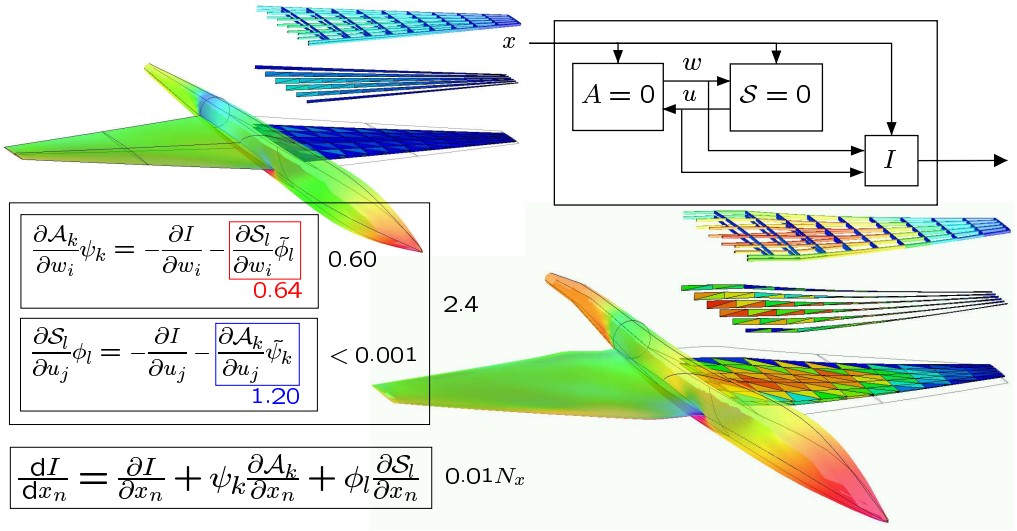
University of Toronto Institute for Aerospace Studies

AER1415
Optimization Concepts and Applications



Joaquim R. R. A. Martins

Course Information

Schedule: Tuesdays 2–4pm

Pre-requisites: advanced calculus and linear algebra, programming (Python [1] or Matlab preferred).

Grade will be based on 6 problem sets. Due dates:

Problem set 1:	January 27^{th}
Problem set 2:	February 10^{th}
Problem set 3:	February 24^{th}
Problem set 4:	March 9^{th}
Problem set 5:	March 23^{rd}
Problem set 6:	April 6^{th}

Handouts and problem sets will be available in the class webpage:
`http://mdolab.utias.utoronto.ca/aer1415.html`

Recommended books and articles will be given for each lecture . . . but feel free to find additional material!

Course Outline

1. Introduction

   1.1 What is "MDO"?
   1.2 Terminology and Problem Statement
   1.3 Classification of Optimization Problems
   1.4 Methods of Solution
   1.5 Practical Applications

2. Single Variable Optimization

   2.1 Optimality Conditions
   2.2 Line Search Methods

3. Gradient-Based Optimization

   3.1 Optimality Conditions
   3.2 Steepest Descent and Conjugate Gradient Methods
   3.3 Quasi-Newton Methods

4. Handling Constraints

   4.1 Karush–Kuhn–Tucker (KKT) Conditions
   4.2 Penalty and Barrier Methods
   4.3 Reduced Gradient and Gradient Projection Methods
   4.4 Sequential Quadratic Programming (SQP)
   4.5 Constraint Agglomeration

5. Sensitivity Analysis

   5.1 Finite Differences
   5.2 Complex-Step Derivative Approximation
   5.3 Algorithmic Differentiation
   5.4 Semi-Analytic Methods

6. Gradient-Free Optimization

   6.1 Genetic Algorithms and Pareto Optimality
   6.2 Collective Intelligence
   6.3 Nelder–Mead Simplex
   6.4 Trust Region Methods
   6.5 Response Surfaces and Design of Experiments

7. MDO Architectures

7.1 Collaborative Optimization (CO)
7.2 Concurrent Subspace Optimization (CSSO)
7.3 Bi-Level Integrated System Synthesis (BLISS)
7.4 Coupled-Sensitivity Analysis

1 Introduction

1.1 What is "MDO"?

Engineering Design Optimization

The "DO" in MDO.

In industry, problems routinely arise that require making the best possible design decision.

However, optimization is still underused in industry. . .

Aerospace is one of the leading applications of engineering design optimization. Why?

Conventional Versus Optimum Design Process

Conventional	Optimal
1. Specification	1. Specification
2. Baseline design	2. Baseline design
3. Analysis (or experiment)	3. Analysis
4. Check performance or failure criteria	4. Check constraints
5. Is design satisfactory? (If yes, then stop)	5. Does design satisfy the optimality conditions? (If yes, then stop)
6. Change design parameters based on intuition and heuristics, return to 3.	6. Change design parameters using an optimization strategy, return to 3.

Multidisciplinary Design Optimization (MDO)

Most modern engineering systems are *multidisciplinary* and their analysis is often very complex, involving hundreds computer programs, many people in different locations. This makes it difficult for companies to manage the design process.

In the early days, design teams tended to be small an were managed by a single chief designer who knew most about the design details and could make all the important decisions.

Modern design projects are more complex and problem has to be decomposed and each part tackled by a different team. The way these teams should interact is still being debated by managers, engineers and researchers [3, 2, 5, 10].

1.2 Terminology and Problem Statement

Objective Function

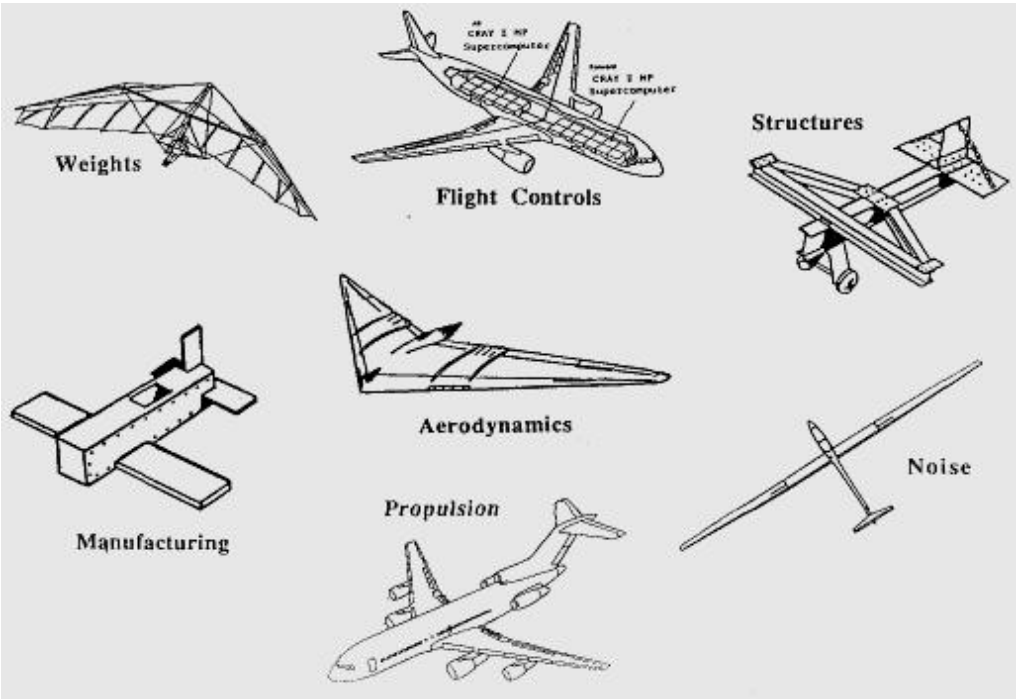
What do we mean by "best"?

Objective function is a "measure of goodness" that enables us to compare two designs quantitatively. Need to be able to estimate this measure numerically...

If we select the wrong goal, it doesn't matter how good the analysis is, or how efficient the optimization method is. Therefore, it's really important to select a good objective function. Underrated.

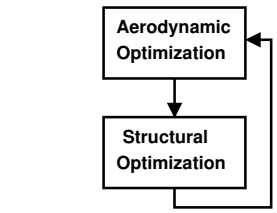
Objective function may be linear or nonlinear and may or not be given explicitly. We will represent it by the scalar f.

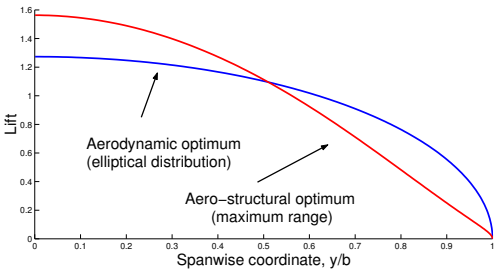
Is there *one* aircraft which is the fastest, most efficient, quietest, most inexpensive?

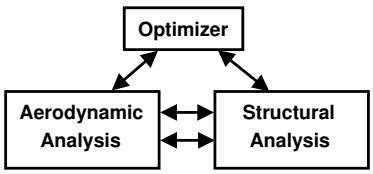


"You can only make one thing best at a time."

Example: Trade-off Between Aerodynamics and Strutures







Sequential optimization does not lead to the true optimum.

Aero-structural optimization requires coupled sensitivities.

Add structural element sizes to the design variables.

Including structures in the high-fidelity wing optimization will allow larger changes in the design.

Design Variables

Design variables are also known as design parameters and will be represented by the vector x. They are the variables in the problem that we allow to vary in the design process.

Optimization is the process of choosing the design variables that yield an optimum design.

Design variables should be as independent of each other as possible.

Design variables can be *continuous* or *discrete*. Discrete variables are sometimes *integer variables*.

Constraints

Few practical engineering optimizations problems are unconstrained.

Constraints on the design variables are called *bounds* and are easy to enforce.

Like the objective function, constraints can be linear or nonlinear and may or not be given in an explicitly form. They may be *equality* or *inequality* constraints.

At a given design point, constraints may be *active* of *inactive*. This distinction is particularly important at the optimum.

Optimization Problem Statement

$$
\begin{array}{rll}
\text{minimize} & f(x) & \\
\text{by varying} & x \in \mathbb{R}^n & \\
\text{subject to} & h_p(x) = 0, & p = 1, 2, \ldots, N_h \\
& g_m(x) \geq 0, & m = 1, 2, \ldots, N_g
\end{array}
$$

f : objective function, output (e.g. structural weight).

x : vector of design variables, inputs (e.g. aerodynamic shape); bounds can be set on these variables.

h : vector of equality constraints (e.g. lift); in general these are nonlinear functions of the design variables.

g : vector of inequality constraints (e.g. structural stresses), may also be nonlinear and implicit.

Classification of Optimization Problems

Constraints: constrained vs. unconstrained

Design Variables: single-variable vs. multivariable; continuous vs. discrete.

Linearity: linear objective function and linear constraints ⇒ linear programming (LP).
If either objective of constraints are nonlinear functions ⇒ nonlinear programming (NP).

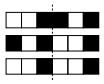
Time: dynamic vs. static optimization.

Data: deterministic vs. stochastic optimization.

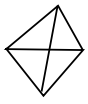
1.3 Optimization Methods

Intuition: decreases with increasing dimensionality.

Grid or random search: the cost of searching the design space increases rapidly with the number of design variables.

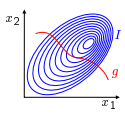


Genetic algorithms: good for discrete design variables and very robust; but infeasible when using a large number of design variables. Multi-objective optimization.



Nelder–Mead algorithm: simple and robust but inefficient for more than a few design variables.

Response surfaces: good for noisy functions, still requires a large number of evaluations to create fit.

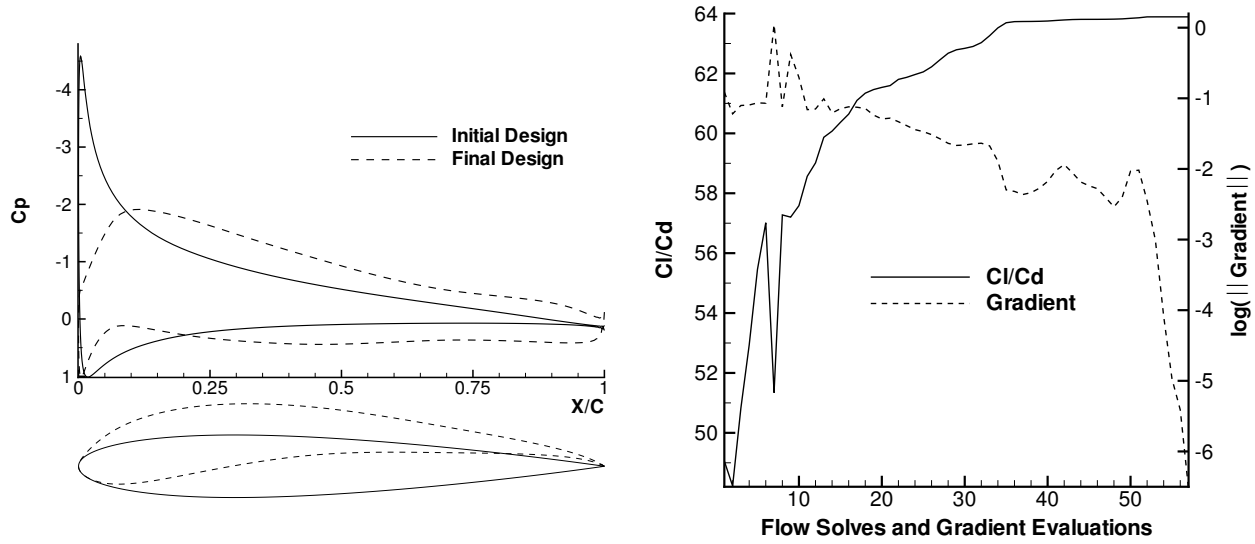


Gradient-based: the most efficient for a large number of design variables; assumes the objective and constraints are smooth functions.

1.4 Practical Applications

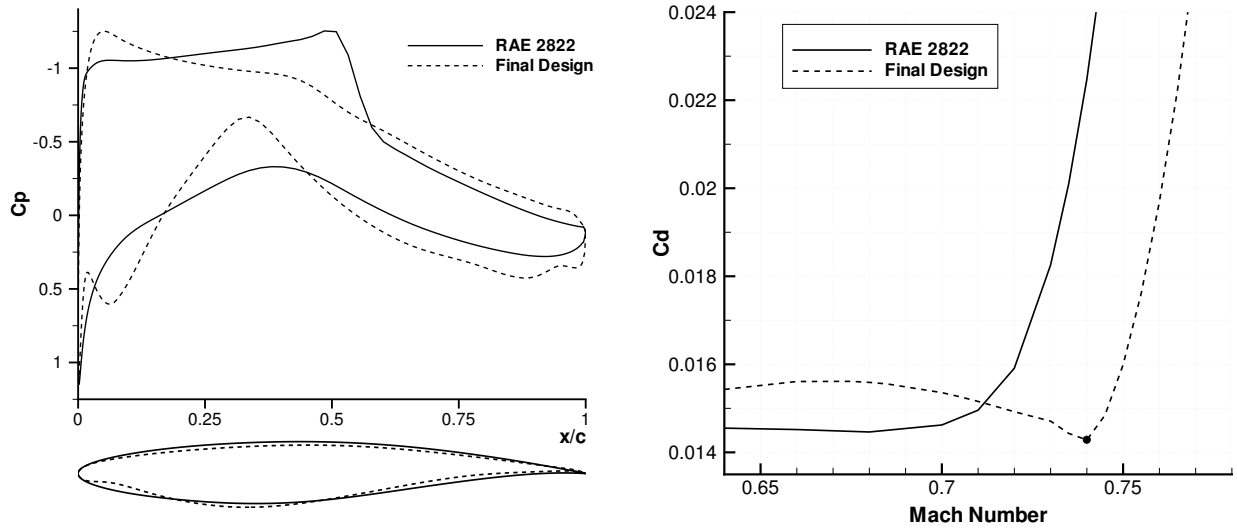
Airfoil Design (D. Zingg, UTIAS / M.Nemec, NASA) [9]

Lift-to-drag ratio maximization



$M = 0.25$, $Re = 2.88 \times 10^6$

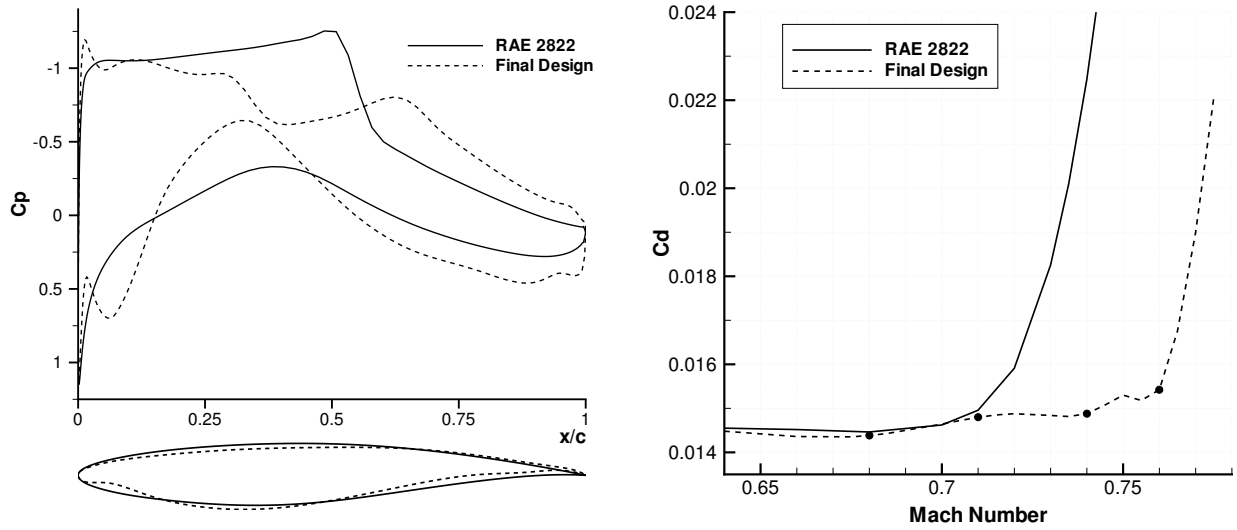
Drag minimization with fixed lift



Single flight condition, $M = 0.74$.

Baseline: RAE2822, $\alpha = 2.9°$. Final: $\alpha = 1.9°$. Drag reduced by 36.4%.

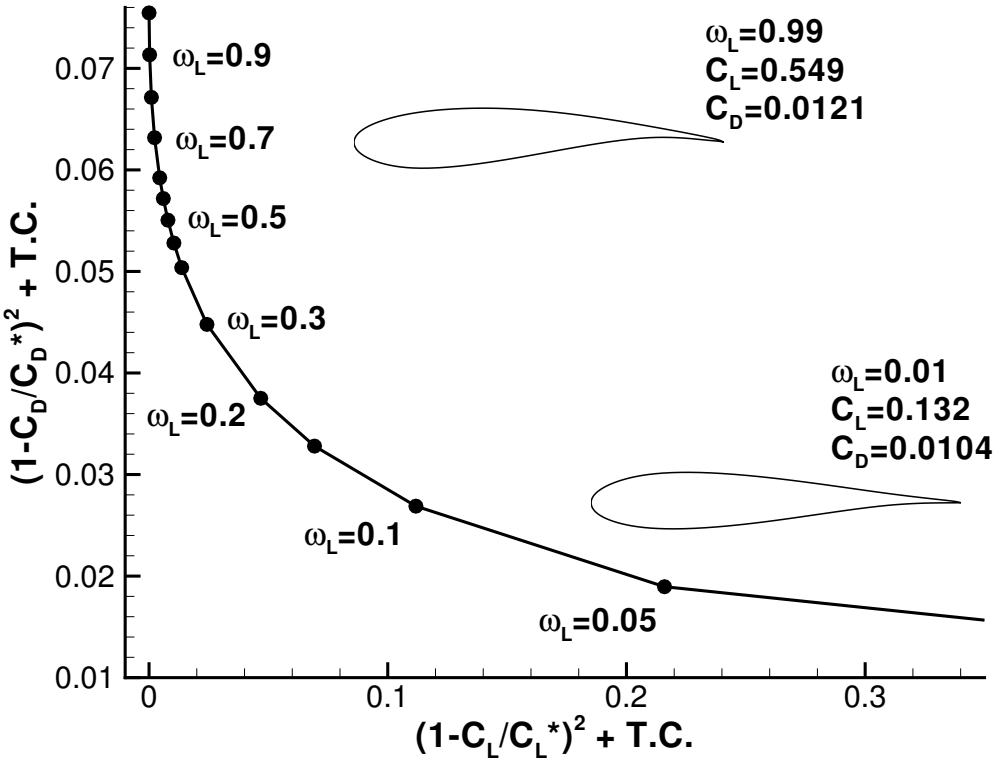
Drag minimization for fixed lift with multiple flight conditions



Four flight conditions: $M_1 = 0.68$, $M_2 = 0.71$, $M_3 = 0.74$, $M_4 = 0.76$.

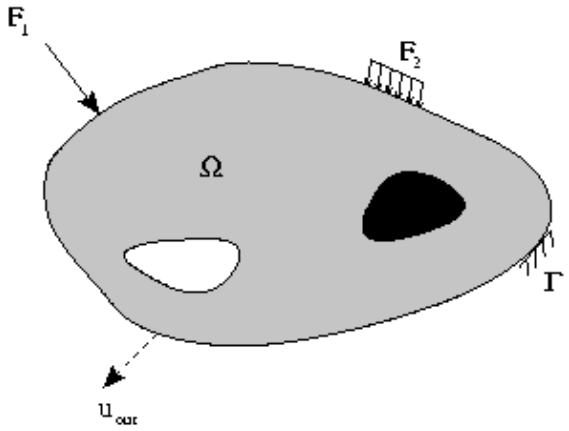
For $M = 0.74$, $\alpha = 1.65°$, drag was reduced by 33.8%.

Trade-off between Lift and Drag Coefficients



Structural Topology Optimization (B.Bailey, UTIAS)

Find the shape and *topology* of a structure that has the minimum compliance (maximum stiffness) for a given loading condition.



Aircraft Design with Minimum Environmental Impact (N.Antoine and I.Kroo, Stanford)

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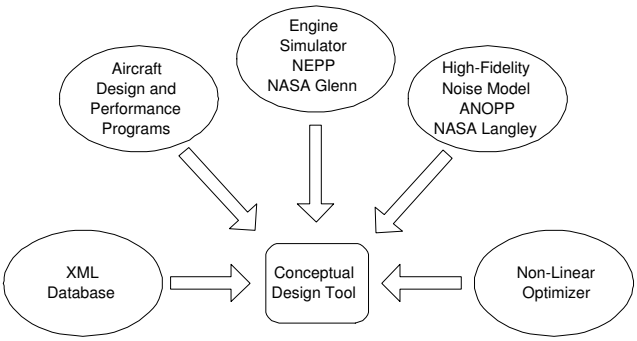
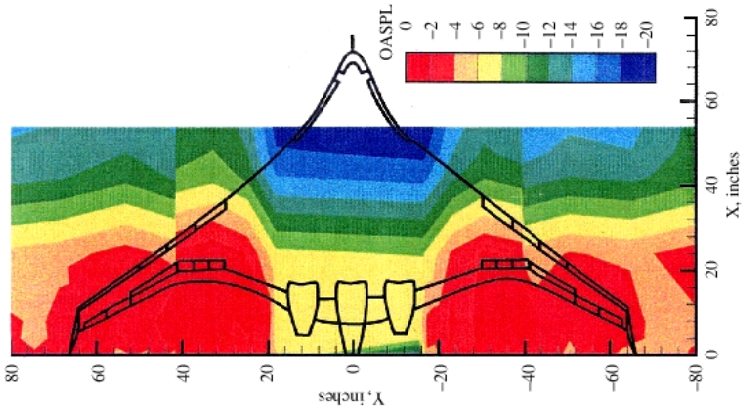
Blended-wing-body (BWB)

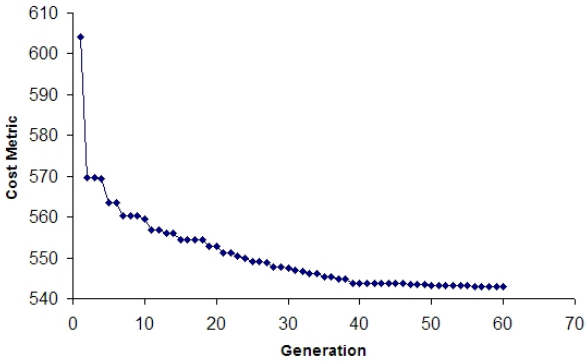
Analysis and Optimization Framework



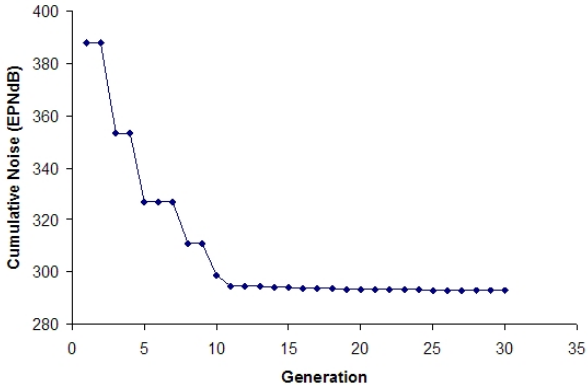
Noise Contours for BWB



Optimization Results Using a Genetic Algorithm



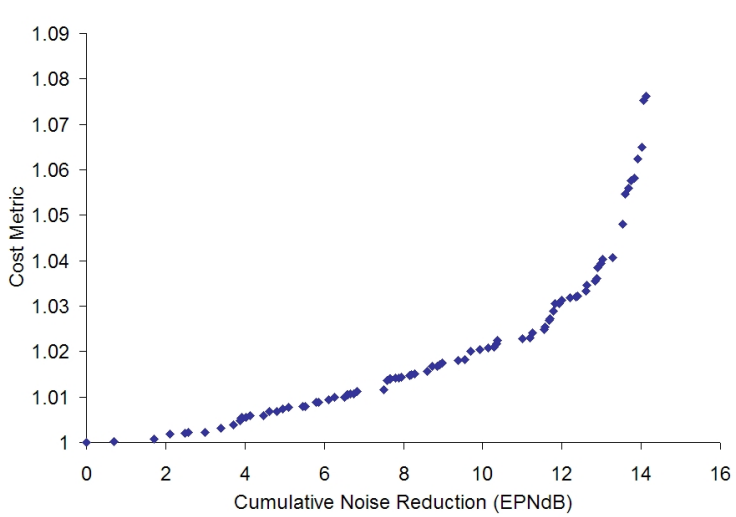
Cost minimization



Noise minimization

Iteration histories for two optimization problems, using a single-objective GA. The objective is either to minimize the operating cost or the cumulative certification noise of the airplane. This is done by changing 12 variables (Bypass ratio, thrust, take-off weight, wing area, wing sweep, wing t/c, wing location, taper, cruise Mach, initial cruise altitude, final cruise altitude, approach angle)

Pareto front representing the trade-off between noise and cost



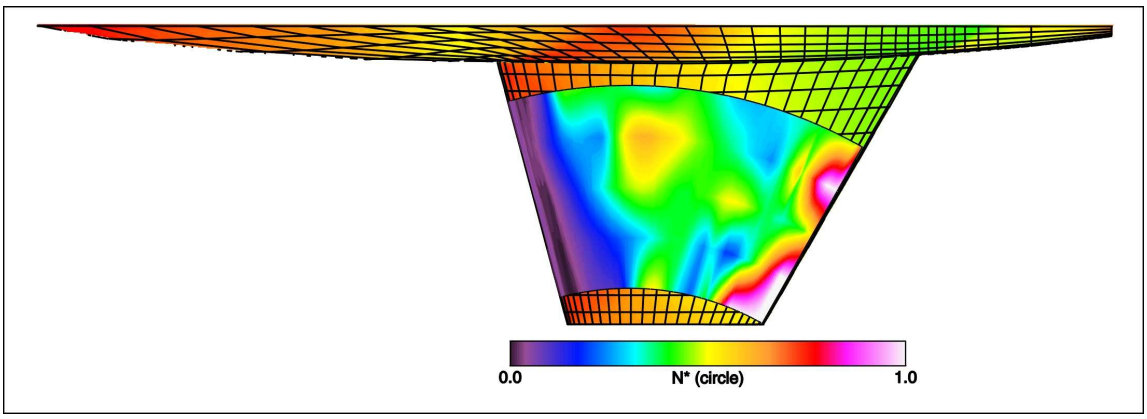
A multi-objective optimizer allows us to determine the *optimum trade-off* between various objectives. The idea, in this case, is to minimize the cost of reducing noise. I can use the curve to determine how much it would cost to reduce the aircraft's noise by, say, 6 dB. We also know what design changes are necessary to get there.

Aerodynamic Design of a Natural Laminar Flow Business Jet (P.Sturdza and I.Kroo, Stanford) [4]

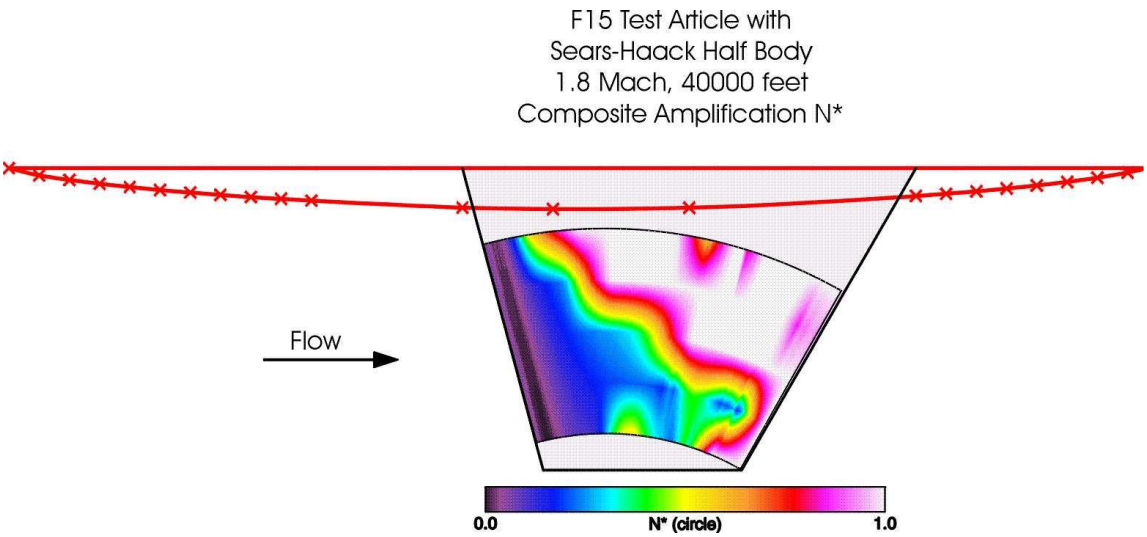
ASSET Corporation configuration. $M = 1.5$; Cruise Altitude: 50,000 feet; Payload: 6-8 Passengers; Range: 5,000 nautical miles; Weight: 100,000 lbs; L/D = 9–10; Wings designed with low sweep for natural laminar flow.

A CFD Euler code was combined with a boundary-layer solver to compute the flow on a wing-body. The fuselage spoils the laminar flow that can normally be maintained on a thin, low sweep wing in supersonic flow. The goal is to reshape the fuselage at the wing-body junction to maximize the extent of laminar flow on the wing.

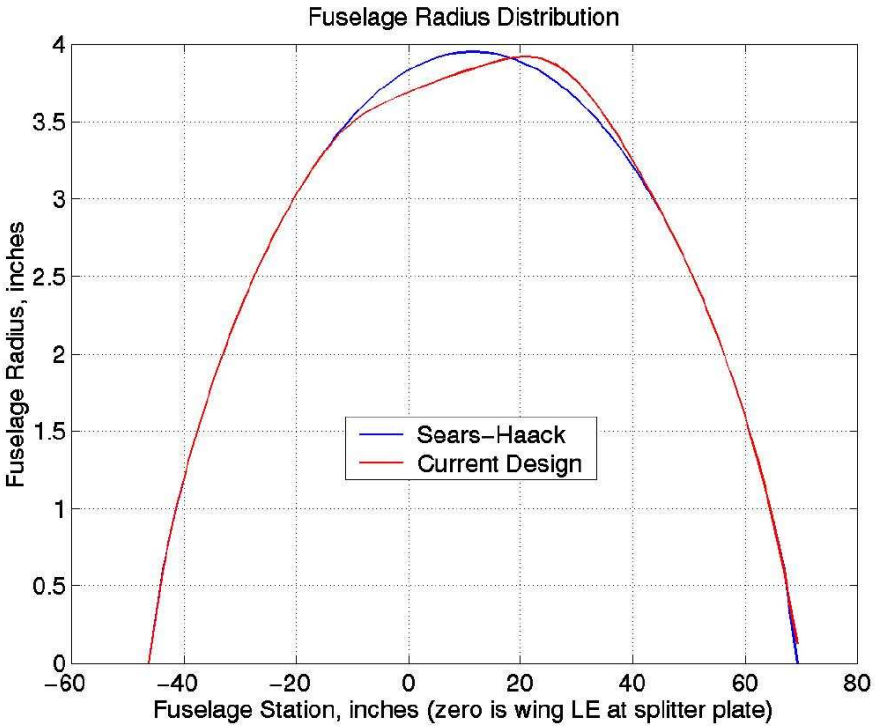
Three design variables were used initially, with quadratic response surfaces and a trust region update algorithm.



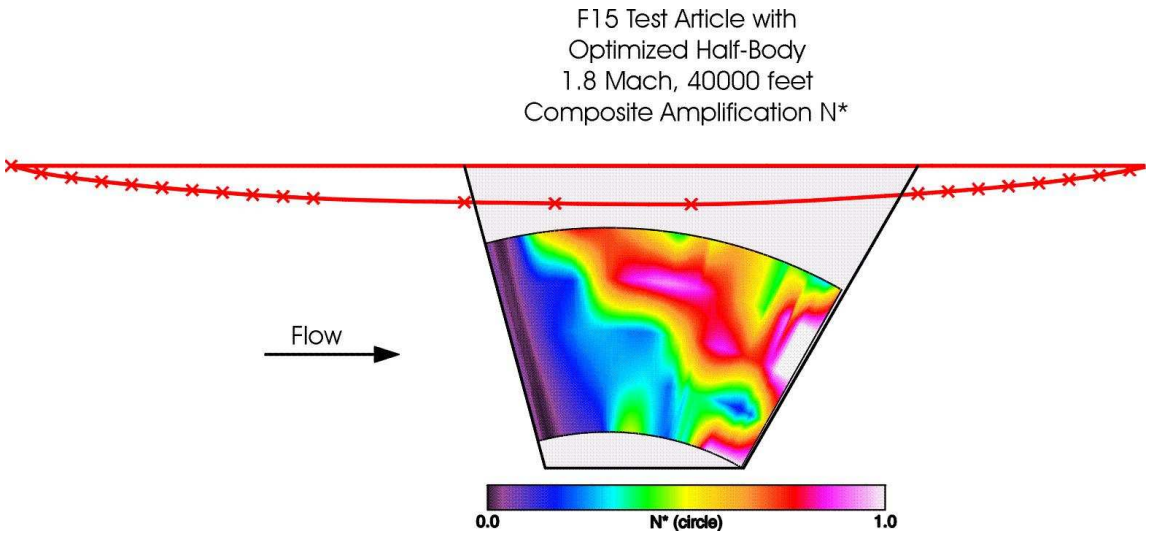
The boundary-layer solution appears superimposed on the inviscid Euler pressures on the surface grid.



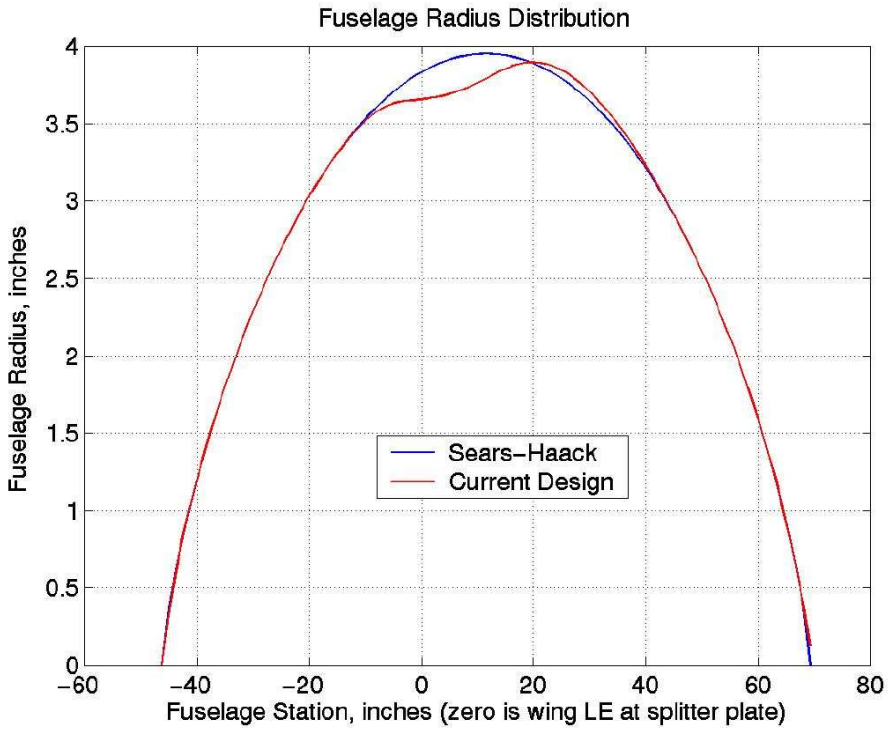
Baseline design: a Sears–Haack body with wing results in early transition (the white areas in the boundary-layer solution). N^* is the measure of laminar instability, with 1.0 (white) being the prediction of transition. The flow is then turbulent from the first occurrence of $N^* = 1$ to the trailing edge irrespective of further values of N^*.



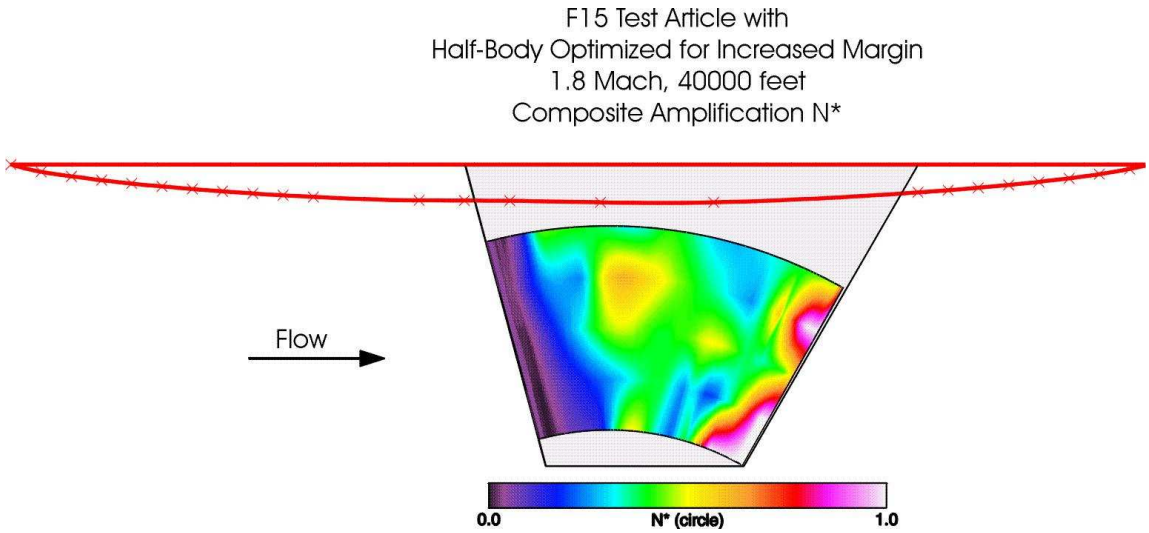
From the nose at left, to the tail at right, this is the radius of the original (blue) and re-designed (red) fuselage after two iterations.



With only 3 design variables (the crosses on the fuselage outline that sit on the wing) and two iterations (not even near a converged optimization) the improvement is dramatic.



With five design variables, and a few more trust-region update cycles, a better solution is found.



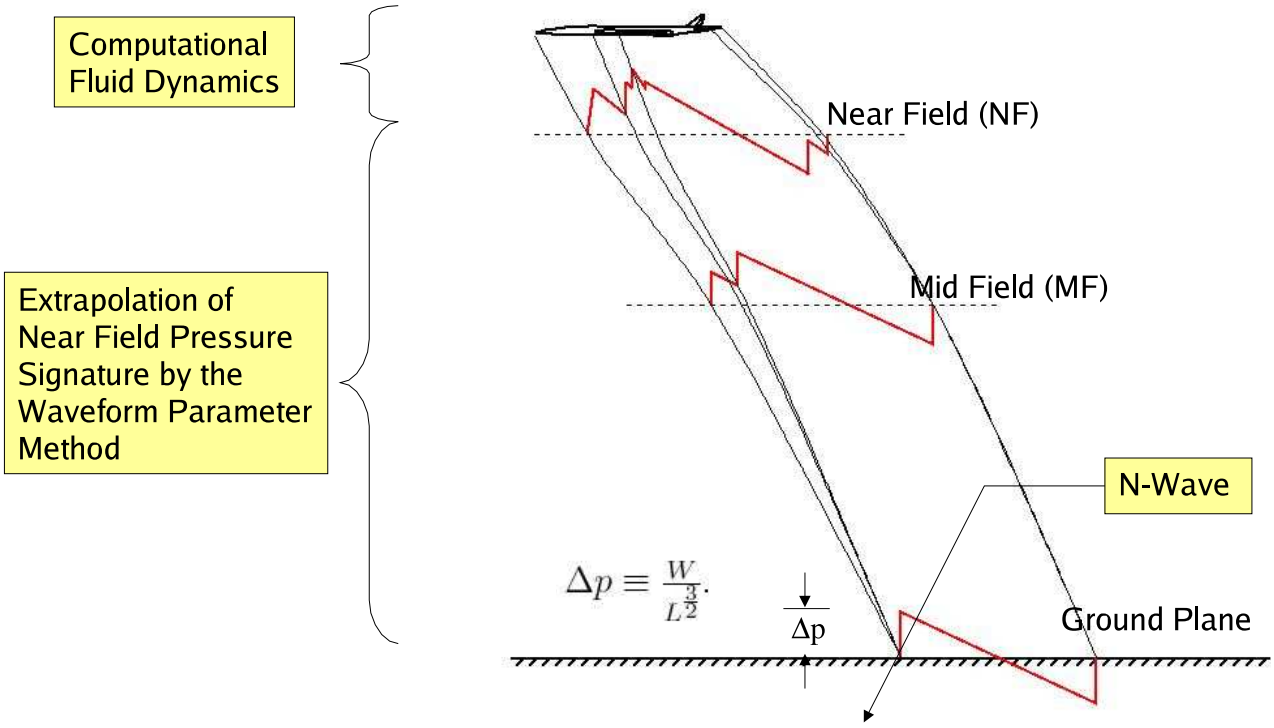
The boundary layer is much farther from transition to turbulent flow as can be seen by comparing the green and yellow colors on this wing with the red and violet colors two figures ago. Also notice how subtle the reshaping of the fuselage is.

Low-Boom Supersonic Aircraft Design (S.Nadarajah, McGill / J.Alonso and A.Jameson, Stanford) [8]

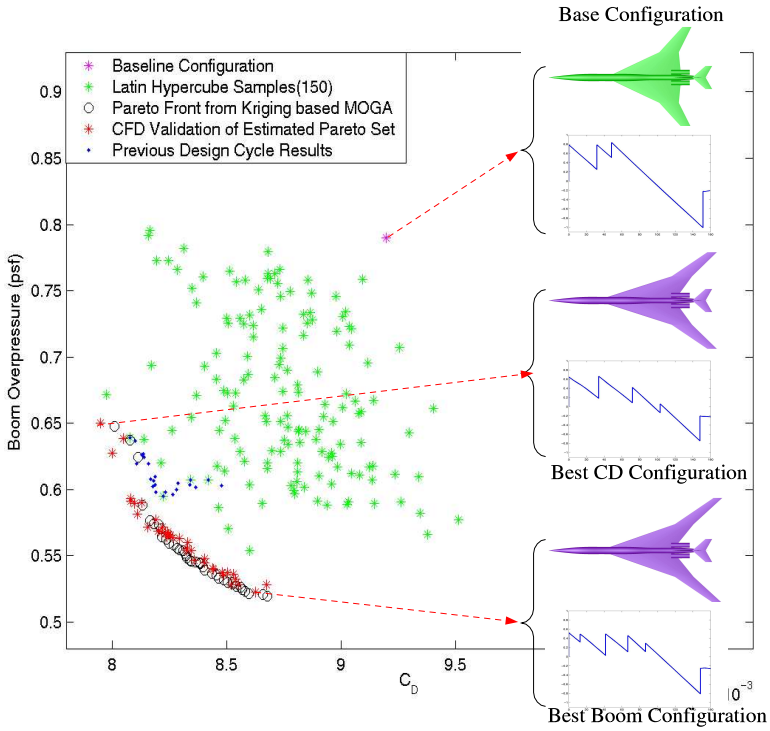
Gulfstream Aerospace configuration. $M = 1.8$; Altitude, 55,000 feet; Payload: 8-10 Passengers; Range: 5,000 nautical miles; Weight: 100,000 lbs; L/D = 9–10; Swing-wing concept.

Objective: reduce sonic boom strength on the ground by changing shape and weight of the aircraft.

Pressure Signature Analysis



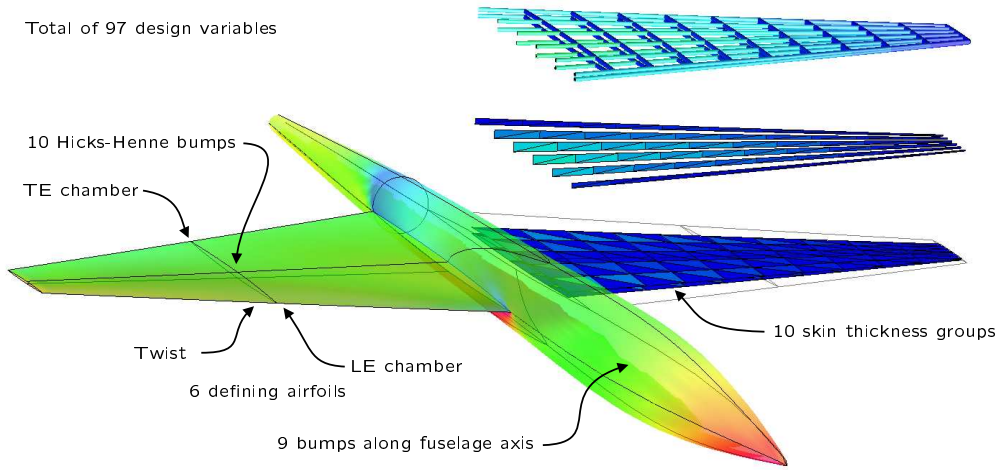
Optimization Results



Two conflicting goals: minimum drag and minimum boom overpressure.

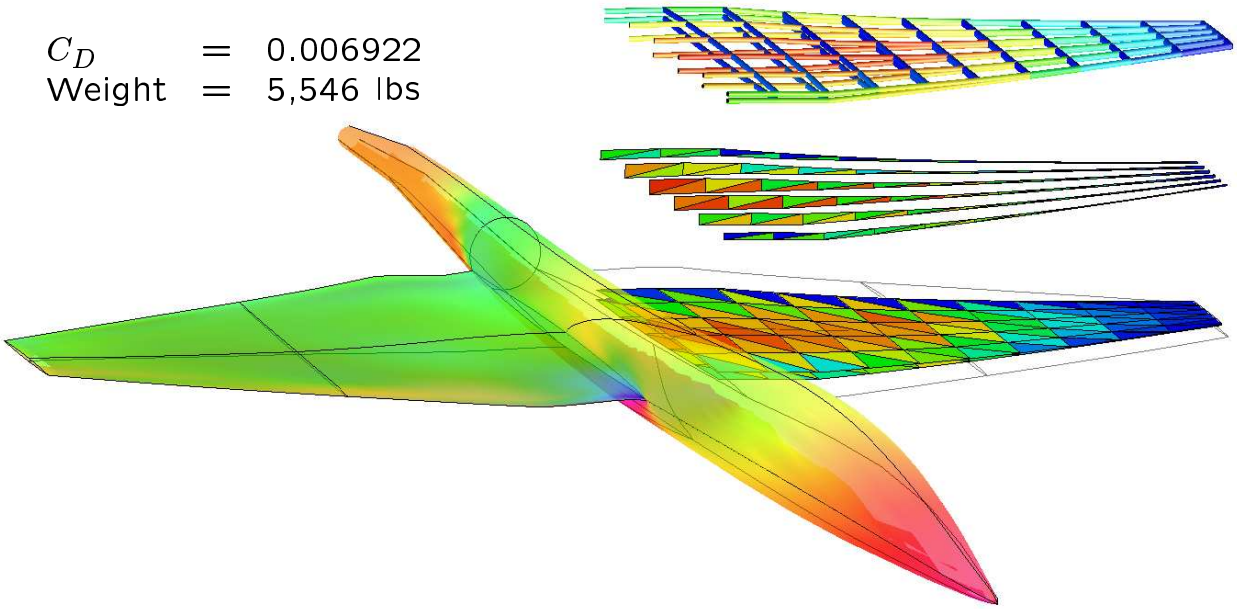
Aero-Structural Design of a Supersonic Business Jet (J.Martins, UTIAS / J.Alonso, Stanford / J.Reuther, NASA) [6, 7]

Baseline configuration and design variables



Optimized design

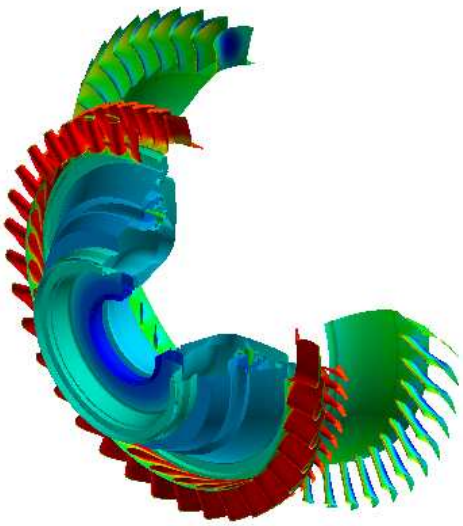
C_D = 0.006922
Weight = 5,546 lbs

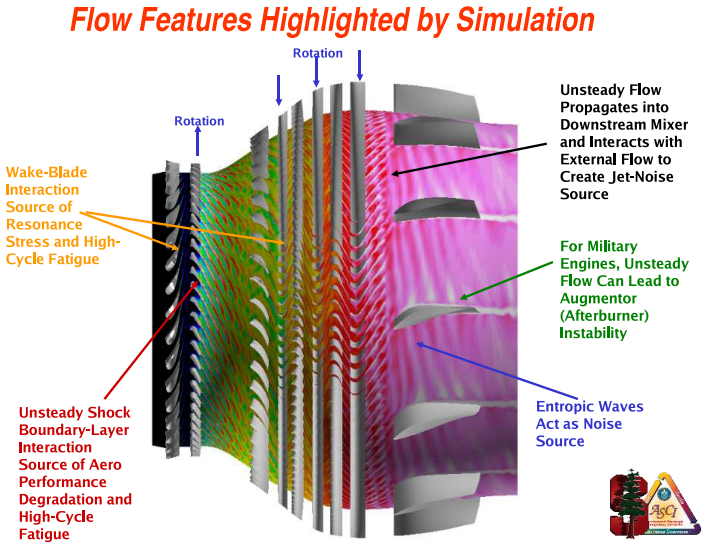


Surface density (cruise)
0.5 1.4

Von Mises stress (maneuver)
0.0 1.0

High-Fidelity MDO of Turbomachinery (J.Alonso, Stanford)





Analysis of complete engines still not mature.

Large potential for an application of MDO.

References

[1] http://www.python.org.

[2] N. Alexandrov and M. Y. Hussaini, editors. *Multidisciplinary Design Optimization: State-of-the-Art*. SIAM, 1997.

[3] N. M. Alexandrov and R. M. Lewis. Comparative properties of collaborative optimization and other approaches to MDO. In *Proceedings of the First ASMO UK / ISSMO Conference on Engineering Design Optimization*, July 1999.

[4] I. Kroo, R. Tracy, J. Chase, and P. Sturdza. Natural laminar flow for quiet and efficient supersonic aircraft. *AIAA Paper* 2002-0146, Jan. 2002.

[5] I. M. Kroo. Decomposition and collaborative optimization for large scale aerospace design. In *Multidisciplinary Design Optimization: State of the Art*. SIAM, 1996.

[6] J. R. R. A. Martins. *A Coupled-Adjoint Method for High-Fidelity*

Aero-Structural Optimization. PhD thesis, Stanford University, Stanford, California, Oct. 2002.

[7] J. R. R. A. Martins, J. J. Alonso, and J. J. Reuther. Complete configuration aero-structural optimization using a coupled sensitivity analysis method. *AIAA Paper* 2002-5402, Sept. 2002.

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