APPLIED MECHANICS GROUP

THE RIGIDITY OF RIB-REINFORCED COVER PLATES

By J. Harvey, B.Eng. (*Graduate*)*, and Professor J. P. Duncan, M.E. (*Member*)†

An experimental approach to the problem of rib-reinforced cover plate design is outlined.

Exact tractable methods for calculating the deflections of radially ribbed plates are not available. Outlines of formal mathematical treatment which use several approximations are presented to highlight the theoretical difficulties.

The technique of Fizeau interferometry was employed to study the deflections of a series of laterally loaded ribbed plates of varying proportions. From the results, values of the rigidity of the plates were found together with the best combination of parameters to ensure optimum stiffening with a minimum of localized curvature for plates of fixed total weight.

The nature and character of the localized curvatures developed in an over-stiffened ribbed plate was revealed by photographic records obtained by means of the Salet-Ikeda slope-contouring technique.

It was concluded that, for optimum design on a basis of rigidity of plates of given total weight, 40 per cent of the total weight of a ribbed plate should be in the ribs or webs and 60 per cent in the basic plate or flange. Localized curvature with attendant stress concentrations were found not to be serious under such a degree of stiffening. Frozen stress photoelastic studies have been initiated to elucidate stress distributions in the vicinity of the ribs.

INTRODUCTION

AN ALTERNATIVE to a thick cover plate of uniform thickness for closing circular openings in high-pressure vessels is a relatively thin plate reinforced by ribs or webs. Where the deflection of a laterally loaded plate must be limited to ensure adequate sealing, a suitably ribbed plate may offer a significant saving in material over the relatively massive plain plate otherwise required. Ribs also allow plates of a maximum available thickness to be stiffened and strengthened.

Currently, the design of rib-reinforced circular plates seems to be empirical and features of their design, such as the best layout, shape and relative size of the webs, appear to be determined by approximate or arbitrary methods. The paper reports on a systematic rational and experimental investigation of some of these problems.

The chosen primary criterion of the relative merits of plates of different proportions was their general rigidity, because this is often of more practical importance in designing a satisfactory gasketed flange than the further need of adequate strength. A related secondary criterion was the avoidance of high localized curvatures in the basic plate.

Rib-systems commonly used in practice can be divided into two main categories—orthogonal and radial. Relatively exact mathematical solutions can be found for the flexure of plates reinforced with orthogonal ribs, but flexural theory can offer no exact yet tractable method for calculating the deflections of radially ribbed plates. An orthogonally ribbed square plate was used, therefore, to prove the experimental method used subsequently to examine circular plates with radial ribs.

THEORETICAL SOLUTION OF RADIALLY RIBBED PLATES

A common configuration of ribs for circular plates consists of a central circular rib from which a number of straight radial ribs radiate to the outer edge of the plates as shown in Fig. 1. The ribs may be of constant depth or may taper to a reduced depth at the boundary.

A ribbed plate of this type has no simple aeolotropy; it is

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* *Rolls-Royce Ltd, Derby. Formerly United Steel Companies Research Scholar, University of Sheffield.*

† *Professor of Mechanical Engineering, University of Sheffield.*

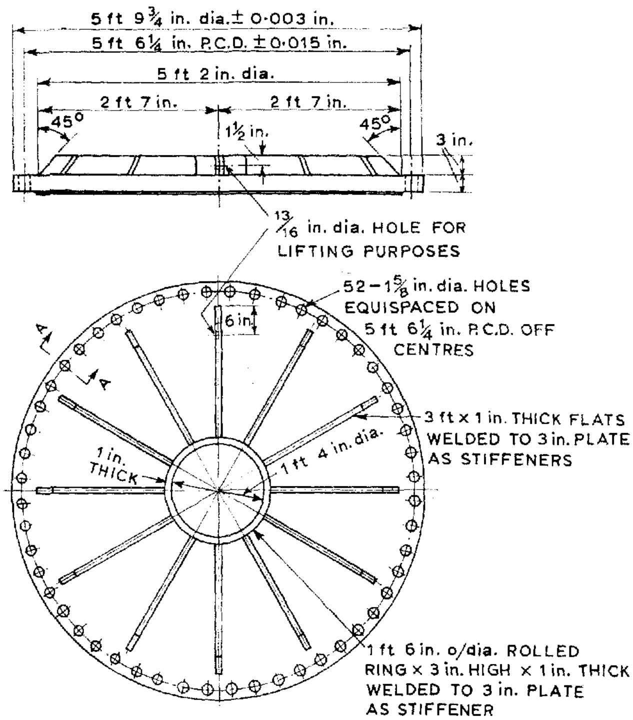


Fig. 1. Design for a large rib-reinforced cover plate

neither isotropic nor orthotropic as a flexural component, and is thus intractable to exact simple flexural theories. It could be analysed exactly by the three-dimensional mathematical theory of elasticity. Even there, the boundary conditions would present unmanageable difficulties. Frozen stress photoelasticity would undoubtedly provide detailed information on stress distribution*, but further integration would then be required to arrive at deflections. Many models would be needed to accumulate sufficient data to determine optimum proportions based on both rigidity and strength.

In cases where the rigidity of the rib-system of a reinforced plate taken alone is very high compared with the rigidity of the basic† flat plate, there is a possibility of an approximate solution. Under these conditions the assumption may be made that the basic plate acts as a system of flexible sectorial plates, serving only to 'collect' the distributed load and transfer it to the supporting rib-system.

THEORETICAL SOLUTION OF ORTHOGONALLY RIBBED PLATES

An approximate method of calculating the deflections in laterally loaded slabs, presented by Ewell, Okubo and Abrams (1)‡ can readily be adapted for the treatment of orthogonally ribbed plates.

The plate is first imagined to be divided into orthogonal T-section strips which are considered as the beams of an analogous gridwork. Using calculated values of the resistance to bending and to twisting of the individual beams, factors are determined for the distribution of unbalanced moments at each grid-beam joint. Each singular point of the gridwork is, in turn, displaced vertically, with the introduction of fixed-end moments in the beams. Using the distribution factors, the fixed-end moments are then distributed throughout the gridwork as bending and torsional moments in each beam. The reactions at all the joints are then found in terms of the displacement of each singular point. By equating these to the externally applied forces at each joint, the induced displacements can be computed.

A distributed external load can be applied only in the form of equipollent concentrated forces at the points of intersection of the beams. This approximation to a uniformly hydrostatic load distributed by means of a diaphragm was shown by experiment to lead to very little error.

Although this method was found to give a reliable prediction of the deflection of a given ribbed plate subjected to hydrostatic load, only by tedious repetition does it offer a feasible method of determining the optimum proportions and layout of the webs. A computer would render this approach tractable and relatively exact as far as rigidity is concerned§.

APPROXIMATE THEORETICAL SOLUTIONS OF RADIALLY RIBBED PLATES

Assuming that no strength is gained from sectors of relatively thin plate spanning relatively massive radial ribs, the deflection of the rib system alone can be calculated approximately. The radial ribs, acted upon by a distributed load along their lengths, each exert a twisting moment on the central circular rib or ring. The effect of the circular plate within the ring was also taken into consideration in this treatment. The behaviour of a ring under systems of concentrated twists has been studied by Malkin (2). The radial ribs function as simple beams subjected to a bending moment at one end, a supporting force at the other end, and a load along their length assumed here to be linearly distributed. The deflection of the sectors of plate spanning the ribs presents a more difficult problem. An assumption can be made that the 'edges' of each sector bounded by the rib-system are rigid and encastré. Later the calculated deflections of the rib-system found as above may be added to those obtained for the sectors of plate to effect a first order correction giving ultimately the actual deflections of the sectors of the basic plate.

Several papers have been published on the subject of deflections in sectorial plates (3)–(7), but all assume boundary conditions different from those met here. For a mathematical solution similar to that used by Deverall and Thorne (3), whose paper approaches nearest to this problem, the tangential bending moment or curvature of the sectorial plate along its two straight edges must be known, rather

* *Studies designed to reveal such stress distributions in typical cases are in progress under one author's guidance.*

† *The basic plate is defined as the plate of uniform thickness to which the ribs may be regarded as attached.*

‡ *A numerical list of references is given in Appendix III.*

§ *If a torsionless solution is used—and this appears to give satisfactory results—the only calculations necessary are a number of evaluations of the I-value of various beams of T-sections. Since the distribution pattern remains the same, deflections vary solely with the inverse of the I-value.*

than the value of the tangential slope, as is known here. An estimate of the bending moment may be substituted and later varied until the boundary conditions derived from the solution compare with those known. This is a long and difficult process.

The authors derived an approximate theoretical solution for the rigidity of a given plate of common proportions by the method outlined, but not unexpectedly it compared poorly with that found by experiment. If the strengthening effect of the circular plate within the ring was taken into consideration, the theoretical value of the central deflection was approximately $1\frac{1}{2}$ times the actual. If the strengthening effect of the basic plate was entirely neglected, the theoretical deflection was twice the actual. These large errors stem largely from the fact that even with a relatively thin basic plate the ribs function as T-beams of variable stiffness in which portions of the basic plate adjacent to the ribs act as flanges.

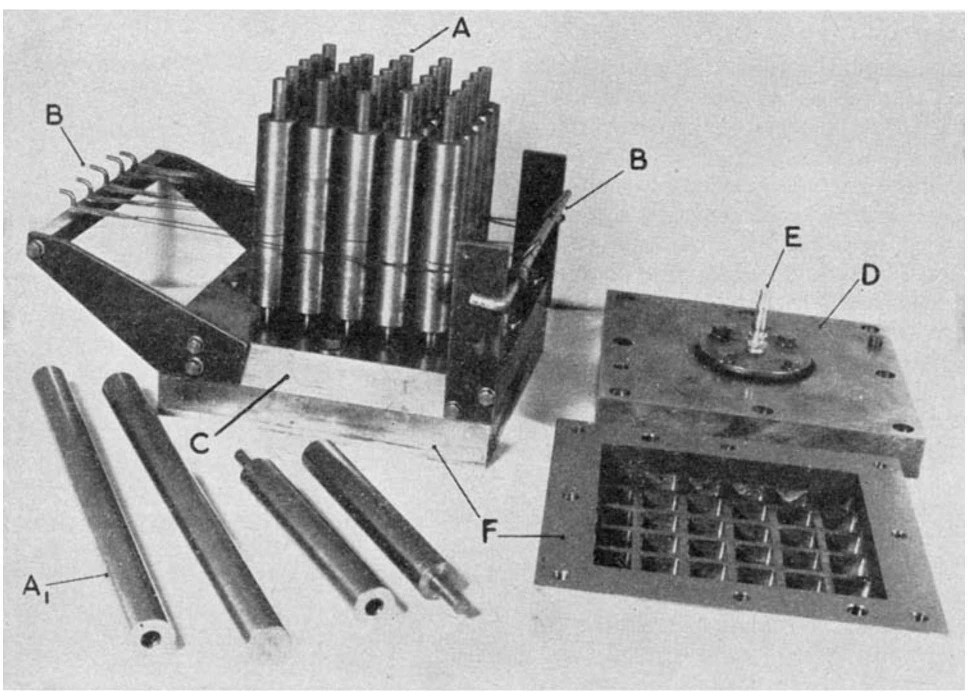
EXPERIMENTAL STUDIES OF RIBBED PLATES

Because theoretical analysis of the problem of optimum design of radially ribbed plates presented so many difficulties and uncertainties, an experimental approach was adopted. An interferometric technique as developed and applied by Duncan and Michejda (8) (9) was used for recording the deflections of the reinforced plates. This method gave a complete deflection pattern for the whole of the surface and revealed qualitatively the local curvatures of the basic plate, if present, at the ribs (see also Appendix I).

The accuracy of the experimental method was first proved by tests on an open gridwork of orthogonally interconnected beams for which accurate theoretical solutions were calculated. The gridwork, which was identical with that studied by Duncan and Michejda (9)*, was hydrostatically loaded, using a very thin metal shim in mechanical contact with the gridwork as a means of distributing the hydrostatic pressure. Subsequently the same gridwork was also subjected to concentrated loads at the intersections of the constituent beams. These loads were provided by a system of revolving dead-weights. Very little difference was noticed in the deflection patterns for equivalent loads of the two types, and both agreed very closely with the theoretical values. Fig. 2, and Fig. 3, Plate 1, show equipment and results.

Similar tests were conducted on an orthogonally ribbed square plate, with built-in edge conditions. It was found that, owing to the relatively greater strength of the ribbed

* *Details of this gridwork and some of the present authors' results are embodied in reference (9).*



A Columnular dead weights.
A_1 Supplementary columnular dead weights.
B Settlement device for revolving all dead weights simultaneously.
C Guide plate positioning columnular dead weights over junctions of orthogonal gridwork F.
D Sealing plate screwed to F to permit pressure loading of gridwork on a sealing diaphragm in contact with the gridwork.
E Tapping for pressure applied via mercury manometer.
F Solid slab from which gridwork was machined as an integral encastré filigree. The boundary rim of F when secured to the boundary plate C or D is much more rigid than the gridwork proper.

Fig. 2. Orthogonal gridwork model with concentrated and hydrostatic loading apparatus

plate compared with the previous open gridwork, the boundary clamping frame which had proved satisfactory for the open gridwork now suffered appreciable deflections and dishing. When allowance was made for the effect of this boundary resilience, the deflections of the plate agreed closely with those calculated from theory.

After the preliminary work of proving the experimental method, tests were carried out on a series of radially ribbed plates of varying proportions. The investigation was planned to cover fully only one common type of rib configuration consisting of one circular and eight radial ribs. It was thought that the optimum web proportions in relation to the basic plate for this layout would be similar for plates with six ribs, and that tests on the former could thus assist the subsequent testing of the latter.

The tests were conducted on small steel models suiting an 8-in. diameter field interferometer. An unsuccessful attempt to fabricate the models by furnace brazing showed that there was a danger that the joints were not always sound.

Plates machined from the solid yielded consistent results and were adopted. Four plates were made each with deep ribs or webs of a different thickness. In the sequence of tests the depth of the webs was progressively reduced, providing plates with more than a dozen different combinations of web shape and size, as shown in Fig. 4, Plate 1.

LOADING CONDITIONS

It was decided that the radially ribbed plates should be tested with boundary conditions that would give the lowest value of overall rigidity. Inflexible simple supports or built-in edges would both lend strength to the plate, which would then not develop a wave of deflection round the boundary as may be permitted in practice by a resilient gasket. The supporting system adopted was one giving a uniform shearing force along the boundary of the plate, independent of the boundary deflection. This was achieved by employing a boundary support in the form of a circular knife edge separated from the basic plate by means of a soft rubber washer. (See Fig. 10, Appendix II.)

The loading in the form of a hydrostatic pressure differential was induced by means of a partially evacuated steel dome, whose rim provided the circular knife edge. This system, illustrated in Fig. 5, was thus completely self-sealing and needed no cumbersome clamping arrangements. A vacuum pump drew air continuously out of the dome and the required pressure was obtained by leaking air in from atmosphere through an adjustable needle valve. This balanced system proved very stable and afforded a simple but accurate control over the loading. The pressure was measured by means of a mercury manometer.

ANALYSIS OF INTERFEROGRAMS

A series of six interferograms, of which Figs 6 and 7, Plate 2, are typical examples, were taken for each plate under various pressure loads, including zero load. For each interferogram and in each sector, the number of interference orders of deflection was counted between the centre

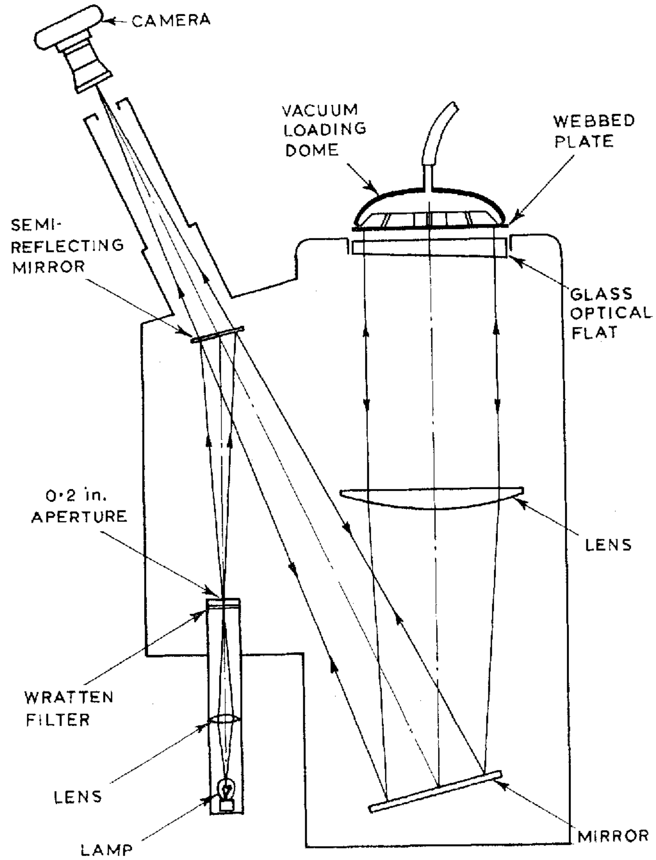


Fig. 5. Diagram of optical system of interferometer

and the support circle along each of the eight radial lines corresponding to rib positions and along the eight radial lines passing through the mid-points of the circular boundary of the sectors. The average values of each set of eight counts was used thus eliminating effects of slightly asymmetrical viewing or elastic behaviour, or surface preparation.

For each plate graphs were plotted of the average number of fringe orders of deflection in each of the above two sets of characteristic radial directions against the pressure differential across the plate. These plots were linear and from the gradient of the lines of best fit the mean deflections of the boundary at its intersection with the members of each of the two groups of characteristic radial directions relative to the centre of the plate were determined.

The values of deflection for each characteristic radial direction gave the maximum and minimum measurement of the boundary deflection relative to the centre for each plate in every machined condition.

BASIS OF COMPARISON

In the assessment of the relative efficiency of the various designs resulting from machining down of the four models, account was taken of the variation in the weight of the plates. A direct comparison between deflections of two plates of unequal mass would be unfairly biased in favour of the design of the heavier plate. To eliminate this, the deflections

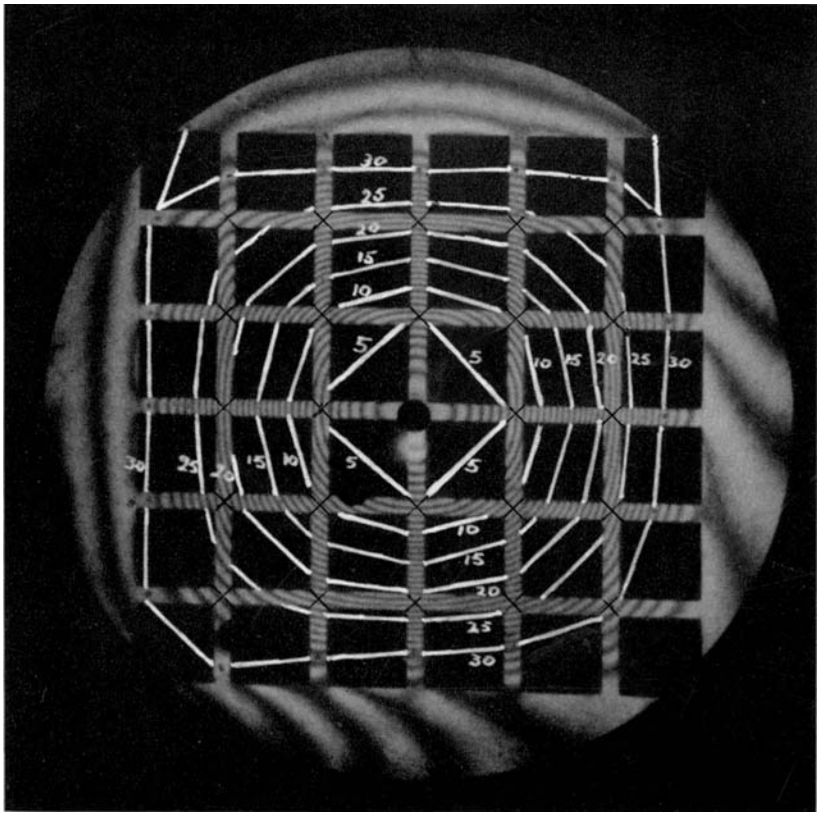
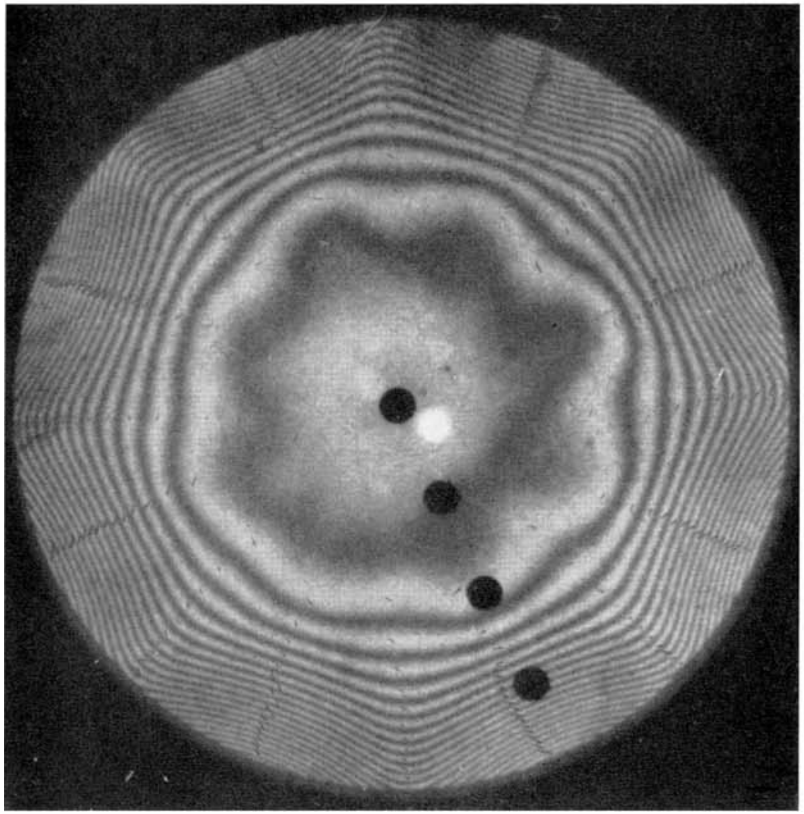


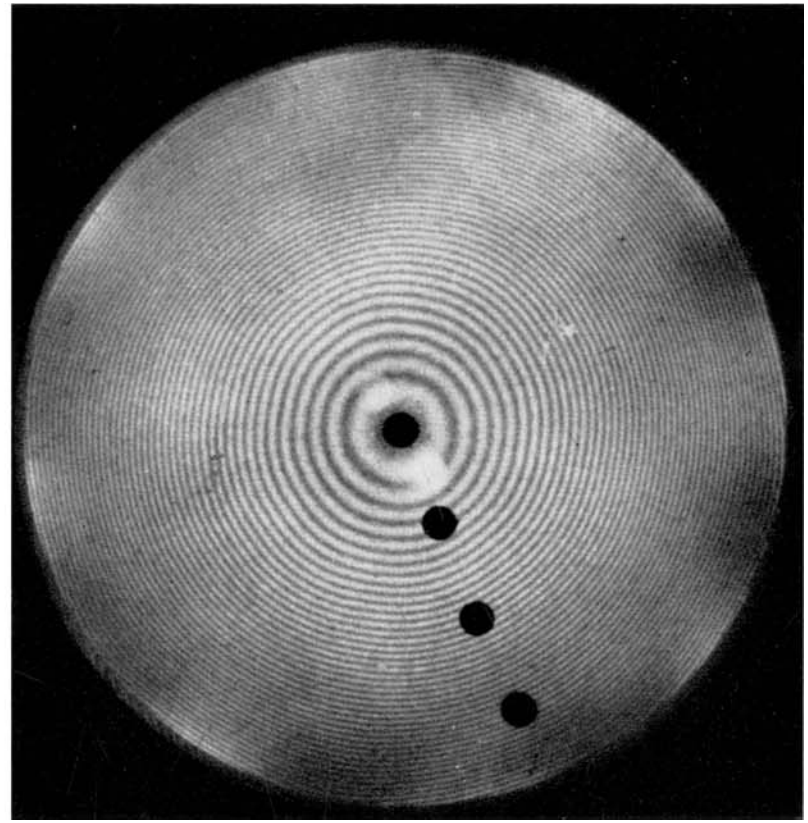
Fig. 3. Fizeau interferogram of a deflected open orthogonal encastré gridwork

Fig. 4. The eight-ribbed circular mild steel cover plates which were progressively machined to provide data shown in Figs 8 and 9



(The black dots are holes through the glass reference flat)

Fig. 6. Interferogram of a deflected over-stiffened circular ribbed plate



(The black dots are holes through the glass reference flat)

Fig. 7. Interferogram of a deflected under-stiffened circular ribbed plate

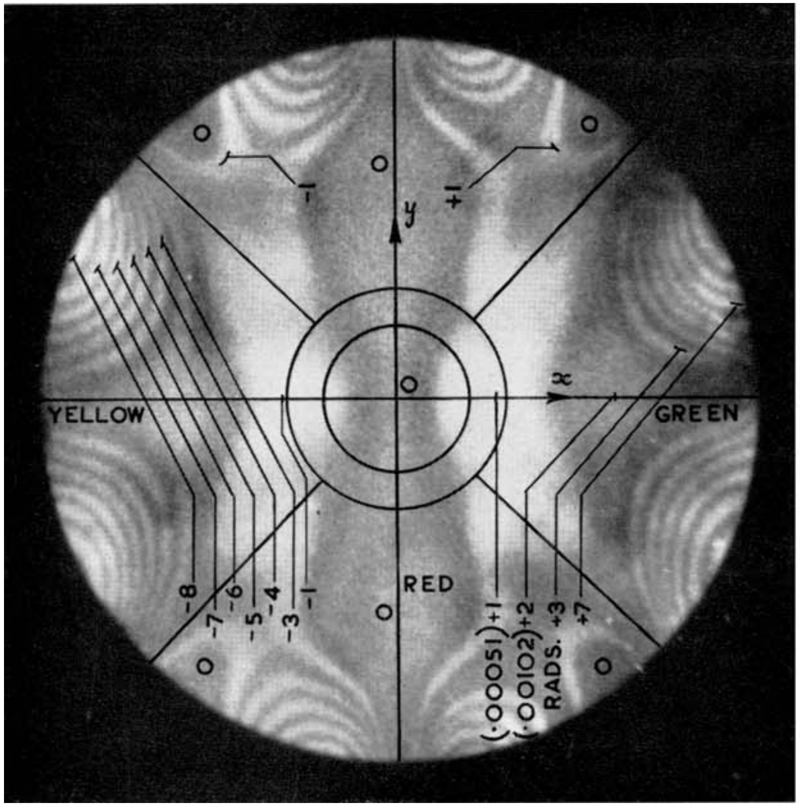


Fig. 11. Contours of partial slope in direction x in hydro-statically loaded eight-ribbed cover plate by the Salet-Ikeda technique

Photograph by P. G. Sabin)

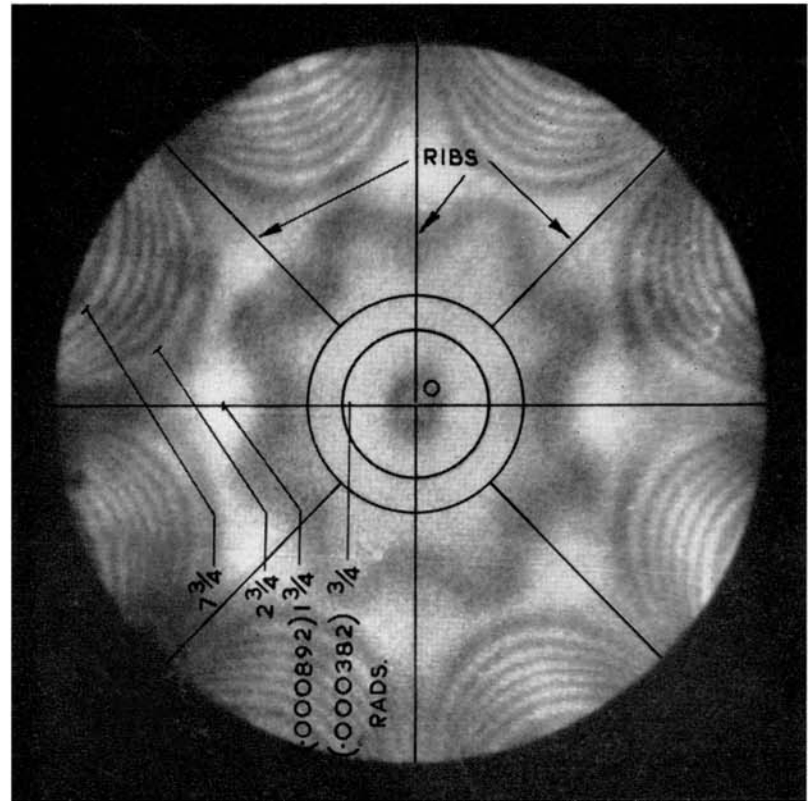


Plate material, mild steel, machine lapped.
Basic plate thickness, 0·078 in.
Web depth, 0·360 in.
Web thickness, 0·358 in.
Support circle diameter, 7·5 in.
Vacuum, 28 in. Hg. = 13·75 lb/in^2.
Initial mean central-deflection-to-boundary error (by Fizeau interferometry), 0·00011 in. in 4-in. radius.

Mean average initial slope, $\frac{0\cdot00011}{4} = 0\cdot00003$ radian.

There is a $\frac{1}{4}$-order phase difference between white orders in Figs 11 and 12 owing to features of central geometry in Salet-Ikeda targets used.

Fig. 12. Contours of principal slope magnitude in hydro-statically loaded eight-ribbed cover plate by the Salet-Ikeda technique

(Photograph by P. G. Sabin)

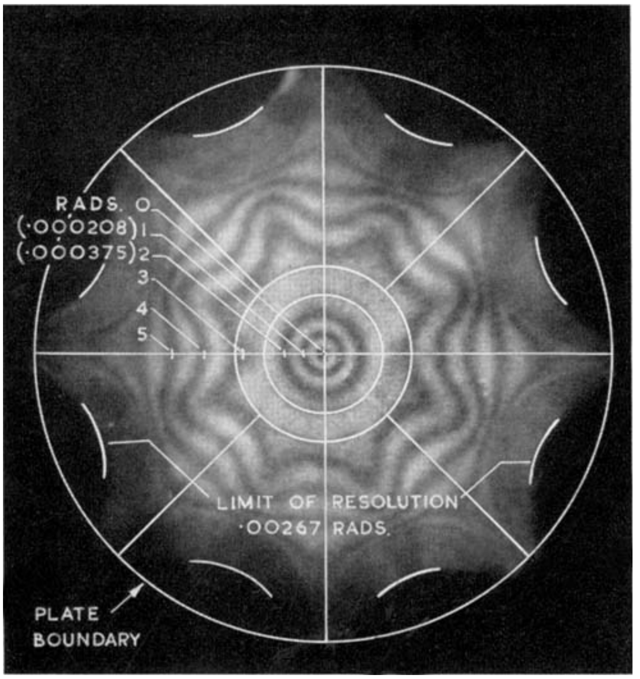


Fig. 13. Contours of principal slope magnitude under identical loading conditions with those in Fig. 12 using fine resolution of slope

(Photograph by P. G. Sabin)

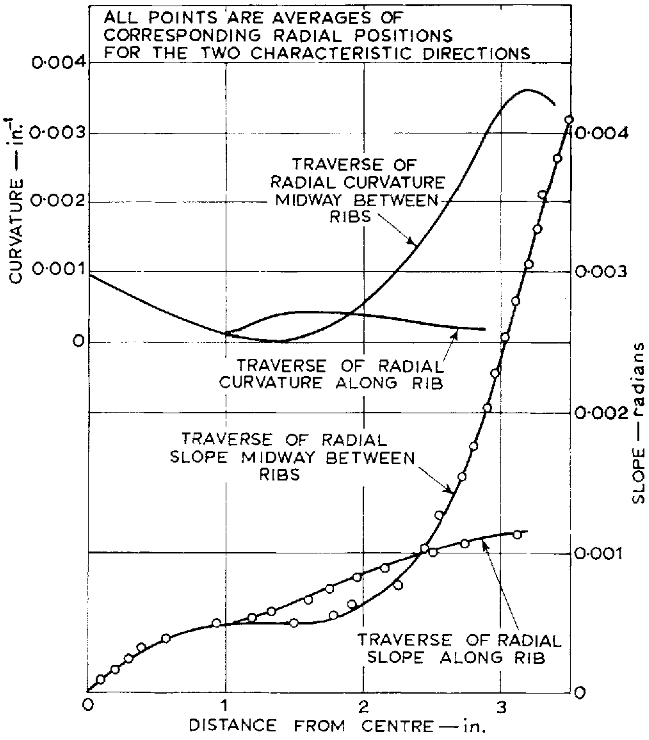


Fig. 14. Curves of average slope and curvature for the two principal directions in Figs 12 and 13

of the plates were assessed relative to the theoretical deflections of plain plates of equal mass. The theoretical plain plate was assumed to be simply supported at its boundary and subjected to loading similar to that sustained by the ribbed plate. The outer diameter of the plain plate was taken to be that of the *effective* span of the reinforced plates, in this case $7\frac{1}{2}$ in. as compared with their outside diameter of $7\frac{3}{4}$ in.

The plates were compared in terms of boundary to centre deflection ratios (see Appendix II), defined as follows:

$$\text{Deflection ratio of ribbed plate} = \frac{\text{Deflection of ribbed plate}}{\text{Deflection of equivalent plain basic plate}}$$

For each plate, maximum and minimum values of the deflection ratio were found from the corresponding deflection measurements. The average of the two figures gave the mean-boundary-to-centre deflection ratio, and their difference gave the double amplitude of the wave of deflection along the boundary as a proportion of the boundary to centre deflection of the equivalent plain plate.

RESULTS

The graphs shown in Fig. 8 give the variation of the mean deflection ratios with the amount of rib reinforcement for a variety of rib proportions, depth/width. From these curves the following may be observed:

(1) For ribs of constant proportions, a definite optimum condition exists for the size of the rib cross-section relative to the basic plate. This optimum condition occurs when the quantity of material in the rib system constitutes about 40 per cent of the whole component.

(2) Deep slender webs are, not unexpectedly, considerably more effective in reducing deflections than the squat designs. There appears to be no optimum level in this case, but high local stresses along the ribs would obviously soon impose a limit.

A curve (Fig. 9) was also drawn to show the variation in the amplitude of the boundary wave with web size. From this it can be seen that reinforced plates incorporating less than 25 per cent rib material have small amplitudes of the boundary deflection wave, whereas in more heavily reinforced plates, above 50 per cent ribs, the local deflections increase rapidly in magnitude until they compare with the central deflection.

CONCLUSIONS

Design of eight-ribbed plates

For the most effective use of material, the ribs should be as deep and slender as possible, within practical limits imposed by localized stresses at the ribs. Four-to-one or eight-to-one cross-sectional proportions seem reasonable.

The size of the rib-system should be adjusted to about 40 per cent of the total mass of the plate. For stronger designs, the thickness of the basic plate should be increased with the webs to maintain this proportion.

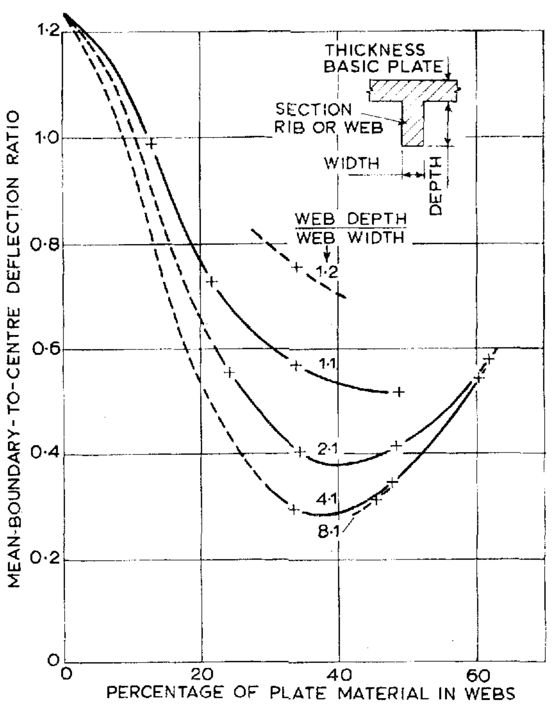


Fig. 8. Variation of mean-boundary-to-centre deflection ratio with material distribution, for webs of constant proportions

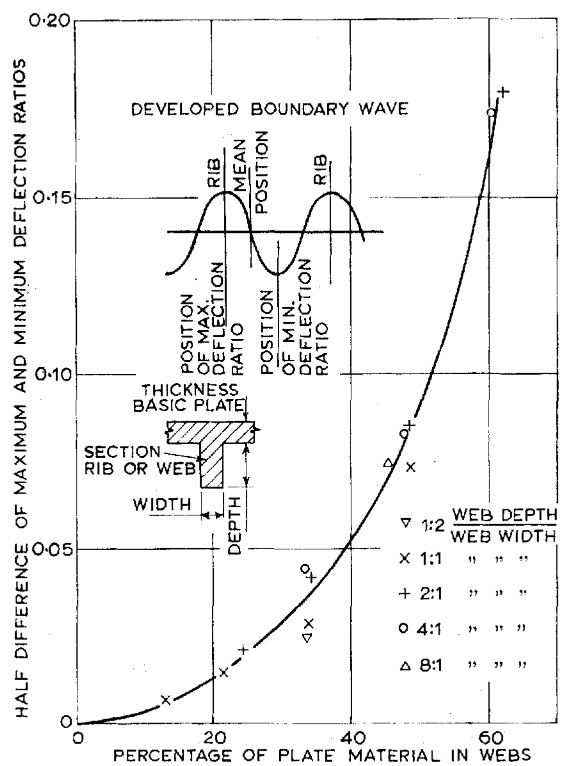


Fig. 9. Amplitude of boundary wave as measured by half the difference of maximum and minimum values of boundary-to-centre deflection ratio

A measure of the rigidity of any eight-ribbed design of the type studied can be obtained as a deflection calculated by multiplication of the appropriate deflection ratio, obtained from the graphs, Figs 8 and 9, and the theoretical deflection of the equivalent basic plain plate obtained by way of standard theory in reference (7) leading to the equation:

$$\text{Central deflection} = \frac{3pa^{10}}{16E}\left(\frac{\pi\rho}{W}\right)^3 (5-4\nu-\nu^2)$$

where a = support radius
ρ = density of material
E = modulus of elasticity
ν = Poisson's ratio
p = load pressure
W = weight of plate

Design of plates with other numbers of ribs

If the thickness of the basic plate is controlled by other considerations, it may be advantageous to use more or less than eight ribs, for two reasons:

(1) It may not be possible to adopt the 30–50 per cent rib proportions if, for example, a very heavily reinforced system is required. In this case, the rib size relative to the basic plate could be adjusted to within the optimum range by increasing the number of ribs.

(2) If the boundary conditions permit uneven deflections, a heavy eight-rib reinforcement leads to a substantial wave of deflection around the plate boundary. This undesirable effect could be reduced by increasing the number of ribs and so shortening the unsupported spans of plate.

Although short of further direct testing there is no direct relation between the strength of plates of unequal numbers of ribs, it seems reasonable to assume that a plate of, say, 12 ribs of the same size relative to the basic plate as one of the eight-ribbed plates tested, would have a similar deflection ratio. On this assumption, results obtained for eight-ribbed plates could be adopted to give guidance in determining the rigidity of plates of other designs.

The limit in the adoption of relatively deep ribs must be judged by three-dimensional stress analysis of stresses in the basic plate near its junctions with the ribs, and at the extreme fibres of the ribs themselves. The frozen stress photoelastic method is being used to pursue this point.

APPENDIX I

REVELATION OF LOCALIZED SLOPES AND THE DERIVATION OF CURVATURES IN OVER-STIFFENED CIRCULAR RIBBED COVER PLATES BY THE SALET-IKEDA TECHNIQUE (10)

The Salet-Ikeda technique, developed by one of the authors for the experimental study of flexure of elastic plates, is an optical method which results in a direct photographic field-plot of the slopes of the flexed plate rather than its deflections as found by Fizeau Interferometry (8).

Since slope in a two-dimensional field has the character of a tensor quantity, rather than a scalar (like lateral deflection), it is necessary to determine slopes in two mutually perpendicular directions at each point of the field. In the case of the eight-ribbed plates studied in this paper resolution of slope over the whole field in a direction parallel to any one diametral rib direction x will give complete information along and across typical rib centre lines. By superposition of partial slopes in the x direction at corresponding points on orthogonal ribs coinciding with the x- and y-axes, slopes may be determined completely at all points of all ribs. Slope gradients then give curvatures. Similarly, by resolving slope along the bisector of the angle between any two ribs, full information about slope along all bisectors can be obtained.

An eight-ribbed mild steel plate similar to the type studied in this paper was loaded by evacuating the ribbed side as in Fig. 10 and the resulting topography of the deflected lapped surface recorded in terms of slope contours by the Salet-Ikeda technique. Figs 11 and 12, Plate 3, and 13, Plate 4, resulted. In Fig. 11 the centre of each black order (or band) differs in slope in the x-direction from the centre of each adjacent white order by 0·00025 radian in the *actual* steel plate. In Fig. 12 the orders represent the location of the maximum *principal* slope of value appropriate to that order. Again the principal slope difference between centres of adjacent black and white orders is 0·00025 radian in the actual place. Because of known symmetry along and midway between ribs, the principal slope contours enable radial slope versus radius curves to be plotted for the two characteristic directions. These are shown in Fig. 14, Plate 4. The central slopes may be recorded in much finer detail by resolving slopes there on a finer scale, as shown in Fig. 13.

APPENDIX II

BASIS OF CALCULATION OF DEFLECTION RATIOS AND RIGIDITIES

With the aid of the experimental data of Figs 8 and 9, the rigidity of any reinforced plate of similar layout to those tested may be calculated in three stages as follows:

(1) Using the total mass of the ribbed plate and the free

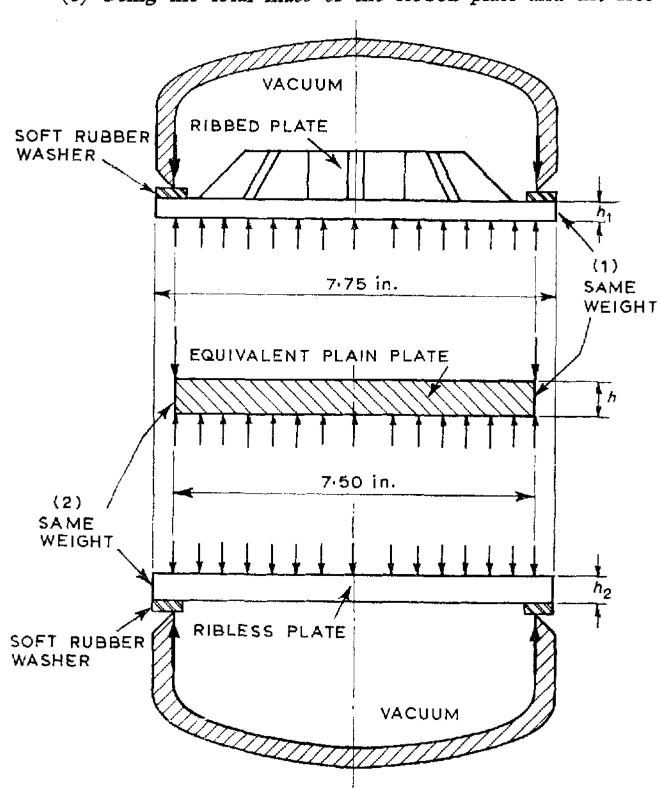


Fig. 10. Diagram illustrating method of loading and boundary conditions of circular ribbed cover plates

span within its circular support, the rigidity of an 'equivalent plain plate' is calculated as:

$$\frac{\text{total uniformly distributed load}}{\text{central deflection}}$$

(2) From the selected or specified cross-sectional proportions of the ribs a value of the appropriate deflection ratio is obtained from the experimental curves of Fig. 8.

(3) The quotient of these two quantities, (1) ÷ (2), gives the rigidity of the reinforced plate.

Although the definition of the 'equivalent plain plate' is purely arbitrary, it lends itself to simple calculations in stage (1) above.

The 'equivalent plain plate' is defined as a flat plate of mass equal to that of the ribbed plate under consideration simply supported at an extreme edge of diameter equal to the diameter of the free span of the reinforced plate. This point is illustrated in Fig. 10.

All the plates tested, including one plain (ribless) plate, had an outer diameter of 7·75 in. and were supported concentrically at a diameter of 7·50 in., which was the free span. A constant margin of $\frac{1}{8}$ in. beyond the support radius was thus a feature of all the experimental plates.

This margin had very little effect on the strength of the remainder of the plate, but obviously it must reduce slightly the overall efficiency with which the total weight of material in the plate is used structurally. Hence, as expected, the ribless plate actually tested experimentally deflected more than its 'equivalent plain plate', defined as above, and had a correspondingly higher deflection ratio than unity as shown in Fig. 8 in zero abscissa.

The observed extent of this effect was accurately predicted by the following calculation.

For plate thicknesses studied it may be demonstrated that the material between the supporting radius of 3·75 in. and the outer radius of 3·875 in. makes a relatively small contribution to the strength of the plate.

Therefore the experimental ribless plate has a rigidity approximately equal to a plain plate of the same thickness but of 7·50 in. diameter and hence of weight $\left(\frac{7\cdot50^2}{7\cdot75^2}\right) W = 0\cdot937W$, where W = weight of the plate of 7·75 in. diameter.

For the 'equivalent plain plate' corresponding to the ribless plate, the central deflection under a uniformly distributed load is proportional to the inverse of the cube of the thickness and hence to the inverse of the cube of the weight. Hence the deflection ratio of the experimental ribless plate of 7·75 in. diameter will be

$$\left(\frac{1}{0\cdot937}\right)^3 = 1\cdot22.$$

It was shown by experiment that the flat ribless plate had a deflection ratio of 1·23.

APPENDIX III

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Communication

Mr G. F. Lake, M.A. (*Member*)—The paper provides valuable information concerning the increase in rigidity obtained from various dispositions of rib reinforcement. There appears, however, to be some risk that rigidity may be regarded as synonymous with strength, or at least that it may be thought that any increase in rigidity is necessarily accompanied by a reduction in stress. This may not be so and there appears to be a possibility of an increase in stress over that in the plain basic plate under the same conditions of loading. The following argument will serve to illustrate this point.

The authors conclude that for the effective use of material

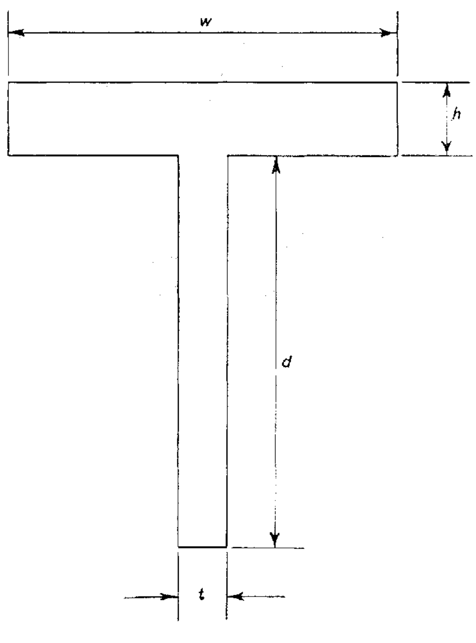

Fig. 15

the size of the rib system should be adjusted to about 40 per cent of the total mass of the plate. Consider therefore a section of a portion of the plate and rib, as indicated in Fig. 15, in which $dt = \frac{2}{3}wh = 0{\cdot}4(dt+wh)$.

The thickness of a corresponding plain plate of equal mass would be $1\frac{2}{3}h$. Let the proportion of the ribs be 8/1, i.e. $\frac{d}{t} = 8$. Then according to Fig. 8 the ratio of the deflection of the ribbed plate to that of the plain plate of equal mass is $0{\cdot}24$.

For the same curvature the stresses in the plates will be proportional to the distances of the extreme fibres from the neutral axes. These distances are $\frac{5}{6}h$ for the plain plate and approximately $0{\cdot}9d$ for the rib extremity of the ribbed plate. Under the same loading conditions the curvature of the ribbed plate would be expected to be of the order of $0{\cdot}24$ times that of the plain plate, so that

$$\frac{S_r}{S_p} = 0{\cdot}24 \times \frac{0{\cdot}9d}{\frac{5}{6}h} = 0{\cdot}259\,\frac{d}{h}$$

$$= 0{\cdot}259\,\frac{d}{t}\cdot\frac{t}{h}$$

$$= 2{\cdot}07\,\frac{t}{h},$$

where S_r = stress in ribbed plate and S_p = stress in plain plate.

Now in practice t is unlikely to be much less than h, and will usually be not less than $\frac{h}{2}$. This suggests that the stress in the ribbed plate will be greater than the stress in the equivalent plain plate under the same loading conditions. I would welcome the authors' views on this deduction from their experimental results.

Authors' Reply

Mr J. Harvey and **Professor J. P. Duncan**—We are quite aware that our paper deals with rigidity rather than strength, a fact implied by the title. As mentioned in the footnote on p. 116, further necessary studies of strength and stresses are in progress. Since writing the paper we have also learned of a new form of analysis being applied to the complete problem of strength and rigidity of ribbed plates by Mr L. M. F. Coulthard of the University of Birmingham.

Our chosen criterion for comparing the effective use of material in ribbed plates was overall rigidity. We do not disagree with Mr Lake's point, demonstrated by his calculation for a specific case, that not all proportions of rib and web are practicable even when the ribs constitute the recommended 40 per cent of the total mass of the plate. Failures of improperly designed ribbed structures at the free end of the web of T sections are, unfortunately, all too common.

Mr Lake has effectively illustrated the underlying reason for such failures in terms of an *analogue* of our case in which he has assigned a definite value to flange width, w, and has then made the *area*, dt, 40 per cent of the *area* $(wh+dt)$. When considering a circular, radially ribbed plate, one cannot say what the effective value of w is at any particular radius and for this reason our 40 per cent optimum figure was determined from the *masses* of the ribbed system and the basic plate. This figure is influenced by the number of ribs as well as by t, d and h in Fig. 15. The effective value of w will be some indeterminate function of radius and of the number, and hence spacing, of the ribs.

Despite those differences of premises, Mr Lake's calculation demonstrates qualitatively the relevant point that deep, thin and widely spaced ribs attached to a flange increase only slightly the modulus of section of the flange and the position of the neutral axis. Thus, curvatures developed by a given bending moment alter little from one case to the other though extreme fibre stresses at the tip of the deep rib increase markedly.

We would be surprised to find the proportion $d/t = 8$ actually adopted in practical applications and this is the main ratio which will determine Mr Lake's point. In other cases where t is equal to or even greater than h, stresses in the sectorial plates spanning the ribs are likely to govern permitted loads and hence limit the curvature developed along the ribs themselves.

We feel that the analysis used by Mr Lake overestimates the extreme fibre stresses in the case of the ribbed plate. The character of the deflection plot across the diameter of a loaded plate is considerably altered by the addition of radial ribs. The relative concentration of reinforcement at the centre reduces the high curvatures normally experienced in this region to about the mean level for the whole surface. Consequently, the plate tends to deflect into a spherical rather than a parabolic form.

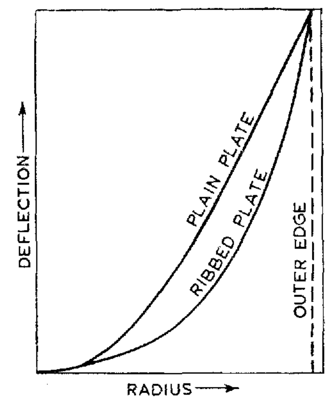

Fig. 16. Comparison of deflection plots along a radial line for ribbed and plain plates

This is illustrated in Fig. 16, where a ribbed plate is compared with a plain plate for equal central deflection. The reinforced plate used for this comparison has a rib depth/thickness ratio of 4, and 48 per cent of the total material in the webs. The deflection plot has been taken along a rib line.

From the two curves, it can be seen that for a given central deflection, the maximum value of radial curvature in the case of the ribbed plate is only about two-thirds that of the plain plate. Hence the estimation made by Mr Lake for the maximum curvature of the ribbed plate can be expected to be excessive by about 50 per cent.

A secondary source of error arises from the assumption of a 40 per cent rib size at the critical section. For small radii, where the highest curvatures would be expected to occur, the flange width of a typical T section would be much less than the average value as illustrated in Fig. 15. This would have the effect of reducing the distance of the extreme fibres from the neutral axis, and thus lowering the maximum stresses for any value of curvature.

For example, if the flange width were reduced by a half, the distance of the extreme fibres would be reduced from $0{\cdot}9d$ to approximately $0{\cdot}75d$. This would have the effect of reducing the maximum fibre stresses by about a fifth.