Performance of Soft Metrics for Convolutional Coded Asynchronous Fast FHSS-MA Networks Using BFSK Under Rayleigh Fading

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***Abstract*—Performance of robust soft decoding metrics is examined for use in convolutional coded asynchronous fast frequency-hop spread-spectrum multiple-access networks using binary frequency-shift keying under Rayleigh fading channels. For comparison, maximum-likelihood metrics based on a Gaussian approximation are derived. Significant gains are observed compared to hard decision decoding by using appropriate soft metrics.**

***Index Terms*—Convolutional codes, fading channels, frequency hop communication, multiaccess communication.**

I. Introduction

FORWARD error correction codes provide an effective means of combating the effects of multiple-access interference (MAI) in frequency-hop spread-spectrum multiple-access (FHSS-MA) networks. Much of the previous work on coded FHSS-MA networks has concentrated on block codes with hard decision decoding, while convolutional codes have received relatively little attention [1]. Performance of convolutional codes for asynchronous fast FHSS-MA (AFFHSS-MA) networks with errors and erasures decoding with perfect side information (PSI), errors only decoding with no side information (SI), and errors and erasures decoding with Viterbi ratio thresholding under nonfading channels has been investigated in [1]. However, since convolutional codes with Viterbi decoding are especially well suited for soft decision decoding, further improvement should be attainable by using appropriately chosen soft metrics.

In this letter, various soft metrics are investigated to find those suitable for AFFHSS-MA networks employing binary convolutional coding with orthogonal binary frequency-shift keying (BFSK). Each frequency-hop slot is assumed to experience independent and flat Rayleigh fading, and we focus on robust soft metrics that can be computed from the outputs of the receiver matched filters only and those with additional SI on the fading amplitude of the desired user.

In general, robust soft metrics may either be based on the square-law detector (SLD) or the envelope detector (ED) outputs. Simulation results showed that soft metrics based on ED outputs consistently outperform those based on SLD outputs under MAI[1]. In this letter, therefore, we concentrate on robust soft metrics based on ED outputs.

For comparison purposes, maximum-likelihood (ML) metrics based on a Gaussian approximation of the MAI (MLGA) requiring the knowledge of the signal-to-noise ratio (SNR) and the number of hit users for each hop are derived. This is clearly not a lower bound on the performance of soft metrics and is used as a reference for lack of a provable lower bound. We find that appropriately chosen robust soft metrics offer performance far superior to traditional metrics considered in previous literature.

II. System Model

AFFHSS-MA networks considered in this letter are identical to those described in [4] and [5], with the exception that information bits are encoded using binary convolutional codes before being BFSK modulated. Each of the K active users employs a Markov hopping pattern to select a hopping slot out of q available frequency slots for each coded symbol. Assuming that the dehopping pattern of the receiver is perfectly synchronized with that of the corresponding transmitter, and signals from different transmitters experience independent Rayleigh fading, complex matched-filter outputs at the reference receiver $(k=0)$, given that the hop is hit by K' users given by [4], [5]

$$\begin{aligned} U_l &= H_0\delta_{m_0,l} + \sum_{k=1}^{K'} H_k e^{j\theta(l,m_k,p_k)} A(l,m_k,p_k) + \mu_l \\ &= H_0\delta_{m_0,l} + \sum_{k=1}^{K'} H_k^l A(l,m_k,p_k) + \mu_l,\ l=0,1 \qquad (1) \end{aligned}$$

where l is the matched-filter index (the filter matched to coded symbol 0 or 1), $\delta_{m,l}=1$ for $m=l$ and zero otherwise, $m_k \in \{0,1\}$ is the equally likely coded symbol transmitted by the kth user, p_k is the normalized delay of the kth interfering user assumed to be independent and identically distributed (i.i.d.) and uniformly distributed on $(-1,1)$, and $\mu_l = N(0,\gamma^{-1})$[2] are i.i.d. with $\gamma = \overline{E}_s/N_0$ where $\overline{E}_s$ is the average received energy per coded symbol and $N_0/2$ is the two-sided power spectral density of the additive white Gaussian noise (AWGN). Also,

[1]The reason for this seems to be similar to the case of partial-band jamming, where the SLD metric of a severely jammed symbol dominates the decoder [2], in which case, some form of limiting operation on the SLD outputs significantly improves the performance of the decoder [3]. The square-root nonlinearity inherent in ED in effect provides a form of soft limiting.

[2]$N(\mu,\sigma^2)$ denotes a complex Gaussian random variable with complex mean μ and i.i.d. real and imaginary parts, each with equal variance σ^2.

Paper approved by N. C. Beaulieu, the Editor for Wireless Communication Theory of the IEEE Communications Society. Manuscript received November 29, 2000; revised February 15, 2002. This work was supported by the HY-SDR Research Center, Hanyang University, Seoul, Korea, under the ITRC Program of MIC, Korea.

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Digital Object Identifier 10.1109/TCOMM.2002.807600

TABLE I
ROBUST SOFT METRICS BASED ON ED OUTPUTS. LI = LINEAR, VR = VITERBI RATIO, SN = SELF-NORMALIZED, AND PR = PRODUCT

Name	Soft metrics
LI	$(-1)^{x_0}(\lvert U_0\rvert - \lvert U_1\rvert)$
VR	$(-1)^{x_0}\frac{\max\{\lvert U_0\rvert,\lvert U_1\rvert\}}{\min\{\lvert U_0\rvert,\lvert U_1\rvert\}}(\lvert U_0\rvert - \lvert U_1\rvert)$
SN	$(-1)^{x_0}\frac{\lvert U_0\rvert-\lvert U_1\rvert}{\lvert U_0\rvert+\lvert U_1\rvert}$
PR	$(-1)^{x_0}[\log(\lvert U_0\rvert) - \log(\lvert U_1\rvert)]$
$\mathrm{LI}_{\lvert H_0\rvert}$	$(-1)^{x_0}\lvert H_0\rvert(\lvert U_0\rvert - \lvert U_1\rvert)$
$\mathrm{VR}_{\lvert H_0\rvert}$	$(-1)^{x_0}\lvert H_0\rvert\frac{\max\{\lvert U_0\rvert,\lvert U_1\rvert\}}{\min\{\lvert U_0\rvert,\lvert U_1\rvert\}}(\lvert U_0\rvert - \lvert U_1\rvert)$
$\mathrm{SN}_{\lvert H_0\rvert}$	$(-1)^{x_0}\lvert H_0\rvert\frac{\lvert U_0\rvert-\lvert U_1\rvert}{\lvert U_0\rvert+\lvert U_1\rvert}$
$\mathrm{PR}_{\lvert H_0\rvert}$	$(-1)^{x_0}\lvert H_0\rvert[\log(\lvert U_0\rvert) - \log(\lvert U_1\rvert)]$

$H_k = N(0,1)$ are i.i.d. and represent the combined effects of the independent Rayleigh fading and the uniform random channel phase for the kth user's signal, $H_k^l = H_k e^{j\theta(l,m_k,p_k)}$ with $\theta(l,m,p) = -\pi p(m+l)$, $A(l,m,p) = (1-p)$ for $l = m$ and is equal to $-|p|\mathrm{sinc}(\pi(l-m)p)$, otherwise where $\mathrm{sinc}(x) = \sin(x)/x$. Note that $H_k^l = N(0,1)$ as well but H_k^0 and H_k^1 are dependent [5]. The ED outputs are then given by $|U_l|$ where $|x|$ denotes the magnitude of x.

III. SOFT METRICS CONSIDERED

A. Robust Soft Metrics Based on Matched-Filter Outputs Only

Robust soft metrics considered in this letter that can be computed from the ED outputs only are the linear (LI) [2], Viterbi ratio (VR) [6], self-normalized (SN) [7], and the product (PR) metrics [8]. We also consider the case when the fading amplitude of the desired user ($|H_0|$) is known to the receiver, in which case the metrics are further weighted by $|H_0|$. Such metrics shall be identified by a subscript $|H_0|$, e.g., $\mathrm{LI}_{|H_0|}$. It should be noted that in practice, obtaining an accurate estimate of $|H_0|$ may be difficult without a pilot channel and/or when the channel is fast varying. Table I summarizes the robust soft metrics considered where $x_0 \in \{0,1\}$ is the transmitted symbol assumed by the receiver in calculating the metric. The PR metric in Table I is obtained by taking the logarithm of the product combining method proposed in [8].

B. ML Metrics Based on a Gaussian Approximation of the MAI

It was shown in [5] that H_k^ls in (1) can safely be assumed to be i.i.d., thus rendering U_ls independent under Rayleigh fading. We then apply a Gaussian approximation to the MAI term $\mathrm{MAI}_{K'} \triangleq \sum_{k=1}^{K'} H_k^l A(l,m_k,p_k)$ where $\mathrm{MAI}_{K'}$ is modeled as $N(0,\eta K')$ with $\eta \triangleq E\{A^2(l,m,k)\} \approx 0.1919$. When $|H_0|$ is unknown, U_l can be approximated as $N(0,\delta_{m_0,l}+\eta K'+\gamma^{-1})$. Thus, $|U_l|^2$ follows the central χ^2 distribution with two degrees of freedom (DOF) [9], resulting in the MLGA metric given K' and γ as follows:

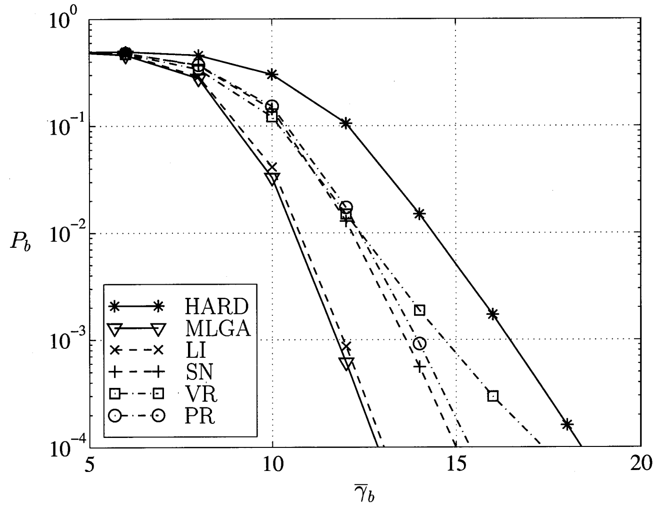

Fig. 1. Decoded BER performance of soft metrics without the knowledge of $|H_0|$ versus $\overline{\gamma}_b$. $K = 1$, $q = 100$.

$$\begin{aligned}
&\ln\left(\frac{p\left(|U_0|^2, |U_1|^2 \middle| m_0 = x_0\right)}{p\left(|U_0|^2, |U_1|^2 \middle| m_0 \neq x_0\right)}\right)\\
&= \ln\left(\frac{p\left(|U_0|^2 \middle| m_0 = x_0\right)p\left(|U_1|^2 \middle| m_0 = x_0\right)}{p\left(|U_0|^2 \middle| m_0 \neq x_0\right)p\left(|U_1|^2 \middle| m_0 \neq x_0\right)}\right)\\
&\propto \frac{(-1)^{x_0}}{(\eta K' + \gamma^{-1})(1+\eta K' + \gamma^{-1})}\left(|U_0|^2 - |U_1|^2\right). \quad (2)
\end{aligned}$$

Note that for $K' = 0$ (no MAI), (2) simplifies to the square-law linear combining metric, which is the exact ML metric without MAI [2].

When $|H_0|$ is known to the receiver, U_l can be approximated as $N(H_0\delta_{m_0,l}, \eta K' + \gamma^{-1})$ and $|U_l|^2$ now follows the noncentral χ^2 distribution with two DOF [9]. The MLGA metric given K', γ, and $|H_0|$ can easily be shown to be

$$\begin{aligned}
(-1)^{x_0}&\left[\ln\left(I_0\left(\frac{|H_0|}{\eta K' + \gamma^{-1}}|U_0|\right)\right)\right.\\
&\left.-\ln\left(I_0\left(\frac{|H_0|}{\eta K' + \gamma^{-1}}|U_1|\right)\right)\right] \quad (3)
\end{aligned}$$

where $I_0(x)$ is the zeroth-order modified Bessel function of the first kind [9]. We shall refer to this metric as $\mathrm{MLGA}_{|H_0|}$.

IV. SIMULATION RESULTS

For simulations, $q = 100$ hopping slots and rate $r = 1/2$, constraint length 7 binary convolutional code with maximum free Hamming distance with generators $(133)_8$ and $(171)_8$ [9] is used. As a reference, we also plot results for the hard decisions metric (HARD) and hard decisions and erasures metric ($\mathrm{HARD_{PSI}}$), where PSI is used to erase the hit symbols. First, we compare the performance of soft metrics as a function of $\overline{\gamma}_b = \overline{E}_b/N_0$, where $\overline{E}_b = r\overline{E}_s$ is the average received bit energy. Fig. 1 shows the decoded bit error rate (BER) (P_b) versus $\overline{\gamma}_b$ for the single user case ($K = 1$). The MLGA metric

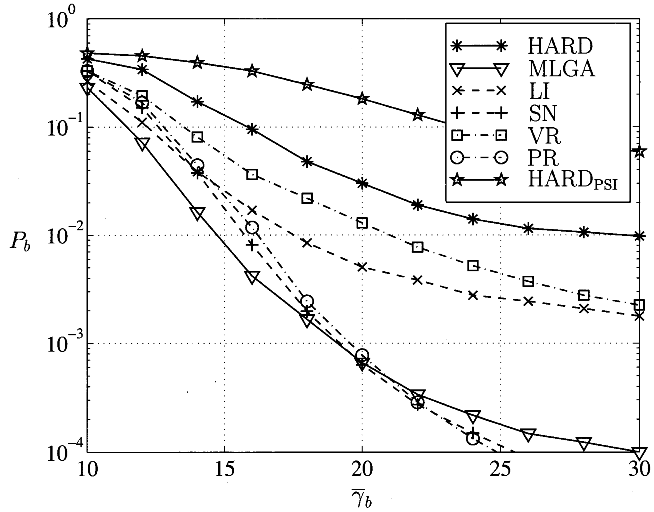

Fig. 2. Decoded BER performance of soft metrics without the knowledge of $|H_0|$ versus $\overline{\gamma}_b$. $K = 30$, $q = 100$.

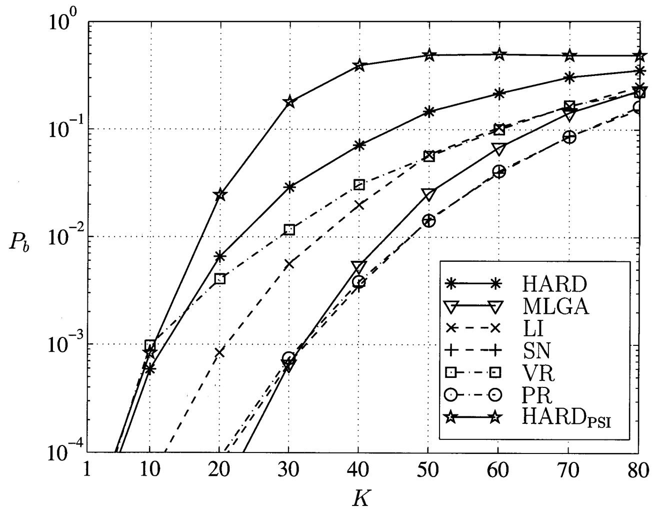

Fig. 3. Decoded BER performance of soft metrics without the knowledge of $|H_0|$ versus K. $\overline{\gamma}_b = 20$ dB, $q = 100$.

(true ML for $K = 1$) performs approximately 4.7 dB better than the HARD metric at $P_b = 10^{-3}$, while the LI metric performs within 0.2 dB of the MLGA metric. Among the robust soft metrics considered, the VR metric shows the worst performance with only 1.7 dB gain over the HARD metric at $P_b = 10^{-3}$. Fig. 2 shows the decoded BER versus $\overline{\gamma}_b$ when $K = 30$, which shows that under the presence of MAI, performance of the LI metric deteriorates drastically while the SN and PR metrics achieve $P_b = 10^{-3}$ within 0.1 and 0.5 dB of the MLGA metric, respectively. Once again, the VR metric is the worst choice among the robust soft metrics considered. Although not shown in this letter, we have also verified similar trends for soft metrics when $|H_0|$ is known.

Decoded BER performance using soft metrics versus K for $\overline{\gamma}_b = 20$ dB is shown in Fig. 3. We first observe that SN and PR metrics exhibit very similar performance for all values of K considered, and provide the best performance among all robust soft metrics considered, whereas the VR metric offers performance even worse than that of the LI metric. By using the

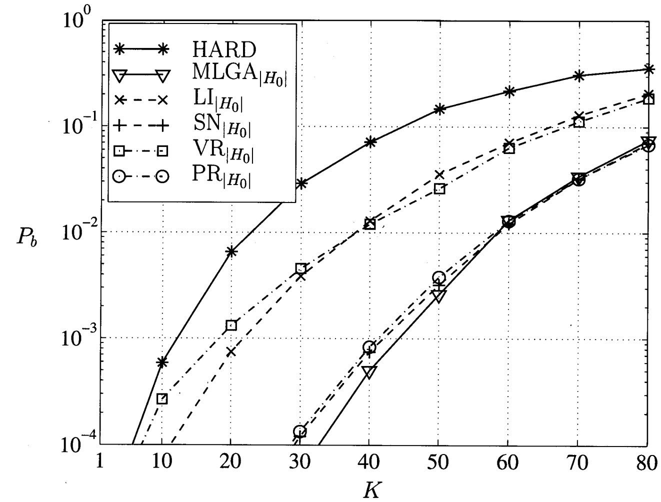

Fig. 4. Decoded BER performance of soft metrics with $|H_0|$ known versus K. $\overline{\gamma}_b = 20$ dB, $q = 100$.

SN or the PR metric, we obtain close to a 170% increase in the number of active users that may be supported by the network at $P_b = 10^{-3}$ compared to the HARD metric. We also observe in Fig. 3 that the performance of the HARD_{PSI} metric deteriorates much faster with increasing K than other metrics. This is due to the fact that for large K, an excessive portion of the received symbols are hit and erased [4].

Fig. 4 shows the decoded BER of soft metrics when $|H_0|$ is known. At $P_b = 10^{-3}$, $\text{SN}_{|H_0|}$ and $\text{PR}_{|H_0|}$ metrics each support approximately 250% and 240% more active users compared to the HARD metric, whereas the $\text{VR}_{|H_0|}$ metric can only support approximately 50% more users. Comparing results from Figs. 3 and 4, we find that the knowledge of $|H_0|$ enables the network to support approximately 30%–40% additional users at $P_b = 10^{-3}$ for SN and PR metrics at $\overline{\gamma}_b = 20$ dB.

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