Investigating Different Effects on Linear Channel Compensated MMSE Receiver Using Kalman Filter Based Channel Estimation in Time Varying Environment

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Abstract – A recently proposed linear MMSE receiver based on Kalman Filter Channel Estimation gives excellent performance in highly time varying environment. In this paper, the accurate channel tracking capability of this receiver is discussed. Also, various operation modes of this receiver are investigated. The BER performance achieved by employing training with Decision Directed (DD) approach is found to give the best results. The performance of all operation modes improves when the rate of channel variation is reduced. The receiver performance degrades gracefully with the increase in loading and it operates satisfactorily in the presence of increasing Doppler shift.

I. INTRODUCTION

The Code Division Multiple Access (CDMA) technique for digital cellular and personal communication has emerged superior to all other digital multiple access techniques in terms of signal quality, security, power consumption and reliability [1]. The CDMA technique is based on the principle of Multi-User Detection (MUD). Various users transmit their information simultaneously on a common channel utilizing the complete available bandwidth resources of the system, each using a particular code sequence, known as signatures. The receiver is able to optimally demodulate the received signal by using a bank of matched filters followed by threshold detectors if the signatures of the users are completely orthogonal. However, due to various channel impairments and multipath effects, the users' signatures do not remain orthogonal and result in increased Multiple Access Interference (MAI). Various receiver structures have been proposed to achieve sub-optimal demodulation. A historical perspective of their developments is presented in [2]. However, all these receiver designs assume a fixed and time-invariant channel response during transmission. Whereas the channel can change quite rapidly due to Doppler shift caused by movement of the vehicles. This time varying channel has been modeled successfully using Kalman filter estimation in the new receiver design [2]. This receiver, which can be called as linear Channel Compensated MMSE (CCMMSE) receiver, quickly adapts to the time varying channel conditions and results in an improved BER performance and faster convergence. The convergence rate of this receiver has been found to be twice as fast as LMS based linear adaptive receiver. In this paper, we verify the accurate channel tracking capability of this receiver, investigate and compare different modes of operation of this receiver and check the effects of system loading and Doppler shift change on this receiver.

Organization of the rest of our paper is as follows: In section II, we discuss the system model and Kalman filter based channel estimation algorithm of linear CCMMSE receiver. In section III, different modes of operation of this receiver are presented. In section IV and V, simulation results and conclusions are given.

II. SYSTEM MODEL

We consider the Discrete Time Baseband system model in time varying conditions as shown in Figure 1.

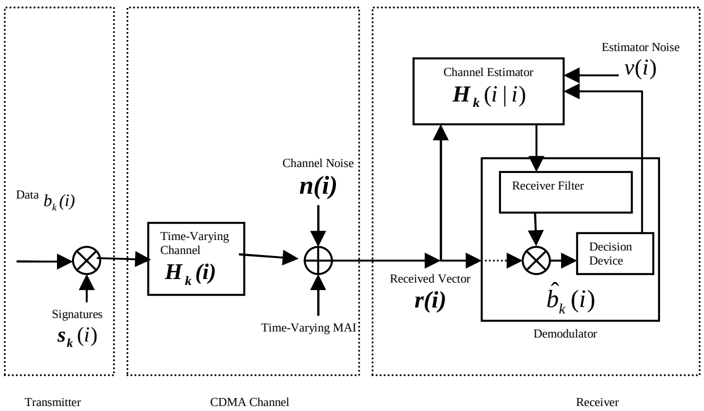

Figure 1: Discrete Time Baseband model

In a synchronous DS-CDMA system with K users and processing gain N, the received vector is given as [3],

$$\boldsymbol{r}(i) = \sum_{k=1}^{K} A_k \boldsymbol{H}_k(i) b_k(i) s_k + \boldsymbol{n}(i) \tag{1}$$

where $b_k(i)$ is the ith symbol transmitted with $E[|b_k(i)|^2]=1$, s_k is the signature vector and A_k is the amplitude, all for user k. Also $\boldsymbol{H}_k(i)$ is the channel matrix and $\boldsymbol{n}(i)$ is the white Gaussian noise vector with covariance matrix $\boldsymbol{\sigma_n^2 I}$.

A. Channel Model

The discrete time-varying channel impulse response is assumed as:

$$\boldsymbol{h}_k(i) = [h_{k,1}(i)\ h_{k,2}(i)\ h_{k,3}(i)\ ..\ h_{k,L_k}(i)\ 0\ ..\ 0\ 0] \tag{2}$$

where L_k is the number of paths spaced at chip duration $T_c=T/N$. (T is chip duration and $L_k < N$) [3].

Thus the time-varying channel matrices are given by:

$$\boldsymbol{H}_k(i) = [\boldsymbol{h}_k^{+0}(i)\ \boldsymbol{h}_k^{+1}(i)\ \boldsymbol{h}_k^{+2}(i)\ ..\ ..\ \boldsymbol{h}_k^{+(N-1)}(i)] \quad (3)$$

where $\boldsymbol{h}_k^{+n}(i)$ is $\boldsymbol{h}_k(i)$ shifted down by n positions and vacant positions filled by zeros.

First-order Auto Regressive (AR) model for time-selective channel, proposed in [4], has been used in which

$$h_{k,l}(i) = \alpha\, h_{k,l}(i-1) + v(i) \quad (4)$$

where noise $\boldsymbol{v}(i)$ is zero-mean complex Gaussian having covariance $\boldsymbol{\sigma}_v^2$ and is statistically independent of $\boldsymbol{h}_k(i-1)$ and α is the AR coefficient which accounts for Doppler effects.

Using Jakes' model [4, 5] the expressions for $\boldsymbol{\sigma}_v^2$ and α are:

$$\sigma_v^2 = 1 - |\alpha|^2 \quad (5)$$

$$E[h_k(i)h_k^*(i-1)] = \alpha = \boldsymbol{J}_0(2\pi f_d T_s) \quad (6)$$

where $E[\cdot]$ stands for expectation and $J_0(\cdot)$ is the 0^{th} order Bessel function and f_d is the maximum Doppler shift.

B. Receiver Model

The proposed receiver [2] comprises of a bank of matched filters followed by linear MMSE detectors. Its output can be expressed by:

$$y(i) = \boldsymbol{R}(i)\boldsymbol{A}\boldsymbol{b}(i) + \hat{\boldsymbol{n}}(i) \quad (7)$$

where $\boldsymbol{R}(i) = \sum_{k=1}^{K} s_k^H \boldsymbol{H}_k^H(i)\boldsymbol{H}_k(i)s_k$ is time varying correlation matrix.

$$\boldsymbol{A} = diag(A_1, A_2, A_3, \ldots\ldots, A_K) \quad (8)$$

$$\boldsymbol{b}(i) = (b_1(i), b_2(i), b_3(i), \ldots\ldots, b_K(i)) \quad (9)$$

and $$\hat{\boldsymbol{n}}(i) = \sum_{k=1}^{K} s_k^H \boldsymbol{H}_k^H(i)\boldsymbol{n}(i) \quad (10)$$

Linear MMSE detection is applied to the outputs of the matched filters bank and we get the following decisions:

$$b_k(i) = \mathrm{sgn}([\boldsymbol{R}(i) + \sigma_n^2\boldsymbol{A}^{-2}]^{-1} y(i))_k \quad (11)$$

where sgn represents the signum function and

$$\sigma_n^2\boldsymbol{A}^{-2} = diag\left\{\frac{\sigma_n^2}{A_1^2}, \frac{\sigma_n^2}{A_2^2}, \frac{\sigma_n^2}{A_3^2}, \ldots\ldots, \frac{\sigma_n^2}{A_K^2}\right\} \quad (12)$$

Thus the receiver filter coefficients are

$$\boldsymbol{c}_k(i) = \boldsymbol{M}^{-1}(i)\boldsymbol{H}_k(i)s_k \quad (13)$$

Here $\boldsymbol{M}(i)$, the received covariance matrix or time varying linear transformation used to eliminate the effects of time varying MAI, is given by:

$$\boldsymbol{M}(i) = \boldsymbol{R}(i) + \sigma_n^2\boldsymbol{A}^{-2} \quad (14)$$

C. Channel Tracking

The channel estimation algorithm used in this receiver is based on Kalman filter approach owing to its good tracking properties in a highly time varying environment by employing active minimization of (estimation error) variance. Thus estimation error vector is:

$$h_k(i+1) = h_k(i+1) - \hat{h}_k(i+1) \quad (15)$$

and the estimation error covariance is:

$$\tilde{\boldsymbol{P}}(i+1) := \mathrm{cov}(\tilde{\boldsymbol{h}}_k(i+1)) \quad (16)$$

The state and observation equations of state space model are given as:

$$\boldsymbol{h}_k(i+1) = \mathbf{A}_k\boldsymbol{h}_k(i) + \boldsymbol{v}(i) \quad (17)$$

$$\boldsymbol{r}(i+1) = \sum_{k=1}^{K} A_k\boldsymbol{H}_k(i+1)b_k(i+1)s_k + \boldsymbol{n}(i+1) \quad (18)$$

where $\boldsymbol{A}_k = \mathrm{diag}[\alpha_{k,1}, \alpha_{k,2}, \alpha_{k,3}, \ldots\ldots, \alpha_{k,Lk}, \ldots.., \alpha_{k,N}]$ (19)

and $\boldsymbol{v}(i) = [v_1(i), v_2(i), v_3(i), \ldots\ldots, v_N(i)]$ (20)

D. Channel Estimation Algorithm

The algorithm takes the following steps:

The algorithm is initialized with initial channel estimates $\boldsymbol{h}_k(i\,|\,i)$ with initial covariance matrix

$$\tilde{\boldsymbol{P}}_k(i) = \boldsymbol{I}_{N\,x\,N}^2 - \boldsymbol{h}_k^H(i) * \boldsymbol{h}_k(i) \quad (21)$$

2. Channel Matrix $\boldsymbol{H}_k(i)$ is formed from $\boldsymbol{h}_k(i\,|\,i)$ by using (3).

3. $\hat{b}_k(i)$ is decoded by using (11) of MF-MMSE detector.

4. The estimates of $\boldsymbol{h}_k(i+1\,|\,i)$ and $\tilde{P}_k(i+1\,|\,i)$ are obtained as:

$$\boldsymbol{h}_k(i+1) = \mathbf{A}_k\boldsymbol{h}_k(i) + \boldsymbol{v}(i+1) \quad (22)$$

and

$$\tilde{\boldsymbol{P}}_k(i+1\,|\,i) = \mathbf{A}_k^2\,\tilde{\boldsymbol{P}}_k(i) + \boldsymbol{v}(i+1)\sigma_v^2\boldsymbol{v}^H(i+1) \quad (23)$$

5. The Channel Matrix $\boldsymbol{H}_k(i + \mathrm{I}\,|\,i)$ is formed from $\boldsymbol{h}_k(i+1\,|\,i)$ by using (3).

6. The measurement error vector and its variance is found by:

$$e(i+1) = \boldsymbol{r}(i) - \sum_{k=1}^{K} A_k\boldsymbol{H}_k(i+1\,|\,i)\hat{b}_k(i)s_k \quad (24)$$

and

$$\boldsymbol{R}_e(i+1) = \boldsymbol{F}_k(i) * \tilde{\boldsymbol{P}}_k(i+1\,|\,i) * \boldsymbol{F}_k^H(i) + \sigma_n^2\boldsymbol{I} \quad (25)$$

where $$\boldsymbol{F}_k(i) = \sum_{k=1}^{K} A_k\hat{b}_k(i)s_k \quad (26)$$

7. Kalman Gain is calculated as:

$$\boldsymbol{G}(i+1) = \boldsymbol{R}_e^{-1}(i+1) * \boldsymbol{P}_k(i+1) * \boldsymbol{F}_k(i) \quad (27)$$

8. The estimates made in step 4 are corrected as:

$$h_k(i+1) = h_k(i+1\,|\,i) + \boldsymbol{G}(i+1) * e^H(i+1) \quad (28)$$

and

$$\tilde{\boldsymbol{P}}_k(i+1) = (I_{N\,x\,N} - G(i+1) * \boldsymbol{F}_k^H(i))\,\tilde{\boldsymbol{P}}_k(i+1\,|\,i) \quad (29)$$

9. This real-time corrected $\boldsymbol{H}_k(i+1)$ is used to detect $\hat{b}_k(i+1)$ in the next iteration. Steps 2-8 are repeated to correct channel matrix for successive iterations.

III. Different Modes of Operation

Training sequence is usually employed to provide information of channel to the receiver. It helps the receiver to quickly adapt itself to the channel conditions. The length of training sequence is kept as small as possible as no useful information is being conveyed during the training mode. This length is usually equal to the number of iterations required by the receiver to converge to its optimum value. The linear CCMMSE receiver has been found to converge in about 45 iterations [2]. Thus a training sequence of length 50 is used for subsequent simulations. It is envisaged that this receiver can operate in four different modes.

A. Decision Directed (DD) mode

In this mode, the receiver operates with Decision feedback only. The initial channel coefficients are assumed arbitrarily. The algorithm finds measurement error vector in (24) by using decision made in the current iteration. There exists a chance of error propagation, as subsequent decisions are heavily dependent upon the decisions made in earlier iterations.

B. Training Directed (TD) and DD mode

For overcoming the possibility of error propagation, training can be employed during the convergence time of the algorithm. In this mode, the receiver operates in Training Directed (TD) mode for 50 iterations and then switches to DD mode for rest of simulation.

C. TD and No Estimation (NE) mode

In this mode, the receiver operates in TD mode for 50 iterations and then switches to No Estimation (NE) mode in which the estimated channel response is kept constant at the last optimized value achieved at the end of TD mode. This mode is similar in analogy to linear adaptive MMSE receiver.

D. Repeated TD and NE mode

In this mode, the receiver is provided training after short intervals of actual data. The receiver operates in TD mode for 50 iterations and then switches to NE mode for 200 iterations. This pattern is repeated till the end of the simulation.

IV. SIMULATION RESULTS

The Monte-Carlo simulations are performed for 2000 iterations for each user. The fading coefficients are regenerated for each trial using Jakes' model [5] for simulating the time varying channel response. Each user generates data with $p(-1) = p(+1) = 1/2$. Orthogonal Gold sequences of length 31 are used as spreading codes. The Doppler spread is assumed to be 60 Hz and 4 multipaths are considered for each user. The number of users is 24 for these simulations.

A. Channel Tracking by Linear CCMMSE Receiver

The performance of linear CCMMSE receiver in highly time varying conditions is mainly dependent on successful estimation and tracking of changing channel behavior. The time varying environment depends on various factors out of which multipath fading is the dominant factor. All users have equal power and undergo independent multipath fading with different multipath gains in their channel impulse responses. The channel coefficients are independent Gaussian random variables and the channel vectors are normalized so that the average power is unity, i.e., $E\{\|\boldsymbol{h}_k\|^2\} = 1$. The same model has been used for combining multipaths gains in [4]. The receiver performs the fading channel response through Kalman filter based estimation. This estimated channel response is used for detection in next iteration and so on. Figure 2 shows the actual and estimated channel responses for each multipath. As is evident from the graph, the receiver is successfully able to closely track the fast fading channel impulse response of each multipath individually and hence results in better BER performance than other receiver types.

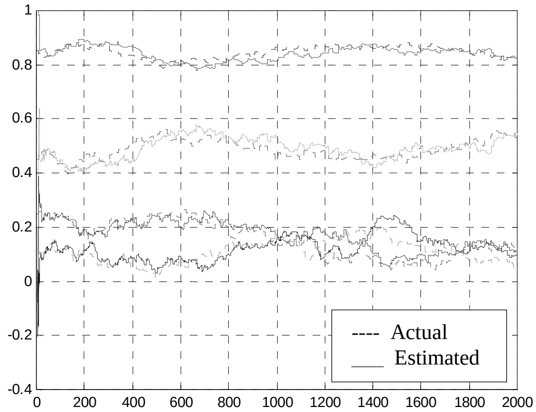

Figure 2: Channel tracking by linear CCMMSE receiver.

B. Comparison of Different Operation Modes of linear CCMMSE Receiver

The different modes of operation of linear CCMMSE receiver are evaluated in highly time varying conditions in which the channel conditions are assumed to change after every 8 iterations. The BER performance results for all operation modes are shown in Figure 3. The following observations are made from these results: -

1. The BER performance achieved by 'TD and DD mode' is better than all other modes.

2. In 'TD and NE mode', the receiver performs poorly.

3. This performance improves marginally when the combination of training mode and constant CE mode is repeated after short intervals.

The above-mentioned observations indicate that the changing behavior of the channel affects the receiver performance severely whenever channel estimation and compensation is not performed. This situation improves when the training is repeated after short intervals. This means that the channel information provided to the receiver by the training sequence in a highly time varying environment is of great help since the channel response is continuously changing. It implies that the receiver continuously needs to perform channel estimation and it cannot use the constant channel response achieved at the end of training. The training is, therefore, required in this receiver design when operating in highly fading conditions. Moreover, when training is not employed, there are chances of error propagation as the receiver performance is highly dependent on initial decisions.

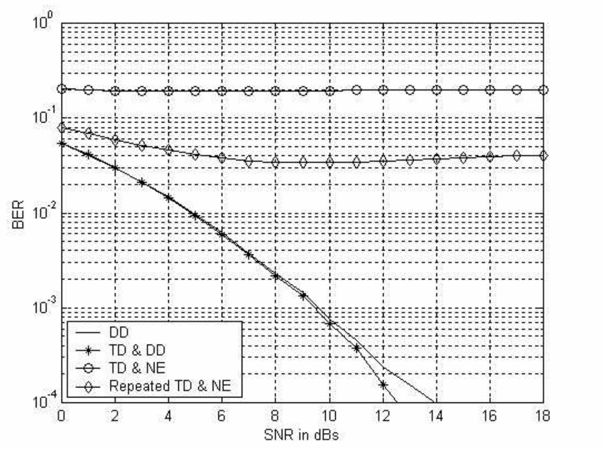

Figure 3: BER performance comparison of Different operation modes of Coded linear CCMMSE Receiver when channel varies after 8 iterations.

In order to further verify our observations made above, the comparison between all operation modes of this receiver is again made by reducing the channel variations by a factor of 2. This means that the channel is assumed to remain constant during 16 iterations. The results are plotted in Figure 4. The performance of all the operating modes has improved when the channel variation is not considered as severe as before.

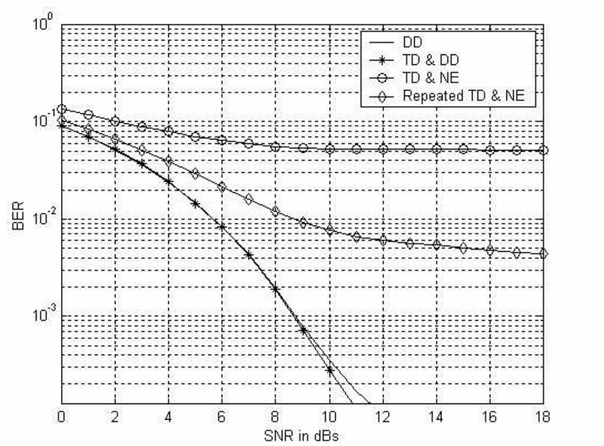

Figure 4: BER performance comparison of Different operation modes of Coded linear CCMMSE Receiver when channel varies after 16 iterations.

C. Effect of Loading

In order to find out the effect of increase in the number of users on performance of linear CCMMSE receiver, the Monte-Carlo simulations are performed for populations of single user, 8, 24 and 31 users respectively. The BER plots are given in Figure 5 for Signal to Noise ratios (E_b / N_o) of 0 dB to 30 dB. It is evident from the plots that the performance of linear CCMMSE receiver degrades gracefully in the presence of time varying channel as the load of the system increases from single user to its maximum capacity. This is due to the fact that as the number of users increases, the time varying Multiple Access Interference (MAI) caused by other users increases. This MAI is given by [2]:

$$\sum_{j\neq k}^{K} s_j^H \boldsymbol{H}_j^H(i)\boldsymbol{H}_j(i)s_jA_jb_j(i) \qquad (30)$$

This performance is found to be quite superior as compared to the degradation observed in other receiver designs.

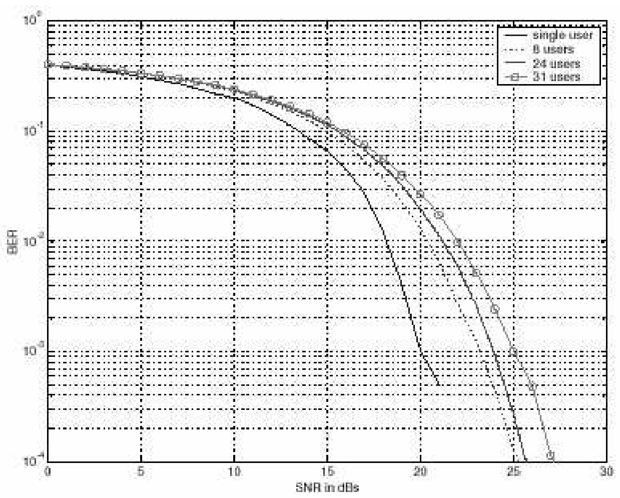

Figure 5: Effect of Loading on BER performance of Uncoded linear CCMMSE receiver.

D. Effect of Change of Doppler Shift

The main cause of time varying channel is fading due to the Doppler shift caused by the speed of the vehicles. The BER performance of linear CCMMSE receiver has been simulated for Doppler shifts of 30 Hz, 60 Hz and 90 Hz. A total of 1000 iterations are performed with 24 users. The BER plots are shown in Figure 5. With the increase in the Doppler shift, the channel variation gets more severe. We see that this receiver is still able to track the channel effectively and provides with consistent BER performance.

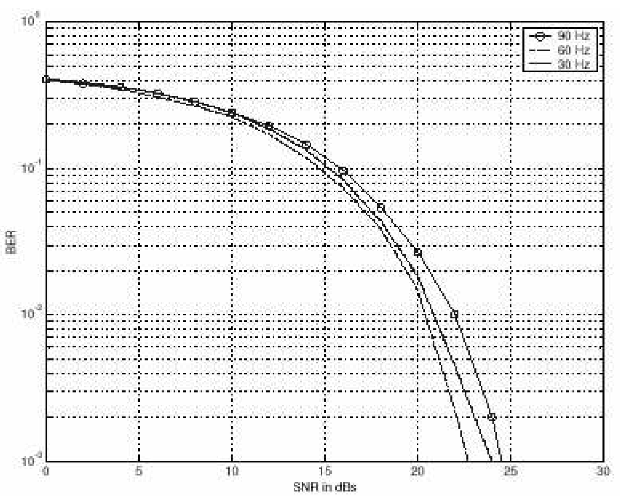

Figure 6: Effect of change of Doppler Shift on BER performance of Uncoded linear CCMMSE receiver.

V. CONCLUSIONS

The channel tracking capability and performance of linear CCMMSE receiver using Kalman filter based channel estimation has been investigated in highly time varying conditions. The comparison of different modes of operation of this receiver shows that its performance is best when training sequence is used initially and then it operates in Decision Directed mode. Also, it is evident that the performance of linear CCMMSE receiver degrades gracefully with the increase in system loading. Moreover, this receiver is able to provide consistent performance successfully in the presence of increasing Doppler shift.

VI. REFERENCES

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