Design of a combined cam–linkage mechanism with an oscillating roller follower by an analytical method

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Abstract: An analytical method for the design of the cam contour of a combined cam–linakge mechanism with an oscillating roller follower to trace precisely a predetermined locus is introduced. It is shown that the composition of the mechanism during kinematic analysis is profitably viewed differently from that during design. Furthermore, as the assembly mode of linkage mechanisms may change during design, a key feature of this paper is the explanation of how to deal with the change in assembly mode of Assur groups during design of such combined mechanisms.

Keywords: combined mechanisms, cam, linkage, locus

1 INTRODUCTION

Cam–linkage combined mechanisms are combinations of linkage mechanisms and cam mechanisms according to certain requirements and have the merits of both of these two kinds of mechanism. This kind of mechanism can generate complex loci and has the potential for many wide-ranging applications. It has been shown in reference [**1**] that there are a significantly large number of cam–link mechanisms that may be used for generating an exact path or for guiding a body through exactly specified positions. In reference [**2**], non-linear programming techniques are applied to the optimum design of systems consisting of a four-bar linkage driven by a disc cam.

Figure 1 shows a cam–linkage combined mechanism, which can produce a complex locus. From the point of view of mechanism composition and kinematic analysis, the oscillating roller follower and the cam constitute a grade II higher-pair Assur group (see Appendix 2). On the one hand, the driving cam determines the motions of the oscillating roller follower and point C on the follower. On the other hand, point A on the driving cam rotates continuously. Links AB and CB therefore constitute an RRR Assur group (the notation indicates that, in an RRR Assur group, the three kinematic pairs are all revolutes). Since the motions of points A and C are determined for any position of the cam, the motions of link CB can be determined from the geometry of triangle ABC. As the lengths of line sections CB and CF are constant, the motion of point F can also be found. Point F may thus be made to trace precisely any predetermined locus by designing the cam contour correctly. Hence, from the point of view of the principle of mechanism composition and kinematic analysis, this combined mechanism is composed of a basic mechanism (consisting of the frame, the driving cam, and the fixed pivot O), a grade II higher-pair Assur group (consisting of the oscillating roller follower, the fixed pivot D, and the higher-pair E between the cam and the roller), and an RRR Assur group (consisting of link AB, link CB, and the three revolutes A, B, and C).

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2 BRIEF DESCRIPTION OF THE METHOD FOR DESIGN OF THE CAM CONTOUR

Before designing the cam contour, the following parameters can be predefined according to the overall arrangement of the mechanism by the designer: the coordinates (X_O, Y_O) of the fixed pivot O, the

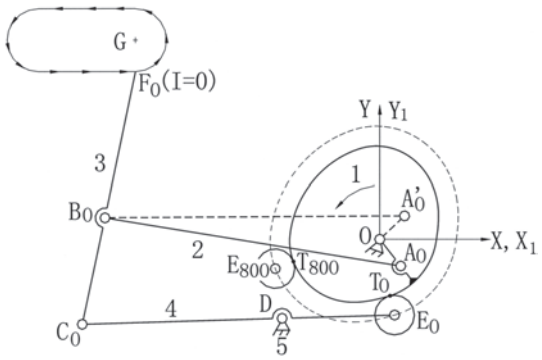

Fig. 1 Combined cam–linkage mechanism at the starting position

coordinates (X_D, Y_D) of the fixed pivot D, the radius r_r, of the roller, and the lengths of some links (L_{DE}, L_{DC}, L_{BC}, and L_{FC}). Obviously, the sum $L_{FC} + L_{DC}$ of the lengths of links FC and DC should be larger than the maximum distance $(L_{DF})_{max}$ between point F and the fixed pivot D. The lengths of links BA and OA should be calculated according to the above dimensions, which will be examined in section 4.

As mentioned above, from the point of view of mechanism composition and kinematic analysis, the oscillating roller follower and the cam constitute a higher-pair Assur group and links AB and CB constitute an RRR Assur group. However, during design of the cam contour, since the locus of point F and the coordinates of the fixed pivot D and the lengths of links FC and DC are predetermined, links FC and DC may be treated as the first RRR Assur group. For every position of point F, the kinematic parameters (θ_{FC}, ω_{FC}, θ_{DC}, ω_{DC}) of links FC and DC can be calculated from the formulae derived in Appendix 2. The kinematic parameters of point B on link FC and of point E on link DC can then be calculated as follows

$$\begin{aligned} X_B &= X_F + L_{FB}\cos\theta_{FC}, \quad Y_B = Y_F + L_{FB}\sin\theta_{FC} \\ V_{BX} &= V_{FX} - L_{FB}\sin\theta_{FC}\omega_{FC} \\ V_{BY} &= V_{FY} + L_{FB}\cos\theta_{FC}\omega_{FC} \end{aligned} \tag{1}$$

$$\begin{aligned} X_E &= X_D + L_{DE}\cos(\theta_{DC} + \pi) \\ Y_E &= Y_D + L_{DE}\sin(\theta_{DC} + \pi) \\ V_{EX} &= V_{DX} - L_{DE}\sin(\theta_{DC} + \pi)\,\omega_{DC}, \\ V_{EY} &= V_{DY} + L_{DE}\cos(\theta_{DC} + \pi)\omega_{DC} \end{aligned} \tag{2}$$

Since the position of point B and the coordinates of the fixed pivot O and the lengths of links BA and OA are known, links BA and OA constitute the second RRR Assur group, and, for every position of point F, their kinematic parameters can be calculated using the formulae derived in Appendix 2.

In this way, the corresponding positions of link OA and point E can be found for every position of point F. The angular displacement of link OA is the same as that of the cam. Therefore, the kinematic relationship between the cam and the roller centre E, or the locus of the roller centre E relative to the cam, can be found. The locus is the pitch curve of the cam, and, as the cam contour is a curve parallel to the pitch curve, it can be designed after the pitch curve is determined. Detailed design steps are described below.

From the above it can be seen that, during design of the cam contour, links FC and DC are treated as the first RRR Assur group while links BA and OA form the second RRR Assur group. The composition of this mechanism during design is therefore considered differently from that during kinematic analysis.

3 PARAMETRIC EQUATIONS FOR A PREDETERMINED LOCUS

Assume a stationary coordinate system XOY in which, for the sake of convenience of design, the origin O coincides with the rotating axis of the cam, as shown in Fig. 1. Thus, the coordinates (X_F, Y_F) of point F on the predetermined locus can be expressed in parametric equations as follows

$$\begin{aligned} X_F &= f_1(I), \quad I = 0, 1, 2, \ldots, N \\ Y_F &= f_2(I), \quad I = 0, 1, 2, \ldots, N \end{aligned} \tag{3}$$

where N is the maximum index value. To increase the accuracy of the design, the maximum index N should be set as 10 times the number of points needed to draw the cam contour.

During design, the X and Y velocity components (V_{FX} and V_{FY}) of point F are needed. They can be calculated from the following

$$\begin{aligned} V_{FX} &= \frac{dX_F}{dI} \\ V_{FY} &= \frac{dY_F}{dI} \end{aligned} \tag{4}$$

A suitable point F_0 on the locus is selected as the starting point of point F. The parametric equations should be so created that, when index I increases from 0, point F will trace a path from the starting point F_0 in the required direction along the

predetermined locus. When index I reaches its maximum value N, point F will return to the starting point F_0. Each value of I corresponds to a point on the locus.

4 DETERMINATION OF THE LENGTHS OF LINKS OA AND BA

Before design commences, the coordinates of points F and D and the lengths L_{FC} and L_{DC} of links FC and DC are specified. Thus, links FC and DC can be treated as an RRR Assur group. In the first phase of design, let index I increase from 0 to N in unit steps. The coordinates (X_F and Y_F) of point F can be calculated for every value of I using parametric equation (3). Then, the positions θ_{FC} and θ_{DC} of links FC and DC can be calculated from the formulae derived in Appendix 2 (where the three letters A, B, and C should be replaced by F, C, and D respectively, as in Fig. 1). The position of point B on link FC can be calculated from equation (1) after the orientation of link FC, θ_{FC}, is found.

Next, the distance L_{OB} between the fixed pivot O and point B for every value of I can be calculated. From all values of L_{OB}, it is possible to find the maximum distance $(L_{OB})_{max}$ and the minimum distance $(L_{OB})_{min}$ as well as the corresponding key values I_{max} and I_{min}. When L_{OB} reaches its minimum value $(L_{OB})_{min}$, or I reaches its key index I_{min}, links OA and BA will be collinear and overlap, so that $(L_{OB})_{min} = L_{BA} - L_{OA}$. Similarly, when L_{OB} reaches its maximum value $(L_{OB})_{max}$, or I reaches its key index I_{max}, links OA and BA will be extended and collinear, so that $(L_{OB})_{max} = L_{OA} + L_{BA}$. Figure 1 shows the starting position of the mechanism corresponding to index $I = 0$. Point F moves along the locus in an anticlockwise direction, as shown by the arrows in Fig. 1. Obviously, $0 < I_{min} < I_{max} < N$.

Hence, the lengths of links OA and BA can be calculated by the following formulae

$$L_{OA} = \frac{(L_{OB})_{max} - (L_{OB})_{min}}{2}, \quad L_{BA} = \frac{(L_{OB})_{max} + (L_{OB})_{min}}{2} \tag{5}$$

5 CALCULATION OF THE KINEMATIC PARAMETERS OF LINK OA AND POINT E

Suppose that a rotating coordinate system X_1OY_1 is fixed to the cam and coincides with the stationary coordinate system XOY at the starting position when $I = 0$ (Fig. 1). For every index I, the kinematic parameters (θ_{FC}, ω_{FC}, θ_{DC}, ω_{DC}) of links FC and DC can be calculated by the formulae derived in Appendix 2. Then, the kinematic parameters (X_B, Y_B, V_{BX}, V_{BY}, X_E, Y_E, V_{EX}, V_{EY}) of points B and E on links FC and CD in the stationary coordinate system XOY corresponding to index I can be easily calculated using equations (1) and (2).

Since link OA is a crank, the assembly mode of the RRR Assur group consisting of links OA and BA will change at the two positions where link OA is extended or overlapping collinear with link BA, or where I is equal to its key index I_{max} or I_{min} respectively. During design, it is necessary to determine the assembly mode before using the formulae derived in the Appendix 2 to calculate the appropriate kinematic parameters of the links in the RRR Assur group. A detailed method is described as follows.

Suppose that the starting positions (when $I = 0$) of links OA and BA are the solid lines OA_0 and A_0B_0 shown in the Fig. 1 and the three revolutes B_0, A_0, and O are arranged anticlockwise. (Note that, if the starting positions of links OA and BA corresponding to index $I = 0$ are the dashed lines OA'_0 and $B_0A'_0$ shown in Fig. 1, then the designed cam contour will be different.)

When $0 \leqslant I < I_{min}$ or $I_{max} < I \leqslant N$, the three revolutes B, A, and O are arranged anticlockwise. When using the formulae in Appendix 2 to calculate the kinematic parameters of links BA and OA, the three letters A, B, and C should be replaced by B, A, and O respectively in Fig. 1. The correct kinematic parameters of links BA and OA will then be calculated.

When $I_{min} < I < I_{max}$, the three revolutes O, A, and B are arranged anticlockwise (see Fig. 2). Therefore, when using the formulae in Appendix 2 to calculate the kinematic parameters of links OA and BA, the three letters A, B, and C should be replaced by O, A, and B respectively in Fig. 2.

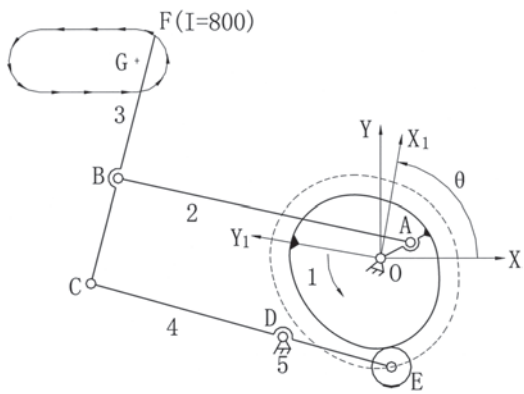

Fig. 2 Combined cam–linkage mechanism when $I = 800$

However, when $I = I_{min}$ or $I = I_{max}$, link OA is overlapping or extended collinear with link BA. In these two cases, the formulae in Appendix 2 cannot be used to calculate the angular velocity ω_{OA} of link OA, which is needed in the next calculation. Although the Lagrange method can be used to calculate the angular velocity ω_{OA} of link OA at these two critical positions, it is rather complicated and tedious since derivatives for both numerator and denominator need to be determined. To overcome this obstacle, the maximum index N is set to 10 times the number of points needed to draw the cam contour. During the second phase of the design, let index I increase from 0 to N with a step size of 10. In this way, the accuracy of the design can be kept acceptable and the obstacle avoided.

6 DETERMINATION OF THE COORDINATES OF THE PITCH CURVE OF THE CAM RELATIVE TO THE ROTATING COORDINATE SYSTEM X_1OY_1 FIXED WITH THE CAM

At the starting position when $I = 0$ (Fig. 1), the rotating coordinate system X_1OY_1 coincides with the stationary coordinate system XOY. The angular position of link OA at this position is θ_{OA0}. For other values of index I, the angular position of link OA will be θ_{OA} and the angular displacement of link OA will be $\theta = \theta_{OA} - \theta_{OA0}$. Since the rotating coordinate system X_1OY_1 is fixed to link OA, the angle between the rotating coordinate system X_1OY_1 and the stationary coordinate system XOY corresponding to index I is also θ, as shown in Fig. 2. The coordinates (X_E, Y_E) of point E in the stationary coordinate system XOY corresponding to any index I have been calculated in section 5. Therefore, the coordinates (X_{E1}, Y_{E1}) of point E relative to the rotating coordinate system X_1OY_1 fixed to the cam corresponding to index I can be calculated by the following coordinate transformation formulae

$$\begin{aligned} X_{E1} &= X_E \cos\theta + Y_E \sin\theta \\ Y_{E1} &= -X_E \sin\theta + Y_E \cos\theta \end{aligned} \tag{6}$$

The locus of the roller centre E on the rotating coordinate system X_1OY_1 fixed to the cam is the pitch curve of the cam. Formulae (6) are therefore those of the coordinates of the pitch curve of the cam relative to the rotating coordinate system X_1OY_1, or those of the coordinates of the pitch curve of the cam relative to the stationary coordinate system at the starting position.

7 DETERMINATION OF THE COORDINATES (X_{T1} AND Y_{T1}) OF THE CAM CONTOUR RELATIVE TO THE ROTATING COORDINATE SYSTEM X_1OY_1 FIXED WITH THE CAM

The cam contour of the cam with an oscillating roller follower is the envelope of the family of the roller circles with roller radius r_r and centred on the pitch curve of the cam, namely the pitch curve and the cam contour are two parallel curves with normal distance r_r. Since the cam rotates in an anticlockwise direction, the pitch curve will be generated around the centre O of the camshaft in the opposite direction, i.e. clockwise in this example (Fig. 3). In the rotating coordinate system X_1OY_1, the direction of the tangent line of the pitch curve at point E_1 is from E_1 to K, as shown in Fig. 3. The slope of the tangent line E_1K is

$$\mathrm{tg}\varphi = \frac{dY_{E1}}{dX_{E1}} = \frac{dY_{E1}/dt}{dX_{E1}/dt} = \frac{V_{EY1}}{V_{EX1}}$$

where V_{EX1} and V_{EY1} can be derived by differentiating X_{E1} and Y_{E1} respectively in equation (6) with respect to time

$$\begin{aligned} V_{EX1} &= V_{EX}\cos\theta - \omega_{OA}X_E\sin\theta + V_{EY}\sin\theta \\ &\quad + \omega_{OA}Y_E\cos\theta \\ V_{EY1} &= -V_{EX}\sin\theta - \omega_{OA}X_E\cos\theta + V_{EY}\cos\theta \\ &\quad - \omega_{OA}Y_E\sin\theta \\ V_{E1} &= \sqrt{V_{EX1}^2 + V_{EY1}^2} \end{aligned} \tag{7}$$

Setting $V_{E1} = \sqrt{V_{EX1}^2 + V_{EY1}^2}$, then $\sin\varphi = V_{EY1}/V_{E1}$ and $\cos\varphi = V_{EX1}/V_{E1}$. Point T_1 is the tangent point between the roller and the cam contour and E_1T_1 is a common normal to the pitch curve and the cam contour. Therefore, $\angle E_1T_1D = \varphi$. Thus, the formulae for the coordinates (X_{T1}, Y_{T1}) of the cam contour relative to the rotating coordinate system X_1OY_1

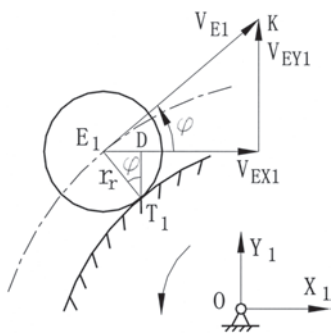

Fig. 3 Derivation of the cam contour

can be derived from [3]

$$X_{T1} = X_{E1} + r_r \sin\varphi = X_{E1} + r_r \frac{V_{EY1}}{V_{E1}}$$
$$Y_{T1} = Y_{E1} - r_r \cos\varphi = Y_{E1} - r_r \frac{V_{EX1}}{V_{E1}} \tag{8}$$

Formulae (7) and (8) are also those of the coordinates of the cam contour relative to the stationary coordinate system XOY at the starting position ($I = 0$).

8 EXAMPLE

Suppose that a combined cam–linkage mechanism is to be designed to trace a curve such as a 400 m race track as shown in Fig. 1. The centre of the right-end semi-circle is G. The lengths of the two straight sides are 100 mm. The radii of the two semi-circles are $100/\pi$. Therefore, the length of each semi-circle is also 100 mm. Some parameters are determined according to the overall arrangement of the mechanism by the designer as follows: $X_O = 0$, $Y_O = 0$, $X_D = -100$ mm, $Y_D = -80$ mm, $L_{BF} = 150$ mm, $L_{CF} = 260$ mm, $L_{CD} = 205$ mm, $L_{DE} = 115$ mm, radius of roller $r_r = 20$ mm, $X_G = -250$ mm, and $Y_G = 200$ mm. There are four sections in the desired locus. A total of 400 points are needed to draw the curve, 100 points for each section. Therefore, the maximum index is $N = 10 \times 400 = 4000$. The parametric equations of the locus are

$$X_F = X_G + \frac{100}{\pi}\sin\left(\frac{\pi}{1000}I\right), \quad I = 0\text{–}1000$$
$$Y_F = Y_G - \frac{100}{\pi}\cos\left(\frac{\pi}{1000}I\right), \quad I = 0\text{–}1000$$
$$X_F = X_G - \frac{I - 1000}{10}, \quad I = 1001\text{–}2000$$
$$Y_F = Y_G + \frac{100}{\pi}, \quad I = 1001\text{–}2000$$
$$X_F = X_G - 100 - \frac{100}{\pi}\sin\left[\frac{\pi}{1000}(I - 2000)\right], \quad I = 2001\text{–}3000$$
$$Y_F = Y_G + \frac{100}{\pi}\cos\left[\frac{\pi}{1000}(I - 2000)\right], \quad I = 2001\text{–}3000$$
$$X_F = X_G - 100 + \frac{I - 3000}{10}, \quad I = 3001\text{–}4000$$
$$Y_F = Y_G - \frac{100}{\pi}, \quad I = 3001\text{–}4000 \tag{9}$$

In the first phase of the design, the lengths of links OA and BA are, calculated by the method outlined in section 4. The results are $(L_{OB})_{min} = 272.383$ mm, $I_{min} = 416$, $(L_{OB})_{max} = 341.069$ mm, and $I_{max} = 2442$. According to formulae (5), $L_{OA} = 34.3428$ mm and $L_{BA} = 306.726$ mm.

The angular position θ_{OA0} of link OA at the starting position (when $I = 0$) is calculated first. At the starting position, X_{F0} and Y_{F0} can be calculated from equation (9) as $X_{F0} = -250$ mm and $Y_{F0} = 168.169$ mm. Links FC and DC constitute an RRR Assur group and their angular positions θ_{F0C0} and θ_{DC0} corresponding to $I = 0$ can be calculated by the formulae in Appendix 2. The calculated values of θ_{F0C0} and θ_{DC0} corresponding to $I = 0$ are $\theta_{F0C0} = -102.193^\circ$ and $\theta_{DC0} = -178.332^\circ$. The coordinates of point B on link FC are calculated from equation (1) as $X_{B0} = -281.681$ mm and $Y_{B0} = 21.5527$ mm.

Links BA and OA constitute another RRR Assur group, and their angular positions θ_{B0A0} and θ_{OA0} corresponding to $I = 0$ can be calculated from the formulae in Appendix 2. The resulting value of θ_{OA0} corresponding to $I = 0$ is $\theta_{OA0} = -51.9412^\circ$.

The kinematic parameters for other values of index I can be calculated similarly. Attention should be paid to the relationship between the letters in the combined cam–linkage mechanism and those in Fig. 5 when using the formulae in Appendix 2 to calculate the kinematic parameters of links OA and BA. When $0 \leqslant I < I_{min}$ or $I_{max} < I \leqslant N$, the three revolutes B, A, and O are arranged anticlockwise (see Fig. 1). When using the formulae to calculate the kinematic parameters of links OA and BA, therefore, the three letters A, B, and C should be replaced by B, A, and O respectively in Fig. 1. When $I_{min} < I < I_{max}$, the three revolutes O, A, and B are arranged anticlockwise (see Fig. 2). Hence, when using the formulae to calculate the kinematic parameters of links OA and BA, the three letters A, B, and C should be replaced by O, A, and B respectively.

Taking as an example the value $I = 800$ (see Fig. 2), X_F and Y_F are calculated from equation (9) as $X_F = -231.29$ mm and $Y_F = 225.75$ mm. The X and Y velocity components (V_{FX} and V_{FY}) of point F can be calculated from equation (4) as $V_{FX} = -0.080917$ and $V_{FY} = 0.0587795$. Links FC and DC constitute an RRR Assur group, and their kinematic parameters can be calculated by the formulae in Appendix 2 when A, B, and C are replaced by F, C, and D respectively in Fig. 2. The calculated kinematic parameters of links FC and DC corresponding to $I = 800$ are $\theta_{FC} = -104.789^\circ$, $\omega_{FC} = 0.000359992$, $\theta_{DC} = 164.621^\circ$, and $\omega_{DC} = -0.000176495$. The kinematic parameters of point B on link FC are calculated from equation (1) as $X_B = -269.58$ mm, $Y_B = 80.7212$ mm, $V_{BX} = -0.0286919$, and $V_{BY} = 0.0449944$. From equation (2), the kinematic parameters of point E on link DC are calculated as $X_E = 10.8823$, $V_{EX} = -0.00538271$, $Y_E = -110.498$, and $V_{EY} = -0.0195702$.

Links OA and BA constitute another RRR Assur group. Since $I_{\min} = 416 < I = 800 < I_{\max} = 2442$, the three revolutes O, A, and B are arranged anticlockwise such that, when using the formulae, the three letters A, B, and C are replaced by O, A, and B respectively. The calculated kinematic parameters of link OA corresponding to $I = 800$ are $\theta_{OA} = 28.1442^\circ$ and $\omega_{OA} = 0.00168933$.

Since $\theta_{OA0} = -51.9412^\circ$ when $I = 0$ and $\theta_{OA} = 28.1442^\circ$ when $I = 800$, the angular displacement θ of link OA during this period is $\theta = \theta_{OA} - \theta_{OA0} = 28.1442^\circ - (-51.9412^\circ) = 80.0853^\circ$. Since the rotating coordinate system X_1OY_1 is fixed to the link OA, the angle between the rotating coordinate system X_1OY_1 and stationary coordinate system XOY corresponding to index $I = 800$ is also equal to $\theta = 80.0853^\circ$, as shown in Fig. 2.

The coordinates ($X_E = 10.8823$, $Y_E = -110.498$) calculated above are the X and Y coordinates of point E relative to the stationary coordinate system XOY when $I = 800$. Now, using the coordinate transformation formulae (6), the coordinates of point E relative to the rotating coordinate system X_1OY_1 corresponding to $I = 800$ can be calculated as follows

$$\begin{aligned} X_{E1} &= X_E \cos\theta + Y_E \sin\theta \\ &= 10.8823\cos 80.0853^\circ - 110.498\sin 80.0853^\circ \\ &= -106.9738 \\ Y_{E1} &= -X_E \sin\theta + Y_E \cos\theta \\ &= -10.8823\sin 80.0853^\circ - 110.498\cos 80.0853^\circ \\ &= -29.7455 \end{aligned}$$

According to equation (7)

$$\begin{aligned} V_{EX1} &= V_{EX}\cos\theta - \omega_{OA}X_E\sin\theta + V_{EY}\sin\theta + \omega_{OA}Y_E\cos\theta \\ &= -0.00538271\cos 80.0853^\circ - 0.00168933 \\ &\quad \times 10.8823\sin 80.0853^\circ - 0.0195702\sin 80.0853^\circ \\ &\quad - 0.00168933 \times 110.498\cos 80.0853^\circ \\ &= -0.0704547 \end{aligned}$$

$$\begin{aligned} V_{EY1} &= -V_{EX}\sin\theta - \omega_{OA}X_E\cos\theta + V_{EY}\cos\theta \\ &\quad - \omega_{OA}Y_E\sin\theta \\ &= 0.00538271\sin 80.0853^\circ - 0.00168933 \\ &\quad \times 10.8823\cos 80.0853^\circ \\ &\quad - 0.0195702\cos 80.0853^\circ \\ &\quad + 0.00168933 \times 110.498\sin 80.0853^\circ \\ &= 0.182647 \end{aligned}$$

$$\begin{aligned} V_{E1} &= \sqrt{V_{EX1}^2 + V_{EY1}^2} = \sqrt{(-0.0704547)^2 + 0.182647^2} \\ &= 0.195764632 \end{aligned}$$

Then, using equation (8), the coordinates (X_{T1}, Y_{T1}) of the cam contour relative to the rotating coordinate system X_1OY_1 corresponding to $I = 800$ can be calculated as

$$\begin{aligned} X_{T1} &= X_{E1} + r_r\frac{V_{EY1}}{V_{E1}} \\ &= -106.9738 + 20\frac{0.182647}{0.195764632} \\ &= -88.3199 \\ Y_{T1} &= Y_{E1} - r_r\frac{V_{EX1}}{V_{E1}} \\ &= -29.7455 - 20\frac{-0.0704547}{0.195764632} \\ &= -22.5475 \end{aligned}$$

These coordinates (-88.3199, -22.5475) are also those of the tangent point T_{800} between the roller and the cam contour corresponding to $I = 800$ relative to the stationary coordinate system XOY when the mechanism is at the starting position (see Fig. 1). Note also that, relative to the cam, point E_{800} in Fig. 1 corresponds to point E in Fig. 2.

9 GRAPHICAL SYNTHESIS

The process of graphically generating the pitch curve and the cam contour is closely parallel to the analytical method. Certain parameters (X_O, Y_O, X_D, Y_D, X_G, Y_G, L_{FC}, L_{DC}, L_{FB}, and L_{DE} and the radius of the roller, r_r), are initially determined according to the overall arrangement of the mechanism by the designer. The process is as follows. For each point F on the locus, construct links FC, DC, and point B. Measure all distances L_{OB} between the fixed pivot O and point B. (The distance between the fixed pivot O and point B_0 in Fig. 1 is $(L_{OB})_0$, corresponding to $I = 0$, while the distance between the fixed pivot O and point B in Fig. 2 is $(L_{OB})_{800}$, corresponding to $I = 800$.) From all values of $(L_{OB})_i$, find the maximum distance $(L_{OB})_{\max}$ and the minimum distance $(L_{OB})_{\min}$ and the corresponding key points $F_{\max}$ and $F_{\min}$.

Having determined the location of point B, the lengths of links BA and OA may then be calculated using equation (5), and the initial positions of link OA_0 and the roller centre E_0 corresponding to $I = 0$ can be constructed (Fig. 1). Subsequent points F on the locus are treated similarly for the graphical construction of links FC, DC, DE, BA, and OA. Care must be taken when constructing the RRR Assur group formed by links BA and OA to take account of the correct closure. For those points between $F_{\min}$ and $F_{\max}$, the revolutes O, A, and B must be arranged in anticlockwise order (see Fig. 2) while, for the

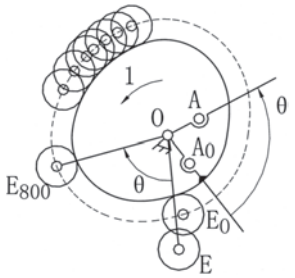

Fig. 4 Graphical synthesis

remaining points, it is the revolutes B, A, and O that must be arranged anticlockwise (see Fig. 1).

To summarize, then, take, for example, point F ($I = 800$) in Fig. 2. The positions of links FC and DC are determined graphically first. The positions of point B on link FC and of point E on link DC are then simply found. Finally, the positions of links OA and BA are constructed, as shown in Fig. 2. The angle θ between link OA_0 and the link OA (see Fig. 4) is then measured. Rotating point E clockwise around the fixed pivot O through angle θ will determine point E_{800}, as shown in Fig. 4.

The pitch curve of the cam is obtained by drawing a smooth curve through all points E_i, as shown by the dashed curve (Fig. 4). Following a standard procedure, roller circles with roller radius r_r are drawn centred on the pitch curve. Since the roller is tangent to the cam contour at all times, the cam contour is the envelope of the family of the roller circles, and the cam contour is found by drawing a smooth curve tangent to all roller circles, to develop the solid curve shown in Fig. 4.

10 CONCLUSION

This paper introduces an analytical method for the design of the cam contour of combined cam–linkage mechanisms with an oscillating roller follower to trace precisely a predetermined locus. Based on the concept of Assur groups, the method has the merits of accuracy and convenience. As an aid to the design, the composition of the mechanism during kinematic analysis is approached differently from that during the design. Since the assembly mode of an Assur group may change during design, this paper describes in detail how to deal with this potential problem.

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APPENDIX 1

Notation

a_{AX}, a_{AY}	linear acceleration of point A in the X and Y directions
r_r	radius of roller
I	index
L_{AB}	distance between points A and B
N	maximum value of I
V_{AX}, V_{AY}	linear velocity of point A in the X and Y directions
X_A, Y_A	X and Y coordinates of point A
θ_{AB}	angular position of link AB
φ_{AB}	angular position of link AB
ω_{AB}	angular velocity of link AB

APPENDIX 2

Concept of Assur groups

The great majority of linkage mechanisms are composed of a series of simple 'building blocks' which were classified in groups by Assur (see, for example, reference [**3**]). Each Assur group is composed of links connected, for example, by revolute or sliding joints. More complicated mechanisms may accordingly be analysed by the sequential analysis of appropriate Assur groups that they contain. In the example examined here, the Assur groups are taken in series either from the input end (the cam) or the output end (the locus of point F). For the purposes of this example, it is necessary to examine only one of the simplest, the all-revolute RRR Assur group.

Kinematic analysis of the RRR Assur group

In the RRR group shown in Fig. 5, the kinematic parameters (X_A, Y_A, V_{AX}, V_{AY}, a_{AX}, a_{AY}, and X_C, Y_C, V_{CX}, V_{CY}, a_{CX}, a_{CY}) of the two outer points A and C and the lengths L_{AB} and L_{CB} of the two links AB and CB are assumed to be known. The kinematic parameters (θ_{AB}, ω_{AB}, ε_{AB}, and θ_{CB}, ω_{CB}, ε_{CB}) of links AB and CB can then be calculated as follows.

When X_A, Y_A, X_C, Y_C, L_{AB}, and L_{CB} are known, there are two assembly modes for this group, as shown in Fig. 5, one in solid lines and the other in dashed lines: on link CB, $X_B = X_C + L_{CB}\cos\theta_{CB}$ and $Y_B = Y_C + L_{CB}\sin\theta_{CB}$; on link AB, $X_B = X_A + L_{AB}\cos\theta_{AB}$

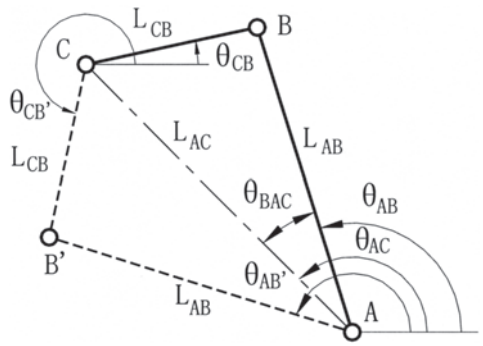

Fig. 5 RRR Assur group

and $Y_B = Y_A + L_{AB} \sin \theta_{AB}$. Combining these two sets of equations yields

$$X_C + L_{CB} \cos \theta_{CB} = X_A + L_{AB} \cos \theta_{AB}$$
$$Y_C + L_{CB} \sin \theta_{CB} = Y_A + L_{AB} \sin \theta_{AB} \tag{10}$$

There are two unknowns, θ_{AB} and θ_{CB}, in this set of equations and, since there are two assembly modes for this group; there will be two sets of solutions. Although some mathematical skill can be used to solve the above trigonometric non-linear equations to obtain two sets of formulae for θ_{AB} and θ_{CB}, the calculation process is tedious and the formulae derived would be very complicated. Furthermore, it is difficult to judge which set of formulae corresponds to a specific assembly mode. The following steps outline a simple method to overcome this difficulty.

Step 1.

$$L_{AC} = \sqrt{(X_C - X_A)^2 + (Y_C - Y_A)^2}$$

Step 2.

$$\cos \theta_{AC} = \frac{X_C - X_A}{L_{AC}} \quad \text{and} \quad \sin \theta_{AC} = \frac{Y_C - Y_A}{L_{AC}}$$

The derived formulae may be used for any combinations of the positions of points A and C. Note that $\sin \theta_{AC}$ may not be equal to $\sqrt{(1 - (\cos \theta_{AC})^2)}$ since $\sin \theta_{AC}$ may be negative. The magnitude of θ_{AC} can be calculated according to the values of both $\cos \theta_{AC}$ and $\sin \theta_{AC}$ using, for example, the ANGLE function in TRUE BASIC. Note again that θ may not be equal to ATN($\sin\theta/\cos\theta$) since θ may be greater than $\pi/2$ and less than $3\pi/2$, whereas the value obtained from the ATN function lies between $-\pi/2$ and $+\pi/2$. (Note that θ_{AC} is the angle from the positive X axis to the vector AC, measured positive anticlockwise. Therefore, in this notation, θ_{AC} differs from θ_{CA} by 180°.)

Step 3.

$$\cos \theta_{BAC} = \frac{L_{AB}^2 + L_{AC}^2 - L_{CB}^2}{2 L_{AB} L_{AC}}$$

Since $180^\circ > \theta_{BAC} > 0^\circ$, $\sin \theta_{BAC} = \sqrt{(1 - (\cos \theta_{BAC})^2)}$. If $L_{AC} > (L_{AB} + L_{CB})$, then $\cos \theta_{BAC} > 1$. This means that the distance L_{AC} between the two outer points is greater than the sum of L_{AB} and L_{CB}. If $L_{AC} < |L_{AB} - L_{CB}|$, then $\cos \theta_{BAC} < (-1)$ and the distance L_{AC} is less than the difference between L_{AB} and L_{CB}. In these cases, the RRR dyad cannot be assembled. The calculation of $\sqrt{(1 - (\cos \theta_{BAC})^2)}$ will fail and computation will be halted.

Step 4. As mentioned above, there are two assembly modes for the RRR group. The following formulae consider only the assembly mode shown by solid lines where $\theta_{AB} = \theta_{AC} - \theta_{BAC}$. During motion of the mechanism, the assembly mode does not change as a result of change in position.

Step 5.

$$X_B = X_A + L_{AB} \cos \theta_{AB} \quad \text{and} \quad Y_B = Y_A + L_{AB} \sin \theta_{AB}$$

Step 6.

$$\cos \theta_{CB} = \frac{X_B - X_C}{L_{CB}} \quad \text{and} \quad \sin \theta_{CB} = \frac{Y_B - Y_C}{L_{CB}}$$

The magnitude of θ_{CB} can be calculated according to the values of both $\cos \theta_{CB}$ and $\sin \theta_{CB}$ by, for instance, the ANGLE function in TRUE BASIC.

Velocity analysis can be progressed only after the position analysis has been completed. The angular velocities, ω_{AB} and ω_{CB}, of links AB and CB can be found by differentiating equations (10) with respect to time

$$V_{CX} - L_{CB} \sin \theta_{CB} \omega_{CB} = V_{AX} - L_{AB} \sin \theta_{AB} \omega_{AB}$$
$$V_{CY} + L_{CB} \cos \theta_{CB} \omega_{CB} = V_{AY} + L_{AB} \cos \theta_{AB} \omega_{AB} \tag{11}$$

Both velocity and acceleration equations of a grade II Assur group are dualistic linear equations. The explicit expressions for ω_{AB} and ω_{CB} can be found easily by solving the two equations simultaneously as follows

$$\omega_{AB} = \frac{(V_{AX} - V_{CX}) \cos \theta_{CB} + (V_{AY} - V_{CY}) \sin \theta_{CB}}{L_{AB} \sin (\theta_{AB} - \theta_{CB})}$$
$$\omega_{CB} = \frac{(V_{AX} - V_{CX}) \cos \theta_{AB} + (V_{AY} - V_{CY}) \sin \theta_{AB}}{L_{CB} \sin (\theta_{AB} - \theta_{CB})}$$

By differentiating equations (11) with respect to time, another set of dualistic linear equations with two unknowns, i.e. the angular accelerations ε_{AB} and ε_{CB} of the links AB and CB, is derived. Explicit expressions for ε_{AB} and ε_{CB} can be found by solving the two equations simultaneously.

Since the formulae derived above consider only the assembly mode shown in solid lines in Fig. 5, care should be taken when using these formulae. The sequence of the three revolute letters A, B, and C must be arranged in anticlockwise order, where A and B are outer revolutes.