A Fiction of Tax Design from the Principle of Maximum Guaranteed Welfare *

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Abstract

Our fiction addresses an important but narrowly defined issue of wealth circulation in the society. In the form of three persons' game we made an inquiry into the welfare effects on taxpayers. Bargainer Nr.1 on legal and moral grounds personalizes taxpayers to have a preference for social services. Bargainer Nr.2 personalizes taxpayers to have a preference for the provision of public goods. Quite the opposite, player Nr.3, the taxpayers' third mind, is the one who gives the consumption a preference over social services and public goods; i.e., taxpayers try to reduce their obligations scheduled at bargainers' account. Voters-taxpayers actually arrive at their expectations for consumption, e.g. at unanimous consent, by a threat of whether to reject or to acknowledge the bargaining agreement. The government, presided over the State, assesses and rules on wealth circulation by poverty line parameter. We provide an evidence for claim that 50% median income is an ideal solution.

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1. Introduction. Public economy trends

In optimal taxation theory and its associate – the distributive taxation, the government maximizes "total welfare" of private households endowed by divergent productivity. This paper distinguishes itself from these studies by different postulate of "guaranteed welfare" – a standard of living guaranteed for the poorest households in the economy. The materialization of the idea behind the guaranteed welfare is an obligation worth considering on legal and moral grounds. Empirical evidence consistent with legal obligations may be found in the literature of social policy: *"...Henderson poverty...line was initially set (in 1966) equal to the level of the minimum wage plus family benefits for one-earner couple with two children,"* cit. Saunders (1993). Hypothesis consistent with moral obligations may be found in the literature of economic politics, Eichenberger and Oberholzer-Gee (1996), Feld and Frey (2002).

Musgrave (1959) examined two basic approaches to taxation: the "benefit approach", which puts the taxation into Pareto efficiency context, and the "ability-to-pay approach", which puts the taxation into equity context. As we shall see, the poverty line in the context of benefit approach would enable guaranteed welfare proposal at a reasonable price. The poverty line in the context of ability-to-pay is set by the government to decide who is living in poverty and to legalize the wealth circulation from the rich to the poor.

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One would perhaps agree with wealth circulation in classical public finance interpretation echoed from Adam Smith (1776): *"…the income of the poor is needed to defray the basic cost of living, a tax on such income or the necessary commodities purchased with such income cannot be paid by the poor but must be passed on to others,"* cit. Musgrave (1959). But professionals, those who advocate for widespread view of total welfare optimization along the lines of Mirrlees n-man productivity (1971), may overlook the obvious just for that very reason that our result lies not on the same page. We formulate entirely novel Welfare Belief Statement: (i) – *equilibrium*, the long-term prospective of welfare expenses stabilization; (ii) – *rationality*, the issue of what is a reasonable boundary between incompatible demands for the social services and for the public goods; and at last, (iii) – *self-interest*, there is a consensus on consumer perception of income tax and obligations reduction – all three together constitute a cascade of three canonical principles embedded into Welfare State Institutions.

Equilibrium. Welfare expenses used to be referred in common parlance as government spending on welfare programs, i.e. not immediately connected with purchasing of public goods and services. Specific expenses in our technology assume a pool of money to reimburse households', those who have a misfortune, through social benefits. To be precise, social benefits for households with low incomes and limited assets provide an adequate chance to improve their disposable income. Households eligible for benefits do not have many assets, they are not flexible on the labor market, and their income lies under certain poverty line, what befall them. Therefore those, who decide to claim the benefits, will be better off under the social security administration. On the other extreme, some old clients, to whom the administration submits a request that the benefits have been revoked by the result of implemented declines in welfare level and services, might find themselves worse off and might decide to be flexible on labor market once again. That is to say, the emphasis on welfare materialization may manifest itself in hidden welfare ambiguity, f.e. as a result of economic growth, decline, wage stagnation, demographical shift or political change,… by sending insecurity waves into social services. But now, while the social services improve or decay, the households, whose disposable income is close to the poverty line, may activate this time an inverse waves of social costs and thus a feedback effect into tax burden for all.

So, our first step in the cascade portrays a social institution to solve the long-term stabilization problem of welfare expenses in cooperation with other state institutions – known issue in public finance, as it cannot be solved by the market mechanism.

Long-term stabilization problem of welfare expenses is two-folded: 1) social services and public goods to be financed from a variety of tax pools must be stable and no excludible – available for everyone, 2) despite the government legislates taxes, the distortions that welfare may introduce in the behavior of private households may "destabilize" income tax schedules.

According to ability-to-pay, known terms of warranty how to counteract instability of tax polices rely on exogenous taxes enforced by the government on households' productivity. A variant of classical public finance and the like, Berliant and Gouveia (1994), is the concept when a household of a given productivity does not shift his/her labor supply after all the adjustments to the tax formula has been implemented: Optimal taxation enforces optimal labor supply. Yet another *"treatment of governmental stabilization,"* closely related to societal stability, entails equity of pre- and post-tax positions. Confusion does not arise in this connection as we continue to rely on governmental stabilization policy. However, our postulate of taxpayers' behavioral adjustments is indigenous: Taxpayers demarcate themselves as being "rich or poor". Comparable view to demarcate between households in taxpayers' community attracts attention: in the distributive taxation literature, Cremer and Gahvari (1997); in poverty measurements, Sen (1976), Atkinson (1987), Ebert (2002), Hunter et al. (2002); horizontal inequalities seem to occupy a place in Stewart's (2000) paper, which reviews the connection between income distribution and economic growth; income redistribution, which operates as a form of social insurance, Peñalosa and Wen (2004); "*the poverty line acts as a threshold with households falling below the poverty line considered poor and those above poverty line considered nonpoor,*" Tarp et al. (2002).

Rationality. Furthermore we argue, that unless we end up in the study for long-term stabilization of welfare expenses, it will be difficult to explain how the government assesses and rules on wealth circulation through social and public institutions perusing their own interests that benefits all of the society. However, provided the stabilization is feasible, entering the bargaining step of our cascade, we demonstrate that the government might end up being capable of keeping incompatible demands for the social services and for the public goods together.

In a sense *"Meaningful tax reform can be, and should be, examined and discussed as a "positive-sum game" and not as the "zero-sum game" that is often made out to be"*, emphasized by Buchanan and Brennan (1980). Therefore, by Kalai's (1977) asymmetric variant of Nash (1950) bargaining solution, we predict over the allotment of taxpayers' obligations between two Welfare State institutions – social agencies on one side and public agencies on the other. But, it might appear that we have a commitment to interpret the Nash solution also by virtue of players' strategic interaction.

It is not quite unrealistic to imagine that our bargaining game actors, the social and public agencies, play the alternating offers drama between themselves and taxpayers, all together, until they reach an agreement. Even though, we emphasize here, once and for all, that our motive is not to disclose by what bargaining procedure do the bargainers actually arrive at an outcome, but to demonstrate, at any rate, a calculus of Nash solution. The procedure of bargaining, whether it is under complete or incomplete information, or whether the utilities are interpersonally comparable or not, is secondary for us. Nonetheless, it makes a difference later, whether the procedure is under the hypothesis of individual or cooperative rationality. The literature dealing with these serious matters of bargaining occupy a wide range of publications, Alvin E. Roth (1985).

Self-interest. At last, only on the third step taxpayers' subjective well-being has been taken into consideration. Only on this, the third step, it is suitable for the government to improve consumer perception and behavior, to meet the momentum of taxpayers' against big taxes and to match welfare demands in exchange for reasonable amount of income tax per-capita scheduled at bargainers' account. Otherwise, destabilized welfare expenses or disagreeing social and public agencies on previous steps could block the further turn of the government in making favorable decision for taxpayers. Therefore, we must be crystal clear that taxpayers in achieving their consumer's objective "*…might exploit their mutual gains with our players…*" to reject or to acknowledge the bargaining agreement, and nothing else, threatening the bargainers by a binary choice only. Continuing the quote from Osborn and Rubinstein (1990), we read: "*…if we perturb a bargaining game of alternating offers by introducing a small exogenous probability of breakdown then, ...the outcome is close to Nash solution.*" In case of *breakdown* the disagreement poverty proposal is implemented. Needless to say that the agreement, whether it is bad or good, has to be rejected or acknowledged by a referendum provided the voters-taxpayers could correctly perceive the referendum results. Yet again, it must be emphasized here, but now we do not disclose by which voting schemes the voters-taxpayers actually arrive at their consumer's objective. Rather than a voting tool of collecting data about who would approve or oppose what, we design a debating and voting platform for the government decisions to be made, Table 1. As noted by Roberts (1977), *"the point is not whether choices in the public domain are made through a voting mechanism but whether choice procedures mirror some voting mechanism."* However, *"Since all can benefit from the provision of a public good, the obvious voting rule for providing it would seem to be unanimous consent,"* cit. Mueller (1979, p.19). This passage heavily points towards the conclusion that the solution, which minimizes the welfare expenses (social and public costs), Table 1, is the unanimous consent on Fig. 2, cf. e.g. Buchanan and Tullock (1962).

In what direction we move in next sections? We recommend continuing with Section 2, our pleasant story, in hope that someone will feel more comfort in further readings. In Section 3, in our unpleasant story, we move along the cascade of three canonical principals mentioned earlier. On the first step, we disclose the long-term stabilization strategy of welfare expenses, which meets specific feasibility criteria for setting up a bargaining game payoff's surface. This surface is an essential instrument for the government to assess and rule on wealth circulation. On the second step, we make an effort to embrace from the angle of bargaining game between social and public agencies the ambivalence and multifaceted welfare perception of taxpayers. Our work here associates the bargaining with poverty proposals related to players' utility anticipations at so-called bargaining bifurcation frontier. The proof of bifurcation frontier criteria is found in the appendix. Finally, only on the third step, having disclosed a favorable poverty proposal for both of them, the Welfare State bargainers (the social and public agencies) might come to an agreement. Worthless in general, this agreement is not necessarily the best response for taxpayers. However, in circumstances the conditions for minimizing a function are smooth enough to calculate the local minimum for welfare expenses, the favorable poverty proposal reveals the fundamental issue that among asymmetric bargaining solutions a specific one would possibly enable the least burden of taxation. Section 4 brings the study to final judgment. Our "wealth expedition" ends in Section 5.

In general, it might appear that nothing is really wrong in our Welfare State yet nothing is right either. Indeed, the professionals might mutiny against and raise an objection that there is no evidence, no matter what, how to associate the government decisions directly with poverty proposals as the game prescribes. Nevertheless, we argue that something is still right here! Observe that our hypothetical proposal, minimizing marginal tax rate [1] in Table 1 (the best welfare price), has a perfect match with an empirical poverty line (50% of median income) found useful by Tsumori, Saunders and Hughes (2002) as well as by Hunter, Kennedy and Biddle (2002).

All over the paper, along the goals of rationalization in our welfare deeds, we "equilibrate" between two delicate issues: a situation, when agents' economic and moral stimulus is under investigation, and the mathematical purity, when normative observations revolve around analytical functions of variables. The balance should be clear from the context.

2. Sweet cake allotment. The pleasant story

Someone thinking about standards humans make their decisions may ask: *"What ought to be a fair bargain to divide a sweet cake between two individuals: HE – soft negotiator, not very keen on sweets, SHE – tough negotiator, but likes sweets?"* An inspired "game theory purist" will immediately tell a pleasant story that the solution over the sweet cake allotment is the product maximization of players' gains above the *disagreement point* $d = (d_1, d_2)$:

$$\arg\max_{(u,g) \geq (d_1,d_2)} f(x,\alpha) = (u(x) - d_1)^{\alpha} \cdot (g(x) - d_2)^{1-\alpha} .$$

This is the asymmetric variant of Nash Bargaining (1950) solution, Kalai (1977).

Slightly confused, the thinker will ask the next question: *"What is x, α, $u(x)$ and $g(x)$? What is the disagreement point (d_1, d_2) and in which direction to use the $\arg\max$ procedure?"* Expecting such like a question the pure theoretician answers that:

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[1] Marginal tax rate (read also "welfare price") makes a difference a propos welfare expenses equal to taxpayers obligations or liabilities. For those familiar with control theory we keep an account upon the marginal tax rate by "the system state" variable τ next to welfare expenses parameter r, see later.

x is HIS fraction of the cake and α is HIS bargaining power, which is less than 1,

$u(x)$ is HIS desirability function, for example $u(x) = x$, of HIS fraction x of the cake,

$1-\alpha$ is HER bargaining power,

$g(x)$ is HER desirability function of HER fraction $1-x$ of the cake, for example $g(x) = \sqrt{1-x}$.

In widely accepted vocabulary we call $s = (u(x), g(x))$ the utility pair. Disagreement – the threat point $d = (d_1, d_2)$ – is what HE and SHE collect if they disagree how to divide a cake. For obvious reason, disagreeing, players collect nothing – the sweet cake disagreement point $d = (d_1, d_2) = (0,0)$. Further, the game theoretician confirms the thinker expectation that for HER the cake is more valuable indicating at HER desirability $g(½) = \sqrt{1-½} = 0.707$, which is greater than HIS desirability $u(½) = 0.5$.

Now, in connection with $arg\,max$ procedure maximizing $f(x,\alpha)$ the thinker may ask a new question: *"What standards HE – the sweet cake bargainer – will make HIS decision on to obtain the half of the cake?"* That is to ask, for example, what standards inherit HIS negotiating power α to obtain the half of the cake? The game theoretician might be slightly confused in finding the specific answer, but one technically minded person can shed us lights on the solution $\alpha := root\left(f_1'(½,\alpha),\alpha\right)$. An explanation from technical viewpoint is simple. First replace $u(x)$ by x, $g(x)$ by $\sqrt{1-x}$ and take the derivative of the result $f(x,\alpha)$ with respect to the first variable x evaluating $f_1'(x,\alpha)$. Then, replace $x = ½$, and finally solve the equation $f_1'(½,\alpha) = 0$ for α; the equation $f_1'(½,\alpha) = 0$ resolves for $\alpha = 1/3$.

On the whole, the thinker might feel some comfort in following passage. "*In HIS opinion to count on the half of the cake is a realistic attitude although SHE is twice as tough negotiator since HE knows that SHE likes sweets and will make a concession sooner rather than later.*" This is en example of standards the bargainers ought to behave.

3. Tax cake allotment. The unpleasant story

Equilibrium – tax cake stabilization. So far, so good with our pleasant story but now comes the less pleasant matter. The root of that more serious matter seems to be in dividing the tax cake – tax obligations between social and public agencies in order to meet incompatible demands for social services and public goods. Is there a resemblance over the sweet cake allotment? From the first glance it seems reasonable to follow the same pattern in dividing the cake between social agencies redistributing wealth to the poor on one side, and public agencies, on the other side, materializing for example, public safety, environment protection, education and health services, national defense, etc. Yes indeed, it makes sense since social agencies bargaining power is intuitively lower than public agencies. Nonetheless, a difficulty could make the difference – the size of the cake might influence the fair bargain; the sweet cake remained fixed when it was divided between HIM and HER. *"Isn't it true that the size of the cake if mutating too fast might confuse the bargainers in making their decisions?"* the thinker asks once again.

What standards the State makes its decisions on this confusing matter? Game theoretician might suggest personalizing people social, public and consumer wants by taking households' incomes into account. Thus, for the game theory minded thinker, the situation looks like a three persons' game between social and public agencies in dividing the cake, and taxpayers financing the cake materialization. But, it is not enough, because as we already mentioned, the size of the cake could mutate too fast introducing in our rational bargainers' behavior. Surely, while the government is reviewing the demands for social services and public goods, the cake could smooth out the balance of income taxes with welfare expenses. Therefore, as long as the government does not stabilize the size of the cake, the fair bargain between social and public agencies is not feasible. So, what might be the feasible – a stable decision?

In answering the question let first define the government decision without any kind of warranty to stability if the government wishes to be in a position capable to assist the poor on legal and moral grounds. Poverty line ξ might be set by the government to decide who is living in poverty to transfer the wealth from the rich: Poor people with income σ to the left from the poverty line ξ, $\sigma \le \xi$, rich people σ to the right, $\sigma > \xi$.

It is reasonable to believe that income distribution density $P(\sigma)$ is the best ground for the government to decide upon ξ in financing the wealth transfer: *"Income distribution is extremely important for development, since it influences the cohesion of society, determines the extent of poverty for any given average per capita income and the poverty-reducing effects of growth, and even affects people's health,"* Stewart working paper (2000). In particular, the importance of income distribution lies in tax pools upholding. But, income distribution is undetectable for period of time when taxes being put at and costs being taken from bargainers' accounts. However, to make taxes somewhat visible, the government without accurate knowledge of $P(\sigma)$ can still estimate tax schedules in the form of $\tau \cdot T$, social and public costs as R and g. Later we deal exclusively with proportional tax rate τ and it is also assumed that the government is able to enforce tax schedules to be equal to tax obligations.

Yet another important issue, one must be concerned about, involves the quantification. We consider only normalized $P(\sigma)$, where the average income equals 1. Hence the average income becomes a fundamental quantity – one unit of wealth, which admits only linear transformations $\sigma' = a \cdot \sigma$, $a > 0$ in establishing an *interval* scale for all variables and functions.

Now, according to our first-fold of public finance, the government is ready to transfer the wealth from the rich $\sigma > \xi$ to the poor $\sigma \leq \xi$. The government is ready to finance social services, for example to refund the costs of social security incomes R via tax obligations $\tau \cdot T$ provided it keeps a balance between the part $x \cdot \tau \cdot T$ of obligations $\tau \cdot T$ put into social agencies account in one period, but now the actual costs R taken out in the other. The government can shift the cost R to $R(\xi)$ and the obligations $\tau \cdot T$ to $\tau \cdot T(\xi)$. In doing so, the government discloses its strategy to keep $R(\xi) = x \cdot \tau \cdot T(\xi)$ in balance.

In our vocabulary the balance $R(\xi) = x \cdot \tau \cdot T(\xi)$ specifies an implemented decision ξ, implemented poverty line ξ to decide who is living in poverty, implemented social policy, welfare reform, pact, program, etc.; one will likely feel more comfort in using these terms. Note that the balance, despite it is valid, might break in time slots ahead of the beginning: Decision ξ happens to be long-term dependent. Nevertheless, we assume here and later on that the implementation of public finance regulation is more practicable if it provides a long-term decision ξ, which will balance $R(\xi) = x \cdot \tau \cdot T(\xi)$ in the long run. This is but first embodiment. The second embodies the welfare distortions as follows.

Recall the main effect of welfare mission towards the poor. By the result of government commitments, just after implementing improvements in welfare level u, some additional clients in our economy claiming the benefits end up being better off. On the other extreme, the outcome of implemented declines in welfare level u could worse off some old clients to whom the request has been submitted that their benefits have been revoked. Thus, the welfare level u *tidal waves* may introduce in the behavior of those who claim the benefits or whose benefits have been revoked. These waves in turn may cause the income distribution feedback shift into undisclosed position $P(\sigma,\xi)$ in relation to $P(\sigma)$. Accordingly the decision ξ will shift $R(\xi)=x\cdot\tau\cdot T(\xi)$ into long-term feedback instability $R_f(\xi)\neq x\cdot\tau\cdot T_f(\xi)$.

But, in compliance with equilibrium principle, economy must be immune against long-term swings in welfare. That is to say, immune decision ξ once taken do not need further adjustments. Likewise, to say that if the government has implemented the decision ξ under the “immune” or “navigable” ξ-protocol twice, is all one as to say implemented only once. For this reason we assume that in our two-folded economy, when the initial *tidal push or pull* upon the economy was stabilized by the decision ξ implementation, the feedback instability $R_f(\xi)\neq x\cdot\tau\cdot T_f(\xi)$ had already been eliminated in the first fold: $R(\xi)=R_f(\xi)$ and $T(\xi)=T_f(\xi)$. In short, we assume that at the moment the decision ξ was taken the ξ-protocol had already emerged, and being given a chance to became implemented as a long-term independent.

In this mode, the government develops the ξ-protocol that outlines the long-term stabilization strategy of welfare expenses by which it might be brought to conclusion. It thus has been concluded that expenditures account $x\cdot\tau\cdot T(\xi)$ meant for social spending, where x is the fraction of tax obligations $\tau\cdot T(\xi)$, must be in balance with social costs $R(\xi)$ not only when the particular decision ξ was taken but in the whole spectrum of current and future events, i.e. the government decisions ξ to be made in the long-run. In focus on taxpayers’ postpositions $\pi(\sigma,\tau)=(1-\tau)\cdot(\sigma-\phi)+\phi$, cf. e.g. Malcomson (1986) where ϕ is the tax relief, the balance $R(\xi)=x\cdot\tau\cdot T(\xi)$ is a relationship in bringing about functional dependency $\tau(\xi,x)$ between ξ and fraction x variables so that taxpayers’ postposition became a function $\pi(\sigma,\tau(\xi,x))$.

At last, this lead us to ask an obvious but very delicate question, by which composition of social costs $R(\xi)$ and obligations $\tau \cdot T(\xi)$, these functions will form the more accurate predictor for the poverty proposal. An empirical data might be the best indicator. Yet another *"technical indicator"* might be that these functions must allow for easy computations by classical method of function maximization (minimization) with constraints. Generally, this composition is responsible for the origin of our two-folded public finance regulation.

Summarizing, the government, upon the name of the State, assesses and rules on wealth circulation by allocation $\langle u,g,r \rangle\, x,\tau,\alpha \Leftarrow \xi,\phi$, where u,g,r embody the competing objectives, x,τ,α – state variables, and ξ,ϕ – decision (or control) variables (parameters); ϕ allocates the tax relief constant:

- u – demand level of social services, the guaranteed welfare,
- g – public goods demands, public costs,
- r – tax obligations, welfare expenses, in case of proportional taxes $r = \tau \cdot T$;
- x – fraction of tax cake (of tax obligations) scheduled at social agencies account,
- τ – marginal tax rate, the welfare price;
- α – negotiating power of social agencies;
- ξ – poverty line to decide who is living in poverty, the choice or control parameter,
- ϕ – tax relief constant, an external control parameter, $0 < \phi < \xi$.

Without proof. Although the parameter $\phi > 0$ importance lies in consumer problem $r_{min} > 0$ solution the reader may probably wonder that the parameter is not relevant at all. Yes it does, otherwise welfare expenses $r(\xi)$ are not smooth enough to satisfy $r'(\xi) = 0$ minimum condition, see below.

Three constraints must hold:

- the welfare spending constraint, all taxes are spent on welfare (social + public), welfare spending equals to taxpayers' liabilities;
$$\tau \cdot T(\xi) = R(\xi) + g \qquad (1)$$
- the balance of social costs with the fraction of taxpayers' obligations scheduled at social agencies account, and
$$R(\xi) = x \cdot \tau \cdot T(\xi) \qquad (2)$$
- the stabilizer, constraint counteracting households' long-term welfare ambiguity,
$$u = (1-\tau)\cdot(\xi - \phi) + \phi \qquad (3)$$

  We distinguish here between arbitrary welfare demand level $u = \pi(\xi,\tau)$ and net income $\pi(\sigma,\tau)$!

where $R(\xi)$ – social costs mean value shifted by the government decision ξ,

and $T(\xi)$ – amount of welfare for the price τ per one wealth unit shifted by decision ξ; this definition of welfare amount circulating in the society does make economic sense only in case of proportional tax rates.

Following justifies welfare ambiguity constraint (3) in more rigorous vocabulary.

Observation 1. *Let the government has implemented a decision ξ' assessed by allocation with the marginal tax τ' and the welfare level $u' = \pi(\xi', \tau')$ but is trying* *to implement an improved* *welfare level $u > u'$. Constraint (3) is necessary to counteract welfare ambiguity.*

Proof. Suppose not. In other words, let the government attempts to implement $u > u'$ making a decision $\xi'' > \xi'$. At once, for some highly pragmatic households σ an occasion *to claim the social benefits* becomes visible and thus to be *better off* as a consequence of inequality $u \geq \pi(\sigma, \tau') > u'$. These pragmatic households σ through social costs *increase* $R(\xi'') > R(\xi')$ can slip $R(\xi') = x \cdot \tau' \cdot T(\xi')$ into *deficit* $R(\xi'') > x \cdot \tau' \cdot T(\xi')$. The only option available for the government, due to the balance (2), remains to adjust from τ' to $\tau(\xi'', \xi', x) = \dfrac{R(\xi'')}{x \cdot T(\xi')} > \tau'$; the variable x is not under government jurisdiction, otherwise by keeping the old decision ξ' in tact, the government could eliminate the *deficit* through x *decrease*. Regardless of ξ'', if still $u > \pi(\xi'', \tau(\xi'', \xi', x))$, the government must continue the adjustment policy but this time upon the decision ξ'', i.e. generating a sequence $\xi', \xi'', \ldots$. To sum up, in making *an improvement* $u > u'$ lies in sequence $\xi', \xi'', \ldots$ generation unless the next decision ξ'' is not equal to its the previous implementation ξ'. ■

The line of reasoning with $u < u'$ is similar. Replace: *"to implement an improved"* by *"to make a decline in"*, *"better off"* by *"worth off"*, *"an improvement"* by *"a declinement"*, *"to claim the social benefits"* by *"that the social benefits have been revoked"*, "$\geq, >$" by "$\leq, <$", "*deficit*" by "*surplus*"; finally transpose "*increase*" and "*decrease*".

Corollary. *Rational policy ξ for the government without additional adjustments ξ', ξ'',… how to counteract welfare ambiguity, relies on resolving $u = \pi(\xi, \tau(\xi, x))$ for ξ excluding the chance for certain households σ to misbalance $R(\xi) = x \cdot \tau \cdot T(\xi)$.*

Finally, encapsulating constraint (2) into (3), the stable decision ξ must match the root of welfare ambiguity constraint $u = \pi(\xi, \tau(\xi, x))$ in the form:

$$L(\xi, x, u) := (\xi - \phi) \cdot R(\xi) - x \cdot (\xi - u) \cdot T(\xi) = 0. \quad (4)$$

Below, we call the last by welfare volatility constraint. In general, due to constraints (1-3), in *tax cake minimization* step, we make use of functions $\tau = \tau(\xi, x, ...)$, $x = x(\xi, g, ...)$ indirect notification. But, regarding social costs $R(\xi)$ and welfare amount $T(\xi)$ in (1,2), a notice is appropriate. Amount $T(\xi)$ and expenditures $R(\xi)$ mean values, as we just have seen, are the key elements in our economy. In further considerations we only accept functions $R(\xi)$, $T(\xi)$ with positive derivatives $R'(\xi) > 0$, $T'(\xi) > 0$. Thus, our economy would have been opposed by voters-taxpayers could they fully anticipate the progressive costs of an excessive public spending caused by large increase in poverty line ξ.

Rationality – tax cake bargaining. Next comes the most unpleasant issue. Let both the rich and the poor pay their taxes τ (necessarily proportional taxes), and the government legislates the poverty line ξ by calculating the wealth estimates $R(\xi)$ and $T(\xi)$. How the social and public agencies are going to bargain upon the tax cake – tax obligations? Notice, that in accordance with the foregoing observation, when the cake *"mutate too fast"* [2] the feasible bargaining outcomes could be at risk unless the welfare volatility constraint $L(\xi, x, u) = 0$.

Let first personalize the objectives by players' appointment to their roles:

Objectives of players No. 1,2 and 3:

Player 1,	u	–	welfare level, legal and moral objective (the lowest disposable net income per capita, guaranteed welfare), a standard to be legislated by the government;
Player 2,	g	–	public goods per capita, public objective that benefits all of society;
Player 3,	$\tau \cdot T(\xi)$	–	estimated welfare expenses, income tax schedule, the size of tax cake, i.e. an estimated amount per capita of tax obligations, the less the better. We consider only the case of proportional tax rates, which substantially simplify the method of function maximization (minimization) with constraints, see later.

[2] Be aware of a phenomenon that the stabilization process $\xi', \xi'', ...$ of tax obligations has a limit $\xi = \lim \xi', \xi'', ...$ whereby welfare level $u = u(\xi)$ and public goods $g = g(\xi)$ functions are also some "stable" limits. Proof of this phenomenon in parallel with the observation 1 is beyond the scope of current presentation; some ideas may be found downloading http://www.datalaundering.com/download/monogame.pdf.

Before advancing any further let recollect what the bargaining is about. Assume that our players bargain over all possible allotments (x,y) of tax obligations $\tau \cdot T(\xi)$, $x+y \le 1$. Below we follow Nash's (1950) axiomatic approach. An arbitrary set S of utility pairs $s=(s_1,s_2)$, $s_1=u$, $s_2=g$ evaluated only for feasible decisions ξ (decisions immune against welfare volatility) can be the outcome of the game. A disagreement occurs at the point $d=(d_1,d_2)$ where both players $i=1,2$ obtain the lowest welfare they count on. A *bargaining problem* is a pair $\langle S,d \rangle$ and there exists $s \in S$ such that $s_i > d_i$ for $i=1,2$ and $d \in S$. A *bargaining solution* is a function $f(S,d)$ that assigns a unique element of S to every bargaining problem $\langle S,d \rangle$. We expect for players to act from the *strong individual rationality* hypothesis SIR. The bargaining solution f, to which the players are committed to implement, satisfies SIR if $f(S,d) > 0$ for every bargaining problem $\langle S,d \rangle$. The solution must comply with other Nash axioms: INV) invariance under the linear change of scale of utilities; IIA) independence of irrelevant alternatives, and PAR) Pareto efficiency.

Note that following PAR our highly pragmatic players might object an outcome s when there is available an outcome s' where both of them are better off. With the respect to government decision ξ, it means that our players might never agree upon ξ when a decision ξ' was due to be implemented for which an outcome $s(\xi')=s'$, $s'>s$, $s=s(\xi)$ is better off for both of them. Therefore, as we already noticed, the poverty line ξ is the choice or control parameter characterizing bargaining situation. Throughout the paper we refer also to poverty proposal ξ, when players' $i=1,2$ try to agree over their allotment assessment. To place a proposal ξ which would enable tax obligations with the least amount seems to reveal the constraint that mediate consumers perception of welfare expenses reduction offering hereby an appealing interpretation to the otherwise puzzling welfare anticipation $s(\xi)=(u,g)$.

Before continuing, we need to clarify a standpoint of confusion. It is widely accepted that fair bargain occurs on what the game theoretician calls "the negotiation set". In our case we call it bargaining bifurcation frontier – a curve induced by geometric space of agreement points from all feasible utility sets S available. Typical example of bargaining bifurcation frontier $S_b = u(g)$ is shown below on Fig. 3. The frontier $S_b = u(g)$ represents government decisions solving the equation (6) within closed interval $[\xi_1,\xi_n]$ all points of which are stabilized by (4). But, the standard definition of bargaining problem consists of a compact convex set $S \subset \Re^2$ and a disagreement (a threat) point $d \in S$. However, from Osborn and Rubinstein, p. 24, we read *"The compactness and the convexity of S are important only insofar as they ensure that the Pareto frontier of S is well defined and concave. Rather than starting with the set S, we could have imposed our axioms on a problem defined by a nonincreasing concave function* (*and a disagreement point d).*"

However, it seems that Osborn's and Rubinstein's alternative definition of bargaining problem falls short of being convincing for our purposes. The bad thing is that the players $i = 1,2$ may approach the bargaining frontier $S_b = u(g)$ within the range of poverty proposals (inevitably higher levels) where both of them, if continuing the bargaining, cannot find any other options <u>at all</u> as *"one agent's welfare necessitates a sacrifice"* for opponents advantage. *"Thus, one would expect the agents to negotiate (or haggle) over these candidates – a process that may in turn expend resources and delay agreement,"* cit. from W. Samuleson, p. 322. So, within these higher levels of poverty proposals (in our terms within the poverty prohibitive range, outside the bargaining frontier; rigorous definition in the appendix), it might be called in question whether any pragmatic governmental institutions will ever negotiate prohibited proposals. Therefore we look at the bargaining interaction of players only in the normal range of poverty (inside the region with bargaining frontier as boundary, where the players cooperate). One would, perhaps, recognize here the Laffer curve terminology, but for different domain, similar to: first, poverty lines being proposed in the normal range of collective rationality; next, "by passing through their last bifurcation" the poverty proposals will be assessed and reviewed in the range of prohibited (individual) rationality. We paraphrase here from Canto, Joins and Laffer (1981), and from Malcomson (1986) as well as Rapoport's (1989, p. 291) *"bifurcation of rationality into 'individual' when the collective rationality vanishes…".*

The alternative definition acknowledges the bargaining in the normal range of poverty where the players' $i = 1,2$ might arrive at the bargaining frontier S_b improving their payoffs in cooperation. Nevertheless, in our case the players might decide to continue the bargaining on individual basis by crossing through the frontier putting the entire agreement at risk. This gives us a reason to call the cross passage by bargaining bifurcation frontier.

The good thing, however, is that it is almost certain where to put the disagreement point d. The intuition tells us that a reasonable candidate for the disagreement point d is the anticipation (u_1, g_n) composed from the utility pairs (u_1, g_1) and (u_n, g_n) at the end points of the closed interval $[\xi_1, \xi_n]$; to sharpen the meaning we interpret this point d as a threat – the threat not to cross the bifurcation boundary into the region of individual rationality without f.e. any commitment at all to keep to the agreements. But, it is not immediately apparent that the threat point is a feasible outcome of the game. Following observation rules out such doubt for the Nash solution to be well defined.

Observation 2. *Suppose that bargaining bifurcation frontier* $S_{\mathrm{b}} = u(g)$ *is a non-increasing function within a closed interval* $[\xi_1, \xi_n]$ *of government decisions* ξ *resolving the bifurcation constraint (6). Then, for the utility point* $d = (u_1, g_n)$ *to be a feasible outcome of the bargaining game it is necessary and sufficient that there exists a decision* δ *resolving the equation:*

$$(\delta - \phi)\cdot(R(\delta) + g_n) - (\delta - u_1)\cdot T(\delta) = 0 \tag{5}$$

where the inequalities $g_1 > g_n$ *and* $u_1 < u_n$ *hold for utility pairs* (u_1, g_1) *and* (u_n, g_n) *at the endpoints of the interval* $[\xi_1, \xi_n]$*; condition* $\delta \notin [\xi_1, \xi_n]$ *is necessarily required.*

Proof. The statement is correct as soon as we can find a stable decision δ for utility pair (u_1, g_n) implementation. Really, let replace in the formula for welfare spending constraint (1) the variable g by g_n. Then, take out from (1) the expression for $\tau = \dfrac{R(\delta) + g_n}{T(\delta)}$ and then substitute it into $(1-\tau)\ldots$ of welfare ambiguity constraint (3), where u should be replaced by u_1 in advance. Simplifying we end up with the statement of our observation. ■

Recall that $T(\delta)$ is the amount of welfare circulating in the society and $R(\delta)$ are the social costs.

Corollary. *For the lowest possible utility pair* $d = (u_1, g_n) \notin S_{\mathrm{b}}$ *at end points* ξ_1, ξ_n *of the bargaining bifurcation frontier* $S_{\mathrm{b}} = u(g)$ *and for any other utility pair* $s \in S_{\mathrm{b}}$ *we have* $s > d$*. Then, for the bargaining frontier* S_{b}*, the pair* $\langle S_{\mathrm{b}}, d \rangle$ *is the bargaining problem.*

Example. There is no reason why the equation (5) has a solution in general, but it is easily checked for any particular case. Let us check it on the basis of hypothetical income distribution density $P(\sigma,\xi)$ in Section 4 that the lowest possible utility pair may be the outcome of the game. Let

$$d=(d_1,d_2)=(u_1,g_n),\ u_1=0.184,\ u_1<u_n,\ u_n=0.734,\ \xi_1=0.243;$$
$$d_1=u_1,\ d_2=g_n,\ g_1=0.314,\ g_1>g_n,\ g_n=0.124,\ \xi_n=1.215\ .$$

Then, the equation (5) may be resolved in that particular case by δ :

$$(\delta-\phi)\cdot(R(\delta)+d_2)-(\delta-d_1)\cdot T(\delta)=0\text{, where } \delta=0.202\notin[0.243,1.215].$$

So, it seems that we passed by all the troubles with the disagreement poverty proposal δ . Not at all: *"a serious objection can be raised to Nash's bargaining scheme, and it is that it does not take threats into account."* Continuing from Owen (1982), *"the threat point d is effective if it is believable."* We call to readers' attention that the threat in our scheme emanates not from the bargainers, but from the third party strategic interaction. In order to reach their objective voters-taxpayers rejecting or acknowledging the agreement ξ on the bargaining bifurcation frontier S_{b} emanate the threat δ by binary choice only. Implementing the proposal δ materializes the threat for the bargainers. However, such a negative effect δ of bargainers' contemplation can still be arbitrated by Nash solution in a standard way.

We will see soon, in the next subsection, that the bargaining bifurcation frontier $S_{\mathrm{b}}=u(g)$ became a curve $(u(\xi),g(\xi))$ parameterized by the poverty proposals $\xi\in[\xi_1,\xi_n]$. Provided the curve is smooth enough the arbitrated point $(u(\theta),g(\theta))$ has to be found in a unique crossing point between the geometric line, initiating from the threat point $d\notin S_{\mathrm{b}}$ with the negative tangent slope, Owen, p.138, and the curve $(u(\xi),g(\xi))$.

Indeed, it is hereby the arbitrated decision θ of the government to be found (*an arbitrated poverty* θ) within the interval $[\xi_1,\xi_n]$ by resolving the equation

$$u'(\theta)\cdot(g(\theta)-d_1))+g'(\theta)\cdot(u(\theta)-d_2)=0$$

for θ, $\dfrac{u'(\theta)}{g'(\theta)}=$tangent at the point $(u(\theta),g(\theta))$.

On the basis of income distribution in Section 4, the arbitrated decision $\theta\in[0.243,1.215]$ occupies a designated column in Table 1. Unlike the asymmetric incomes κ, minimizing tax obligations (welfare expenses) and λ the tax rate (the best welfare price) accordingly (or the 50% median income $\mu/2$, or the symmetric income η), the arbitrated income θ, as it name implies, is well enough for bargainers not to be voted for, but, on the contrary, to be implemented by the government ex ante decision θ, observe Table 1.

To close the section we return to our temporarily interrupted way of presenting the bifurcation frontier criteria. The feasible allotments (x,y) being a consequence of feasible (stable decisions ξ), as we already noticed, do not allow too fast mutation of our tax cake $r=\tau\cdot T$ in size. Notice also that from this moment and further on we demand for the decisions ξ to be materialized in feasible poverty proposals ξ, and exclusively in such feasible boundaries ξ demarcating the limits of the poverty – the welfare volatility constraint $L(\xi,x,u)=0$ must be in force, see (4). Arriving at the bargaining bifurcation frontier within normal range of poverty ξ, one can find a set of utility pairs (u,g) where neither social agencies nor public agencies can improve their positions without decreasing counterpart positions on the basis of cooperative rationality only: The players are just to enter the prohibitive range of poverty.

Summarizing. In compliance with $Q(\xi,\tau,g)=0$ and $L(\xi,x,u)=0$, the welfare spending (1) and welfare volatility (4) constraints accordingly, an allocation $\langle u,g,r\rangle\, x,\tau,\alpha\Leftarrow\xi,\phi$ on the bargaining bifurcation frontier must satisfy the bifurcation constraint (proof in the appendix):

$$D(\xi,x,u):=\frac{\partial}{\partial\xi}[(\xi-\phi)\cdot R(\xi)-x\cdot(\xi-u)\cdot T(\xi)]=0. \qquad (6)$$

Self-interest – tax cake minimization. The whole story must be familiar for those who use to make the decisions under constraints. However, if someone doubts our bargaining problem $\langle S_{\mathrm{b}}, d \rangle$ solution, we reproduce the calculus (6), for bargaining bifurcation frontier $D(\xi, x, u) = 0$ as we just noticed, in the appendix, but this detail is irrelevant at the moment. Indeed, the tax obligations (welfare expenses) minimization (M)-problem is a consumer problem, and (N)-problem is the bargaining problem:

(M) $\min_{\xi, g} \tau(\xi, x(\xi, g)) \cdot T(\xi)$,
where the tax parameter is a function $\tau = \tau(\xi, x(\xi, g))$ and fraction x of the tax cake (fraction of tax obligations) itself is a function $x = x(\xi, g)$;

(N) $\max_{u, g} f(u, g) = (u - d_1)^{\alpha} \cdot (g - d_2)^{1-\alpha}$ with some unknown α.

Both (M) and (N) are subject to

$$\left. \begin{aligned} &Q(\xi, \tau, g) = 0 \text{ – welfare spending constraint;} \\ &L(\xi, x, u) = 0 \text{ – welfare volatility constraint;} \\ &D(\xi, x, u) = 0 \text{ – bifurcation constraint.} \end{aligned} \right\} \tag{7}$$

In particular case of proportional tax rates, these three constraints Q, L and D all together (following a succession of collections on sub-expressions and simplifications) imply that

$$g(\xi) = \frac{R(\xi) \cdot [T(\xi) \cdot (T(\xi) - R(\xi)) + (\xi - \phi) \cdot (R(\xi) \cdot T'(\xi) - R'(\xi) \cdot T(\xi))]}{R(\xi) \cdot T(\xi) + (\xi - \phi) \cdot (R'(\xi) \cdot T(\xi) - R(\xi) \cdot T'(\xi))},$$

$u(\xi) = \xi - \dfrac{(\xi - \phi) \cdot (R(\xi) + g(\xi))}{T(\xi)}$, where

$R'(\xi) = \dfrac{d}{d\xi} R(\xi) > 0$, and $T'(\xi) = \dfrac{d}{d\xi} T(\xi) > 0$.

Thus, as one can see from the above, the public goods $g(\xi)$ and the welfare level $u(\xi)$ functions should hereby only depend on the government poverty proposals (decision) ξ. The consumer problem becomes to minimize (M), and the bargaining calculus becomes to maximize (N), but this time without constraints:

(M) $\min_{\xi} r(\xi) = R(\xi) + g(\xi)$

(N) $\max_{\xi} f(\xi, \alpha) = (u(\xi) - d_1)^{\alpha} \cdot (g(\xi) - d_2)^{1-\alpha}$.

At last, we arrived at the idea of major resemblance between our pleasant story, where two highly pragmatic individuals try to divide a sweet cake, and our unpleasant story, where social and public agencies try to divide a tax cake.

Really, the thinker knows that the consumer problem (M) solution is the root κ of the equation $\frac{d}{d\xi} r(\xi) = 0$, provided $r(\xi)$ is smooth enough. The thinker also anticipates, at present and in the long-run perspective, that the decision κ is the best decision in taxpayers' favor (the least amount of welfare expenses) and for that reason the decision κ might be chosen by the taxpayers in unanimous consent. Moreover, in favor of diminishing the tax evasion and filing false returns, the decision κ (the consumer poverty line) might be the most desirable decision also for the government. So, the thinker may ask once again (e.g., the procedure over sweet cake allotment): *"What standards might tune social agencies negotiating power on financing welfare level $u(\kappa)$ with a sufficiently suitable tax package?"*

Indeed, the idea appears to be clear. The standards HE tuned HIS power on over sweet cake allotment can now be the standards for social agencies to tune their negotiating power α on resolving the equation $\alpha := root\left(f_1'(\kappa,\alpha),\alpha\right)$. Recall that first we must take the derivative of $f(\xi,\alpha)$ with respect to ξ evaluating $f_1'(\xi,\alpha)$, then we must replace ξ by κ (do not replace $\xi = ½$ this time as in case of sweet cake). At last, negotiating power α of social agencies negotiating welfare level $u(\kappa)$ solves the equation $f_1'(\kappa,\alpha) = 0$ for α.

4. Final judgment. The welfare masquerade

Before we move to our final issue a comment is in place. In natural sciences *"the predictive power of mathematical model"* is relatively familiar to everyone. For example, *"Kepler's inductive inferences of planetary motion to predict planets observed positions could be mathematically deduced from Newton's mechanics provided one has to notify planets snapshot positions and velocities,"* paraphrased from Rapoport (1983), p.12. In social sciences, on the contrary, the problem is common to researches predicting human responses; much worth case to proceed with "welfare experiment" involving bargainers of social and public agencies.

Nevertheless, let try to proceed with welfare masquerade notifying a snapshot position of the State by some income distribution density, for example, similar to exponential:

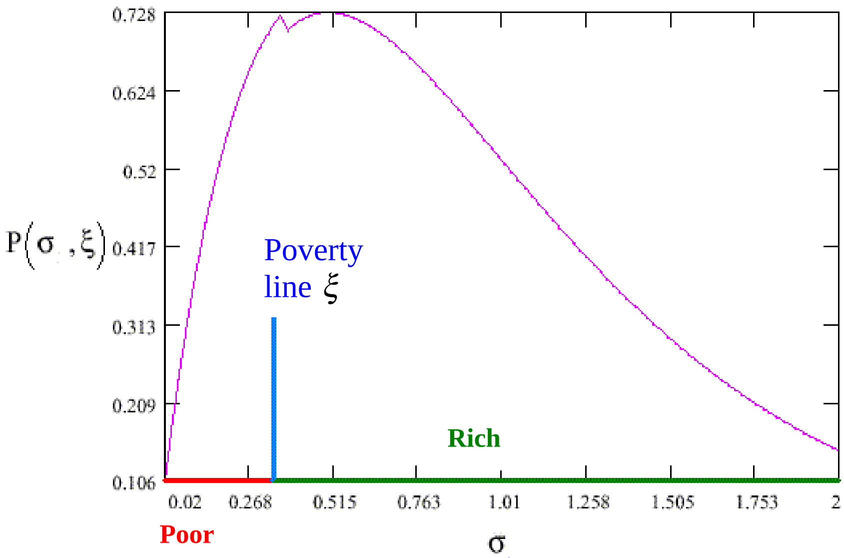
$$P(\sigma,\xi)=\frac{4\cdot\sigma\cdot exp(-2\cdot\sigma)+h}{1+h\cdot\xi} \text{ if } \sigma\le\xi \text{ and}$$

$$P(\sigma,\xi)=\frac{4\cdot\sigma\cdot exp(-2\cdot\sigma)}{1+h\cdot\xi} \text{ if } \sigma>\xi \text{ .}$$

Average income of this distribution $a(\xi)$ equals $\frac{1}{1+h\cdot\xi}$.

We rest on an assumption that people respect the basic morals of not lying at all and thus this particular snapshot requires the obligation to tell the truth. Such, at least, is the case when all households $\sigma<\xi$ claim the social benefits, or visa versa, when no households with higher incomes $\sigma\ge\xi$ receive a benefit, i.e. the request has been submitted to revoke the benefits from those who may be received the benefits in the past. Baseline probability component $P(\sigma)=4\cdot\sigma\cdot exp(-2\cdot\sigma)$ highlights the first-fold in our two-folded public finance regulation when the circulation of wealth from the rich to the poor was just originated (implemented) by the decision ξ, i.e. $P(\sigma)$ is the initial position hiding the tidal wave of welfare ambiguity at the moment the decision ξ was taken. The feedback wave $h\cdot\xi\cdot a(\xi)$ highlights the second fold of finance regulation, where the additional push $h>0$ or pull $h<0$ into the undisclosed position $P(\sigma,\xi)$ was actually caused by the decision ξ feedback factor $h\neq 0$. Notice that the tidal wave hidden in the density $P(\sigma)$, as well as its undisclosed position $P(\sigma,\xi)$, both have characteristic "tail" to the right, which is typical for societies sharply divided into very rich and very poor people, Fig. 1.

Figure 1. Income Distribution Density



Next, we elsewhere emphasized that materializing social needs the government, upon the name of society, assesses and rules on wealth circulation from the rich to the poor not purely on economic, but on legal and moral grounds. For instance, social benefits, materialized in SSI – in the Supplemental Security Incomes $\xi - \sigma > 0$ compensating the unfair subsistence σ of the poor might be eligible. As it name implies SSI supplements the income σ up to the level ξ. Supplemental Income is a supplement to make the minimum expenses for food, clothing and shelter, fuel and lights, etc., the eligible composition of poverty basket for the poor. A more comprehensive assistance rules could be easily incorporated.

The explicit formulas of governmental SSI payments for R, and amount T of welfare circulating in the society, derived on the basis of hypothetical income distribution density, Fig.1, look like:

$$R(\xi) = \frac{½ \cdot h \cdot \xi^2 + G(\xi) - 1}{1 + \xi \cdot h};$$

$$T(\xi) = \frac{G(\xi) - [\phi + \xi \cdot h \cdot (\xi - \phi)]}{1 + \xi \cdot h}, \text{ where}$$

$$G(\xi) = \xi + (1 + \xi) \cdot exp(-2 \cdot \xi).$$

We do not expect that our thinker is necessarily a wise person, but now that the thinker has examined the situation, a chance is given to masquerade “the fresh and green pasture” of the reality by specific question, as the thinker always does: *“Is it none the less true that in the long-run it is going to be difficult to maintain the welfare if the entire payroll tax burden for all taxpayers – including the poor, is decreasing?”* On the surface it may seem that at some point the fairness and equity disappears because poor people will become poorer and rich people richer, the thinker thinks, and the effect is that the social security level tends to drop down dramatically. However, it remains to be seen what fallout may result from such eventu-alities, especially if social agencies try to reach an agreement with public agencies.

Let first recall the list of all the different poverty proposals – incomes k, θ, η, λ, $\mu/2$ on the bargaining bifurcation frontier (bear in mind that δ is outside the frontier) as follows:

- κ poverty line, minimizing welfare expenses (tax obligations);
- θ arbitrated poverty decision;
- η poverty line, when the negotiating power of social and public agencies is equal;
- λ poverty line, minimizing tax rate (the best welfare price) instead of welfare expenses;
- $\mu/2$ 50% of median income μ; μ separates the income distribution into two equal parts;
- δ disagreement (the threat) income – the poverty line to disagree, when the social and public agencies bargainers cannot decide how to divide the tax cake.

After a quick overlook at the list, we mirror the "*tax burden eventualities*" by poverty proposals in the table below, predicted in the way the bargaining game actors ought to behave.

Table 1. Numerical Results of Welfare State Bargaining Game Solutions [3]

(obtained by calculus on the basis of hypothetical income distribution density, Fig. 1)

	Tax consensus, minimizing welfare expenses	**Arbitrated decision**	**Symmetric decision**	**The decision, minimizing welfare price**	**50% of median income**	**The threat decision**
Poverty line	0.369	0.564	0.731	*0.423*	*0.413*	0.202
Poverty rate: percentage of households below the poverty line	17.5%	31.9%	43.8%	*21.4%*	*20.7%*	6.6%
Negotiating power of social agencies	0.124	0.296	0.5	*0.162*	*0.155*	**Not defined**
Welfare level	0.272	0.401	0.502	*0.309*	*0.302*	0.184
Public goods	0.283	0.253	0.225	*0.275*	*0.276*	0.124
Amount of welfare circulating in society	0.941	0.981	1.035	*0.95*	*0.948*	0.927
Welfare Price, Marginal tax rate	32.7%	33.2%	34.8%	*32.6%*	*32.6%*	13.9%
Welfare expenses	0.308	0.325	0.36	*0.31*	*0.309*	0.129

[3] Imagine the column in the table is an option on the Voting Ballot the voter-taxpayer wishes to vote for!

The table gives a reason why, despite its being an obvious fiction, the social and public agencies when engaged in interaction, like between HIM and HER over the sweet cake allotment, can still make a fair bargain in order to help the poor. Indeed, at the time, when the government was reviewing the demands for social services and public goods whether to implement the symmetric poverty line *0.731*, social agencies negotiating power $\alpha = 0.5$ was stronger than *0.124* as everyone can see from Table 1. Nonetheless, the moment with a weakened negotiating power $\alpha = 0.124$ is yet to come, when the non-symmetric agreement over poverty line *0.369* can still be reached; the marginal tax rate *32.7%* will be less than *34.8%* and the welfare expenses *0.308* will be also less than *0.36* . Should the thinker rephrase the sentence, most likely, social agencies weakening their bargaining position, will sooner rather than later still be able to find a consensus with public agencies to maintain a fair welfare level despite the government reduces taxpayers' liabilities, i.e., to debate the welfare expenses in favor for taxpayers unanimous consent, Fig. 2. This is an example of standards the State might make its decisions – the sacrifice the State will have to fulfill its welfare mission.

5. Concluding remarks.

Having returned back from the government welfare deeds it is an appropriate moment to pass in review the knowledge what the expedition left to us. So far we followed the wealth circulation in two directions: the vertical where we revised a way how to estimate the welfare amount circulating in the society and the social spending channeled through the tax system; the horizontal, where households have been split up into two groups depending on poverty line to decide who is living in poverty. Therefore the poverty line was the government choice or decision parameter, which influenced the circulation of wealth and thus has had a feedback effect on taxes. The Social and the public agencies, the bargaining game actors, tried to meet the demands for the social services and for the public goods by navigating the government decision-making process.

But now we reveal a certain implicit idea of coalitions formation among households with higher incomes contra lower incomes. Actually we determined the net income to serve household's self-interest. Without the reader being fully aware of it, in doing so, we characterized some remarkable or feasible coalitions giving credit to their ability to maintain reasonably high living standards – net income levels of guaranteed welfare. The guaranteed welfare was later chosen to be the welfare indicator. Social agencies, whose payoff matched all of the efforts they put in to increase the chosen indicator, were then appointed to the role of coalition formation. So, the root of our thinly veiled scheme is the coalition formation under moderator jurisdiction – under government officials' pursuing their own self-interests that benefits all of society.

Within the limits of the scheme (what was also left apart from the study) it is now a standard exercise to find out that, whereas the poverty line increases, our feasible coalitions with higher incomes are decreasing in size together with their members-households utilities thanks to the marginal tax rate (the welfare price) increase. Anyway, this imperative precondition played a major role in establishing a non-trivial bargaining solution on the bargaining bifurcation frontier. But, after all, this missing detail becomes merely a pretext to the following.

In fact, the goal was to capture attention to *ex mathematica pura* idea of a game implanted into institutional economics environment via our "Welfare Belief Statement". What goes on the expense of the game story transformation into "***genre of economic fiction***", we then deducted from the statement a masquerade of taxpayers interaction with social and public agencies. As a result, searching for the reason behind our pure idea, we developed a guide to how the welfare phenomena ought to be examined more profoundly since now we can measure the wealth circulation in the society. But all this was done on understanding the key issue that it appears to be difficult, if not impossible, to support our "***bargaining game fiction***" by data and empirical research from the field. However, on the behalf of our "***hypothetical income distribution***" we have provided an evidence for claim that the solution being the best welfare price, which thus would enable the best tax proposal, has a perfect match with widely recognized but very low poverty line defined as 50% of median income.

Appendix. The bifurcation frontier criteria

Here we investigate players' 1,2 social services and public goods anticipations (u,g), which guarantee the bargaining bifurcation frontier $S_{\mathrm{b}} = u(g)$ properties. The bargaining occurs in feasible allocations $\langle u,g,r\rangle\, x,\tau,\alpha \Leftarrow \xi,\phi$, where neither the welfare level u nor the public goods g can be improved varying government decision ξ – the feasible poverty proposal turns up on the bargaining bifurcation frontier.

In feasible allocation, the welfare level u, the fraction x, the decision ξ and the tax rate τ variables depend on each other. The fraction $x = x^{\circ}$, if settled as a possible agreement, redirects the welfare level $u = \pi(\xi,\tau(\xi,x^{\circ})$ to become a function $u = u(\xi,x^{\circ})$. Consequently, our maximization problem framework in analyzing the welfare level u looks like:

$$\xi^{\circ} = \arg\max_{\xi} u(\xi,x^{\circ}) \tag{A.1}$$

Observation 3. *Assume that social agencies do not shift from the fraction* $x = x^{\circ}$. *Let the welfare volatility constraint (4) solves for two different decisions* $\xi_1 < \xi_n$. *Let the welfare level* $u = u(\xi,x^{\circ})$ *be a differentiable and strictly convex function of* ξ *within closed interval* $[\xi_1,\xi_n]$, *the derivatives*

$$\left.\frac{\partial}{\partial\xi}u(\xi,x^{\circ})\right|_{\xi=\xi_1} > 0,\ \left.\frac{\partial}{\partial\xi}u(\xi,x^{\circ})\right|_{\xi=\xi_n} < 0 \text{ and } \frac{\partial^2}{\partial\xi^2}u(\xi,x^{\circ}) < 0,$$

then, there exists a unique interior decision ξ° *maximizing* u *at*

$$\left.\frac{\partial}{\partial\xi}u(\xi,x^{\circ})\right|_{\xi=\xi^{\circ}} = 0.$$

Corollary. *The poverty proposal* ξ *higher than* ξ° *can only decline the welfare level* $u(\xi^{\circ},x^{\circ})$. The proof is elementary; we omit it. We say that if $\xi \le \xi^{\circ}$ the poverty proposal ξ for the fraction x° is in the normal range, if $\xi > \xi^{\circ}$ the proposal ξ is in the prohibitive range, i.e. the welfare level $u(\xi,x^{\circ})$ is single $\cap$-peaked.

We came to the standpoint of the main outlook to be considered in this section – social and public agencies, the players 1,2 bargaining bifurcation frontier.

Observation 4. *Assume that the welfare volatility constraint (4) is a differentiable of its variables. The welfare level* $u = u(\xi, x^\circ)$ *is differentiable and strictly convex with respect to decision* ξ *variable within some closed interval* $[\xi_1, \xi_n]$. *For the feasible allocation* $\langle u, g, r \rangle\, x, \tau, \alpha \Leftarrow \xi, \phi$ *to become a point on the bargaining bifurcation frontier* $S_{\mathrm{b}} = u(g)$ *it is necessary and sufficient that the decision* ξ° *resolves the equation:*

(**i**) $\left.\dfrac{\partial}{\partial \xi} L(\xi, x^\circ, u^\circ)\right|_{\xi=\xi^\circ} = 0$ where $u^\circ = u(\xi^\circ, x^\circ)$ provided that

(**ii**) $\left.\dfrac{\partial}{\partial u} L(\xi^\circ, x^\circ, u)\right|_{u=u^\circ} \neq 0$.

Proof. Necessity. In the feasible allocation the maximization procedure (A.1) varying the decision variable ξ is equivalent to $\max_\xi \pi(\xi, \tau(\xi, x^\circ))$. Let the allocation $\langle u^\circ, g^\circ, r^\circ \rangle\, x^\circ, \tau^\circ, \alpha \Leftarrow \xi^\circ, \phi$ on the bargaining bifurcation frontier $S_{\mathrm{b}} = u(g)$ maximizes (A.1) at $u^\circ = u(\xi^\circ, \tau(\xi^\circ, x^\circ))$. Substituting $u = u(\xi, \tau(\xi, x^\circ))$ into constraint (4), we obtain an identity $L(\xi, x^\circ, \pi(\xi, \tau(\xi, x^\circ))) \equiv 0$. Within some proximity for ξ°, u° we exhibit that

$$\frac{\partial}{\partial \xi} L(\xi, x^\circ, u^\circ) + \frac{\partial}{\partial u} L(\xi^\circ, x^\circ, u) \cdot \frac{\partial}{\partial \xi} \pi(\xi, \tau(\xi, x^\circ)) = 0, \tag{A.2}$$

from which we can deduce the necessity statement for $\xi = \xi^\circ$ and $u = u^\circ$.

Sufficiency. Let the equation (**i**) have a solution ξ° for some feasible allocation $\langle u^\circ, g^\circ, r^\circ \rangle\, x^\circ, \tau^\circ, \alpha \Leftarrow \xi = \xi^\circ, \phi$ provided that the condition (**ii**) is fulfilled. Now, combining (**i**) and (A.2), we conclude that

$$\left.\frac{\partial}{\partial \xi} \pi(\xi, \tau(\xi, x^\circ))\right|_{\xi=\xi^\circ} = 0$$

The sufficiency statement for (A.1) holds, since the welfare level $u = u(\xi, x^\circ)$ is a strictly convex function of ξ. This concludes the proof. ■

N.B. Suppose that public agencies pursuing their own self-interest try to increase the payoff g° to $g' > g^\circ$ promising to keep the welfare level u° unchanged. Observe that this increase may be achieved exclusively by tax increase $\tau' = \tau(\xi', x') > \tau(\xi^\circ, x^\circ)$ just sloping along the contour $u(\xi^\circ, x^\circ) = \text{const} = u(\xi', x')$. However, it can be shown that individual move g' of public goods, due to constraints (7), must diminish the fraction $x' < x^\circ$ but to increase poverty proposal $\xi' > \xi^\circ$ thanks to the tax increase. Thus, for x' the poverty proposal ξ' is in the prohibitive range (outside the bargaining bifurcation frontier). By promising a lower tax rate $\tau'' < \tau'$ this public agencies move could potentially wipe out mutual opportunities for social agencies and for voters-taxpayers: $u'' = \max_\xi u(\xi, x') > u(\xi', x')$ and $\tau'' < \tau'$.

Figure 2. As it follows from the graph 32.7% tax consensus between social and public agencies seems to be unanimous at 0.308 level of welfare expenses. On the contrary, the best welfare price 32.6% might course a disconsent just for the reason that higher tax price might finance higher social, but lower public services at the same level of welfare expenses 0.310, Tab. 1

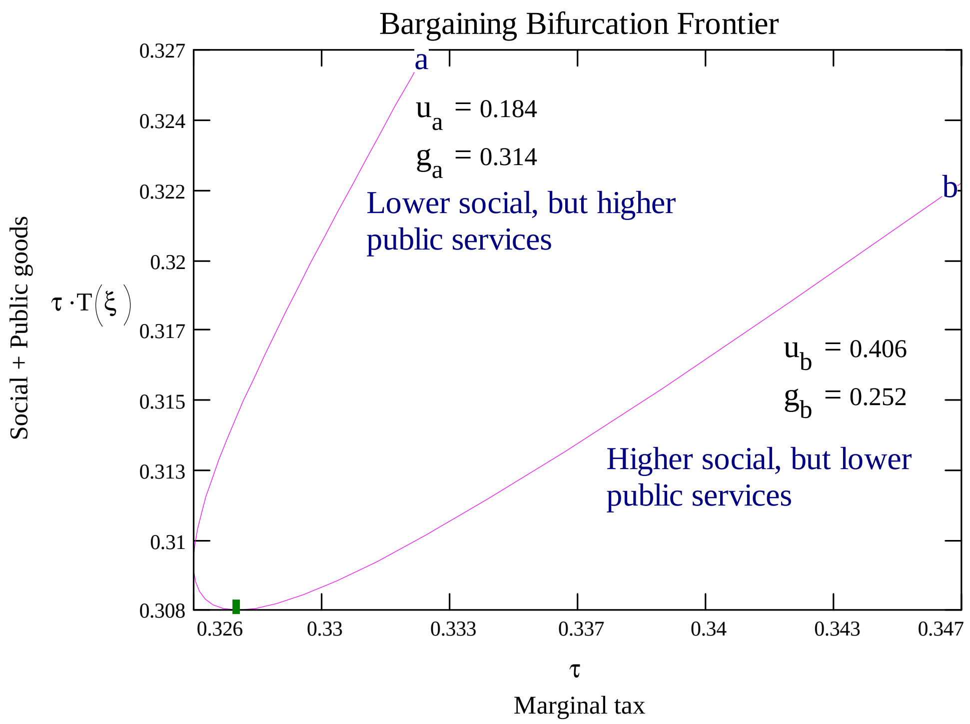
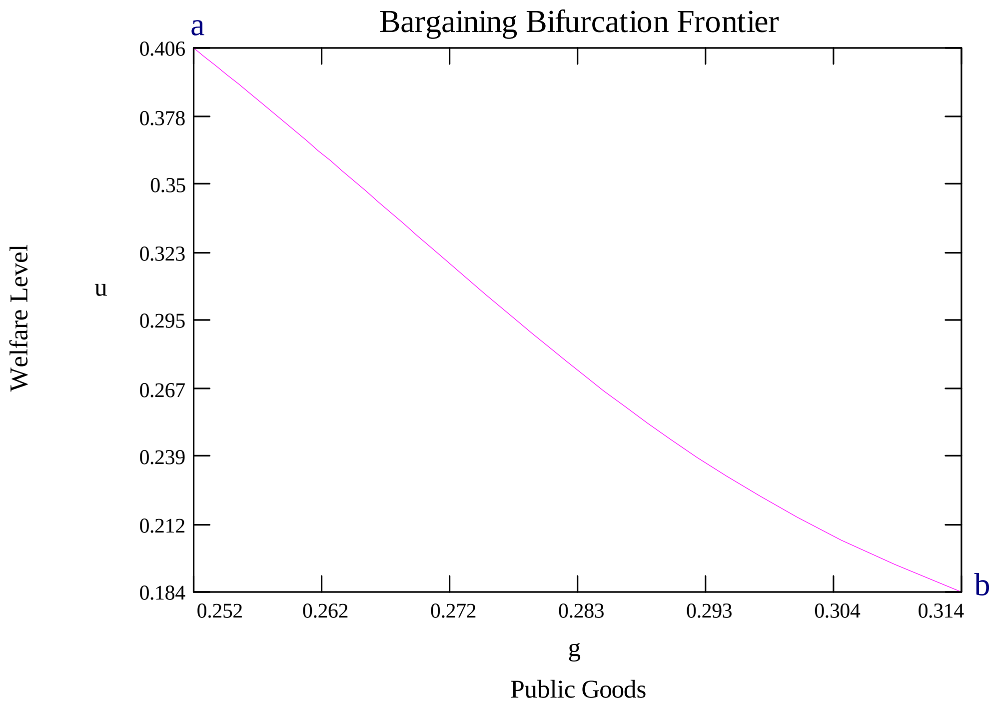


Figure 3. On the vertical and horizontal axes are depicted the utilities of social and public agencies. The graph represents bargaining bifurcation frontier S_b = ***u(g)*** of guaranteed welfare sloping down from the left-top **a** towards right-bottom **b** . It is the projection found resolving the bifurcation constraint (6)



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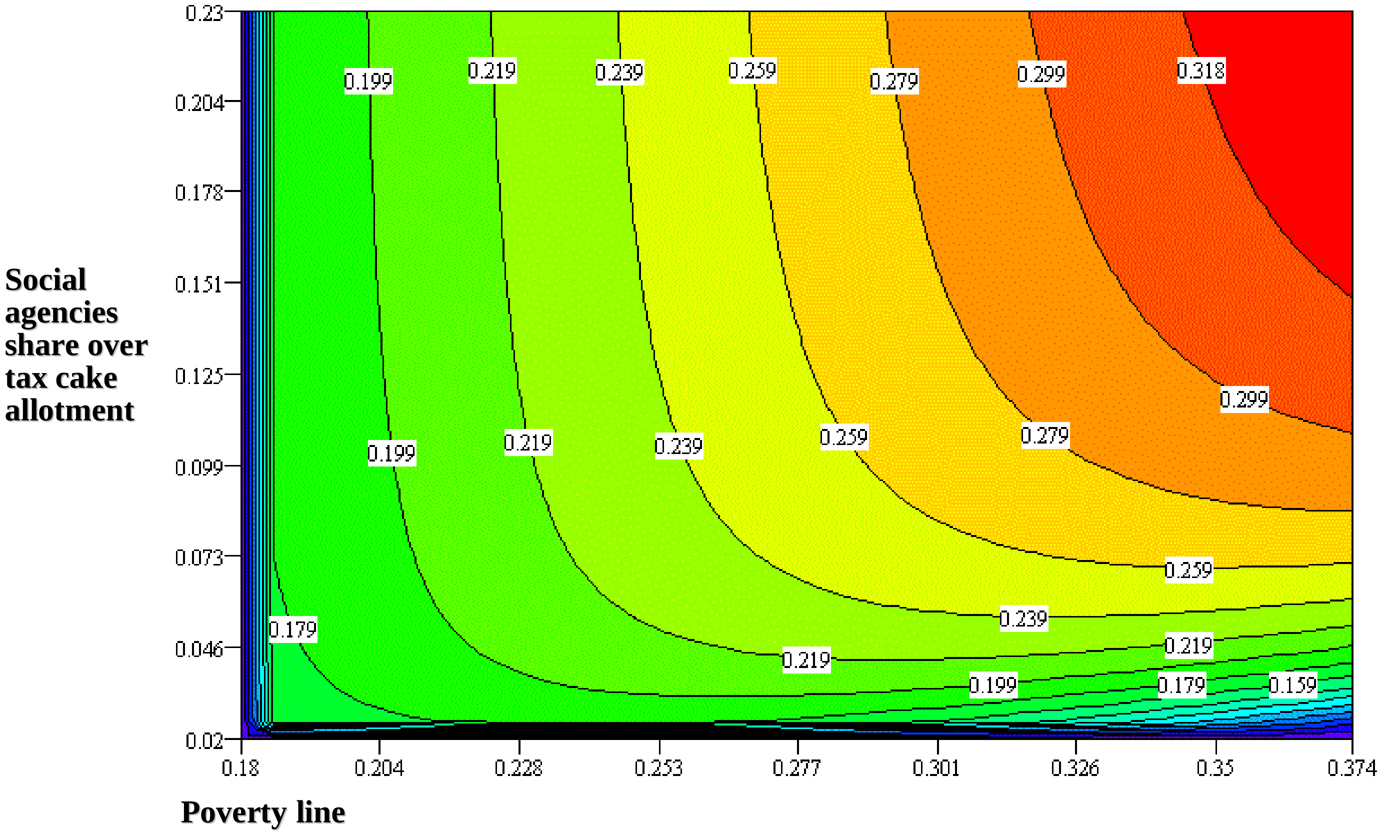
Contour Plot: Demands for Social Services



Contour Plot: Demands for Public Goods