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Classification of an Unstable Particle and Its Propagator

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The classification of an unstable particle according to the values of the complex wave function renormalization constant is discussed. The propagator of the unstable particle is analytically continued to an unphysical Riemann sheet, on which there exist poles of the propagator corresponding to the unstable particle. The complex renormalization constant is defined as a limiting value of the unphysical propagator at high energies. Equations to determine the complex coupling constant and the complex mass are obtained. The classification thus obtained is similar to the case of a stable particle Extension to the multi-channel formalism is also discussed.

§ 1. Introduction

Recently many papers have been appeared on the elementarity or the compositeness of strongly interacting particles.[1] In his work, Ida classified hadrons into four categories: superelementary, elementary, intermediate and composite according to the number of free parameters left undetermined in principle.[2] Hayashi et al.[3] treated a pion and examined which category the pion belongs to. According to their results the pion should be placed in the intermediate type. The case may be the same for the kaon.[4] The nucleon seems to belong to the same type, too.[5] Therefore it may be possible to extend the conclusion to the baryon multiplet with spin 1/2 and the pseudoscalar meson multiplet. Of course this conclusion is restricted by the specific forces between the constituent particles and by the elastic unitarity. However, the fact that the pion is of the intermediate type seems not to depend very strongly on the restrictions, for the pion mass is too small to be consistent with the composite model. Thus in nature there seems to be at least some hadrons which do not belong to the composite type.

If some of the hadrons are of the intermediate type, this means that our information from the present system of hadrons is not enough to determine completely the dynamical parameters which indicate the mass levels and the structure of the interactions. Therefore in order to find the laws existing in the system of hadrons we should go beyond the observed system. It is at present an open question what exists behind the phenomena or what dynamics rules the phenomena. On the other hand if all the hadrons belong to the com-

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posite type, the dynamics behind the present system of hadrons is not necessary, and then we will probably lose a clue to go beyond the present theory of elementary particles. It is, however, questionable that all the hadrons are of a unique type. For examples, the vector meson octet may belong to the composite type or the spin $3/2^+$ baryon decuplet may do so. Thus it is interesting to test which types these multiplets belong to. These multiplets, however, are unstable for strong interactions and have higher spins. Hence, as the first step, the classification should be reexamined in the case of the unstable particle. This is the aim of this paper.

Several years ago theories on the unstable particle were extensively proposed, but the renormalization procedure was thought ambiguous.[6),7)] Araki et al.[6)] and Nakanishi[8)] considered the renormalization procedure for the unstable particle and obtained some important results by using the Lee model. One of the results is that one can eliminate divergences by the ordinary renormalization procedure, but it is impossible to put the interpretation that the renormalization constant Z_V is a probability for finding the bare V particle in the physical V state, because of the non-reality of Z_V. Our interest, however, is in the physical meaning of the fact that $Z_V=0$. Therefore we shall compare the results of the Lee model, in which $Z_V=0$, and the separable potential model discussed by Vaughn et al.,[9)] in which only the direct $NN\theta\theta$ interaction is contained, and show that the Lee model coincides with the separable potential model when $Z_V=0$.

In this paper an explicit representation for the wave function renormalization constant is given by using the Lee model and the relativistic scalar meson model. The complex renormalization constant is introduced in the same way as in the stable case, but a point of difference is that quantities such as the propagator should be analytically continued from the physical sheet to the unphysical Riemann sheet on which the unstable particle pole exists. The results are almost parallel to the stable case.

In § 2 the Lee model is discussed so as to show how the renormalization constant is introduced and what meaning the fact $Z=0$ has. The analytic continuation of the propagator for the relativistic scalar particle is treated in § 3. Explicit representations for the renormalization function and constant are given in § 4. We discuss the classification of the unstable particle in § 5. In § 6 extension of the theory to the multi-channel reaction is carried out.

§ 2. Preliminary discussions using the Lee model

2–1 *The Lee model*

The N-θ scattering amplitude considered on the complex z plane cut along the real axis from μ to ∞ is given by

$$T(z)=g_0^2/h(z) \tag{2·1}$$

and

$$h(z)=E_V^0-z-\frac{1}{\pi}\int_\mu^\infty d\omega\frac{g_0^2A(\omega)}{\omega-z}, \tag{2·2}$$

where g_0^2 is the unrenormalized coupling constant in Gaussian units,

$$E_V^0=m_V^0-m_N \tag{2·3}$$

and

$$A(\omega)=\sqrt{\omega^2-\mu^2}\,|\,u(\omega)\,|^2. \tag{2·4}$$

$u(\omega)$ is a cutoff function, which we assume is analytic on the cut plane and can be continued to the 2nd Riemann sheet through the cut. The branch of square root function $(z^2-\mu^2)^{1/2}$ is taken to be that corresponding to positive root when $z=\omega+io$ and negative when $z=\omega-io$, where $\omega>\mu$. Then $A(z)$, defined as $(z^2-\mu^2)^{1/2}u^*(z)u(z)$ which gives Eq. (2·4) as the positive boundary value $A(\omega+io)$, has the following property:

$$A(z)=-A^*(z^*). \tag{2·5}$$

The reality condition holds for $T(z)$ and $h(z)$;

$$T^*(z)=T(z^*),$$
$$h^*(z)=h(z^*). \tag{2·6}$$

From this reality condition and the unitarity for T, we can continue T or h on to the 2nd sheet through the cut and obtain

$$T_{II}(z)=g_0^2/h_{II}(z),$$

where

$$h_{II}(z)=h(z)+2ig_0^2A(z). \tag{2·7}$$

The existence of the unstable V requires that $T_{II}(z)$ has a pole at $z_0=x-iy$, $(x>\mu$ and $y>0)$, or equivalently $h_{II}(z)$ has a zero at z_0. From the reality condition it follows further that $h_{II}(z_0^*)=0$.

The independent free parameters contained in Eq. (2·7) are g_0^2 and E_V^0, where m_N, μ and the parameters in $u(z)$ are assumed to be given. Eliminating E_V^0, we have

$$h_{II}(z)=(z_0-z)\left[1+\frac{1}{\pi}\int_\mu^\infty d\omega\frac{g_0^2A(\omega)}{(\omega-z_0)(\omega-z)}-2ig_0^2\frac{A(z)-A(z_0)}{z-z_0}\right]$$
$$=(z_0-z)H(z), \tag{2·8}$$

and therefore

$$T_{II}(z)=g_0^2/(z_0-z)H(z). \tag{2·9}$$

$H(z)$ should satisfy the following equation:

$$H(z_0^*)=1+\frac{1}{\pi}\int_\mu^\infty d\omega\frac{g_0^2A(\omega)}{(\omega-z_0)(\omega-z_0^*)}+2g_0^2\frac{\mathrm{Re}\,A(z_0)}{y}=0, \tag{2·10}$$

because of the reality condition on T or h. Therefore the number of free parameters is *two* in Eqs. (2·8) and (2·9).

Now, we introduce a complex renormalization constant Z and a complex coupling constant Γ which is the residue of $T_{II}(z)$ at $z=z_0$;

$$\Gamma=g_0^2/H(z_0). \tag{2·11}$$

Z is defined as the ratio of Γ to g_0^2 in the same way as in a stable V particle:

$$\Gamma=Zg_0^2, \tag{2·12}$$

and we have from Eq. (2·8)

$$Z=1-\Gamma\left[\frac{1}{\pi}\int_\mu^\infty d\omega\frac{A(\omega)}{(\omega-z_0)^2}-2iA'(z_0)\right], \tag{2·13}$$

where $A'(z_0)=dA(z)/dz|_{z=z_0}$. Since we have Eq. (2·10) as a subsidiary condition on $H(z)$, we get another expression for Z,

$$Z=\Gamma\left[-\frac{1}{\pi}\int_\mu^\infty d\omega\frac{A(\omega)}{(\omega-z_0)(\omega-z_0^*)}-2\frac{\mathrm{Re}\,A(z_0)}{y}\right], \tag{2·14}$$

by substituting Eq. (2·12) into Eq. (2·10).

When $Z=0$ in Eqs. (2·13) and (2·14), we obtain

$$\Gamma=\left[\frac{1}{\pi}\int_\mu^\infty d\omega\frac{A(\omega)}{(\omega-z_0)^2}-2iA'(z_0)\right]^{-1} \tag{2·15}$$

and

$$\frac{1}{\pi}\int_\mu^\infty d\omega\frac{A(\omega)}{(\omega-z_0)(\omega-z_0^*)}+2\frac{\mathrm{Re}\,A(z_0)}{y}=0. \tag{2·16}$$

From these equations only *one* out of the four real parameters contained in Γ and z_0 is left undetermined.

2–2 *Separable potential model*

The Hamiltonian for this model is

$$H=\int\omega_k\,a_k^*\,a_k\,d^3k+H_{\mathrm{int}},$$

$$H_{\mathrm{int}}=\frac{4\pi\xi}{(2\pi)^3}\int\frac{u(k')\,u(k)}{\sqrt{4\omega_{k'}\omega_k}}a_{k'}^*\,a_k\,d^3k'\,d^3k. \tag{2·17}$$

The N-θ scattering amplitude obtained from this Hamiltonian can be written as

$$T_S(z) = -\xi \Big/ \left[1 + \frac{1}{\pi} \int_\mu^\infty d\omega \frac{\xi A(\omega)}{\omega - z} \right]. \tag{2·18}$$

By analytic continuation as in § 2–1, we get

$$T_S^{II}(z) = -\xi \Big/ \left[1 + \frac{1}{\pi} \int_\mu^\infty d\omega \frac{\xi A(\omega)}{\omega - z} - 2i\xi A(z) \right], \tag{2·19}$$

and since $T_S^{II}(z)$ should have two poles at $z=z_0$ and $z=z_0^*$, we can write

$$T_S^{II}(z) = [(z_0 - z) H_S(z)]^{-1}, \tag{2·20}$$

where

$$H_S(z) = \frac{1}{\pi} \int_\mu^\infty d\omega \frac{A(\omega)}{(\omega - z_0)(\omega - z)} - 2i \frac{A(z) - A(z_0)}{z - z_0}, \tag{2·21}$$

and the subsidiary condition on $H_S(z)$ is

$$H_S(z_0^*) = \frac{1}{\pi} \int_\mu^\infty d\omega \frac{A(\omega)}{(\omega - z_0)(\omega - z_0^*)} + 2 \frac{\mathrm{Re}\, A(z_0)}{y} = 0. \tag{2·22}$$

Let us write the residue of the pole at $z=z_0$ as Γ_S, where Γ_S is written as

$$\Gamma_S = \left[\frac{1}{\pi} \int_\mu^\infty d\omega \frac{A(\omega)}{(\omega - z_0)^2} - 2iA'(z_0) \right]^{-1}. \tag{2·23}$$

Comparing Eqs. (2·22) and (2·23) to Eqs. (2·15) and (2·16), we can conclude that the Lee model in the limit $Z=0$ coincides with this separable potential model which does not contain the V field at all in its Hamiltonian.

Thus, although we cannot accept the interpretation that Z is the probability for finding the bare V in the physical V, the physical meaning for $Z=0$ in the Lee model is definite.

2–3 *The V-particle propagator*

In this subsection we shall discuss the propagator of the unstable V particle. The renormalized propagator of the V particle is defined as

$$S_V'(t-t') = \langle 0 | T(\psi_V(z)\, \psi_V^*(t')) | 0 \rangle, \tag{2·24}$$

where

$$\psi_V = Z^{-1/2} \psi_V^0 \quad \text{and} \quad \psi_V^* = Z^{*-1/2} \psi_V^{0*}. \tag{2·25}$$

Therefore, the relation between the renormalized S_V' and unrenormalized $S_V'^0$ is

$$S_V'(t-t') = |Z|^{-1} S_V'^0(t-t'). \tag{2·26}$$

In the Lee model the scattering amplitude $T(z)$ is written as

$$T(z) = g_0^2 S_V'^0(z),$$

or by using further Eq. (2·26)

$$T(z) = |\Gamma| S_V'(z). \tag{2·27}$$

On the 2nd sheet we have $T_{II}(z) = |\Gamma| S_V'^{II}(z)$, so that the residue of $S_V'^{II}(z)$ at $z = z_0$ is not 1 but $\Gamma/|\Gamma| = e^{i\varphi}$.

Now, we introduce the renormalization function $Z_{II}(z)$ by the following equation:

$$Z_{II}(z) = |Z| H(z),$$

hence, $Z_{II}(z)$ is expressed as

$$Z_{II}(z) = e^{-i\varphi} + \frac{z-z_0}{\pi}\int_\mu^\infty d\omega \frac{|\Gamma| A(\omega)}{(\omega-z_0)^2(\omega-z)} - 2i|\Gamma|\left[\frac{A(z)-A(z_0)}{z-z_0} - A'(z_0)\right]. \tag{2·28}$$

From the definition, $Z_{II}(z_0^*) = 0$ should hold and therefore $Z_{II}(z_0^*) = 0$ becomes the subsidiary condition for Eq. (2·28),

$$Z_{II}(z_0^*) = e^{-i\varphi} + \frac{z_0^*-z_0}{\pi}\int_\mu^\infty d\omega \frac{|\Gamma| A(\omega)}{(\omega-z_0)^2(\omega-z_0^*)} + 2|\Gamma|\left[\frac{\mathrm{Re}A(z_0)}{y} + iA'(z_0)\right] = 0. \tag{2·29}$$

This equation gives two conditions for the four real parameters, $|\Gamma|$, φ, $\mathrm{Re}\, z_0$ and $\mathrm{Im}\, z_0$. The absolute value of Z is given by Eqs. (2·28) and (2·29) as

$$|Z| = e^{-i\varphi} - |\Gamma|\left[\frac{1}{\pi}\int_\mu^\infty d\omega \frac{A(\omega)}{(\omega-z_0)^2} - 2iA'(z_0)\right]. \tag{2·30}$$

If we use the subsidiary condition (2·29), another expression for $|Z|$ is obtained:

$$|Z| = |\Gamma|\left[-\frac{1}{\pi}\int_\mu^\infty d\omega \frac{A(\omega)}{(\omega-z_0)(\omega-z_0^*)} - 2\frac{\mathrm{Re}\, A(z_0)}{y}\right]. \tag{2·31}$$

When $Z=0$, we get three conditions for $|\Gamma|$; φ and z_0 from Eqs. (2·30) and (2·31), the results being the same as obtained in § 2–1.

§ 3. The propagator of a relativistic scalar particle

We consider the system in which a scalar "ρ" meson, with complex mass $z_0 = (M - i\gamma/2)$, decays into two scalar "π" mesons, with mass μ, where $M > 2\mu$. The discussion is developed along the lines of the Lee model.

The existence of a field $\varphi(x)$ of ρ is assumed. According to the Lehmann prescription,[10] we get the definition for the propagator of the unstable ρ meson;

$$\Delta_F'(x-y) = \langle 0|T(\varphi(x), \varphi^\dagger(y))|0\rangle, \tag{3·1}$$

or in the momentum space

$$\Delta_F'(s) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} ds' \frac{\sigma(s')}{s'-s}, \tag{3·2}$$

where

$$\sigma(s) = \pi(2\pi)^3 \sum_{P_n^2=-s} |\langle 0|\varphi(0)|P_n\rangle|^2. \tag{3·3}$$

Since in our model only the $\rho-2\pi$ channel is considered,

$$\sigma(s) = \rho_0(s)|F(s)|^2, \tag{3·4}$$

and

$$F(s) = (2\pi)^3(4p_0q_0)^{1/2}\langle 0|\varphi(0)|p, q \text{ in}\rangle|_{(p+q)^2=-s}, \tag{3·5}$$

where p and q are the 4-momenta of two π's, and $\rho_0(s)$ is a phase factor defined for real s as follows

$$\rho_0(s) = \tfrac{1}{4}\sqrt{(s-4\mu^2)/s} > 0. \tag{3·6}$$

$F(s)$ can be factorized into a free propagator and a form factor in the case of a stable particle, but in the unstable case the form factor is not well defined because of the lack of a free propagator. On the other hand $F(s)$ can also be factorized into a clothed propagator and a proper vertex function as

$$F(s) = \Delta_F'(s)\Gamma(s). \tag{3·7}$$

We regard Eq. (3·7) as the defining equation for the proper vertex function $\Gamma(s)$ of the unstable particle. $F(s)$ satisfies the following unitarity relation:

$$\text{Im}\, F(s) = F(s)\rho_0(s)T^*(s) \quad \text{for } s>4\mu^2, \tag{3·8}$$

where $T(s)$ is the S-wave scattering amplitude of the two π mesons. From Eqs. (3·7) and (3·8) the unitarity relation for $\Gamma(s)$ is

$$\text{Im}\,\Gamma(s) = \Gamma(s)\rho_0(s)U^*(s), \tag{3·9}$$

where $U(s)$ is the one ρ-meson irreducible part of $T(s)$ and therefore T can be decomposed as

$$T(s) = \Gamma(s)\Delta_F'(s)\Gamma(s) + U(s). \tag{3·10}$$

Now, let us consider the analytic continuation of the above quantities from the

*) From Eq. (3·1), $\Delta_F'(x-y) = |Z|^{-1}\langle 0|T(\varphi^0(x), \varphi^{0\dagger}(y))|0\rangle = |Z|^{-1}\Delta_F'^0(x-y)$, where the quantities with ^{0} are the unrenormalized ones, so the Z factor appearing in Δ_F' is the absolute value.

physical s plane to the unphysical plane which is reached through the elastic cut. $T(s)$ is assumed to be analytic except for cuts along the real axis from $s=4\mu^2$ to ∞ and from $s=0$ to $-\infty$, and $\Delta_F'(s)$ and $\Gamma(s)$ are also analytic except for the physical cut. Then the following reality conditions hold;

$$\begin{aligned} T^*(s) &= T(s^*), \\ \Delta_F'^*(s) &= \Delta_F'(s^*), \\ \Gamma^*(s) &= \Gamma(s^*). \end{aligned} \tag{3·11}$$

$\rho_0(s)$ is to be the positive boundary value of the function

$$\rho(s) = \tfrac{1}{4}[(s-4\mu^2)/s]^{1/2}, \tag{3·12}$$

where we define the roots $(s-4\mu^2)^{1/2}$ and $s^{1/2}$ such that their imaginary parts are non-negative in the cut plane with the branch lines $s>4\mu^2$ and $s<0$ respectively. Note that we have

$$\rho^*(s) = -\rho(s^*). \tag{3·13}$$

We can now continue $\Delta_F'(s)$, Γ, T and U through the cut $s>4\mu^2$ using the unitarity and the reality conditions.

Equation (3·9) can be written in the form

$$\Gamma(s+io) - \Gamma(s-io) = 2iU(s+io)\rho(s+io)\Gamma(s-io);$$

solving for $\Gamma(s-io)$ we obtain

$$\Gamma(s-io) = \Gamma(s+io)/[1+2i\rho(s+io)U(s+io)], \tag{3·14}$$

where the right-hand side is the boundary value of an analytic function

$$\Gamma_{II}(s) = \Gamma(s)/[1+2i\rho(s)U(s)]. \tag{3·15}$$

For $s>4\mu^2$, we have, by construction

$$\Gamma_{II}(s+io) = \Gamma(s-io),$$

and also we have

$$\Gamma_{II}(s-io) = \Gamma(s+io).$$

Hence the cut $s>4\mu^2$ connects just two Riemann sheets.[11] We call this sheet connected to the physical sheet through the cut the 2nd sheet. Let $\Delta_F'^{II}(s)$ be the propagator on the 2nd sheet, so that we have

$$\Delta_F'^{II}(s) = \Delta_F'(s)[1+2i\rho(s)U(s)]/[1+2i\rho(s)T(s)], \tag{3·16}$$

where we have used Eqs. (3·7) and (3·15). We rewrite the above equation as

$$\Delta_F'^{II}(s) = \Delta_F'(s)\tilde{S}(s)S^{-1}(s), \tag{3·17}$$

where

$$\widetilde{S}(s) = 1 + 2i\rho(s)U(s),$$
$$S(s) = 1 + 2i\rho(s)T(s). \tag{3·18}$$

Since U satisfies the same unitarity relation as T does, $\widetilde{S}$ may be called the pseudo S matrix in the sense that it is not the directly observable S matrix. Using Eq. (3·18) we have

$$T_{II}(s) = T(s)S^{-1}(s) \text{ and } U_{II}(s) = U(s)\widetilde{S}^{-1}(s). \tag{3·19}$$

Hence the functions on the 2nd sheet satisfy the same reality conditions as in Eq. (3·11).

The existence of the ρ meson requires the zeros of $S(s)$ at $s = z_0$ and z_0^*. $\Delta_F'(s)$ and $\widetilde{S}(s)$ being regular at these points, $\Delta_F'^{II}(s)$ must have two simple poles. If $\widetilde{S}$ has some zeros, $\Delta_F'^{II}(s)$ should have zeros at those points. Zeros of $\widetilde{S}$ yield the resonance poles in U_{II}, but not in T_{II}, hence these unobservable resonances may be called pseudoresonances. When U has particle poles, which we call the pseudoparticle poles, the situation is somewhat different. In this case $\Delta_F'(s)$ must have zeros at the points because $T(s)$ must not have double poles or pseudoparticle poles. Hence the Lehmann representation Eq. (3·2) does not hold but one subtraction is needed. Because of the zeros of $\Delta_F'(s)$, $\Delta_F'^{II}(s)$ is regular at the ps-particle poles.

§ 4. Renormalization function and Z factor

$T_{II}(s)$ on the 2nd sheet has two poles, which are complex conjugates of one another, at $s = z_0$ and z_0^*. Let us write the residues as g^2 and g^{*2} respectively, in Gaussian units. Ignoring the possible subtraction, we can write $T_{II}(s)$ as

$$T_{II}(s) = \frac{g^2}{z_0 - s} + \frac{g^{*2}}{z_0^* - s} + \frac{1}{\pi}\int_{4\mu^2}^{\infty} ds' \frac{\operatorname{Im} T_{II}(s')}{s' - s} + \frac{1}{\pi}\int_{-\infty}^{0} ds' \frac{\operatorname{Im} T_{II}(s')}{s' - s}. \tag{4·1}$$

On the other hand $T_{II}(s)$ can be expressed in terms of Γ_{II} or $\Delta_F'^{II}$ as

$$T_{II}(s) = \Gamma_{II}(s)\Delta_F'^{II}(s)\Gamma_{II}(s) + U_{II}(s). \tag{4·2}$$

If we normalize $\Gamma_{II}(s)$ as follows,

$$\Gamma_{II}(z_0) = ge^{-i\varphi/2} \text{ and } \Gamma_{II}(z_0^*) = g^*e^{-i\varphi/2}, \tag{4·3}$$

the residues of $\Delta_F'^{II}(s)$ at $s = z_0$ and z_0^* are $e^{i\varphi}$ and $e^{-i\varphi}$, respectively.*)

Then the dispersion relation for $\Delta_F'^{II}$ is given by

$$\Delta_F'^{II}(s) = \frac{e^{i\varphi}}{z_0 - s} + \frac{e^{-i\varphi}}{z_0^* - s} + \frac{1}{\pi}\int_{4\mu^2}^{\infty} ds' \frac{\sigma_{II}(s')}{s' - s} + \frac{1}{\pi}\int_{-\infty}^{0} ds' \frac{\sigma_{II}'(s')}{s' - s}, \tag{4·4}$$

*) φ is not the phase of g itself, in contrast to the Lee model. See Eq. (4·17).

where the subtraction is ignored.

The discontinuities σ_{II} and σ_{II}' are obtained by the following procedure; we introduce a function $A(s)$ which has no branch point at $s=4\mu^2$,

$$A(s)=F(s)\cdot F_{II}(s)=\Delta_F'(s)\Gamma(s)\Delta_F'^{II}(s)\Gamma_{II}(s). \tag{4·5}$$

Using $A(s)$ we have

$$\Delta_F'^{II}(s)=\Delta_F'(s)-2i\rho(s)A(s), \tag{4·6}$$

hence

$$\sigma_{II}(s)=-\rho_0(s)|\Delta_F'^{II}(s)\Gamma_{II}(s)|^2. \tag{4·7}$$

$\sigma_{II}'(s)$ comes from the left-hand cut of $A(s)$:

$$\sigma_{II}'(s)=\frac{1}{2i}[\Delta_F'^{II}(s+io)-\Delta_F'^{II}(s-io)]$$

$$=\rho_0(s)[A(s+io)+A(s-io)] \quad \text{for } s<0.$$

Since $F(s)$ has no branch point at $s=0$, we have

$$\sigma_{II}'(s)=\rho_0(s)|\Delta_F'^{II}(s)\Gamma_{II}(s)|^2 f(s), \tag{4·8}$$

where

$$f(s)=2[1+2\rho_0(s)v(s)], \tag{4·9}$$

$$v(s)=\mathrm{Im}[T(s)] \quad \text{for } s<0. \tag{4·10}$$

In Eq. (4·8) $\rho_0(s)$ tends to ∞ as s tends to $0-o$, but $\Gamma_{II}(s)$ tends to 0 at the same time because of the relation $\Gamma_{II}(s)=\Gamma(s)/[1+2i\rho_0(s)U(s)]$, so that $\sigma_{II}'(0)<\infty$.

Now, introducing the renormalization function $Z_{II}(s)$, we solve for $\Delta_F'^{II}(s)$ from Eq. (4·4):

$$\Delta_F'^{II}(s)=[(z_0-s)Z_{II}(s)]^{-1}. \tag{4·11}$$

From this definition $Z_{II}(s)$ has the following properties,

$$\left.\begin{aligned} Z_{II}(z_0)&=e^{-i\varphi},\\ Z_{II}(z_0^*)&=0. \end{aligned}\right\} \tag{4·12}$$

Hence we obtain

$$Z_{II}(s)=e^{-i\varphi}-\frac{s-z_0}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')|\Gamma_{II}(s')|^2}{(s'-z_0)^2(s'-s)}+\frac{s-z_0}{\pi}\int_{-\infty}^{0}ds'\frac{\rho_0(s')|\Gamma_{II}(s')|^2 f(s')}{(s'-z_0)^2(s'-s)}$$

$$+\sum_n c_n\frac{z_n-z_0^*}{z_n-z_0}\cdot\frac{s-z_0}{z_n-s}+\sum_n c_n^*\frac{z_n^*-z_0^*}{z_n^*-z_0}\cdot\frac{s-z_0}{z_n^*-s}. \tag{4·13}$$

There is a subsidiary condition for $Z_{II}(s)$ in Eq. (4·13):

$$Z_{II}(z_0^*)=e^{-i\varphi}-\frac{z_0^*-z_0}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')\,|\Gamma_{II}(s')\,|^2}{(s'-z_0)^2(s'-z_0^*)}+\frac{z_0^*-z_0}{\pi}\int_{-\infty}^{0}ds'\frac{\rho_0(s')\,|\Gamma(s')|^2 f(s')}{(s'-z_0)^2(s'-z_0^*)}$$

$$+\sum_n c_n\frac{z_0^*-z_0}{z_n-z_0}+\sum_n c_n^*\frac{z_0^*-z_0}{z_n^*-z_0}=0. \tag{4·14}$$

Here the c_n's, the coefficients of the pole terms, are determined by the condition that the ps-resonances do not become true resonances. For this purpose $\Gamma(s)$ is assumed to be given by the inverse of the denominator function $\mathcal{D}(s)$ of U, where U is written as $U=\mathcal{N}/\mathcal{D}$, through the unitarity for $\Gamma(s)$, Eq. (3·9):

$$\Gamma(s)=G/\mathcal{D}(s), \tag{4·15}$$

where G is a real constant. On the 2nd sheet, therefore, we have

$$\Gamma_{II}(s)=G/\mathcal{D}_{II}(s), \tag{4·16}$$

G is, then, given by g and φ through Eq. (4·3) as

$$G=g\mathcal{D}_{II}(z_0)e^{-i\varphi}. \tag{4·17}$$

Substituting Eq. (4·16) into Eq. (4·2), we have

$$T_{II}(s)=\frac{G^2}{(z_0-s)\,Z_{II}(s)\,\mathcal{D}_{II}^2(s)}+\frac{\mathcal{N}(s)}{\mathcal{D}_{II}(s)}, \tag{4·18}$$

so that

$$c_n=G^2/[(z_0-z_n^*)\,(z_0^*-z_n)\,d_n(z_n)\,\mathcal{N}(z_n)], \tag{4·19}$$

where $d_n(s)$ is defined by the following equation:

$$\mathcal{D}_{II}(s)=(z_n-s)\,d_n(s).$$

Thus in our solution for $\Delta_F'^{II}(s)$ there are four parameters, G, φ and z_0, or equivalently g and z_0, on which there are two conditions, Eq. (4·14).

Now, $|Z|$ is defined as the limiting value of $Z_{II}(s)$ at $s=\infty$, so we have

$$|Z|=e^{-i\varphi}+\frac{1}{\pi}\int_{4\mu^2}^{\infty}ds\frac{\rho_0(s)\,|\Gamma_{II}(s)\,|^2}{(s-z_0)^2}-\frac{1}{\pi}\int_{-\infty}^{0}ds\frac{\rho_0(s)\,|\Gamma_{II}(s)\,|^2 f(s)}{(s-z_0)^2}$$

$$-\sum_n c_n\frac{z_n-z_0^*}{z_n-z_0}-\sum_n c_n^*\frac{z_n^*-z_0^*}{z_n^*-z_0}. \tag{4·20}$$

Eliminating the φ-term, we obtain another expression for $|Z|$ by using Eq. (4·14):

$$|Z|=\frac{1}{\pi}\int_{4\mu^2}^{\infty}ds\frac{\rho_0(s)\,|\Gamma_{II}(s)\,|^2}{(s-z_0)\,(s-z_0^*)}-\frac{1}{\pi}\int_{-\infty}^{0}ds\frac{\rho_0(s)\,|\Gamma^{II}(s)\,|^2 f(s)}{(s-z_0)\,(s-z_0^*)}-\sum_n(c_n+c_n^*). \tag{4·21}$$

Therefore if $Z=0$, we have three equations

$$\frac{1}{\pi}\int_{4\mu^2}^{\infty} ds \frac{\rho_0(s)}{(s-z_0)(s-z_0^*)|\mathcal{D}_{II}(s)|^2} - \frac{1}{\pi}\int_{-\infty}^{0} ds \frac{\rho_0(s)f(s)}{(s-z_0)(s-z_0^*)|\mathcal{D}_{II}(s)|^2}$$
$$-\sum_n (c_n' + c_n'^*) = 0, \tag{4·22}$$

and

$$g^2 = \mathcal{D}_{II}^{-2}(z_0)\left[-\frac{1}{\pi}\int_{4\mu^2}^{\infty} ds \frac{\rho_0(s)}{(s-z_0)^2|\mathcal{D}_{II}(s)|^2} + \frac{1}{\pi}\int_{-\infty}^{0} ds \frac{\rho_0(s)f(s)}{(s-z_0)^2|\mathcal{D}_{II}(s)|^2}\right.$$
$$\left. + \sum_n c_n' \frac{z_n - z_0^*}{z_n - z_0} + \sum c_n'^* \frac{z_n^* - z_0^*}{z_n^* - z_0}\right]^{-1},$$

where $c_n' = c_n/G^2$.

For arbitrary value of z_0 it is not guaranteed that $|Z|<1$.

§ 5. Equivalence to the ordinary N/D methods and classification

First we construct the denominator function $D(s)$ and numerator function $N(s)$ of the full scattering amplitude $T(s)$ from $\mathcal{D}(s)$ and $\mathcal{N}(s)$ of U.

$$\left.\begin{aligned} T(s) &= N(s)/D(s), \\ U(s) &= \mathcal{N}(s)/\mathcal{D}(s). \end{aligned}\right\} \tag{5·1}$$

Hence on the 2nd sheet we get

$$T_{II}(s) = N(s)/D_{II}(s) = \frac{G^2}{(z_0-s)Z_{II}(s)[\mathcal{D}_{II}(s)]^2} + \frac{\mathcal{N}(s)}{\mathcal{D}_{II}(s)}. \tag{5·2}$$

We now put

$$\begin{aligned} D_{II}(s) &= (z_0-s)Z_{II}(s)\mathcal{D}_{II}(s), \\ N(s) &= G^2/\mathcal{D}_{II}(s) + (z_0-s)Z_{II}(s)\mathcal{N}(s). \end{aligned} \tag{5·3}$$

In Eq. (5·3), $N(s)$ should be the same function on the physical and on the 2nd sheets. It is easy to show this by using the following equations,

$$\begin{aligned} \mathcal{D}_{II}(s) &= \mathcal{D}(s) \times \tilde{S}(s), \\ (z_0-s)Z_{II}(s) &= \Delta_F'^{-1}(s) \times S(s)\tilde{S}(s)^{-1}, \end{aligned}$$

and we get

$$N(s) = G^2/\mathcal{D}(s) + \mathcal{N}(s)/\Delta_F'(s). \tag{5·4}$$

Similarly, $D(s)$ can be written as

$$D(s) = \mathcal{D}(s)/\Delta_F'(s). \tag{5·5}$$

N and D given by Eqs. (5·4) and (5·5) should satisfy the ordinary condition for N and D. This may be shown as follows:

$$\begin{aligned}\mathrm{Im}\, D(s) &= \mathrm{Im}\, \Delta_F'^{-1}\, \mathcal{D}^* + \Delta_F'^{-1}\, \mathrm{Im}\, \mathcal{D} \\ &= -\rho_0 G^2 \mathcal{D}^*/|\mathcal{D}|^2 - \rho_0 \Delta_F'^{-1} \\ &= -\rho_0(s) N(s) \quad \text{for } s>4\mu^2, \end{aligned} \tag{5·6}$$

and

$$\mathrm{Im}\, D(s) = 0 \quad \text{for } s<4\mu^2. \tag{5·7}$$

On the other hand

$$\mathrm{Im}\, N(s) = \frac{\rho_0 G^2 \mathcal{N}}{|\mathcal{D}|^2} - \frac{\rho_0 G^2 \mathcal{N} |\Delta_F' \mathcal{D}^{-1}|^2}{|\Delta_F'|^2} = 0 \quad \text{for } s>4\mu^2, \tag{5·8}$$

$$\mathrm{Im}\, N(s) = \mathrm{Im}\, \mathcal{N}/\Delta_F' = \mathrm{Im}\, T\mathcal{D}/\Delta_F' = \mathrm{Im} TD(s) \quad \text{for } s<0. \tag{5·9}$$

Now, in order to construct N and D from $\mathcal{N}$ and $\mathcal{D}$ we demand some properties for U;

1) U does not contain any CDD poles.
2) U may have some resonances, but no bound states.
3) The integral equations for $\mathcal{N}$ and $\mathcal{D}$ have a unique solution.

From the demand 3) $v(s)$ should be smaller than $(\ln|s|)^{-2-\varepsilon}$ when s tends to negative infinity, where ε is an arbitrary positive constant.[2),12)] From the demand 1) and 2), and from the Levinson theorem it follows that the phase shift of U, say $\delta_0(s)$, should satisfy the asymptotic behavior:

$$\delta_0(4\mu^2) - \delta_0(\infty) = 0.$$

The asymptotic behavior of $\mathcal{D}(s)$ is, then, at most logarithmically divergent as s tends to positive infinity.

We require that $\mathcal{N}$ and $\mathcal{D}$ satisfy the unsubtracted dispersion relation

$$\left.\begin{aligned}\mathcal{N}(s) &= \frac{1}{\pi}\int_{-\infty}^{0} ds' \frac{v(s')\mathcal{D}(s')}{s'-s}, \\ \mathcal{D}(s) &= 1 - \frac{1}{\pi}\int_{4\mu_2}^{\infty} ds' \frac{\rho_0(s')\mathcal{N}(s')}{s'-s}. \end{aligned}\right\} \tag{5·10}$$

In this case it follows that

$$\begin{aligned}&\mathcal{D}(s) \sim \text{const}, \\ &\mathcal{N}(s) \lesssim (\ln s)^{-1-\varepsilon} \quad \text{for } s\to\infty. \end{aligned} \tag{5·11}$$

We can see the asymptotic behavior of $Z_{\mathrm{II}}(s)$ from Eq. (5·11) and a slightly

modified expression for it, which is

$$Z_{II}(s)=|Z|-\frac{1}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')\,|\Gamma_{II}(s')|^2}{(s'-z_0)\,(s'-s)}+\frac{1}{\pi}\int_{-\infty}^{0}ds'\frac{\rho_0(s')\,|\Gamma_{II}(s')|^2 f(s')}{(s'-z_0)\,(s'-s)}$$
$$+\sum_n c_n\frac{z_n-z_0^*}{z_n-s}+\sum_n c_n^*\frac{z_n^*-z_0^*}{z_n^*-s}, \qquad (5\cdot12)$$

the results being

$$\begin{aligned} &Z_{II}(s)\sim\text{const} && \text{for } Z\neq0,\\ &Z_{II}(s)\sim\ln s/s && \text{for } Z=0. \end{aligned} \qquad (5\cdot13)$$

In both cases the unsubtracted dispersion relation holds for $\Delta_F'(s)$. From the above results we obtain

$$\begin{aligned} &D(s)\sim s && \text{for } Z\neq0,\\ &D(s)\sim\ln s && \text{for } Z=0, \end{aligned} \qquad (5\cdot14)$$

and

$$\begin{aligned} &N(s)\sim s(\ln s)^{-1-\varepsilon} && \text{for } Z\neq0,\\ &N(s)\sim\text{const} && \text{for } Z=0. \end{aligned} \qquad (5\cdot15)$$

We shall discuss the case $Z=0$ in detail. The dispersion relations for N and D are as follows;

$$\left.\begin{aligned} N(s)&=N(\infty)+\frac{1}{\pi}\int_{-\infty}^{0}ds'\frac{v(s')D(s')}{s'-s},\\ D(s)&=1-\frac{s-s_0}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')N(s')}{(s'-s_0)\,(s'-s)}, \end{aligned}\right\} \qquad (5\cdot16)$$

where $N(\infty)$ and s_0 are real and $s_0<4\mu^2$. Two parameters $N(\infty)$ and s_0 appear; these can be transformed into the ρ-meson pole z_0, so we obtain

$$\left.\begin{aligned} N(s)&=1+\frac{1}{\pi}\int_{-\infty}^{0}ds'\frac{v(s')D(s')}{s'-s},\\ D_{II}(s)&=-\frac{s-z_0}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')N(s')}{(s'-z_0)\,(s'-s)}+2i[\rho(s)N(s)\\ &\qquad-\rho(z_0)N(z_0)] \end{aligned}\right\} \qquad (5\cdot17)$$

There is a subsidiary condition, namely $D(z_0^*)=0$;

$$D_{II}(z_0^*)=-\frac{2y}{\pi}\int_{4\mu^2}^{\infty}ds\frac{\rho_0(s)N(s)}{(s-z_0)\,(s-z_0^*)}-4\,\mathrm{Re}[\rho(z_0)N(z_0)]=0. \qquad (5\cdot18)$$

The two parameters x and y are, therefore, not independent; the number of free parameters is *one*. The complex coupling constant g^2 is given by

$$g^2=\left[1+\frac{1}{\pi}\int_{-\infty}^{0}ds\frac{v(s)D(s)}{s-z_0}\right]\left[\frac{1}{\pi}\int_{4\mu^2}^{\infty}ds\frac{\rho_0(s)N(s)}{(s-z_0)^2}-2i[\rho(s)N(s)]'_{s=z_0}\right]^{-1}. \tag{5·19}$$

This corresponds to the intermediate type as discussed by Ida[2] for a stable particle.

Similarly, in the case of non zero Z, we have

$$\left.\begin{aligned}N(s)&=N(s_0)+\frac{s-s_0}{\pi}\int_{-\infty}^{0}ds'\frac{v(s')D(s')}{(s'-s_0)(s'-s)},\\ D(s)&=1-b(s-s_0)-\frac{s-s_0}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')N(s')}{(s'-s_0)(s'-s)}.\end{aligned}\right\} \tag{5·20}$$

Three parameters $N(s_0)$, s_0 and b appear, but since there is the condition $D_{II}(z_0^*)=0$ relating them, the number of free parameters is *two*. This case corresponds to the elementary type.

Other dispersion relations for $\mathcal{N}(s)$ and $\mathcal{D}(s)$, which satisfy three demands 1), 2) and 3), are as follows:

$$\left.\begin{aligned}\mathcal{N}(s)&=\mathcal{N}(\infty)+\frac{1}{\pi}\int_{-\infty}^{0}ds'\frac{v(s')\mathcal{D}(s')}{s'-s},\\ \mathcal{D}(s)&=1-\frac{s-s_0}{\pi}\int_{4\mu^2}^{\infty}ds'\frac{\rho_0(s')\mathcal{N}(s')}{(s'-s_0)(s'-s)}.\end{aligned}\right\} \tag{5·21}$$

There are no other dispersion relations which satisfy three demands. In this case the asymptotic behavior of $\mathcal{D}$ and $\mathcal{N}$ is

$$\begin{aligned}\mathcal{N}(s)&\sim\text{const},\\ \mathcal{D}(s)&\sim\ln s.\end{aligned} \tag{5·22}$$

Then we have

$$\begin{aligned}Z_{II}(s)&\sim\text{const} \qquad \text{for } Z\neq 0,\\ Z_{II}(s)&\sim(s\ln s)^{-1} \quad \text{for } Z=0.\end{aligned} \tag{5·23}$$

When $Z=0$, the unsubtracted dispersion relations hold for N and D, because

$$\begin{aligned}N(s)&\lesssim(\ln s)^{-1},\\ D(s)&\sim\text{const}.\end{aligned} \tag{5·24}$$

Hence N and D contain no free parameters, that is g^2 and z_0 are completely

determined. In this case, however, unsubtracted Lehmann representation does not hold for $\Delta_F'(s)$ because $\Delta_F'(s) \sim \ln s$. This corresponds to the composite type. In the propagator method one more condition holds, namely

$$\lim_{s\to\infty} sZ_{II}(s) = 0, \tag{5·26}$$

where the imaginary part of the above equation is equal to $y|Z|$, only the real part gives a new condition. Hence the four parameters g^2 and z_0 are also completely determined.

However, since there are two more parameters $\mathcal{N}(\infty)$ and s_0, g^2 and z_0 seem to be functions of these parameters. This situation existed also in the case of the stable composite particle.[2] If two ps-resonances exist, $\mathcal{N}(\infty)$ and s_0 are not free but to be determined. From the compatibility of Eqs. (5·26) and $Z=0$ it follows that at least one ps-resonance exists, and so number of free parameters is one. Even if this is not the case, the equations for N and D have a unique solution and on the other hand N and D are to be constructed uniquely from $\mathcal{N}$ and $\mathcal{D}$ and the value of Z, so that the propagator method should not depend on the parameters of $\mathcal{N}$ and $\mathcal{D}$. We summarize the results obtained above in Table I.

Table I.

class	super-elementary	elementary	intermediate	composite
$\mathcal{D}(s)$	$\ln s$	const.	const.	$\ln s$
$\mathcal{N}(s)$	const.	$(\ln s)^{-1-\varepsilon}$	$(\ln s)^{-1-\varepsilon}$	const.
$Z_{II}(s)$	const.	const.	$(\ln s)/s$	$(s \ln s)^{-1}$
$\Delta_F'(s)$	s^{-1}	s^{-1}	$(\ln s)^{-1}$	$(\ln s)$
$D(s)$	$s \ln s$	s	$\ln s$	const.
$N(s)$	s	$s(\ln s)^{-1-\varepsilon}$	const.	$(\ln s)^{-1-\varepsilon}$
$\delta(\infty)$	π	π	0	0
no. of free parameters	3	2	1	0

§ 6. Discussion

For an arbitrary value of z_0 it is not guaranteed that the absolute value of Z is not greater than 1 both in the Lee model and in the relativistic scalar ρ-meson model. This situation may be related to the fact that the norm of the "eigenstate" of the total Hamiltonian with the eigenvalue z_0 is zero. The fact that $Z<1$ for the stable particle comes from the normalizability of the eigenstate of the particle to unity. The unstable particle "eigenstate" exists on the 2nd sheet and its norm is zero as shown by Nakanishi.[8] Therefore we can have no restrictions on the Z, which is to play the role of a normalization constant in the case of the stable particle. The physical meanings in the cases $Z=0$ and $Z=1$, however, are definite: in the former the elementary particle becomes

an intermediate one and in the latter the coupling constant vanishes.

The ρ meson discussed in the text is not realistic but mearly a simple model. In order to treat the vector ρ meson it is necessary to construct the higher spin propagator and vertex functions. Further, a multichannel formalism on the unphysical Riemann sheets may also be necessary. The multichannel problem is solved by the following consideration: note that the vertex functions form a column matrix and can be represented by

$$\boldsymbol{\Gamma}(s)=[{}^{t}\mathfrak{D}(s)]^{-1}\boldsymbol{G}, \tag{7·1}$$

where $\boldsymbol{U}(s)=\mathfrak{N}\mathfrak{D}^{-1}$ and all quantities are $n\times n$ matrices. The $n\times n$ scattering amplitude is written as

$$\boldsymbol{T}(s)=\boldsymbol{\Gamma}(s)\,\Delta_F'(s)\,{}^{t}\boldsymbol{\Gamma}(s)+\mathfrak{N}(s)\,\mathfrak{D}(s)^{-1}. \tag{7·2}$$

Now, we introduce a matrix function $\boldsymbol{X}(s)$ which satisfies the following equation,[13]

$$\boldsymbol{X}(s)\,\boldsymbol{G}=\Delta_F'(s)\,\boldsymbol{G} \tag{7·3}$$

and

$$\mathrm{Im}\,\boldsymbol{X}(s)\,\boldsymbol{G}=\boldsymbol{\Gamma}^{\dagger}(s)\,\boldsymbol{\rho}(s)\,\boldsymbol{\Gamma}(s)\,|\Delta_F'|^2\boldsymbol{G}. \tag{7·4}$$

After some calculation we obtain $\boldsymbol{X}^{(i)}(s)$ continued analytically to the i-th unphysical sheet through the cut $s_{i-1}<s<s_i$, where s_i is the threshold energy of the i-th channel,

$$\boldsymbol{X}^{(i)}\boldsymbol{G}={}^{t}\mathfrak{D}\boldsymbol{\rho}_{(i)}^{-1/2}\tilde{\boldsymbol{S}}^{(i)}(\boldsymbol{S}^{(i)})^{-1}\boldsymbol{\rho}_{(i)}^{1/2}({}^{t}\mathfrak{D})^{-1}\boldsymbol{X}\boldsymbol{G}, \tag{7·5}$$

where $\boldsymbol{\rho}_{(i)}^{1/2}$ is a diagonal matrix whose elements are the square root of the phase factor of each channel below the i-th threshold and zero above the i-th threshold. $\boldsymbol{S}^{(i)}$ and $\tilde{\boldsymbol{S}}^{(i)}$ are the unitary matrix if $s<s_i$, and can be written as

$$\begin{aligned}\tilde{\boldsymbol{S}}^{(i)}&=\boldsymbol{1}+2i\boldsymbol{\rho}_{(i)}^{1/2}\,\boldsymbol{U}\,\boldsymbol{\rho}_{(i)}^{1/2},\\ \boldsymbol{S}^{(i)}&=\boldsymbol{1}+2i\boldsymbol{\rho}_{(i)}^{1/2}\,\boldsymbol{T}\,\boldsymbol{\rho}_{(i)}^{1/2}.\end{aligned} \tag{7·6}$$

From Eq. (7·5) it follows that

$$\det\boldsymbol{X}^{(i)}(s)=\det\tilde{\boldsymbol{S}}^{(i)}(s)\,[\det\boldsymbol{S}^{(i)}(s)]^{-1}\det\boldsymbol{X}(s), \tag{7·7}$$

therefore the zeros of this equation give the resonance poles at values of $x's$ which lie in the interval $s_{i-1}<x<s_i$. The renormalization function and the equivalence of the propagator method to the ordinary N/D method are similar to those in the stable case, or the single channel unstable case. The detailed discussion will be given on the problems of the vector mesons.

In § 3, we derived Eq. (3·10), which means that the resonant part can be separated out from the partial wave amplitude. In the stable particle case this is self-evident, and further if the spin of the particle is zero the term written as $\Gamma(s)\,\Delta_F'(s)\,\Gamma(s)$ can be given by a single spectral function in the s channel

in the Mandelstam representation for the total scattering amplitude and the term U has contributions from the double spectral function and the single spectral functions in the t and u channels. In the case of the non composite resonant particle the term $\Gamma(s)\Delta_F'(s)\Gamma(s)$ is given by the single spectral function, if it is a S-wave resonance. The single spectral function $\sigma_s(s)$ can be approximated as follows,

$$\text{Im}\, T(s) = \sigma_s(s) + \frac{1}{2\pi}\int_{-1}^{1} dz \int dt' \frac{\sigma_{st}(s, t')}{t' + 2q^2(1-z)}$$

$$= \rho_0(s)\,|T(s)|^2, \tag{7·8}$$

then we have

$$\sigma_s(s) \approx \rho_0(s)\,|T(s)|^2. \tag{7·9}$$

If we use a one-level formula for $T(s)$, we obtain

$$\sigma_s(s) \approx \rho_0(s)\,\frac{[M\gamma]^2}{(s-M^2)^2 + [\rho_0(s)\,M\gamma]^2}, \tag{7·10}$$

where M is the position of the resonance and γ is its reduced width. Therefore if the S-wave resonance is exchanged in the t channel, we first approximate $\sigma_t(t)$ by Eq. (7·10) and then the double spectral function can be determined through the unitarity so as to give the term $U(s)$. Once $U(s)$ is given, we can obtain the resonance in the s channel. We further conjecture that a single spectral function multiplied by a polynomial in t, the degree of which is l, the angular momentum of the resonance, represents the resonant part of the scattering amplitude, which yields the l-th partial wave amplitude in the form Eq. (3·10).

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