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DISCUSSION

E. Brundrett[4]

The author is to be congratulated on his very thought-provoking re-evaluation of the concept of the equivalent hydraulic diameter. For this writer there are, as with all interesting research, certain questions arising from the analysis for the best characteristic diameter for rectangular ducts as presented by this paper.

This writer would feel more comfortable with the new concept if a greater experimental base could be used for the confirmation of the technique, particularly since a laminar flow analysis is being extended to the turbulent flow regime. Also this writer would feel more comfortable with the concept if some further discussions of the rejected data had occurred for those cases where obvious experimental errors could not be detected.

Also the utilization of the correction parameter ϕ^* on the classical hydraulic diameter, D_e, to yield D_L, does not appear to provide the classical asymptotic solution when the duct becomes quite large. For example with an aspect ratio of 10:1, D_L should be very nearly equal to $2S$ if classical arguments are at all valid. By computation for an aspect ratio of 10:1 $De = 4A/P = (4SW/2(W + S)) = 1.818S$ which of course gives rise to the dissatisfaction of the author, and of this writer with the hydraulic diameter. The author's proposed characteristic dimension at a 10:1 aspect ratio is by equations (5) and (7)

$$D_L = \phi^* \left(\frac{W}{S}\right) D_e = 0.7537\ D_e = 0.753 \times 1.818S = 1.302S$$

This value is still considerably less than $2S$, and shows no ability to converge to $2S$ as the aspect ratio increases. Perhaps as with all currently available characteristic diameters, there is a best range of application of the author's proposed effective diameter.

Finally this writer is inclined to treat with some caution the author's stated accuracy of prediction of ± 5 percent in view of the very extensive amount of data rejection that has occurred. The writer is however, impressed with the extent of the analysis that has been undertaken, and with the very thought-provoking comments, particularly those associated with the effect of roughness upon similarity in rectangular ducts at high aspect ratios.

P. S. Barna[5]

In recent years, the vexing question of friction in noncircular ducts has received renewed attention and it is gratifying that the author, Dr. Owen C. Jones, has so carefully reviewed the

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relevant literature from the early days of Blasius and Nikuradse up to date.

Friction is as complex as it is intricate and the writer had his share of hassle with it. He also had serious misgivings about using the "equivalent" or "hydraulic" diameter D_e in the past especially for turbulent flow between flat plates for which, for the case of infinite aspect ratio, $D_e = 2h$, where h is the distance between the plates.

For fully developed flow one may explain the discrepancy in friction with increasing aspect ratio, as discussed in the paper, by simply considering the contributions by the four walls on which the boundary layer is formed. For a square duct these contributions must be equal because of symmetry, but for other rectangular ducts they become unequal. In the case of moderate aspect ratio, say up to three, while the contribution to friction of the *near* walls increases, of the *far* walls it decreases and presumably these may compensate for each other. It may then be anticipated that the friction factor based on the equivalent hydraulic diameter may indeed agree with the results based on a circular pipe. With increasing aspect ratio, however, the contributions of the far walls to the overall friction may become negligible while the increasing influence of the near walls become predominant.

For the parallel plates with no far walls present one has had to contend with $D_e = 2h$ and with this value-upon substitution one calculates a lower friction and higher Reynolds number than the value $D_e = h$ would otherwise predict when using the relevant formulas. Therefore, it is not surprising to learn from the paper that high aspect ratio ducts yield higher friction, while so many earlier investigators found good agreement with circular pipe-friction for lower and even intermediate aspect ratios when using the hydraulic diameter. It is then refreshing to learn that the author has found through his studies an explanation for the discrepancies, a sort of reconciliation between the different views, and was able to establish a method which appears satisfactory. By replacing the hydraulic diameter D_e with the "laminar equivalent" diameter $D_e\phi^*$, the author concludes that by calculating a modified Reynolds number $R_e^* = vD_e\phi^*\rho/\mu$ an excellent similarity between round tubes and rectangular ducts in both laminar and turbulent flow may be obtained for all aspect ratios. A critical examination appears to prove his point.

The painstaking work is highly commendable and the paper is an excellent contribution to literature for which the author must be wholeheartedly congratulated.

N. Madsen[6]

The author of this paper has performed an extremely useful service for engineers interested in turbulent flow through smooth rectangular ducts. The collection of the extensive literature data, the contribution of original data, and the empirical correlation are all valuable additions to the arsenal of the designer. The ingenious use of an "equivalent laminar" diameter for calculation of a modified Reynolds number in turbulent flow seems to fit the chosen data very well indeed.

As with the introduction of any new concepts, new questions arise. The author has discussed some of these; however, a very basic question remains. According to the reasoning presented under the heading "Modified Reynolds Number and Laminar Equivalent Diameter" the concept of an "equivalent laminar" diameter should be applicable to all uniform, noncircular ducts for which an equation analogous to the Cornish equation exists.

An equation for pressure drop during laminar flow can always be derived for a given uniform noncircular cross section, either analytically or numerically, and it can be represented by an equation $f = 64/\mathrm{Re}^*$. It is thus always possible to calculate a modified Reynolds number and an "equivalent laminar" diameter for any uniform cross section, whatever; and by force of the arguments presented in the paper, equation (10) should apply to turbulent flow through a smooth duct with the given cross section.

Of course, purely speculative ideas of this kind especially invented to match the requirements of a specific data set, can have no broad scientific value until they acquire support from theoretical evidence or at least are tested with other duct cross sections. Extension of the concept to other cross sections and the application to rough ducts, as suggested by the author, deserve further investigation. He correctly points out that the procedure does not allow for pressure drop caused by secondary flow.

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Author's Closure

The author wishes to thank Professors Brundrett, Madsen, and Barna for their kind remarks concerning this work, and agree completely with Professors Brundrett and Madsen in their desire for a larger data base and extension to other geometries. Other, independent data on rectangular ducts, especially at high aspect ratios is needed. Sufficient data in the laminar regime must be obtained to insure that adequate experimental procedures are followed. This can be especially troublesome for high aspect ratios in ducts of small gap, due to problems of dimensional control and stability. As for other geometries, this method has now been successfully extended to the case of smooth, concentric annuli and will be the subject of a forthcoming paper [31].

The experiments were reviewed in detail with the results documented in reference [1] of this paper. Any attempt to reject data for cause, except in a few cases, was a hopeless task. Having abandoned that attempt, the alternative was to accept data based on a culling procedure without prejudice to those data not accepted. This, of course, was done. It is certain that some perfectly valid data were not accepted. It is almost equally certain that no bad data were accepted, (with the exception of the questionable data of Washington and Marks [17] which deviated from the laminar theory above Re = 1000). It is the latter point which led directly to development of the method.

In answer to Professor Brundrett's remarks pertaining to the recommended 5 percent uncertainty, the author is convinced that a well-controlled experiment on an accurately known rectangular geometry will yield results within this uncertainty band. This is because uncertainties such as those associated with errors of flow, geometry, and thermodynamic state have hopefully been minimized in development of the correlation. Deviation of the friction factor outside of the recommended 5 percent range due to variations of similar factors within a given design tolerance must be considered by the designer in whatever context is required and over which the correlator has no control.

Finally, regarding the proper limiting value of the laminar equivalent diameter, the asymptotic value is seen to be correct when the variations of the laminar coefficient are considered. Since $f = 64/\mathrm{Re}^*$, always, and since $f \rightarrow 96/\mathrm{Re}$ as $w/s \rightarrow \infty$, it is easily seen that $D_L \rightarrow 2D_e/3$ as $w/s \rightarrow \infty$. The limiting value of D_e is twice the spacing, $2s$, so that $D_L \rightarrow 4s/3$ as $w/s \rightarrow \infty$ in agreement with (5), (6), and (7).

Additional Reference

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