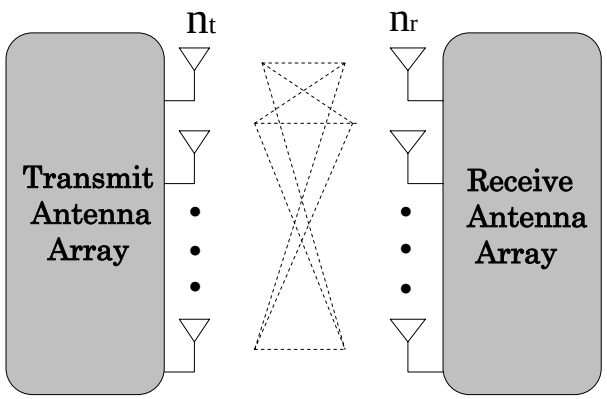
Distributed Space-Time Codes for Cooperative Communication

P. Vijay Kumar
ECE, IISc, Bangalore

DRDO-IISc Workshop on Mathematical Engineering

June 9, 2007

Point-to-Point Communication Channel Model



- operate within channel coherence time
- $Y = HX + W$ channel model
- $H = [h_{ij}]$ represents channel fading coefficients,
- Receiver knows H
- fading coefficients assumed to be Rayleigh

Diversity-Multiplexing Gain (D-MG) Tradeoff

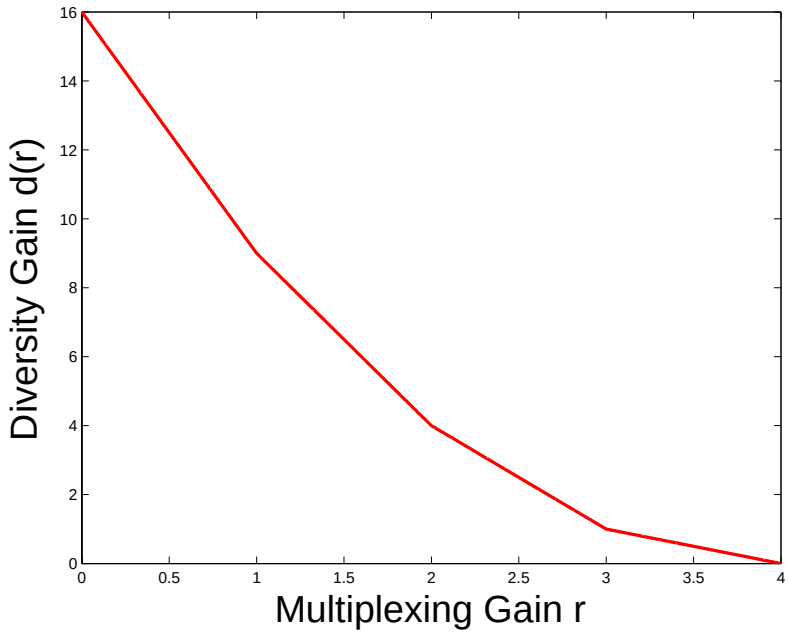
- Multiplexing gain r defined by :

$$\text{rate} \;=\; r \log \text{SNR} \;\; \text{bits/channel use}$$

- Diversity gain $d(r)$ defined by

$$P_e(r) \;\doteq\; \text{SNR}^{-d(r)}, \quad \text{as SNR} \to \infty \quad .$$

Diversity-Multiplexing Gain (DMG) Tradeoff



- L. Zheng and D. Tse (May 2003) introduced notion of DMG tradeoff
- Elia, Kumar, Pawar, PVK and Lu (Sep. 2006) First, explicit, general and optimal code construction (based on cyclic division algebras)

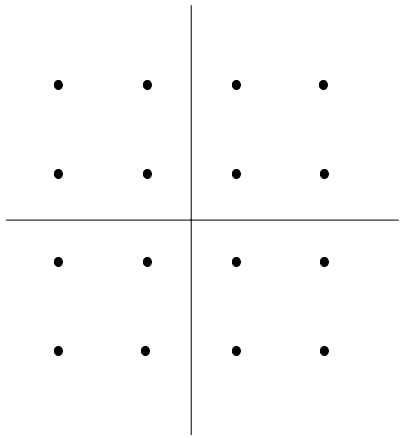
References

- L. Zheng and D. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels," *IEEE Trans. Info. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.
- H. Yao and G. W. Wornell, "Achieving the full MIMO diversity-multiplexing frontier with rotation-based spacetime codes," in *Proc. Allerton Conference*, Oct. 2003.
- H. El Gamal, G. Caire and M. O. Damen, "Lattice Coding and Decoding Achieve the Optimal DiversityMultiplexing Tradeoff of MIMO Channels," *IEEE Trans.Inform Theory*, June 2004.
- P. Elia, K.Raj Kumar, S. A. Pawar, PVK, Hsiao-feng Lu, "Explicit, Minimum-Delay Space-Time Codes Achieving the D-MG Tradeoff ", *IEEE Trans.Inform Theory*, Sep. 2006.

The Cyclic Division Algebra Construction

Code matrices are of the form

$$X = \begin{bmatrix} \ell_0 & \gamma\sigma(\ell_2) & \gamma\sigma^2(\ell_1) \\ \ell_1 & \sigma(\ell_0) & \gamma\sigma^2(\ell_2) \\ \ell_2 & \sigma(\ell_1) & \sigma^2(\ell_0) \end{bmatrix}$$



where the ℓ_i are derived from

$$\overbrace{\begin{bmatrix} \ell_0 & \ell_1 & \ell_2 \end{bmatrix}}^{\text{information-bearing symbols}} = \overbrace{\begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}}^{\text{basis vectors}}$$

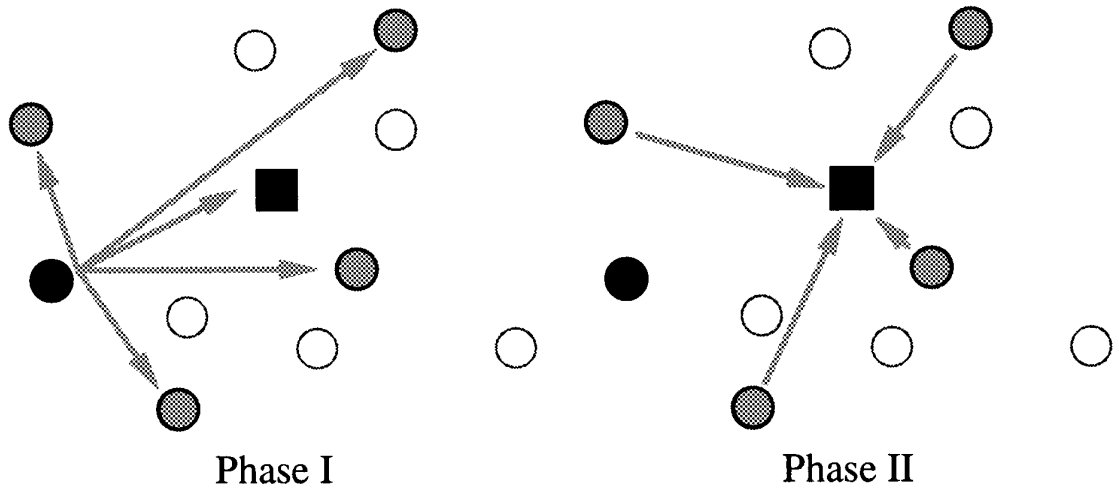
$$\Uparrow$$

$$\overbrace{\begin{bmatrix} \ell_{0,1} & \ell_{1,1} & \ell_{2,1} \\ \ell_{0,2} & \ell_{1,2} & \ell_{2,2} \\ \ell_{0,3} & \ell_{1,3} & \ell_{2,3} \end{bmatrix}}^{\text{QAM message symbols}}$$

References

- B. A. Sethuraman, B. Sundar Rajan, and V. Shashidhar, "Full-Diversity, High-Rate, Space–Time Block Codes From Division Algebras," *IEEE Trans. Info. Theory*, vol. 49, no. 10, pp. 2596–2616, Oct. 2003.
- J. -C. Belfiore and G. Rekaya,"Quaternionic Lattices for Space-Time Coding," in *Proc. IEEE Information Theory Workshop*, Paris, France, Mar-Apr. 2003, pp. 267270.
- P. Elia, K. Raj Kumar, S. A. Pawar, P. Vijay Kumar, and H-F. Lu, "Explicit, Minimum-Delay Space-Time Codes Achieving The Diversity-Multiplexing Gain Tradeoff," *IEEE Trans. Info. Theory*, vol. 52, no. 9, pp. 3869–3884, Sept 2006.
- J.-C. Belfiore, G. Rekaya and E.Viterbo, "The Golden code: a 2×2 full-rate space-time code with non-vanishing determinants," *IEEE Trans. Inform. Theory*, vol.51, No. 4, April 2005, pp. 1432-1436.
- P. Elia, B. Sethuraman, and P. Vijay Kumar, "Perfect Space-Time Codes with Minimum and Non-Minimum Delay for Any Number of Antennas," submitted to *IEEE Trans. Info. Theory*, Dec. 2005.

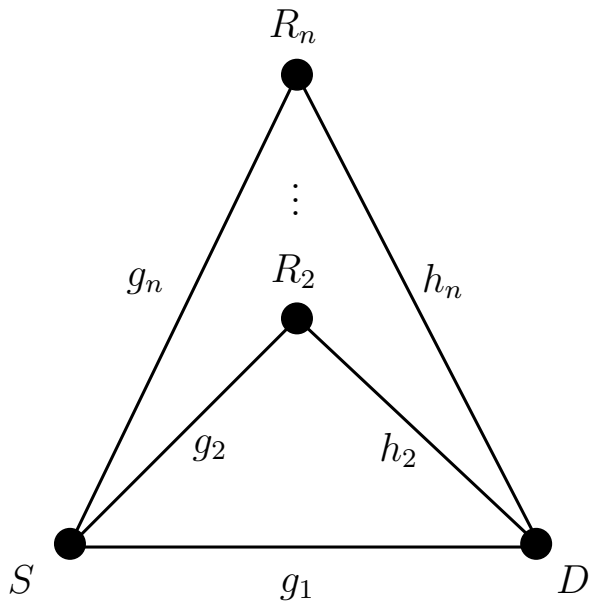
Cooperative Relay Communication Concept



- Phase I: Broadcast
- Phase II: Cooperation

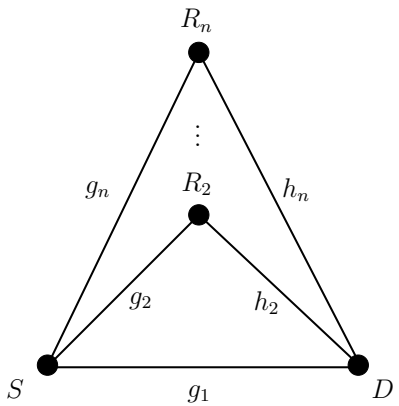
Figure from: J. N. Laneman and G. W. Wornell, "Distributed Space–Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. Info. Theory*, pp. 2415–2425, Oct. 2003.

Notation



- S, D are source and destination nodes
- $\{R_i\}$ are the cooperating relays
- $\{g_i, h_i\}$ fading coefficients as shown

Assumptions



- operate within channel coherence time
- all nodes have a single transmit and receive antenna,
- all nodes operate in half-duplex mode
- Rayleigh fading
- Receiver knows its channel-fading coefficient

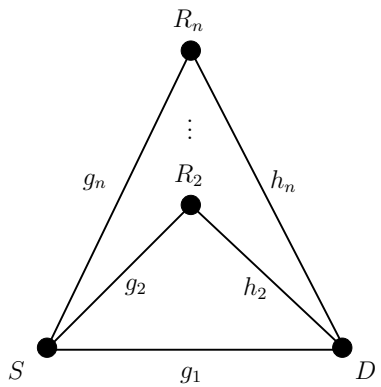
Orthogonal and Non-orthogonal Protocols

	Broadcast (Phase I)				Cooperation (Phase 2)			
Time	1	2	$\cdots$	p	1	2	$\cdots$	q
Source	Tx	Tx	$\cdots$	Tx				
Relays					Tx	Tx	$\cdots$	Tx

(Tx indicates transmission)

- The protocol is orthogonal if source does not transmit in cooperation phase (as above)
- non-orthogonal otherwise

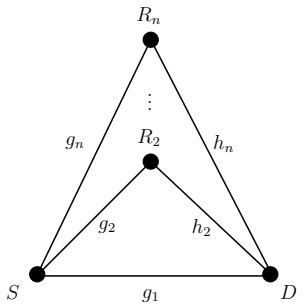
Decode-and-forward versus Amplify-and-forward



	Orthogonal	Non-orthogonal
Amplify & Forward	OAF	NAF
Decode & Forward	OSDF	NSDF

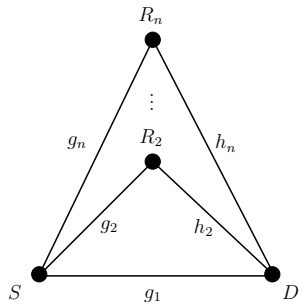
OSDF = Orthogonal Selection Decode and Forward
NSDF = Non-Orthogonal Selection Decode and Forward

Decode-and-Forward Protocol Variations



- In OSDF, a relay decodes only when the corresponding source-channel is good enough
- FDF (fixed decode-and-forward) has poorer performance
- in Dynamic Decode -and-forward (DDF), a relay transmits only after it has received enough information from the source

Equivalent MISO Channel (AF)

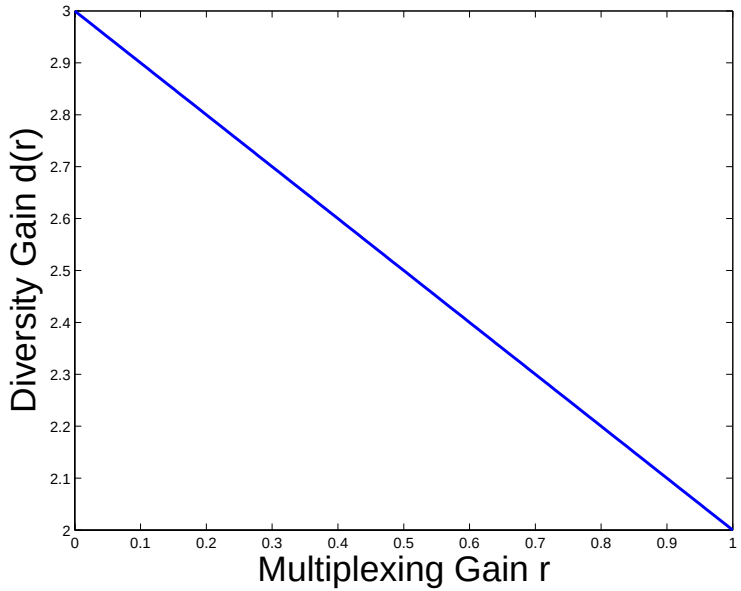


MISO = multiple transmit, but single receive antenna

$$Y = HX + W \Rightarrow \left[\overbrace{y_1\ y_2 \cdots y_p}^{\text{Phase I}} \;\middle|\; \overbrace{y_{p+1}\ \cdots y_{p+q}}^{\text{Phase II}} \right]$$

$$= \begin{bmatrix} g_1 & g_2 h_2 & \cdots & g_n h_n \end{bmatrix} \underbrace{\left[\begin{array}{c|c} \underline{x}_1^T & \underline{x}_2^T \\ & \vdots \\ & \underline{x}_w^T \end{array}\right]}_{\text{Source Tx} \quad \text{Relay Tx}} + \underline{n}^t,$$

DMG Tradeoff of a Cooperative Relay Channel



- The MISO channel model tells us that with $(n-1)$ relay nodes, DMG is upper bounded by the *n transmit-antenna, single-receive-antenna bound*

$$d(r) \leq n(1-r)$$

- $n = 3$ in plot above

References

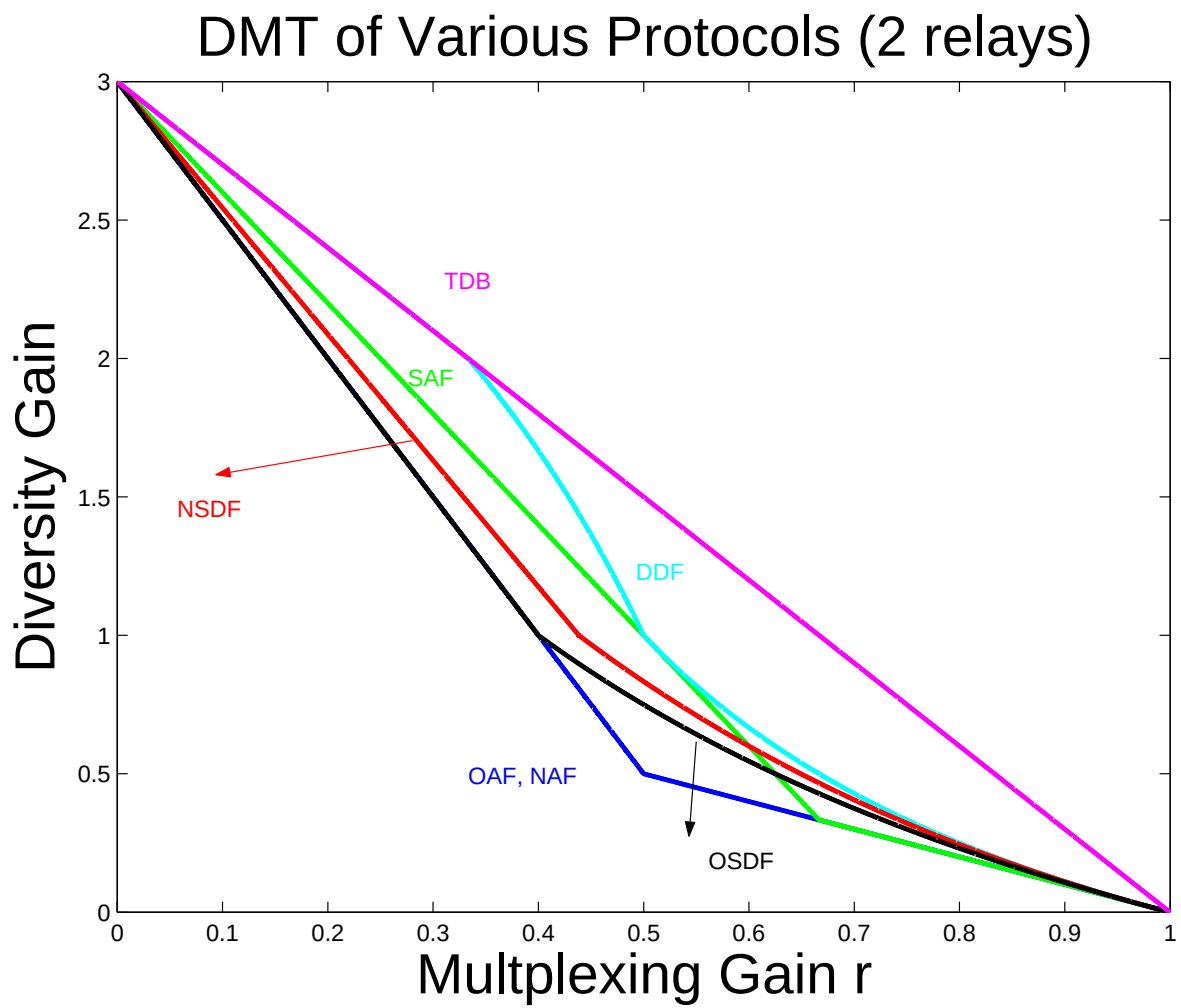
- A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity–Part I: System Description," *IEEE Trans. Commun.*, Nov. 2003.
- J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Info. Theory*, Dec. 2004.
- J. N. Laneman and G. W. Wornell, "Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. Inform. Theory*, Oct. 2003.
- J. N. Laneman and G. W. Wornell, "Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. Inform. Theory*, Oct. 2003.
- J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Info. Theory*, Dec. 2004.
- K. Azarian, H. El Gamal, and P. Schniter, "On the Achievable Diversity-Multiplexing Tradeoff in Half-Duplex Cooperative Channels," *IEEE Trans. Info. Theory*, Dec. 2005.
- S. Yang and J.-C. Belfiore, "Optimal Space-Time Codes for the MIMO Amplify-and-Forward Cooperative Channel," submitted to *IEEE Trans. Wireless Comm.*, Sept. 2005.
- S. Yang and J.-C. Belfiore, "Towards the Optimal Amplify-and-Forward Cooperative Diversity Scheme," submitted to *IEEE Trans. Wireless Comm.*, Mar. 2006.

Standardization Activity

- Cooperative communication of the type discussed here is under consideration for example, by the IEEE 802.16j Relay Task Group and a standard is expected to be completed and approved by 2008.

K. Sivanesan and D. Mazzarese, " Cooperative Techniques in the IEEE 802 Wireless Standards: Opportunities and Challenges: Explicit Macro Cooperation in Practice," a chapter in the book, *Cooperation in Wireless Networks: Principles and Applications*, Frank H. P. Fitzek and Marcos D. Katz (eds.), 2006 Springer, pp. 497-514.

DMT of Several Protocols



Our Results in Cooperative Communication

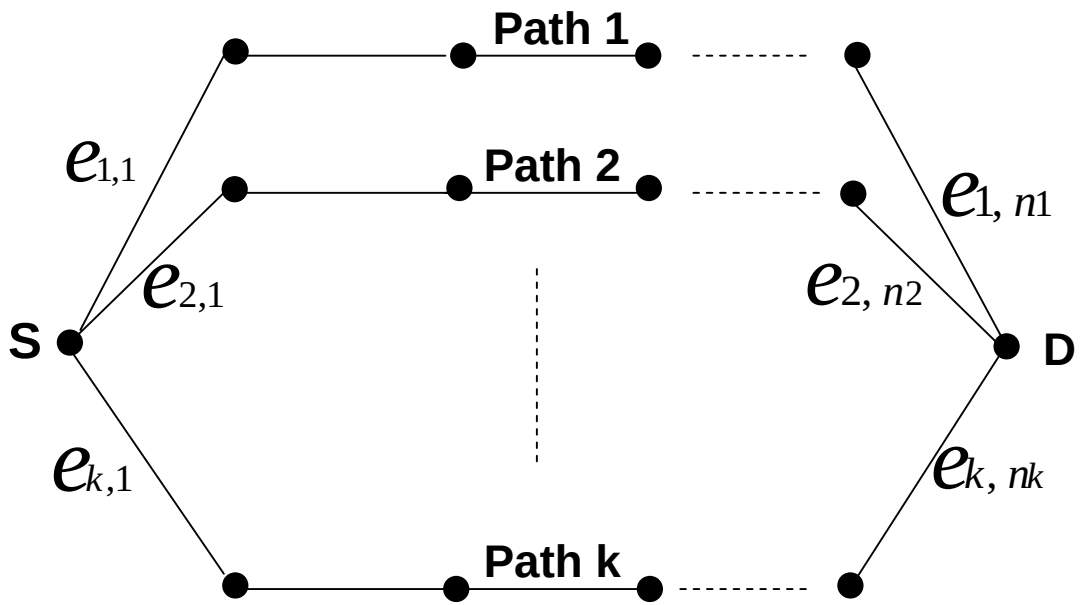
1. DMG of the OAF protocol and optimal code construction
2. DMG of the OSDF protocol and optimal code construction
3. DMG of the OSDF protocol and optimal code construction
   (items 1-3 above represent joint work with P. Elia, K. Vinodh and M. Anand)
4. and most recently, DMG-optimal code for the DDF protocol of Azarian, El Gamal, and Schniter.
   (joint work with P. Elia)

Journal Publications in this Area

1. P. Elia, K. Vinodh, M. Anand and PVK, "D-MG Tradeoff and Optimal Codes for a Class of Static AF and DF Cooperative Communication Protocols," submitted to the *IEEE Trans. Inform. Theory*, Jan. 2007.
2. S. A.Pawar , K.R. Kumar , P. Elia , PVK , B. A. Sethuraman, "Codes Achieving the Diversity-Multiplexing Gain Delay Tradeoff of the MIMO-ARQ Channel," submitted to the *IEEE Trans. Inform. Theory*, Dec. 2005.
3. P. Elia, B. A. Sethuraman, PVK, "Perfect Space-Time Codes with Minimum and Non-Minimum Delay for Any Number of Antennas, " to appear in the *IEEE Trans. Inform. Theory*.
4. P. Elia; F. Oggier and PVK, "Asymptotically optimal cooperative wireless networks with reduced signaling complexity," *IEEE J. Selected Areas in Communication*, Feb. 2007
5. P. Elia, K.R. Kumar, S. A. Pawar, PVK H-f Lu,"Explicit, Minimum-Delay Space-Time Codes Achieving the Diversity Multiplexing Gain Tradeoff ", *IEEE Trans. Inform Theory*, Sep. 2006.

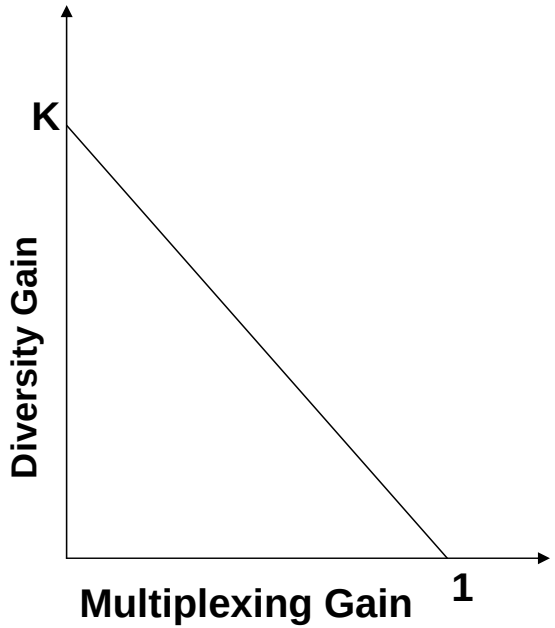
Recent Results on Multi-hop Networks

We consider the case of k edge-disjoint, parallel paths between source and destination.



k-path DMG Tradeoff

For several interference models, we determine the DMG tradeoff. The best possible tradeoff is shown below.



(joint work with K. Sreeram, S. Birenjith, K. Vinodh and M. Anand)

Thanks!