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Equations for the Z-factor and Compressibility of Nigerian Natural Gas

Ohirhian, P.U.
Department of Petroleum Engineering, University of Benin, Benin City, Nigeria
okuopet@yahoo.com

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Abstract. New equations for calculating the Gas Deviation Factor (z) and the Compressibility of Nigerian Natural Gas have been developed. The equation for the Gas Deviation Factor was developed from a plot of the logarithm of the ratio of Pressure to Temperature versus the logarithm of the ratio of the Pressure to the Gas Deviation Factor.. The plot gave a linear relationship that gave the Gas Deviation Factor equation. A differentiation of the Gas Deviation Factor equation produced the new equation for calculating the Gas Compressibility. Values from the new equation for calculating the Gas Deviation Factor were compared with experimental Pressure, Volume, Temperature (PVT) measurements and the following statistical measurements of accuracy were obtained: Average percent error of 3.050, average absolute percent error of 3.649 and standard deviation of 3.910. Over 75% data tested showed error less than 5 %. A big advantage of the new equations is that they are simple and do not require pseudo critical pressure and pseudo critical temperature needed to obtain z from the current popular equations of state used in the Oil Industry. The data used to develop the model in this work came from Gas associated with produced oil in the Niger Delta. API gravity of the oil, ranged from 17.4 to 44. Pressure ranged from 15 psia to 4015 psia and Temperature ranged between 127 and 205 degree Fahrenheit. This range of data covers at least, ninety five percent of data used in the upstream sector of the Nigeria Oil and Gas Industry.

Introduction

There are several equations that have been proposed to calculate the volumetric changes of gas with changes in pressure and temperature. Some of these equations are complex and tedious to solve when applied to Natural Gas. This is because Natural Gas is a mixture of hydrocarbon gases with minor impurities of non-hydrocarbon gases such as carbon dioxide (CO_2), hydrogen sulphide (H_2S) and Nitrogen(N_2). When the major non-hydrocarbon gas present in Natural Gas is CO_2, it is termed sweet. The Nigerian Natural Gas that provided the data used in this work is a sweet Natural Gas. The CO_2 impurity usually ranges between 0.002 to 0.016 mole percent.

A popular equation used in the oil industry to predict volumetric changes of gas with changes in pressure and temperature (equation of state) is given as

$$PV = z\,n\,RT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In equation (1) z is called the Gas Deviation Factor. It is defined as:

$$\mathbf{z} = \frac{\textbf{Actual volume of n moles of gas at a given pressure and temperature}}{\text{Ideal volume of n moles } \textbf{of gas at same pressure and temperature}}$$

The calculation of z has also drawn the attention of several researchers. Standing and Katz [1], through experimental measurement came up with a chart that can be used to

estimate the gas deviation factors (z) of Natural Gas. According to Ikoku [2] ," the Standing-Katz, z-factor chart is generally reliable for sweet Natural Gases with small amounts of non-hydrocarbon components, say, less than 5% by volume".
To obtain z from the Standing and Katz chart we need values of pseudo critical pressure (Pc) and pseudo critical temperature (Tc). Both pseudo critical pressure and temperature can be obtained when the composition of the Natural Gas is known. Determination of the composition of a Natural Gas is an expensive procedure. When the gas composition is not available, the value of the gas gravity (Gg) can be used to estimate values of Pc and Tc from correlations available in the literature. One of the earliest (and still very popular) correlation of obtaining Pc and Tc when Gg is known is that of Brown. The calculation of the gas gravity also requires knowledge of the gas composition. When the composition is not given, the gas gravity can be measured in the laboratory.

Rogers and Mayhew [3] have observed that even in cases where the pseudo critical properties are determined accurately, the use of the Standing and Katz chart can still yield errors as high as 10 % , when applied to a particular gas. This fact has been confirmed by Agbala [4] who used the Standing and Katz chart to determined the z-factor of Nigerian Natural Gas, He used accurately determined of pseudo critical properties. Most of the samples tested had percentage error below 10% but 1 sample of a gas associated with a low API degree crude oil had an error of about 14%. Whether by determination of pseudo critical properties or by laboratory measurement of the gas gravity, the current practice of obtaining the z-factor is a tedious process. The need is there to generate a less cumbersome method of obtaining the z-factor

In this work a simple and fairly accurate equation for calculating the Gas Deviation Factor is presented. The equation does not require pseudo critical properties or the Gas Gravity. The simple z- factor equation generated in this work, has also been differentiated to produce another simple equation for calculating the Compressibility of Nigerian Natural Gas.

Natural Gas associated with oil produced from forty-four Oil Reservoirs was used to generate the new equations. The American Petroleum Institute (API) gravity of the oil ranged from 17.4 – 44 degree. Pressure varied from 15 to 4015psia and temperature varied from 127 to 235.4 degrees Fahrenheit. Comparison of values from the new equation with corresponding experimental pressure, Volume, Temperature (PVT) Data, tested the accuracy of the z- factor equation. The following statistical tools for error estimation were obtained.

Minimum absolute Error: 0.000%
Maximum absolute Error: 9.025%
Average Error : 3.05%
Average absolute Error: 3.649%
Standard Deviation: 3.910.

Over 75% data tested showed error less than 5 %. This is the type of error obtained from the Standing-Katz chart (from Agbala's work). Thus the new simple z-factor equation

can be used in the place of the Standing –Katz chart, at least, within the range of data considered in this work.

Development of the Z - Equation

Figure 1 shows a plot of the logarithm of the ratio of pressure (in psia) to the temperature (degree Rankine) versus the logarithm of the ratio of the pressure to the compressibility factor. The data came from 44 samples of gas associated with Nigerian Oil Reservoirs. The values of the pressure, temperature, and z-factor, were obtained from the PVT data of renowned companies, part of which are shown in Table 1.

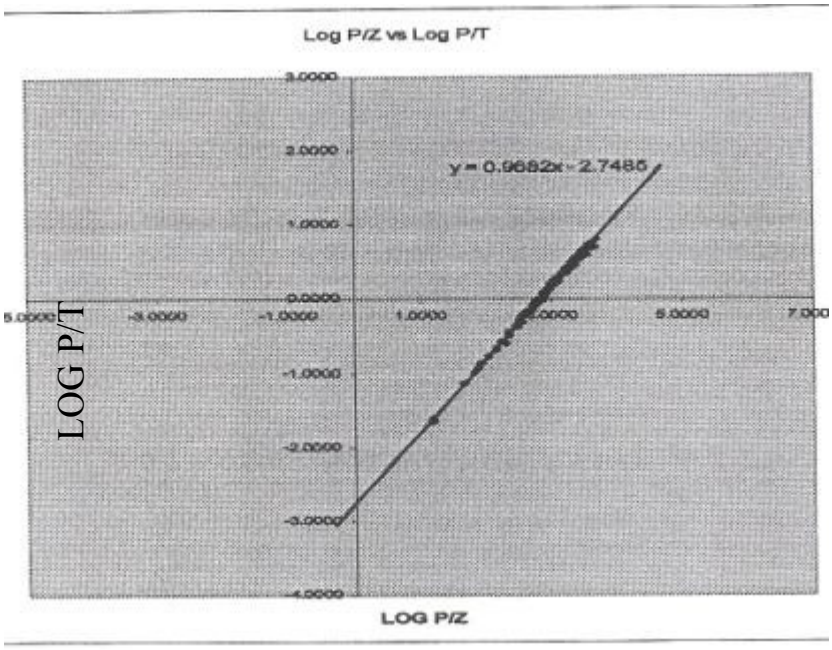

Figure 1: plot of log (p/T) versus log (p/z)

The plot shown in figure 1 suggested a linear relationship between log (P/T) versus log (P/z). That is:

$$\text{Log } (P/T) = a + b \log (P / z) \quad \text{----------(2)}$$

A line of best fit gave:

$$a = -2.7427 \text{ and } b = 0.96824$$

Equation (2) can be simplified as

$$z = P^{(1 - 1/b)} \times T^{1/b} \times 10^{a/b} \quad \text{----------(3)}$$

Substitution of a = - 2.7427 and b = 0.96824 into eqn. (3) gave

$$z = 0.00147 \times P^{-0.0328} T^{1.0328} \quad \text{----------(4)}$$

When equation (3) was used to calculate z, it was found that the errors can be reduced by a correction factor (C).

The correction factor followed a pattern that can be described as

$$C = 7.7716 - 2.43076 \log T \quad \text{----------(5)}$$

Thus the z-equation became:

$$z = 0.00147P^{-0.0328} T^{1.0328} \times (7.77176-2.43076 \log T) \text{ ----------(6)}$$

In equation (6), unit of P is psia, and T is expressed in degrees Rankine
Comparison of results obtained from equation (6) with experimental PVT data showed that an average percent error = 3.050, average absolute percent error = 3.649 and standard deviation =3.910. Over 75% data tested showed error less than 5 %.

Example 1
Calculate z for a Nigerian Natural Gas at a pressure = 1015 psia and a temperature = 180 ^{0}F
Solution
T = 180^{0}F = (460+180)^0 R= 640^{o}R
Substitution of given values into eqn..(6) gives.
z = 0.8809

Table 1 also shows part of the comparison of values from the new z-equation (eqn. 6) with experimental PVT data used to generate the equation.

In table 1,the error is defined as

$$\text{Error (\%)} = \frac{(Z_{\text{Calc.}} - Z_{\text{PVT}})}{Z_{\text{PVT}}} \times 100$$

Other statistical measurements of accuracy were also applied. They are defined as follows:

$$\text{Mean error} = \frac{Error(\%)}{N}$$

Where N = total number of data points

$$\text{Mean Absolute Error} = \left| \text{Error}\% \right|$$

$$\text{Standard Deviation} = \sqrt{\frac{\left[\left(\sum \mathrm{D}\mathbf{i}^{2}\right)\right]}{\mathrm{N}-1} - \frac{\left(\sum \mathbf{D}\mathbf{i}\right)^{2}}{\mathrm{N}}}$$

Where Di is the absolute Error
The mean Error for the 260 data points used in the work is 3.05 % , the Mean Absolute Error is 3.649 and the standard deviation is 3.910. Over 75% of he Data had error below 5%. The last column of table 1 shows correction factor that can be applied to the calculated z, in order to obtain the PVT z.. This correction factor is useful when the new z equation is applied to an associated Gas with known oil API gravity and Reservoir Temperature.

Table 1: PVT Data used in this work and Comparison of calculated z with PVT z

WELL'S API/TEMP	DATA NO.	PRESSURE	TRUE VALUE	ESTIMATED VALUE	ERROR	CORRECTION FACTOR
	1	165.0	.957	.937	2.090	1.021
17.4/604	2	135.0	.960	.944	1.667	1.017
	3	105.0	.964	.951	1.349	1.014
	4	75.0	.971	.962	.927	1.009
	5	45.0	.985	.978	.711	1.007
	6	15.0	.998	1.014	-1.603	.984
	7	915.0	.953	.886	7.030	1.076
21.0/587	8	715.0	.961	.893	7.076	1.076
	9	565.0	.968	.900	7.025	1.076
	10	365.0	.979	.913	6.742	1.072
	11	215.0	.987	.929	5.876	1.062
	12	15.0	.999	1.014	-1.502	.985
	13	1815.0	.826	.866	-4.843	.954
22.5/594	14	1415.0	.854	.874	-2.342	.977
	15	1015.0	.886	.883	.339	1.003
	16	745.0	.910	.892	1.978	1.020
	17	315.0	.958	.918	4.175	1.044
	18	15.0	1.000	1.014	-1.400	.986
	19	2215.0	.861	.859	.232	1.002
25.1/635	20	1715.0	.874	.866	.915	1.009
	21	1315.0	.902	.874	3.104	1.032
	22	915.0	.917	.884	3.599	1.037
	23	415.0	.953	.908	4.722	1.050
	24	15.0	.996	1.012	-1.606	.984

Table 1: Continued

	25	3015.0	.863	.851	1.390	1.014
33.9/625	26	2365.0	.853	.858	-.586	.994
	27	1715.0	.859	.867	-.931	.991
	28	1065.0	.886	.882	.451	1.005
	39	515.0	.933	.902	3.323	1.034
	30	15.0	.998	1.013	-1.503	.985
	31	3015.0	.806	.850	-5.459	.948
36.4/641	32	2315.0	.824	.857	-4.005	.961
	33	1415.0	.854	.871	-1.991	.980
	34	615.0	.902	.895	.776	1.008
	35	215.0	.950	.927	2.421	1.025
	36	15.0	1.000	1.011	-1.100	.989
	37	2515.0	.875	.854	2.400	1.025
40.6/651	38	2015.0	.853	.860	-.821	.992
	39	1515.0	.856	.868	-1.402	.986
	40	1015.0	.870	.880	-1.149	.989
	41	515.0	.903	.899	.443	1.004
	42	15.0	.997	1.010	-1.304	.987
	43	3315.0	.887	.849	4.284	1.045
44.0/626	44	2615.0	.885	.855	3.390	1.035
	45	1915.0	.938	.864	7.889	1.086
	46	1215.0	.964	.877	9.025	1.099
	47	515.0	.979	.902	7.865	1.085
	48	15.0	.997	1.013	-1.605	.984

Example 2
What correction factor should be applied to the z computed in Example 1 in order to obtain a z closest to the PVT z , given that Oil associated with the Gas has an API gravity of 36 ?

Solution
Refereeing to table 1.the closest well has API degree = 36.4 and a temperature of 641 ^{o} R=181 ^{0}F. The correction factor ≈ 0.994. Value of z closest to PVT z = 0.8809 × 0.994 = 0.876
A comparison of z determined with equation (6) and corresponding values from the Standing and Katz chart was made for a particular associated Gas. The composition of the Gas was obtained from a PVT report. The reservoir existed at a temperature of 184^{0}F. The composition of the Gas is at 15 psia is shown in Table 2. The P_C of the Gas was computed as 601.887 psia while its T_C was 639.734 .

Table 2 : Composition of an associated Gas at 15 psia and T = 184 ^{o} F

Come	Yi	Pci	Tci	Yi Pci	Yi Tci
N_2	0.0002	493.0	227.6	0.0986	0.0455
Co_2	0.0033	1071.0	547.9	3.5343	1.8081
H_2s	0.0000	1306.0	672.7	-	-
CH_4	0.2042	667.8	343.37	136.3648	70.1162
C_2	0.1745	707.8	550.09	123.3111	95.9907
C_3	0.2571	616 .3	666.01	158.4507	171.2312
i C_4	0.0874	529.1	734.98	46.2433	64.2373
N C_4	0.1035	550.7	765.65	56.9975	79.2448
i C_5	0.0427	490.4	829.1	20.9401	35.4026
N C_5	0.0267	488.6	845.7	13.0456	22.5802
C_6	0.0272	436.9	913.7	11.8837	24.8526
C_7	0.0732	421.0	1014.0	30.8172	74.2248

$P_C = \sum Yi\ \mathbf{Pc\ i} = \mathbf{601.887}$

$T_C = \sum Yi\ \mathbf{Tc\ i} = \mathbf{639.784}$

Table 3 gives a comparison between z from the Standing and Katz chart and z from the new equation. The z from the Standing and Katz chart was obtained from pseudo critical properties computed from PVT compositional analysis. The new equation performed as well as the Standing and Katz chart in this case. In many cases, the new equation does as well as the Standing and Katz chart and even better in some cases. More values for comparison can be obtained from Agbala's work.

Table 3: Comparison between standing-Katz z and z of this work

P	Pr	Tr	Z_{PVT}	Z_{KATZ}	ABSOLUTE ERROR (%)	Z_{OHIR}	ABSOLUTE ERROR(%)
3015	4.5389	1.6822	0.806	0.859	6.576	0.850	5.459
2315	3.4826	1.6888	0.824	0.853	3.519	0.857	4.005
1415	2.1271	1.6493	0.854	0.871	1.991	0.871	1.991
615	0.9250	1.6130	0.902	0.931	3.215	0.895	0.776
215	0.3261	1.4498	0.950	0.962	1.263	0.926	2.526
15	0.0249	1.0067	1.000	1.005	0.500	1.011	1.100

THE COMPRESSIBILITY EQUATION

The compressibility of a gas is defined as :

$$Cg = \frac{\mathbf{1}}{\mathbf{p}} = \frac{\mathbf{1}}{\mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{p}} \Big)_{\mathbf{T}} \text{ ----------(7)}$$

Where Cg = compressibility
p = pressure in absolute unit
T = temperature in absolute unit

A partial differentiation of eqn. (5) gives

$$\frac{\partial z}{\partial p}\Big)_T = -0.0328\, p^{-1.0328}\, k$$

where $k = 0.00147 \times T^{1.0328}$

Then

$$\frac{1}{z}\frac{\partial z}{\partial p}\Big)_T = \frac{-0.0328\, p^{-1.0328}\, k}{p^{-0.0328}\, k}$$

$$= \frac{-0.0328}{p}$$

Example 3

Calculate the compressibility of the Gas of example 1

Solution

$z = 0.8809$

$$\frac{1}{z}\frac{\partial z}{\partial p}\Big)_T = \frac{-0.0328}{1015} = -0.000032315$$

$$Cg = \frac{1}{1015} + 0.000032315$$

$$= 0.0010175 \text{ psia}^{-1}$$

Differentiating the general form of the z equation (eqn.3) and substituting in eqn.(7) ,after some simplification, yields

$$Cg = \frac{\frac{1}{b}}{p} = \frac{C'}{p} \text{ ---------- (8)}$$

Where b = is the slope from the plot of log (P/T) versus log (P/z) and $C^{/}$ = constant, which is a reciprocal of the slope log (P/T) versus log (P/z).
Equation (8) is a generalized equation for calculating the compressibility of a sweet Natural Gas. The constant for Nigerian (Sweet) Natural Gas is 1.0328

Example 4

Calculate the compressibility of a sweet Natural Gas at 2000 psia and 150^{o} F.

Solution

By use of eqn.,(8) the Cg = 1.0328 / 2000 psia = 0.0005164 psia $^{-1}$
This problem is example 2.11 in " Natural Gas Production Engineering" by Ikoku. By use of charts of Matter, Brar and Aziz [5] that were generated from a computer solution to a complex z-factor equation of Dranchuk et al [6], Ikoku obtained Cg = 0.000527 psia $^{-1}$
Which of the values is more accurate? This question can only be answered by actual laboratory measurement of the Compressibility of the sweet Natural Gas.

Conclusions

1) A simple and fairly accurate equation for calculating the Gas Deviation Factor of Nigerian (Sweet) Natural Gas has been formulated.
2) The new simple equation for calculating the z-factor of sweet Natural Gas has been differentiated to arrive at another simple equation for calculating the Compressibility of a sweet Natural Gas.

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