International Journal of Engineering Research in Africa
ISSN: 1663-4144, Vol. 2, pp 63-72
doi:10.4028/www.scientific.net/JERA.2.63
© 2010 Trans Tech Publications, Switzerland
Online: 2010-06-30

Finite Element Method of Predicting GSM Radio Power Received in a Macrocell Environment

J.O. Emagbetere[1,a] and F.O. Edeko[2,b]

[1,2] Department of Electrical/Electronic Engineering, University of Benin, Benin City, Nigeria.
[a]joyokumo@yahoo.com, [b]frededeko@yahoo.co.uk
[Date received: 2 September 2009. Date accepted: 22 April 2010]

Keywords: Maxwell wave equation, Helmholtz equation, Finite Element Method, Partial differential equation, Base transceiver station, GSM.

Abstract. The numerical solution of the partial differential equation (PDE) of the received signal strength from fixed transmitting stations as derived from Maxwell's wave equation is presented in this paper. The *received signal strength level* (RSSL) at a defined distance from a source point (base transceiver station) was simulated for two real environments described as Sites 1 and 2. The values of RSSL were taken at different nodes and at different radial directions away from the source point using the Finite Element Method (FEM) tool of the MATLAB package. The hexagonal geometry with no describing function was assumed for the cell. A 3-D model of the power received versus distance was obtained, and the numerical solution of the model presented. The numerical results obtained from the 3-D model were compared with the results of the drive test conducted in Sites 1 and 2 for functional GSM radio networks in the areas. The suitability of the method was justified for the two sites with a 5.55dB standard deviation of error for network A in Site 1 and 8.36dB and 3.40dB standard deviations of error for networks A and B respectively for Site 2.

Nomenclature

c = speed of light
u = corresponding value of the received signal strength
f = operating (carrier) frequency
i,j = row/column key indices
k = wave number
K^e = stiffness matrix
M = vector
n = refractive index,
w = weighted function
$\int_\Gamma$ = integrating around the boundaries
λ = wavelength
μ = constant
Ω= bounded domain in the plane

Introduction

Global System for Mobile (GSM) communication is a cellular network. Like all cellular networks, it is made up of a number of cells, with each served by at least one fixed-location transceiver. As a result of the cell sizes they can be classified as macro, micro, pico or umbrella cell in the order of large to small. The approximate coverage area for a macrocell is from 1km to 35km, for microcell it is between 100m to 1km, and for pico and umbrella cells, the coverage areas are smaller [1].

Since GSM inception in Nigeria in 2001, it has recorded tremendous growth in the telecommunication market as well as improved the country's socio-economic standard. Nethertheless, consumers have questioned the reliability of initiating, establishing, and sustaining a call for a period of time (usually 120seconds). Although this problem may not be peculiar to

All rights reserved. No part of contents of this paper may be reproduced or transmitted in any form or by any means without the written permission of Trans Tech Publications, www.ttp.net. (#69797389, Pennsylvania State University, University Park, USA-17/09/16,08:16:21)

Nigeria, it is a worrisome issue to GSM consumers. In other countries, different methods such as; predicting radio models (both deterministic and empirical models) suitable for the environment, radio resource management and optimization techniques, have been proffered to solve this problem [2 - 4]. A propagation model is a set of mathematical expressions, diagrams, or algorithms used to represent the radio characteristics of a given environment [2]. Propagation modeling is an effort to predict what happens to signals from the transmitter to the receiver [5, 6]

The need for this work arose out of a quest to find a suitable model for the Nigerian GSM macrocellular environment. Various methods such as the continuous wave test and drive test are used to estimate the RSSL value for network planning and optimization [7]. All of these methods are very expensive, time consuming, and strenuous. Thus, means of predicting the RSSL without going through the rigors mentioned above while the degree of accuracy is maintained could be an alternative. Thus, in this paper, the deterministic method of predicting the GSM radio signal in terms of the received signal strength level (RSSL) using the Finite Element Method to discretize the propagation domain was employed [8]. The results obtained were then compared with the results of the drive test conducted in Sites 1 and 2, for the functional GSM radio networks in the area.

Methodology

Since the radio wave lies within the electromagnetic spectrum, the interaction of the wave within the air-interface of the GSM radio network can be modeled using the Maxwell wave equation. The two-dimension partial differential Maxwell's wave equation was transformed into a two-dimensional Helmholtz equation, and the solution solved using the *pdetool* of the MATLAB package. The bounded domain (cell) was discretized (Finite Element Method (FEM)), and hence the received signal strength level at each node of selected element of the model was predicted. The Constructive Solid Geometry (CSG) model used for modeling the cell was the hexagonal geometry (conceptual cell shape). This shape was used for its simplicity.

The model developed with the computer aided method, was simulated for different macro cell GSM operators' networks that are functional in two environments discussed as Sites 1 and 2. Site 1 is described with characteristic dense vegetative features mostly palm trees, while Site 2 is a rural area characterized with nucleated buildings [1].

Theoretical background. The general wave equation in two space Dimension (2-D) derived from Maxwell's equation is [9]

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad 1$$

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad 2$$

where u is the corresponding value of the received signal strength expressed in both x and y directions.

The time variable in Equation (2) is removed by considering a harmonic steady state, with only negative values of a constant leading to a solution $u = 0$ at the boundary. That negative constant is assumed as $-v^2$ [9 - 12]. Thus Equation (2) can be written as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -v^2 \qquad 3$$

Equation (3) is simplified as

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+v^2=0 \quad 4$$

Equation (4) is a 2-D Helmholtz equation that describes the spatial scalar wave $u(x_i, y_i)$.

In order to obtain the effect of refractivity of a real macrocell environment v^2 is replaced with k^2n^2, where the refractive index of the environment is n, and k is the wave number in free space which is expressed as [9, 12]

$$k=\frac{w}{c}$$

c is wave speed which is approximately the speed of light, and w is the angular velocity of the wave which is equal to $2\pi f$, where f is the operating (carrier) frequency assumed as operating frequency for GSM1800 [7].

Generalizing the Helmholtz equation, we can write

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+k^2n^2u=\mu \quad 5$$

where μ is a constant, and can also be assumed as a positive value or zero (i.e $\mu \geq 0$).

Equation (4) is then expressed as [9, 10, 12]

$$U_{xx}+U_{yy}+k^2n^2u=0 \quad 6$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+k^2n^2u=0 \quad 7$$

Equation (7) is a differential equation that describes the interaction of the electromagnetic wave within the air-interface of the GSM network with no damping term or function, and no external force acting on the system or network. However, considering the environment as a medium with slightly variable refractive index $n=n(x,y)$, the refractive index will follow a Gaussian normal distribution which can be expressed as a harmonic time variation of the form [9, 12]

$$n=n(x,y)e^{-k\sqrt{x^2+y^2}}$$

This Gaussian normal distribution vector takes care of the probabilistic effect in the different spatial dimensions. Equation (7) can be represented in the MATLAB®7 pdetool box as an elliptical type of PDE mode, with PDE specification as $c=1$, $a=k^2n^2e^{-k\sqrt{x^2+y^2}}$, and $f=0$, and solved using the Finite Element Method (FEM). The FEM MATLAB tool box, discretizes the solution region into a finite number of sub-regions or elements, derives the governing equations for a typical element, assembles all elements in the solution region, and solves the system of equations obtained. Theoretically, using the FEM, with; $u=\sum u_j\psi_j(x,y)$ and $w=\psi_i(x,y)$, (where $w=\psi_i(x,y)$ is the weighted function, and i and j are the row/column key indices, the solution of Equation (7) which is $u(x,y)$ is given by the integral

$$\int_{\Omega} w\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 y}{\partial y^2}+k^2n^2u-\mu\right)=0 \quad 8$$

The Ω (bounded domain in the plane) symbol with the integration sign means integration around the irregular domain. Equation (8) can be transformed into the one with less discontinuity using the Green's theorem. Thus it can be expressed as [2, 5, 13]

$$\int_{\Omega}\left(\frac{\partial w \partial u}{\partial x \partial x}+\frac{\partial w \partial u}{\partial y \partial y}+k^2 n^2 wu\right)\partial x \partial y-\oint_{\Gamma} w\left(\frac{\partial u}{\partial x}n_x+\frac{\partial u}{\partial y}n_y\right)ds+\int_{\Omega} w\mu dxdy=0 \qquad 9$$

$$=\int_{\Omega}\left(\frac{\partial w \partial u}{\partial x \partial x}+\frac{\partial w \partial u}{\partial y \partial y}+k^2 n^2 wu\right)\partial x \partial y-\oint_{\Gamma} w\left(\frac{\partial u}{\partial n}\right)ds+\int_{\Omega} w\mu dxdy=0 \qquad 10$$

The presence of a unit normal derivative in Equations (9 and 10) makes the problem a Neumann problem. The unit normal derivative is represented by $q=\frac{\partial u}{\partial n}$ and is called flux in FEM. The flux gives the outflow or change of physical quantity of the signal strength with respect to a unit normal to the boundary. Substituting the expression in Equation (10) for the interpolation function and the corresponding weighted function, the matrix formulation of the FEM will be given as [11]

$$\left[K^e\right]\left[u^e\right]+\mu M=q \qquad 11$$

Equation (11) makes it possible to use the Dirichlet boundary condition ($h*u=r$), when $q=0$. This condition was chosen since we assume the least signal strength at the boundary of the hexagonal macro-cell geometry before call drop, and also for the fact that as one transverses the cell at integral multiples of wavelength ($\lambda=0.167m$ at 1800MHz), the signal strength variation along the radius with respect to a unit normal will be infinitesimally small. Thus, in this paper, the Dirichlet boundary condition ($h*u=r$) and homogeneity of the environment was assumed for simplicity.

Matlab Simulation. To solve the problem in Equation (7), triangulation method using FEM tool of the MATLAB package was employed to reduce the complexity of solving the matrix. The $p\det ool$ on the MATLAB® command line invokes the Graphic User Interface (GUI). The GUI is a self contained graphical environment for PDE solution. This $p\det ool$ requires no knowledge of the mathematics behind the PDE. This MATLAB® tool provides a guide to solving the problem in a stepwise manner. The only requirement is to formulate a PDE problem (i.e draw a domain geometry, fix the boundary conditions, the PDE specification, create the triangular mesh (finite elements), solve the PDE, and plot the solution) [11-12]. Thus in summary, the process involved in solving an elliptic equation such as the one used to describe the Helmholtz equation is; to triangulate the domain, hence build a stiffness matrix of K^e as in Equation (11), solve the linear system of the equation and apply the 3-D tool of the GUI window. Note that all of these steps are done with the MATLAB toolbox.

From the Dirichlet boundary condition used, where h and r are complex-valued functions defined on the domain $\partial\Omega$, $h=1$, and r is the value of the weakest signal strength received in an environment. It is also known as receiver sensitivity [7, 9]. The values of r used in this paper were taken from the drive test measurement reported in [1] for different GSM operator networks in Sites 1 and 2. Thus for Site 1; $r=-105.10dBm$ for network A. For Site 2; $r=-105.80dBm$ for network A, and $r=-97.80dBm$ for network B. The parameter r chosen from the work of [1] was to minimize prediction errors [11-12].

Results and Discussion

Using the boundary conditions for each operator and specifying the partial differential equation (PDE), and the wave number k calculated at 1800MHz, with the value of n = 1.59 assumed for the rural macrocell environments considered in [1], a 3-D model of power received versus distance was developed on the GUI window. Three figures of identical shape, but different characteristics were modelled using the network parameters in Sites 1 and 2. Only Figure 1 is presented since the shape of the model look the same irrespective of the numerical results obtained. However, the numerical results are tabulated in Table 1. The plot in Figure 1 is the 3-D Model generated using the 3-D tool of the menu on the GUI window.

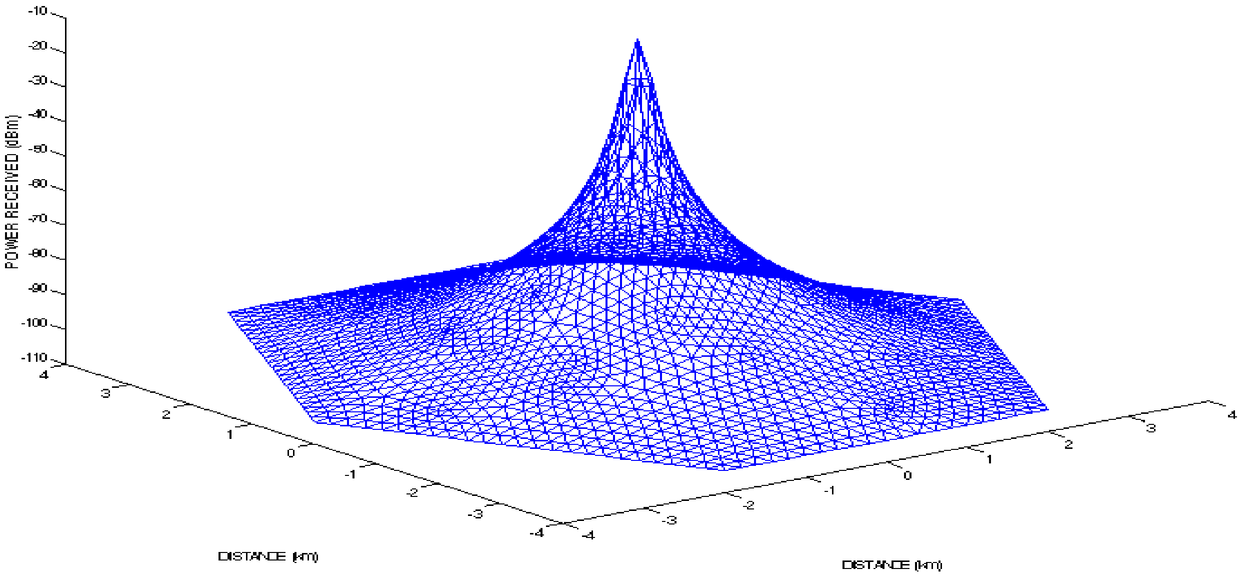

Figure 1 A 3-D Model of the PDE solution.

Table 1: PDE numerical solution obtained from the 3-D model for different simulated environments (Sites 1 and 2)

Site 1, NK A	
D (m)	RSSL (dBm)
104	-23.47
118	-54.26
653	-60.71
900	-68.96
1341	-78.45
1647	-88.87
1926	-87.28
2263	-99.76
2317	-91.81
2615	-94.81
3112	-98.94
3646	-102.8
4000	-105.1

Site 2, NK A	
D (m)	RSSL (dBm)
163	-47.86
299	-41.31
369	-61.82
995	-72.52
1841	-89.46
1941	-88.62
2287	-92.38
2513	-94.37
2854	-97.39
3263	-100.6
3646	-103.4
4000	-105.8

Site 2, NK B	
D (m)	RSSL (dBm)
389	-74.04
999	-82.93
1321	-85.93
1668	-88.41
2033	-90.47
2395	-92.14
2741	-93.53
3074	-94.75
3373	-95.79
3646	-96.67
4000	-97.8

In both sites, the measurement path distance used was 4000 meters, and each point on the distance axis of Figure 1 represents 1000m. To obtain numerical results with minimal scaled error from Figure 1, the 3-D plot was rotated to different Azimuths and Elevations (Az and El) using the rotation icon on the GUI window, by as much as possible maintaining an Azimuth of 44^0 to 47^0. At different nodes of the FEM elements in Figure 1, the value of RSSL corresponding to a particular distance was obtained and recorded as shown in Table 1. In Site 1, the angle from which the PDE solution for Operator A was viewed was Az 44^0, El 87^0. For Site 2, Operator A was Az 46^0, El 87^0, Operator B was Az 47^0, El 86^0.

Comparison of model with measured data. To evaluate the suitability and accuracy of the PDE model for Sites 1 and 2, the data in Table 1 was compared with the experimental data obtained from the radio propagation study conducted in the sites [1] as shown in Tables 2 and 3. The comparison was in terms of their prediction error statistics; relative mean error and standard deviation error. Linear regression method of analysis using the MATLAB tool was also applied to the data in Table 1 so as to ensure uniformity in scale with respect to distance compared to that of the experimental scale. The prediction errors were computed using the expression

Experimental power received (dBm) – PDE RSSL (dBm) = prediction error (dB) 12

With the absolute mean (*AVEDEV*) and standard deviation (*STDEVP*) tools of the Microsoft Excel package, the prediction error statistics (mean error, and standard deviation of error) were determined. The prediction error statistics of the developed model are shown in Table 4.

Table 2**:** Comparing model with experimental data obtained from [1] in Site 1

Operator A in Site 1			
Distance (m)	Experimental Power received, (dBm)	Linear model of PDE numerical solution RSSL (dBm)	Mean prediction error
200	-57.35	-46.00	-11.35
400	-56.69	-58.94	2.25
600	-64.54	-66.52	1.98
800	-70.55	-71.89	1.34
1000	-73.17	-76.06	2.89
1200	-78.42	-79.46	1.04
1400	-80.00	-82.34	2.34
1600	-85.54	-84.83	-0.71
1800	-83.89	-87.03	3.14
2000	-80.42	-89.00	8.58
2200	-80.48	-90.78	10.30
2400	-84.51	-92.40	7.89
2600	-99.33	-93.90	-5.43
2800	-100.00	-95.28	-4.72

Table 3**:** Comparing model with experimental data obtained from [1] in Site 2

	Operator A in Site 2			Operator B in Site 2		
Distance (m)	Experimental Power received (dBm)	PDE model RSSL (dBm)	Mean prediction error	Experimental Power received (dBm)	PDE model RSSL (dBm)	Mean prediction error
200	-63.86	-45.00	-18.86	-64.12	-67.00	2.88
400	-65.91	-58.85	-7.06	-67.83	-74.22	6.39
600	-68.34	-66.95	-1.39	-74.30	-78.45	4.15
800	-71.92	-72.69	0.77	-77.50	-81.45	3.95
1000	-74.81	-77.15	2.34	-79.60	-83.78	4.18
1200	-72.37	-80.79	8.42	-73.42	-85.68	12.26
1400	-75.08	-83.87	8.79	-75.81	-87.28	11.47
1600	-79.95	-86.54	6.59	-80.44	-88.67	8.23
1800	-82.76	-88.90	6.14	-81.35	-89.90	8.55
2000	-81.90	-91.00	9.10	-81.23	-91.00	9.77
2200	-80.06	-92.90	12.84	-78.30	-91.99	13.69
2400	-83.40	-94.64	11.24	-82.69	-92.90	10.21
2600	-86.56	-96.24	9.68	-84.37	-93.73	9.36
2800	-85.29	-97.72	12.43	-79.81	-94.51	14.70
3000	-90.79	-99.10	8.31	-90.82	-95.23	4.41
3200	-91.31	-100.39	9.08	-90.84	-95.90	5.06
3400	-87.59	-101.60	14.01	-87.58	-96.53	8.95
3600	-86.58	-102.74	16.16	-84.96	-97.13	12.17
3800	-87.19	-103.82	16.63	-88.08	-97.69	9.61
4000	-89.53	-104.85	15.32	-90.16	-98.22	8.06

Table 4: Prediction error statistics

Parameter	Operator A Site 1	Operator A Site 2	Operator B Site 2
Mean prediction error	4.03	6.07	2.83
Standard deviation of error	5.55	8.36	3.40

Tables 2 and 3 are presented graphically as shown in Figures 2 and 3 for the different networks in Sites 1 and 2 respectively.

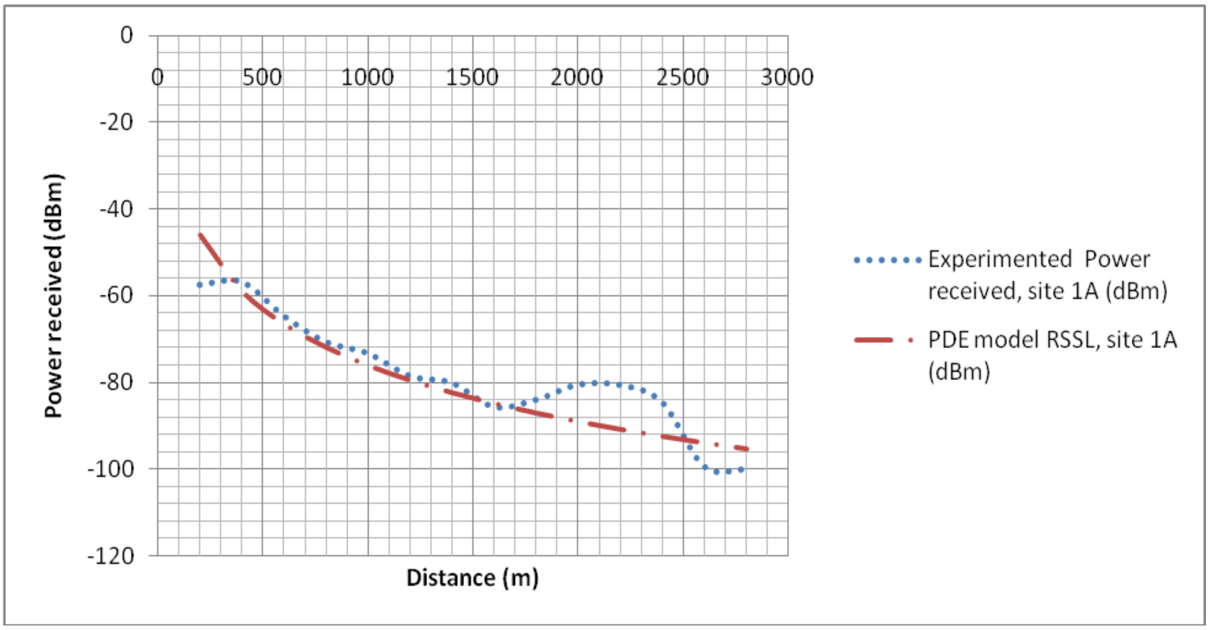

Figure 2: Comparison of power received in Site 1 for network A using experimental and simulated data

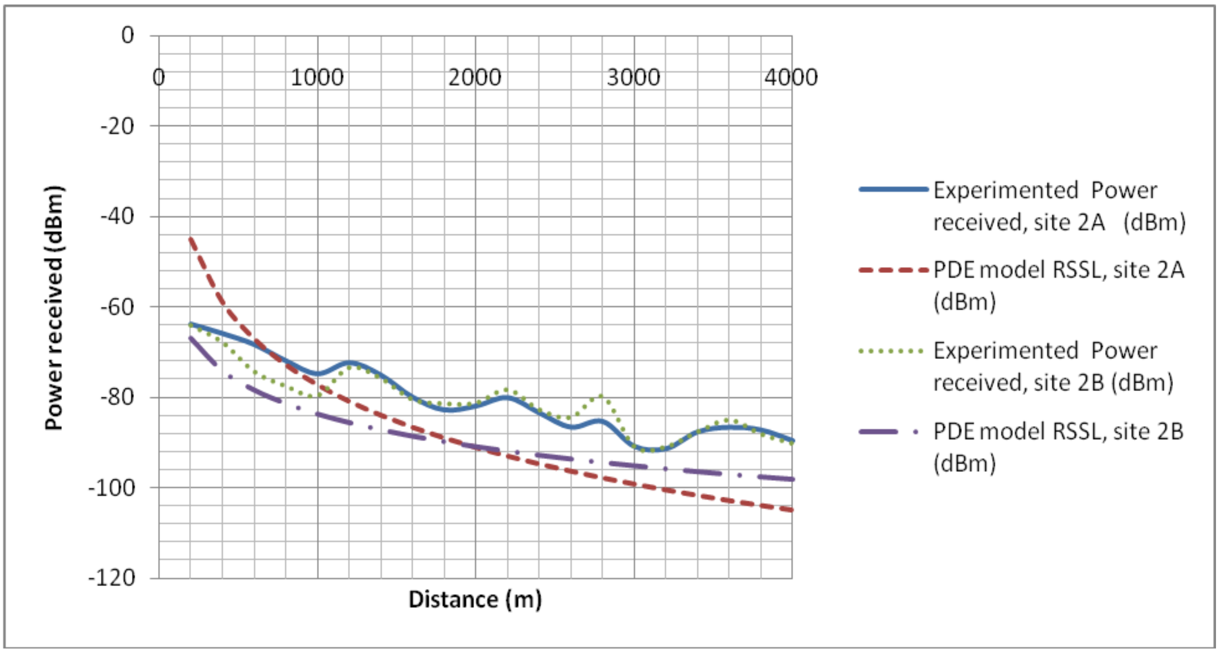

Figure 3: Comparison of power received in Site 2 for networks A and B using experimental and simulated data

Figures 2 and 3 show that power received decreases as distance from source of signal generator increases for both the experimental and predicted values. While the PDE model plots show a smooth exponential decay of power received, the experimental plots decay in a rather irregular manner. The reason for this could be the terrain, as well as the environmental conditions of the physical environment.

Both figures show that the PDE model generally predicts a lower power received. However, we see that the predicted power received is higher than the experimental power received at distances below 400m for network A in Site 1 (Figure 2), 700m for network A in Site 2 (Figure 3). This higher prediction is again repeated for network A in Site 1 at distances beyond 2500m (Figure 2). However, the PDE model is able to predict the power received fairly well. The mean prediction errors and standard deviation of errors are shown in Table 4 which vary from 2.83dB to 6.07dB for the mean prediction errors and 3.40dB to 8.36dB for the standard deviation of error.

Several investigators [7, 14, 15] are of the opinion that the performance of mobile communication networks for outdoor environment will be better when the standard deviation of error in propagation prediction is about 8 – 10dB. Thus in terms of prediction error statistics as presented in Table 4, the model can be applicable for planning a suitable network, for network A in Site 1, and for networks A and B in Site 2.

The planning tool developed in this paper can be useful during the initial design stage of the macro cellular network in similar environments such as Sites 1 and 2. It will be useful for radio network planners of wireless outdoor mobile communication networks. This tool will also be useful in the 3-D simulation of different networks, which enables prediction of signal strength level at different radial directions relative to the source point. Thus, with the knowledge of the signal strength received, accurate prediction for the radio cell size and base station location, can be made possible for effective communication without undergoing tedious and time consuming drive testing method.

Conclusion

A computer aided method of modeling received signal strength from a source point (BTS) using the Finite Element Method (FEM) tool of the MATLAB package has been presented. The computer aided tool was used to solve the 2-D partial differential problem derived from Maxwell's wave equation for simulated macrocellular environments. A 3-D model of the power received in dBm versus source-destination terminal separation distance was developed, and the numerical solution of the model presented. The received signal strength level obtained from the model compared well with drive test results recorded in the referenced macrocellular environments. The received signal strength level obtained can help predict the network characteristics which are important considerations for network performance determination as well as network design process planning.

References

[1] J.O. Emagbetere, Evaluation Of The Performance Characteristics Of GSM Networks In Nigerian Rural Environments: Case Study Of Nifor In Edo State And Oghara In Delta State, Ph.D (Electrical/Electronic Engineering) Thesis, University of Benin, Benin City, Nigeria, 2008
[2] GSM Nigeria, GSM Operators, the Global System for Mobile Communications, MTN Nigeria, http://www.jidaw.com/telecom/gsm.html (2007)
[3] T.L Adebayo and F.O Edeko, Characterization of propagation path loss: A case study of Benin City, Nigeria, Research Journal of Applied Sciences, Vol. 1, Issue 1 (2006) 92-96.
[4] J.O. Emagbetere and F.O. Edeko, Radio coverage analysis of GSM radio networks in Oghara, Journal of Science and Technology Research, Vol. 7, No.4 (2008) 70 – 74.
[5] R.A. Harry, Coverage Prediction for Digital Mobile Systems, Part 1, MRT, 1997
[6] K.S Tapan, J. Zhong, K. Kyungjung, M. Abdellatif and S. Magdalena S, A survey of various propagation models for mobile communications, IEEE Antenna and Propagation Magazine, vol. 45, No. 3 (2003) 11.
[7] R.M. Ajay, Advanced Cellular Networks Planning and Optimization, John Wiley Sons, London, 2007.
[8] S. Jie and W. Li-Lian, Spectral approximation of the Helmholtz Equation with high wave numbers, SIAM J. NUMER. ANAL, Society for Industrial and Applied Mathematics, Vol. 43, No. 2 (2005) 623–644.
[9] C. Matzler, 'Parabolic equations for wave propagation and the advanced atmospheric effects prediction system (AREPS), Research paper, Institute of Applied Physics, Switzerland, Research report No.2004-03 (2004) 1-15.

[10] K. Huang, K. Solna, and H. Zhao, Coupled parabolic equation for wave propagation, Journal of Methods and Application of Analysis, Vol. 11, No. 3 (2004) 399-412.
[11] M.J. Fagan, Finite Element Analysis: Theory and Practice, Longman Group, London, 1992
[12] MathWorks, 'Partial Differential Equation Toolbox User's Guide for Use with MATLAB®, COMSOL AB, Version 1, March 2006
[13] E. Kreyszig, Advanced Engineering Mathematics, 8th Edition, Wiley & Sons, Inc., New York, 1999
[14] T.S. Rappaport, Wireless Communication Principles and Practice, 2nd Edition, University of Texas, Austin, 2003
[15] A. Neskovic, N. Neskovic and G. Paunovic, Modern approaches in modeling of mobile radio systems propagation environment, IEEE Communication Surveys, 2000.