thereof, and that all observations originate from a single, specified distribution. In practice it is often the case that spurious observations, or *outliers*, are present, which can wreck havoc on techniques not designed to accommodate them. While the identification and removal of univariate outliers is usually done by inspection of a plot of the data, this is not possible for multivariate data. *Robust* techniques have been developed to deal with this problem, the most famous being the Minimum Covariance Determinant or MCD, proposed by Rousseeuw (1984) as a robust estimator of location and shape of a multivariate distribution. This involves finding the subset, of a specified size, with the minimum determinant of its covariance matrix. An exhaustive search of all possible subsets is only feasible for small samples, so heuristic algorithms were developed for finding good approximate solutions. An overview of the work of Pesch (2000), who developed exact algorithms for the bivariate case that run in polynomial time, is presented. These provide valuable information on the efficacy of the heuristic algorithms.

SAMS subject classification: 10

[1]The financial assistance of the National Research Foundation (NRF) towards this research is hereby acknowledged.

BINARY CODES OF TRIANGULAR GRAPHS AND PERMUTATION DECODING

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We examine the binary codes obtained from the adjacency matrix of the triangular graph $T(n)$ for $n \geq 5$ and, by finding explicit PD-sets, we show that permutation decoding can be used for these codes.
This is a joint paper with J D Key (Clemson, USA) and J. Moori (University of Natal, Pietermaritzburg).

SAMS subject classification: 2, 14

TOTAL VARIATION AND SMOOTHNESS

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Information is processed, transmitted, stored and interpreted digitally everywhere. We speak of smoothness. For functions we have concepts of smoothness like Continuity, Differentially, Integrability etc, often derived from concepts of sequences, yet concepts of smoothness are required before we can use the accepted *measures* of smoothness like Total Variation. A new, natural, concept of smoothness is discussed and demonstrated to link consistently with large classes of smoothing operators in use, attributes of these, a complementary concept of pulse, as well as with Total Variation.

SAMS subject classification: 16

BANACH SPACE SEQUENCES AND PROJECTIVE TENSOR PRODUCTS II

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We extend recent work in [1] and [2] to the general setting of the projective tensor product of a Banach space U (which has a normalised unconditional basis) with an arbitrary Banach space X. Vector sequence space characterisations of such tensor products are considered. Using basic results from the theory of tensor products of Banach spaces, we establish the isometry of the so called space of strongly U-summable sequences in a Banach space X with the space of nuclear X-valued operators on U^*, as well as with the complete projective tensor product of U with X. We'll also discuss some results about U-summing multipliers. Examples where U is replaced by classical Banach spaces will be considered.

SAMS subject classification: 9

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MARKOV SEMIGROUPS AND THEIR APPLICATIONS

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A transformation $P : L^1 \to L^1$ is called a Markov operator if P is linear and $P(D) \subset D$, where D is the set of all densities. Markov operators and Markov semigroups occur in models of cellular populations [2, 6] and cell cycle [1]. If, for example, f is the desity of the distribution of the size of cells in one generation, then the density of the size of cells in the next generation is given by Pf, where P is a Markov operator. Time and size dependent models of populations can be described by a transport equation of the form

$$\frac{\partial u}{\partial t} - \frac{\partial (V(x)u)}{\partial x} = -u(x,t) + Pu(x,t). \tag{4}$$

Here the function $V(x)$ is the velosity of the growth of the size of a cell and P is a Markov operator describing the process of replication. Equation (1) generates a semigroup of Markov operators on $L^1(\mathbf{R})$.

We present some recent results concerning asymptotic behaviour of Markov operators and semigroups. We consider two properties: stability and sweeping. A Markov semigroup $\{P^t\}_{t\geq 0}$ is called *asymptotically stable* if there is an invariant density f_* such that

$$\lim_{t\to\infty} \|P^t f - f_*\| = 0 \quad \text{for} \quad f \in D.$$

A Markov semigroup $\{P^t\}_{t\geq 0}$ is *sweeping* with respect to a set Z if

$$\lim_{t\to\infty} \int_Z P^t f(x)\, dx = 0 \quad \text{for} \quad f \in D.$$

First we show that most of Markov semigroups satisfy Foguel alternative, i.e. they are asymptotically stable or sweeping with respect to bounded sets [7, 5]. Then we give some sufficient conditions for asymptotic stability of Markov semigroups. These conditions base on a new notion called a Hasminskii function [3]. We apply these results to study asymptotic stability and sweeping of Markov operators and semigroups connected with biological models [6].

SAMS subject classification: 10, 11, 17

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ELLIPTIC CURVES OVER FUNCTION FIELDS — A GROSS-ZEGIER FORMULA

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In this talk we consider elliptic curves which are defined over the rational function field over an finite field. We explain the notions of modularity, Heegner points and L-series and present a formula of Gross-Zagier type for these quantities.

SAMS subject classification: 2, 3

COMPLETE CONTINUITY PROPERTIES OF BANACH SPACES ASSOCIATED WITH SUBSETS OF A DISCRETE ABELIAN GROUP

PAULETTE SAAB

University of Missouri-Columbia, Potchefstroom University and the University of Pretoria.

In this talk we discuss the types I-, II-, and III-Λ-complete continuity property of Banach spaces, where Λ is a subset of the dual group of a compact metrizable abelian group G.

SAMS subject classification: 9

STABILITY PROBLEMS OF SOME PROPERTIES IN BANACH SPACES

ELIAS SAAB

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If E is a Banach space, $(\Omega, \Sigma, \lambda)$ a probability space and $1 \leq p < \infty$, we denote by $L_p(E)$ the space of p-Bochner integrable functions from Ω to E equipped with the norm $||f||_p = (\int_\Omega ||f(\omega)||^p d\lambda(w))^{\frac{1}{p}}$. If $p = +\infty$, then $L_\infty(E)$ will stand for the space of all measurable and essentially bounded functions from Ω to E equipped with the norm $||f||_\infty = \text{ess sup}\, ||f(\omega)||$.

If K is a compact Hausdorff space, $C(K, E)$ will denote the Banach space of all E-valued continuous functions on K endowed with the uniform norm. In case E is the scalar field then $L_p(E)$ will be denoted by L_p, and $C(K, E)$ will be denoted by $C(K)$.

In this talk we discuss the following problem: Given a Banach space E that has a certain property (P) and suppose that L_p or $C(K)$ has this property. We would like to know when does $L_p(E)$ or $C(K, E)$ have the property (P).

The properties that we would like to concentrate on are the properties (u), (V) and (V^*) of Pełczynski, the Dunford-Pettis property, the embedding of c_0, ℓ_1 and L_1 in E as subspaces or as complemented subspaces. Some other related properties will also be discussed. We will also state some problems that are still open.

SAMS subject classification: 9

EXISTENCE OF SOLUTIONS FOR A DOUBLY DEGENERATE STOCHASTIC ECOLUTION PROBLEM

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Let D be a bounded domain in the Euclidean space $\mathbf{R}^n (n \geq 2)$ with a sufficiently smooth boundary ∂D. For $0 < T < \infty$, we denote by Q_T the cylinder $D \times (0, T)$. Let $\left(\mathbf{\Omega}, \mathbf{F}, \{\mathbf{F}_t\}_{0 \leq t \leq T}, \mathbf{P}\right)$ be a standard filtered probability space on which is defined the one-dimensional Wiener process $W(t)$, and denote by ω the sample points from $\mathbf{\Omega}$. In $Q_T \times \mathbf{\Omega}$, we investigate the initial boundary value problem for the doubly degenerate quasilinear stochastic parabolic equation

$$\begin{aligned} & d\left(|y|^{\alpha-2} y\right) - \left[\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(\left|\frac{\partial y}{\partial x}\right|^{p-2} \frac{\partial y}{\partial x_i}\right) - c_1 |y|^{2\mu-2} y\right] dt \qquad (5) \\ = \; & c_2 |y|^{\sigma-1} y dW, \text{ in } Q_T \times \mathbf{\Omega}, \end{aligned}$$

$$y(x, t, \omega) = 0 \text{ on } \partial D \times (0, T) \times \mathbf{\Omega}, \qquad (6)$$

$$y(x, 0, \omega) = y_0(x, \omega) \text{ in } D \times \mathbf{\Omega}, \qquad (7)$$

where c_1, c_2, α, μ, σ and p are some nonnegative numbers satisfying appropriate restrictions. Using the splitting-up method, we establish the existence of a solution of the problem (5)-(7).

In the deterministic case (i.e., $c_2 = 0$), equations of this type model processes ranging from the theory of non newtonian fluids to population dynamics. Initial boundary value problems of equation (5) are naturally expected to describe some stochastic counterparts of the deterministic known models.

SAMS subject classification: 10, 11

COVERING AND SEPARATION PROPERTIES FOR BIFRAMES

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Since any quasi-uniform space has two natural underlying topologies, bitopological spaces arise frequently when such asymmetric notions are discussed. A variety of different definitions for such basic concepts as compactness, sobriety and regularity have appeared in the literature. We discuss such properties in the setting of biframes: these form an analogue of bispaces in the more lattice-theoretic and constructive approach to topology that is frame theory.

SAMS subject classification: 2, 5

RAMSEY AND RAINBOW COLOURINGS

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In this talk we consider edge colourings of graphs. For given graphs $F_1, F_2, \ldots, F_k$, $k \geq 2$, the ramsey number $r(F_1, \ldots, F_k)$ is the smallest integer n such that if we arbitrarily colour the edges of the complete graph of order n with k colours, then there is always a monochromatic copy of some F_i for $1 \leq i \leq k$. We will list the ramsey numbers if the graphs F_i are complete or cycles and report about recent progress on some conjectures of Erdős.

For given graphs G, H the rainbow number $rb(G,H)$ is the smallest number m of colours such that if we colour the edges of G with at least m different colours, then there is always a totally multicoloured or rainbow copy of H. We will list the known rainbow numbers if G is the complete graph and report about recent progress on the conjecture of Erdős, Simonovits and Sós on the rainbow numbers $rb(K_n, C_k)$ for cycles. Finally, new results on the rainbow numbers $rb(Q_n, Q_2)$ for the hypercube Q_n will be presented.

SAMS subject classification: 14

PACKING BOXES DENSELY AND DISJOINTLY

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This is a paper in computational general topology. We effectively construct in the Hilbert cube $\mathbb{H} = [0,1]^\omega$ two sets $V, W \subset \mathbb{H}$ with the following properties:

a) $V \cap W = \emptyset$

b) $V \cup W$ is discrete-dense, i.e. dense in ${[0,1]_D}^\omega$, where $[0,1]_D$ denotes the unit interval equipped with the discrete topology

c) V,W are open in $\mathbb{H}$. In fact, $V = \bigcup_{\mathbb{N}} V_i$, $W = \bigcup_{\mathbb{N}} W_i$, where $V_i = \bigcup_0^{2^{i-1}-1} V_{ij}$, $W_i = \bigcup_0^{2^{i-1}-1} W_{ij}$. V_{ij}, W_{ij} are basic open sets and $(0,0,0,\ldots) \in V_{ij}$, $(1,1,1,\ldots) \in W_{ij}$

d) $V_i \cup W_i, i \in \mathbb{N}$ is point symmetric about $(1/2, 1/2, 1/2, \ldots)$

Instead of $[0,1]$ we could have taken any T_4-space or a digital interval, where the resolution (number of points) increases with i. More information is contained on the web-site www16.brinkster.com/jodis

SAMS subject classification: 5, 14, 16

MAXIMAL NONTRACEABLE GRAPHS

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A graph G is maximal nontraceable if G is not traceable, i.e. G does not contain a Hamiltonian path, but $G + e$ does contain a Hamiltonian path for all $e \in E(\overline{G})$. We present a lower bound for the number of edges of maximal nontraceable graphs of order n.

SAMS subject classification: 14

EXISTENCE AND UNIQUENESS OF A LOEB MEASURE UNDER WHICH THE STOCK PRICE IS A MARTINGALE IN A COMPLETE MARKET

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We investigate and (try to) characterise the existence and uniqueness of a *Loeb* measure under which the stock price is a martingale in a complete market *hypermodel*.

SAMS subject classification: 10

ANOTHER LOOK AT RELATIVE LOCAL CONNECTEDNESS

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We say an onto frame homomorphism $h : L \to M$ is *locally connected relative to* L if the set

$$\{a \in L \mid h(a)\text{connected}\}$$

is a basis for L. In [2], we referred to this notion of connectedness as "*local connectedness with respect to along*". Baboolal and Banascheski [1] proved that dense homomorphisms preserve connectedness and that under suitable conditions the right adjoint $h_* : M \to L$ reflects connectedness. In this talk, we present other conditions under which connectedness is preserved and reflected.

SAMS subject classification: 5

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THE OPTIMAL SYNTHESIS OF A PARALLEL MANIPULATOR VIA THE MINIMIZATION OF THE CONDITION NUMBER OF THE ASSOCIATED JACOBIAN MATRIX

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A fundamentally sound and robust numerical methodology for synthesizing a parallel manipulator for desired workspaces is presented. The procedure includes the determination of the workspace of the manipulator, for a given fixed set of design variables, via a mathematical optimisation algorithm. In addition a constrained optimization formulation is proposed that is aimed at determining a manipulator design so that a prescribed workspace is not only fully enclosed, but that the workspace is also well-conditioned with respect to the condition number of the Jacobian matrix

of the manipulator. The particular manipulator used to illustrate and evaluate the formulation is a simple planar 3-degree of freedom parallel manipulator. Solutions to the manipulator design problems for different prescribed workspaces are found in an efficient and convincing manner.
Keywords: parallel manipulator, workspace, optimization, condition number
SAMS Subject Classifications: 16, 17

WEAK MIXING PROPERTY FOR $C^\star$-DYNAMICAL SYSTEMS

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We introduce mixing properties for $C^\star-$dynamical systems and provide conditions for which weak-mixing implies weak-mixing of all ordes. For classical dynamical systems this result is part of a very deep ergodic theory of multiple recurrence developed by Furstenberg. The results presented form part of a programme to extend classical ergodic theorems to $C^\star-$dynamical system.

SAMS subject classification: 9

THE NUMBER OF UNIT DISTANCES BETWEEN n POINTS ON A SPHERE

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One of the oldest and most difficult questions in combinatorial geometry was asked by ErdHos in 1946: Show that the maximum number of unit distances possible between n distinct points in the plane is at most $O(n^{1+\varepsilon})$ for any $\varepsilon > 0$. It is still far from solved, with the best known upper bound $O(n^{4/3})$ already standing for 20 years.

The same question can be asked for many other metric spaces. For example, in 1966 L. Moser conjectured that the maximum number of unit distances on a sphere in $\mathbf{R}^3$ is $O(n)$. This was disproved by ErdHos, Hickerson and Pach in 1989, who showed that this is at least $\Omega(n \log^* n)$ for spheres of diameter greater than 1, where $\log^* n$ is the iterated logarithm function, i.e., the smallest k for which the sequence a_k defined by $a_0 = n, a_{k+1} = \log\max\{a_k, 1\}$, becomes 0. Since $\log^* n$ grows very slowly, their lower bound is only slightly superlinear. We improve this lower bound to $\Omega(n\sqrt{\log n})$.

This is joint work with Pavel Valtr, Department of Applied Mathematics, Charles University, Prague.

SAMS subject classification: 4, 14

CREDIT RISK MODELS

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We investigate and compare the Merton model and the Shimko- Tejima- van Deventer model for the evaluation of credit risk. These models lead to partial differential equations which can be solved to obtain the credit spreads for risky bonds.

In the Merton model a constant short-term interest rate is assumed whereas in the model by Shimko et al. we consider an interest rate driven by a stochastic differential equation. The models are also applied to South African data.

SAMS subject classification: 11, 17

ON EXCELLENT GRAPHS

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For a graphical parameter P, a graph G is said to be P-excellent if every vertex of G is contained in some set that attains the optimal value of P. P - excellence is considered for selected classes of graphs, where P is the independence number, domination number or strong domination number of a graph.

SAMS subject classification: 14

ON DETERMINANTS IN BANACH ALGEBRAS

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SAMS subject classification: 9

ON INERTIAL MANIFOLDS FOR THE PROBLEM OF AGEOSTROPHIC FLOWS WITH BETA-PLANE APPROXIMATION

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Consider the paradigm example for system of partial differential equations

$$\begin{aligned}\frac{\partial u}{\partial t} + \underline{u}.\bigtriangledown u + (\frac{\beta}{Ro} - y)w - (\frac{1}{Ro} + \beta y)v + \frac{Ek}{Ro}\bigtriangleup^6 u &= -\frac{1}{Ro}\frac{\partial p}{\partial x} + \frac{Ek}{Ro}\bigtriangleup u\\ \frac{\partial v}{\partial t} + \underline{u}.\bigtriangledown v + (\frac{1}{Ro} + \beta y)u + \frac{Ek}{Ro}\bigtriangleup^6 v &= -\frac{1}{Ro}\frac{\partial p}{\partial y} + \frac{Ek}{Ro}\bigtriangleup v\end{aligned}$$

$$\begin{aligned}
\frac{\partial w}{\partial t}+\underline{u}.\bigtriangledown w-(\frac{\beta}{Ro}-y)u+\frac{1}{Ro}+\rho+\frac{Ek}{Ro}\triangle^{6}w &= -\frac{1}{Ro}\frac{\partial p}{\partial z}+\frac{Ek}{Ro}\triangle w\\
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} &= 0\\
\frac{Fr^2}{Ro}(\frac{\partial \rho}{\partial t}+\underline{u}.\bigtriangledown \rho)+w+\frac{Fr^2}{Ro(Ed)}\triangle^{6}\rho &= \frac{Fr^2}{Ro(Ed)}\triangle \rho,
\end{aligned}$$

where the parameter Ro is the Rossby number, Fr is the Froude number, Ek is the Ekman number, β is reference reciprocal Coriolis parameter and Ed is the nondimensional eddy diffusion coefficient which is equal to the eddy diffusivities for heat and salt budget. First, we illustrate that the above system of partial differential equations prescribed appropriate boundary conditions is equivalent to functional differential equations of the form

$$\begin{aligned}
\frac{dp}{dt}+Lp+(\Pi_n)N(U) &= 0\\
\frac{dq}{dt}+Lq+(I-\Pi_n)N(U) &= 0
\end{aligned}$$

where

$$\Pi_n=\frac{1}{2i\pi}\int_{\Gamma}(\eta I-L)^{-1}d\eta$$

is the spectral projection of rank n, N is nonlinear, $p(t)=\Pi_n U(t)$ and $q(t)=(I-\Pi_n)U(t)$. The utility of exponential dichotomies of variational equations and the virtue of a priori estimates give existence proof for the normally hyperbolic invariant manifold.

SAMS subject classification: 11, 12

NONLINEAR INSTABILITY AND STABILITY OF AGEOSTROPHIC FLOWS WITH VISCOELASTIC-TYPE REYNOLDS STRESS

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Consider the prototype system of partial differential equations governing ageostrophic flows with Cushman-Roison constitutive relation for Reynolds stress:

$$\begin{aligned}
\frac{\partial \underline{u}}{\partial t}+\underline{u}.\bigtriangledown\underline{u}-\frac{g}{\rho_o}\rho\underline{k} &= -\frac{1}{\rho_0}\bigtriangledown p+\nu\triangle\underline{u}+\frac{1}{\rho_0}\bigtriangledown .\tau\\
\bigtriangledown .\underline{u} &= 0\\
\frac{\partial \rho}{\partial t}+\underline{u}.\bigtriangledown\rho &= \kappa\triangle\rho\\
\frac{\partial \tau}{\partial t}+\underline{u}.\bigtriangledown\tau+\tau\Omega-\Omega\tau-\tau D-D\tau+\tau &= 2\mu D
\end{aligned}$$

where $\underline{u}$ is velocity of the fluid, τ is Reynolds stress tensor, Ω is vorticity tensor, D is deformation tensor, ρ is deformation tensor and $\underline{f}$ is the earth's rotation. Here the parameter ν is kinematic viscosity, μ is dynamic viscosity, κ is effective eddy diffusivity and g is gravitation. We elucidate nonlinear instability and stability of the rest points for the above system of partial differential equations prescribed appropriate boundary and initial conditions. Our approach will be to transform the system into an ordinary differential equation in an infinite- dimensional phase space and give existence proof for periodic solutions. Abreast of these results, we investigate normally hyperbolic invariant manifolds in the Ed Lorenz ansatz sense. The splendid and elegant techniques of R Camassa *et al* are utilized.

SAMS subject classification: 11, 12

ON SPECTRAL TORSION THEORIES

Alphonse Uworwabayeho
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Let R be an associative ring with unit and $Mod-R$ the category of unitary right R-modules. Recently the lattice of torsion theories on $Mod-R$ has been used to get information about the ring R, its localization ring R_τ at the torsion theory τ and their categories of modules. The purpose of the talk is to introduce the notion of a *spectral* torsion theory and give some characterizations of the ring R and its ring of quotients with respect to τ.

SAMS subject classification: 2

THE PATH PARTITION CONJECTURE FOR DIRECTED GRAPHS

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Let $\vec{\tau}(G)$ denote the number of vertices in a longest directed path of a digraph G. The following conjecture (the DPPC) was first introduced by Bondy in [1]:

For every digraph G, the vertex set $V(G)$ can be partitioned into two subsets A and B such that $\vec{\tau}(G(A)) \leq a$ and $\vec{\tau}(G(B)) \leq b$ for all pairs of positive integers (a, b) such that $\vec{\tau}(G) = a + b$.

We will give a brief overview of this conjecture and the relation between the DPPC, the Path Partition Conjecture for undirected graphs (PPC) and the Path Partition Conjecture of oriented graphs (OPPC).

SAMS subject classification: 14

References

1 J.A. Bondy, Handbook of Combinatorics, eds., R.L. Graham, M. Grötschel, and L. Lovász, The MIT Press, Cambridge, MA, Vol I.

RADICAL IDEALS IN CALUATINO DOMAINS

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SAMS subject classification: 2

XTR PUBLIC KEY SYSTEM

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My Masters thesis will be on XTR public key system introduced by A.K. Lenstra and E.R. Verheul in 2000. It is a variant of the Diffie-Hellman key agreement protocol. I discuss the basic underlining ideas. The main difference between these systems, is the use of the trace function which reduces the number of bits to be exchanged by by a factor 3. This decrease in mount of bits used also reduce the cost of calculations.

SAMS subject classification: 2, 15

A LOWER BOUND ON THE LOTTERY NUMBER $L(49, 6, 6; 3)$

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Suppose a lottery scheme consists of randomly selecting a winning t–set from a universal m–set, while a player participates in the scheme by purchasing a playing set of any number of n–sets from the universal set prior to the draw, and is awarded a prize if k or more numbers in the winning t–set match those of at least one of the player's n–sets in his playing set ($k \leq t, n \leq m$). This talk is devoted to the problem of finding lower bounds on the minimum number of n–sets that a player should play in order to guarantee him/her a prize. Although the methods employed may be generalised, the presentation will focus on the special case where $m = 49$, $t = n = 6$ and $k = 3$ (which coincides with the South African National Lottery, *Lotto*).

SAMS subject classification: 14

SCHACHERMAYER'S PROOF OF THE DISCRETE-TIME ARBITRAGE THEOREM

GUSTI VAN ZYL

The talk gives an overview of my MSc dissertation. The dissertation deals with the proof of one major result, the discrete time arbitrage theorem (Dalang-Morton-Willinger [DMW]). It states that a given discrete time stochastic process (on a general probability space) has the no-arbitrage property if and only if there is an equivalent measure under which the process is a martingale. The sufficiency part of this statement is easy to show, but proving the necessity of existence of the martingale measure requires that a certain set $K - L^0_+$, which represents the possible final values of the portfolio less a random nonnegative amount, be shown to be closed in probability.

To do this, the space $L^0(\Omega, \mathcal{F}, \mathcal{P}; \mathbb{R}^\lceil)$ of all $\mathbb{R}^d$-valued measurable functions is decomposed into two subspaces. These spaces are analysed by Hilbert space (as subspaces of $L^2(\Omega, \mathcal{F}, \mathcal{P}; \mathbb{R}^\lceil)$) orthogonality techniques, to get conclusions which can be extended to L^0. Finally, to show that the above-mentioned set is closed, a substitute for sequential compactness (which is unavailable here due to the generality of the space L^0) is given.

The proof presented here follows that of Schachermayer [Sch]. (For the continuous-time analogue of this result, see [DeSc].)

SAMS subject classification: 9

References

[DMW] R.C. Dalang, A. Morton & W. Willinger *Equivalent martingale measures and no-arbitrage in stochastic securities market models.* Stochastics and Stochastics Reports 29 (1990) pp. 185-201
[DeSc] F. Delbaen & W. Schachermayer *A general version of the fundamental theorem of asset pricing* Math. Ann. 300 (1994), pp.463-520
[Sch] W. Schachermayer *A Hilbert space proof of the fundamental theorem of asset pricing in finite discrete time* Insurance: Mathematics and Economics 11 (1992) pp. 249-257.

THE IMPACT OF MATHEMATICAL COMPETITIONS AND OLYMPIADS IN PROMOTING MATHEMATICS

John Webb
University of Cape Town.

It is internationally recognized that large-scale mathematical competitions are by far the most effective way of promoting interest in mathematics. This presentation will examine the impact of mathematical competitions and Olympiads in South Africa, with particular emphasis on two projects:

- the university of Cape Town Mathematics Competition, a regional event in the Western Cape
- the Old Mutual Mathematical Talent Search, a national programme which leads to the selection of teams to represent South Africa in the Pan African Mathematics Olympiad (PAMO) and the International Mathematical Olympiad (IMO)

Participation

A simple measure of the impact of a mathematics competition is the number of participants. The impact of an event is greatly improved by build-up, follow-up and back-up.

Build-up

- ensuring that schools are aware of forthcoming events, through publications and by mail, fax and internet
- maintaining good contact with mathematics teachers, encouraging teachers to enter their pupils and to run training programmes in their schools
- drawing teachers into the organization of the event

Follow-up

- identification of the most promising pupils
- recognizing their achievement with certificates, prizes, bursaries and early offers of university admission
- offering them opportunities for further achievement, especially participation in regional and international events
- issuing invitations to consistent high achievers to attend mathematical camps, from which teams for international competitions are selected

- obtaining media coverage

Back-up

- publication of inexpensive publications (training manuals, sets of problems from previous years)
- websites
- efficient administrative support, creating confidence in the event

Frequently asked questions

- Do boys do better than girls in maths Olympiads?
- Are the top achievers in Olympiads attracted to careers in science and mathematics?
- Are the South African PAMO and IMO teams "representative"?
- Why should a productive research mathematician get involved in Olympiads?

SAMS subject classification: 18

THE ACCURACY OF CHEBYSHEV DIFFERENCING FOR ANALYTIC FUNCTIONS

Andre Weideman

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The Chebyshev collocation method is one of the most accurate methods for numerical differentiation, particularly when the function under consideration is smooth. When the function is moreover analytic, the speed of convergence is exponential. In this case the truncation error can be estimated by means of a contour integral. The speaker will derive this error estimate, and present applications to the numerical solution of differential equations of boundary-value type.

SAMS subject classification: 8, 16

INVERTIBILITY PRESERVING LINEAR MAPS BETWEEN BANACH ALGEBRAS

Martin Weigt

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Let $\mathcal{A}$ and $\mathcal{B}$ be unital complex Banach algebras and T a linear map from $\mathcal{A}$ into $\mathcal{B}$ with the property that Tx is invertible in $\mathcal{B}$ for every invertible element x in $\mathcal{A}$. We examine an unsolved problem posed by I. Kaplansky: What conditions on $\mathcal{A}$, $\mathcal{B}$ and T imply that $Tx^2 = (Tx)^2$ for every $x \in A$? The problem was motivated by the Gleason-Kahane-Żelazko Theorem and a result of Marcus and Purves.

SAMS subject classification: 9

PERTUBATION THEORY AND STABILITY OF THE INDEX OF MULTIVALUED SEMI-FREDHOLM OPERATORS WITH GENERALISED INVERSES

Diane Wilcox

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We give several perturbation theorems for $\alpha-$Atkinson and $\beta-$Atkinson relations, where a map $T : X \to Y$ is an Atkinson relation between normed linear spaces if T is a multivalued semi-Fredholm operator such that $\overline{R(T)}$ and $N(T)$ are topologically complemented. In particular, we give positive results and discuss the limitations of stability theorems for Atkinson type properties and invariance of the index of such operators under perturbation by bounded, compact, strictly singular and strictly cosingular perturbation.

SAMS subject classification: 9

THE CLASSICAL GRAPH CROSSING NUMBER PROBLEM

Wynand Winterbach

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Two algorithms to find the crossing number of a graph are discussed. The first is branch & bound brute force algorithm which is of theoretical interest, since the problem is NP-complete. The second is an approximation algorithm which is conceptually very simple. The algorithm functions by embedding a graph into a drawing of a larger graph. It then employs optimisation techniques such as Tabu searching on the embedding to derive upper bounds for the crossing number. Finally some conjectures which would imply an increase in computational efficiency for the algorithms are discussed.

SAMS subject classification: 14

MORPHISMS OF MISLIN GENERA INDUCED BY FINITE NORMAL SUBGROUPS

Peter Joseph Witbooi* and P. J. Hilton

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For any group G, by $\chi(G)$ we denote the set of all isomorphism classes of groups H such that $H \times \mathbf{Z} \simeq G \times \mathbf{Z}$. If G is finitely generated with finite commutator subgroup, then $\chi(G)$ has a group structure, see [1]. For a suitable finite normal subgroup F of G, we prove that there is an induced epimorphism of groups $\chi = (G) \to \chi(G/F)$. The significance of this result is demonstrated in [2, Example 5.4].

SAMS subject classification: 2

References

[1] P. J. Witbooi, Generalizing the Hilton-Mislin genus group, J. Algebra 239(1) (2001) 327-339.

[2] P. J. Witbooi, Non-cancellation for groups with non-abelian torsion, (under submission).

SEMI-INFINITE PROGRAMMING APPROACH TO NONLINEAR OPTIMAL CONTROL PROBLEMS

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Consider the class of nonlinear optimal control problems with continuous linear constraints. This class of problems is difficult to solve numerically. Here, a computational method based on a semi-infinite programming approach is considered. This can be done by considering the control as the decision variable while taking the state as a function of the control. After parametrizing the control, an approximate semi-infinite nonlinear programming problem is obtained. To solve this approximate problem, we use dual parametrization method. The dual problem is a max-min problem, which can be solved by optimizing over two levels. The upper level can be solved by simulated annealing and the lower level can be soled by using any unconstrained optimization technique, such as the Quasi-Newton method.

SAMS subject classification: 16

THE PERMUTATION ACTION OF THE SYMMETRIC GROUPS S_n ON ELEMENTARY ABELIAN GROUPS m^n

Kenneth Zimba

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Let Z^+ be the set of non-negative integers. Let $n > 0,\ m \in Z^+$. Here we discuss the permutation actions of symmetric groups S_n on elementary abelian groups m^n and their irreducible ordinary characters from a combinatorics point of view. This information is useful in the construction of the Fischer-Clifford matrices of generalized symmetric groups $m^n{:}S_n$ or $B(m, n)$, and some related groups. As examples, the action of S_5 on the group 3^5 and the action of S_6 on the group 2^6 are discussed.

SAMS subject classification: 16

References

[1] Almestady Mohammed S.O.; The Fischer matrices of the Generalized Symmetric Groups, PhD Thesis, University of Wales, Aberystwyth, Uk (1998).

[2] Cannon, J.J., An introduction to the Group Theory Language Cayley, Computational Group Theory (Academic press, San Diego (1984).

[3] Gorenstein D.; Finite groups, Harper and row publishers, New York (1968).

[4] Stanley R.P..; Enumerative Combinatorics, Vol. 1, Cambridge Studies in Advanced Mathematics **49**, CUP, Cambridge 1997.

[5] Zimba K.; The Fischer-Clifford matrices of the Generalized Symmetric Group $B(2, 6)$, preprint, unin. (October 2000).

[6] Zimba K.; The Fischer-Clifford matrices of the Generalized Symmetric Group $B(3, 5)$, preprint, unin. (October 2001).

MATHEMATICA, A TOOL FOR RESEARCH AND TEACHING

Clemens A. Dempers (Product Manager)
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Technical, Scientific and Business Software Solutions

Mathematica, a high level symbolic, numerics and graphics programming language was released in the mid 80's by a US company Wolfram Research Inc. Today it is used world wide as a teaching and research tool in a wide range of disciplines. This talk will focus on some of the mathematical research applications as well as the use of this tool for teaching and stimulating interest in mathematics. Some illustrative calculations will be performed in real time.

SAMS subject classification: 18

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