On the Performance of Integrated Interleaving Coding Schemes[1]

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I. Introduction

The correction of errors in bursty channels commonly utilizes interleaved blocks of codewords. The Integrated Interleaving coding schemes [1] uses nonuniform redundancy in the interleaves that can be adjusted on-the-fly according to the error statistics of the channel. This gives greater flexibility in the designing of the code and achieves higher reliability for the same code rate. We give a different and substantially tighter theoretical evaluation on the performance of the Integrated Interleaving code than [1]. Based on the tighter theoretical evaluation, we formulate a closed-form analytical expression for its performance in terms of the word-error probability. We also propose a modified decoding algorithm of the Integrated Interleaving codes which achieves a significant gain in performance. We optimize the performance of this scheme within a given class of codes.

II. Main Results

Let $\{C_i\}_{i=1}^2$ be $[n_i, k_i, d_i]$ linear codes over the Finite Field $\mathbb{F}_q$ such that $C_1 \supset C_2$ and $d_2 > d_1$. We define the Integrated Interleaving code [1] C as the set of $m \times n$ matrices whose i^{th} row, denoted by $\mathbf{c}_i$, satisfies

$$C = \{\mathbf{C} \in \mathbb{F}_q^{mn} : \mathbf{c}_i \in C_1, i = 0, ..., m-1 \text{ and } \sum_{i=0}^{m-1} \alpha^{bi}\mathbf{c}_i \in C_2, b = 0, 1, ..., B-1\}$$

where α is a primitive element of $\mathbb{F}_q$, m is the number of rows in the codeword matrix, and B satisfies $B < m \leq q-1$. C has minimum distance $d = \min((B+1)d_1, d_2)$ [1].

We present below a bound on the performance of the Integrated Interleaving code using the decoding algorithm in [1]. This bound is substantially tighter than the one given in [1] and is proved in [2]. Note that in the decoding process, the decoding algorithm in [1] perceives the error weight in an interleave to be τ, which might be different than the true error weight in an interleave, t.

Theorem. *Denote by t_i the weight of the error in the i^{th} row of the codeword matrix and by τ_i the perceived error weight in the i^{th} row of the codeword matrix by the decoder. Let $\tilde{I} = \{i : \frac{d_1}{2} < t_i \leq \frac{d}{2}$ and $\tau_i < \frac{d_1}{2}\}$. Let $\tilde{\tau} = min\{\tau_i : i \in \tilde{I}\}$. Denote by f the number of rows with $\frac{d_1}{2} < t_i \leq \frac{d}{2}$ and by k the number of rows with $\tilde{\tau} \leq t_i \leq \frac{d_1}{2}$. The code C can correct any combination of errors if conditions $t_i < d/2\ \forall i$ and $k + f \leq B$ are satisfied.*

We modify the decoding algorithm in [1] to include an erasure step and obtain a significant improvement in performance. We take advantage of the fact that in bursty channels, errors in the interleave appear in columns of the codeword matrix near each other. The second-level code, C_1, can be used to identify the columns affected by the burst, which can then be used by the first-level code, C_2, as erasure locations. The modified decoding algorithm is presented in detail in [2], and the following theorem gives the performance of the Integrated Interleaving code using the modified decoding scheme and is proved in [2].

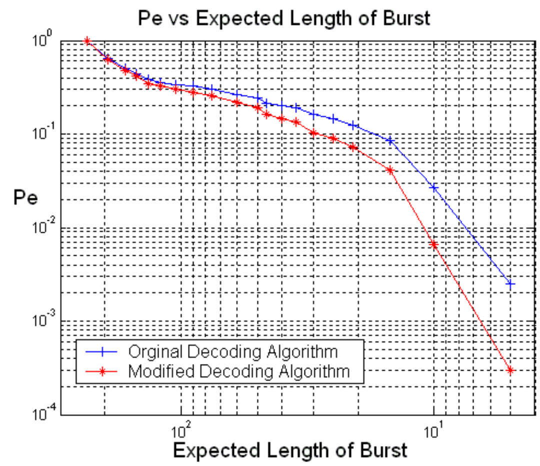

Figure 1: Comparison of the decoding algorithm given in [1] and the modified decoding algorithm. $B = 1$, $m = 3$, rate=0.85, $n = 127$, P_B=0.9, $P_G = 0.01$.

Theorem. *Let $\bar{t}$ be the number of columns of $\mathbf{C}$ in error, and $\bar{t_c}$ be the number of columns where the errors within the columns cancel out when summed. The modified decoding algorithm corrects any combination of errors provided that either of the following condition holds: $t_i < d_1/2$ for all i or $\bar{t} \leq d_1 - 1$ and $\bar{t_c} < d_1 - \bar{t}$.*

To analyze the performance of the Integrated Interleaved codes in bursty channels, we model bursty channels using the Gilbert-Elliot model, which is a discrete, first order, stationary Markov chain with two states. The probability of symbol error in the "bad" and "good" states are denoted P_B and P_G respectively. Figure 1 gives an comparison of the probability of word-error (P_e) between the modified decoding algorithm and the decoding algorithm in [1] for different expected length of burst given that a burst occurred. The figure shows that the modified scheme gives a significant improvement in performance. For the same rate of the code, different values of d_2 result in different performance levels. We use asymptotic expressions to change the problem of finding an optimal d_2 into a continuous setup, and find the optimal d_2 for a given set of parameters.

Future directions of research include the adaptation of the Integrated Interleaved code to be implemented on factor graphs. Methods of implementing, encoding, and decoding the code, as well as the types of graphs that can be implemented on are to be studied.

References

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[2] X. Tang, R. Koetter, "On the performance of Integrated Interleaving Coding Schemes", Asilomar Conf. on Signals, Systems, and Computers, 2002.

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