Dependence Analysis for the Exchange Rate Data using Extreme Value Copulas

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Abstract

This article considers the bivariate generalized extreme value (BGEV) distribution and the bivariate generalized Pareto distribution (BGPD) to model the tail probability and tail dependence of the financial return series based on monthly and daily maxima of BHT/USD, EUR/USD foreign exchange data, respectively. The selection and estimation of the copula is based on the maximum likelihood estimation(MLE) approach which is proposed for nine parametric models of dependence function for both distributions. The copula parameters are estimatedby Inference For Margins(IFM) approach and then select best fitting model by Akaike Information Criterion (AIC) value.

Keyword: bivariate generalized extreme value distribution, bivariate generalized pareto distribution, parameter estimation, extreme value copulas, dependence function, tail probability, tail dependence.

Introduction

Extreme value theory (EVT) works with the extreme deviations from the mean of probability distributions. It is important to describe the shape of the tail part in order to make an accurate estimation of the tail probability, when modeling the distribution of the rarely events such as asset return, not yet seen disasters, etc. Recently, EVT is quick development which based on normal distribution in many situations. It has been widely used in the area of statistics and gradually in the financial, climate, hydrology and other fields;see Joe (1997), Coles (2001), McNeil (2005), etc.

Definitely, "Copula" approach is a great statistic tool which can be combined with EVT in the case of multidimensional variables. In 1999, Nelson proposed copula approach in his monograph, presenting the theory and basic introduction to this nonlinear dependence measure. Since then, copulas are very popular approach and rapid development (see Frees and Valdez (1998), Embrechts et al. (2002)). Especially, copulas reveal to be an excellent powerful tool in financial, insurance and related fields. A copula is a hidden dependence structure that couples a joint distribution with its margins. The fact that the theory of multivariate in EVT can be expressed in terms of copulas has been recently recognized (see McNeil 2005). A class of copulas well-known as extreme value copulas emerges as the class of natural limiting dependence structures for multivariate variables and these provide useful reference structures for modeling the behavior of variables that appear to show tail probability and tail dependence, especially with rare event. This articleworks with the tail behavior of the tails of financial return series from foreign exchange market using the EVT and concentrate on the tail probability and tail dependence analysis based on the extreme value copulas by using "evd" and "copula" package on R program.

This article is organized as follows. Section II presents the definition of univariate and bivariate EVT which can be used to model the maximum series distributions. Section III reviews the concept of copula function and extreme value copulas in accompany with their dependence functions that will be applied. In Section IV, the description of the tail probability which defined as joint survival function and quantifications of the magnitude of the tail dependence. In Section V, parameter estimation based on the MLE approach and the statistical estimation

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of the copula parameters based on IFM approach. Section VI presents the results of an empirical application of foreign exchange return data. However, this also presents the procedure for the selection of the suitable copula which can be described tail probability and tail dependence characteristics. Finally, Section VII summarizes the major findings and introduces directions for further research.

Univariate and Bivariate extreme value theory

In 1928, Fisher-Tippett proposed Block Maxima and then Pickands (1975) and Balkema and De Haan (1974) proposed Threshold Exceedances. These are two kind approaches of EVT mainly.

Univariate Extreme Value Distribution

Block Maxima is the traditional approach for identifying extremes in data, which choose the largest (smallest) value during a certain period (annual, quarter, month, etc.) that constitute the extreme events for the model building which that distribution must be the generalized extreme value distribution or "GEV".

For univariate GEV, let $X_i, i = 1,2,...,n$ be a random sample coming from the distribution F(X). Define $M_n = \max(X_1, X_2,..., X_n)$ as the maxima (also denoted as block maxima). As stated in Fisher and Tippett (1928), if the block maxima of identically and independent distribution (i.i.d) random variables converge in to some non-degenerate distribution function *H* under an appropriate normalization, this distribution is called extreme value distribution, which belongs to one of the three types of class, widely known as Frechet, Weibull and Gumbel family, respectively. Jekinson (1955) unified the three families into a single family of models that have the distribution functions of the form:

$$H(x;\mu,\sigma,\gamma) = \exp\left\{-\left(1+\gamma\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\gamma}\right\} \quad (1)$$

where μ, σ, γ is called location, scale , shape parameter, respectively, and $1+\gamma\left(\frac{x-\mu}{\sigma}\right) > 0$. When $\gamma = 0$, it is Gumbel distribution, i.e., type I distribution. When $\gamma > 0$, it is Frechet, i.e., type II distribution. When $\gamma < 0$ corresponds to type III, also known as Weibull distribution. The generalization of three families into a single one greatly simplifies statistical implementation as shown in the simple form of (1).

For the univariate threshold Exceedances approach concerns about all observed data exceeding a certain threshold in the sample, which that distribution must be the generalized Pareto distribution or "GPD". Let $X_i, i = 1,2,...,n$ be a random sample coming from the distribution F(X). Then, for large enough u, the distribution function of $(X-u)$ conditional on $X > u$, the distribution function of the GPD is approximately

$$H(x;\sigma,\gamma) = \exp\left\{-\left(1+\gamma\left(\frac{x}{\tilde{\sigma}}\right)\right)^{-1/\gamma}\right\} \quad (2)$$

where σ, γ are called scale and shape parameters, respectively. Define on $\{x; x > 0\}$ and $1+\gamma\left(\frac{x}{\tilde{\sigma}}\right)$, where $\tilde{\sigma} = \sigma + \gamma(u-\mu)$. If $\gamma < 0$, the distribution of excesses has an upper bound of $u - \left(\frac{\tilde{\sigma}}{\gamma}\right)$. If $\gamma > 0$, the distribution has no upper limit. The distribution is also unbounded if $\gamma = 0$, which should again be interpreted by taking the limit $\gamma \to 0$ in (2).

Next subsection presents BGEV and BGPD which are adopted to study the tail behavior of foreign exchange data.

Bivariate Generalized Extreme Value Distribution (BGEV)

Let (X,Y) be a bivariate random sample vector represent the componentwise maxima or minima.Under the appropriate conditions the distribution of (X,Y) can be approximated by a bivariate extreme value distribution (BGEV) with margins G_1 and G_2, respectively. By Pickandsdependence function $A(w)$ (see Berlaint et al., 2004),

$$G(x,y) = \exp\left\{\log(G_1(x)G_2(y))A\left(\frac{\log(G_2(y))}{\log(G_1(x))\log(G_2(y))}\right)\right\} \quad (3)$$

where $A(w)$ is called the dependence function between the margins.

Bivariate Generalized Pareto Distribution (BGPD)

Let $(X,Y)=(Z_x-u_x,Z_y-u_y)$ be random vector of exceedances where (Z_x,Z_y) be the observed random variable, (u_x,u_y) a given threshold. Its cumulative distribution function (cdf) as in the paper of Rootzen and Tajvidi (2006),

$$H(x,y)=-\frac{1}{\log G(0,0)}\times\frac{\log G(x,y)}{G(x\wedge 0,y\wedge 0)}, \tag{4}$$

For some BGEV, G with non-degenerate margins and with $0<G(0,0)<1$.

The theory of BGEV and BGPD with the copula methodology are combined, then apply the class of extreme value copulas to explore the extremal dependence function of these data set.

Copula function and the extreme value copulas

Copula function

In 1959, the copula was first proposed by Sklar. In this article, (X,Y) is bivariate random vector and G is the distribution of (X,Y) with marginal distribution $F_x(X),F_y(Y)$. The Sklar's Theorem assures the existence of a distribution function C on $[0,1]^2$ for all $(x,y)\in R\times R$ such that:

$$G(x,y)=C(F_x(X),F_y(Y)) \tag{5}$$

where C is called the copula associated with X and Y which couples the joint distribution G with its margins. Equation (5) is equivalent to $\bar{G}(F_x^{-1}(u),F_y^{-1}(v))=C(u,v)$ as a consequence of the Sklar's Theorem, where $u=F_x(X),\quad v=F_y(Y)$ are marginal distribution of (X,Y),(See Nelsen, (1999)).

The theory of multivariate EVT can be expressed in terms of copulas. Let $M_X=\max(X_1,...,X_n)$ and $M_Y=\max(Y_1,...,Y_n)$ be the maxima of (X,Y) component. The object of interest is the vector of componentwise block maxima: $M=(M_X,M_Y)$ In particular, the possible multivariate limiting distributions for M under certain appropriate normalizations are interested. The outcome is similar to the univariate case, which can find a non-degenerate distribution function so that the bivariate extreme distribution G can be connected by an extreme value copula (EV copula) C_0:

$$G(x,y)=C_0(F_x(x;\mu_x,\sigma_x,\gamma_x),F_y(y;\mu_y,\sigma_y,\gamma_y)), \tag{6}$$

where μ,σ,γ are GEV parameter and F is GEV margin. In 1997, Joe presented the unique copula C_0 of F exists and satisfies:

$$C_0(u^t,v^t)=C_0^t(u,v),\quad t>0. \tag{7}$$

The extreme value copula

In 1981, Pickands pointed out a bivariate copula is an extreme value (EV) copula if and only if it takes the form:

$$C_0(u,v)=P(F_x(x)\le u_x,F_y(y)\le u_y)$$

$$=\exp\left\{\ln(uv)A\left(\frac{\ln v}{\ln(uv)}\right)\right\}, \tag{8}$$

where $A(w)$ is called the dependence function.

According to bivariate case, $A(w)$ is one-dimensional and (8) simplifies to

$$-\log G(x,y)=\left(\frac{1}{x}+\frac{1}{y}\right)A\left(\frac{x}{x+y}\right), \tag{9}$$

where

$$A(w)=\int_0^1\max\{a(1-w),(1-a)w\}S(da).$$

The finite positive measure on interval S is equivalent to

$$\int_0^1 aS(da)=\int_0^1(1-a)S(da)=1.$$

foreach $a \in [0,1]$. The dependence function $A(w)$ involve in an EV copula must satisfy three properties:

(i) $A(0) = A(1) = 1$,

(ii) $\max(w, 1-w) \le A(w) \le 1$ for $0 \le w \le 1$,

(iii) $A(w)$ is convex function in the region $0 \le w \le 1$.

The upper and lower bounds of $A(w)$ have intuitive interpretations. If $A(w) = 1$ for all w, then the copula is independent copula. If $A(w) = \max(w, 1-w)$, then the copula is perfectly dependent.

The following are nine of extreme value copulas commonly used and it is convenient to prove that all of them are satisfied with (7).

(i) Logistic modelor "log" (Gumbel, 1960):

The corresponding copula function is given by

$$C(u,v) = \exp\left\{-\left[(-\ln u)^{1/r} + (-\ln v)^{1/r}\right]^r\right\}, \quad (10)$$

with $0 < r \le 1$ Independence and complete dependence correspond to $r = 1$ and $r = \infty$, respectively. In this model the variables are exchangeable.

(ii) Asymmetric logistic model or "alog" (Tawn, 1988):

The copula function is

$$C(u,v) = \exp\left\{\begin{array}{l} -(1-\theta)\ln u - (1-\phi)\ln v \\ \quad -\left[(\theta \ln u)^{1/r} + (\phi \ln v)^{1/r}\right]^r \end{array}\right\}, \quad (11)$$

with $\theta \ge 0$, $\phi \le 1$, and $0 < r \le 1$. Independence dependence correspond to $\theta = \phi = 1$ and $r = \infty$. Complete dependence correspond to $\theta = 0$ or $\phi = 0$ or $r = \infty$.

(iii) HuslerReiss Model or "hr" (Husler and Reiss, 1989):

The corresponding copula function is

$$C(u,v) = \exp\left(\begin{array}{l} -\ln u \Phi\left\{\frac{1}{r} + \frac{1}{2} r\left[\log\left(\frac{\ln u}{\ln v}\right)\right]\right\} \\ -\ln v \Phi\left\{\frac{1}{r} + \frac{1}{2} r\left[\log\left(\frac{\ln v}{\ln u}\right)\right]\right\} \end{array}\right) \quad (12)$$

where Φ is the standard normal distribution function and $r > 0$ Independence is obtained in the limit as $r \to 0$ and complete dependence is obtained as $r \to \infty$

(iv) Negative Logistic Model or "neglog" (Galambos, 1975):

The copula function is

$$C(u,v) = \exp\left(-\ln u - \ln v + \left[\frac{1}{(\ln u)^r} + \frac{1}{(\ln v)^r}\right]^{-1/r}\right), \quad (13)$$

where $r > 0$ This is a special case of the bivariate asymmetric negative logistic model. Independence is obtained in the limit as $r \to 0$ and complete dependence is obtained as $r \to \infty$

(v) Asymmetric Negative Logistic Model or "aneglog" (Joe, 1990):

The corresponding copula function is given by

$$C(u,v) = \exp\left(-\ln u - \ln v + \left[\frac{1}{(\theta \ln u)^r} + \frac{1}{(\phi \ln v)^r}\right]^{-1/r}\right) \quad (14)$$

where $r > 0$ and $0 < \theta, \phi \le 1$. When $\theta = \phi = 1$ the asymmetric negative logistic model is approaches equivalent to the negative logistic model. Independence is obtained in the limit as either r, θ or ϕ approaches zero. Complete dependence is obtained in the limit when $\theta = \phi = 1$ and $r \to \infty$.

(vi) Bilogistic Model or "bilog" (Smith, 1990):

The copula function is

$$C(u,v) = \exp\left(-\ln u (q)^{1-\alpha} - \ln v (1-q)^{1-\beta}\right), \quad (15)$$

where $q = q(\ln u, \ln v; \alpha, \beta)$ is the root of the equation

$$(1-\alpha)\ln u (1-q)^{\beta} - (1-\beta)\ln v (q)^{\alpha} = 0,$$

$0 < \alpha, \beta < 1$. When $\alpha = \beta$, the bilogistic model is equivalent to the logistic model with dependence parameter $r = \alpha = \beta$. Complete dependence is obtained in the limit as $\alpha=\beta$ approach zero.Independence is obtained as $\alpha=\beta$ approaches one, and when one of α, β is fixed and the other approaches one. Different limits occur when one of α, β is fixed and the other approaches zero.

(vii) Negative Bilogistic Model or 'negbilog" (Coles and Tawn, 1994):

The copula function is

$$C(u,v)=\exp\left(-\ln u-\ln v+\ln u(q)^{1+\alpha}+\ln v(1-q)^{1+\beta}\right), \quad (16)$$

where$q=q(\ln u,\ln v;\alpha,\beta)$ is the root of the equation

$$(1+\alpha)\ln u(q)^{\alpha}-(1+\beta)\ln v(1-q)^{\beta}=0,$$

$\alpha>0,\beta>0$. When $\alpha=\beta$, the negative bilogistic model is equivalent to the negative logistic model with dependence parameter$r=\frac{1}{\alpha}=\frac{1}{\beta}$. Complete dependence is obtained in the limit as $\alpha=\beta$ approach zero. Independence is obtained as $\alpha=\beta$ approaches one, and when one of α,β is fixed and the other approaches infinity. Different limits occur when one of α,β is fixed and the other approaches zero.

(viii) Coles and Tawn Model or "ct" (Coles and Tawn, 1991):

The copula function is

$$C(u,v)=\exp\left\{\begin{matrix}-\ln u[1-Be(q;\alpha+1,\beta)]\\-\ln vBe(q;\alpha,\beta+1)\end{matrix}\right\}, \quad (17)$$

where$q=\alpha\ln v/(\alpha\ln v+\beta\ln u)$ and$Be(q;\alpha,\beta)$ is the beta distribution function evaluated at q with α and β Complete dependence is obtained in the limit as $\alpha=\beta$ tends to infinity. Independence is obtained as $\alpha=\beta$ approaches zero, and when one of $\alpha=\beta$ is fixed and the other approaches zero. Different limits occur when one of $\alpha=\beta$ is fixed and the other tends to infinity.

(ix) Asymmetric Mixed model or "amix" (Tawn, 1988):
The dependence function is

$$C(u,v)=uv\left\{\exp\left((\ln u)(\ln v)d\right)\right\} \quad (18)$$

with $d=\dfrac{\ln u(\alpha+\beta)+\ln v(\alpha+2\beta)}{(\ln u\ln v)^2}$,

where α and $\alpha+3\beta$ are non-negative, and where $\alpha+\beta$ and $\alpha+2\beta$ are less than or equal to one. Complete dependence cannot be obtained. Independence is obtained when both parameter are zero.

The tail probability and the tail dependence

Tail probability

From (3) and (4), the tail probability estimation ofBGEV and BGPDare calculated from the identity:

$$P(X>x,Y>y)=1-H_x(x)-H_y(y)+G(x,y) \quad (19)$$

isdefined as joint survival function, which can obtain tail probability exceeding estimation for those.

Tail dependence

Tail dependence is a kind of dependence measure which can calculate from copulas. In the event, if model the tail dependence, its' structure must be considered also. The tail dependenceof (X,Y) withrespective distribution $G(x,y)$ assuming BGEV distribution and $H(x,y)$ assuming BGPD can be measured as follows:

$$\lambda_u=\lim_{c\to1}\{1-2c+C_0(c,c)\}/(1-c) \quad (20)$$

and

$$\lambda_l=\lim_{c\to0}C_0(c,c)/c,(21) \quad (21)$$

wherec is q[th]-quantile, λ_u and λ_l are measures of upper and lower tail dependence, respectively. If $\lambda_u\geq0$ claims the upper tails of (X,Y) are asymptotically dependent.If $\lambda_l\geq0$ claims the lower tails of (X,Y) are asymptotically dependent. To consider the estimation of λ_u and to obtain the relationship between λ_u and the dependence function $A(w)$ (F. Gabriel et al., 2006):

$$\hat{\lambda}_u=2-2A(1/2). \quad (22)$$

Parameter estimation for EV distributions and copulas

A traditional approach, maximum likelihood estimation (MLE), is used to estimate the parameter in the BGEV and BGPD model by maximizing the log-likelihood function of the distribution. Set $l(\mu,\sigma,\gamma)$ and $l(\sigma,\gamma)$ as the sample log-likelihood function of the BGEV distribution and BGPD, respectively. The maximum log-likelihood estimate of parameter of BGEV and BGPD is as in (24) and (25),

$$(\hat{\mu},\hat{\sigma},\hat{\gamma}) = \arg\max l(\mu,\sigma,\gamma)$$

$$= agr \max \sum_{i=1}^{m} \log h(\mu,\sigma,\gamma)(x_i), \quad (23)$$

where x_i is the ith block maximum from the underlying data, $i = 1,...,m.$ m is the block size based on the original series data. $h(\mu,\sigma,\gamma)$ is the density of the GEV distribution. The maximization must be subject to the parameter constraints that $\sigma > 0$ and $1+\gamma\left(\frac{x_i - \mu}{\sigma}\right) > 0$ for all i.

$$(\hat{\sigma},\hat{\gamma}) = \arg\max l(\sigma,\gamma)$$

$$= agr \max \sum_{i=1}^{k} \log h(\sigma,\gamma)(x_i), \quad (24)$$

where x_i is the ith over threshold data, $i = 1,...,m.$ $m$is the number of excesses over threshold u . $h(\sigma,\gamma)$is the density of the GPD.

To estimate the copulas parameters, there are several popular and widely used approaches: (i) Exact maximum likelihood approach (EML), (ii) Inference functions for margins (IFM), (iii) Canonical maximum likelihood approach (CML) and (iv) Nonparametric approach. In this article, I use IFM to implement model and parameter estimation for extreme value copulas by assuming their margins are GEV distributed and GPD.

Set $l(\theta)$ as the log-likelihood function of copula. The maximum log-likelihood estimate of parameter θ is:

$$\hat{\theta} = \arg\max l(\theta) = \arg\max \sum_{i=1}^{m} \ln\left(c(\hat{u}_i,\hat{v}_i);\theta\right), \quad (25)$$

where $\hat{u}_i,\hat{v}_i$ represents the estimated value of the two margins. $c(\cdot,\cdot)$ is the density function of the copula. In IFM, the estimation of θ depends on the choice of marginal distribution. As the margins are GEV distributed and GPD, (3) and (4) give me two estimates for the distribution $H_x(x), H_y(y)$ Theninstitute m estimated value of into (5) to estimate the parameter θ of the copula function.

Results comparison

The daily closing prices of BTH/USD and EUR/USD foreign exchange data are used to analyze the tail probability characteristic and extremal dependence function using extreme value copulas. The data set is selected from January 3, 2000 to December 30, 2012 with 3391 effective observations for each index (http://www.federalreserve.gov). Define $\{(S_t)\}$ as the daily market closing price and transform it to the continuously compounded return-series (log-returns) as $R_t = (\ln(S_t) - \ln(S_{t-1})) \times 100.$ The results are described into three subsections as follows;

The parameter estimations of the GEV and GPD model

The estimation results of (μ,σ,γ) of GEV model and (σ,γ) of GPD model based on MLEapproach are shown in Tab 1. For GEV model, BHT/USD and EUR/USD correspond to Weibull distribution ($\hat{\gamma}$ <0).

Table 1 the parameter estimation results using the ml approach based on gev and gpd model

Model	BHT/USD		EUR/USD	
	Parameter	ML	Parameter	ML
GEV	$\hat{\mu}$	-0.8922 (0.2326)	$\hat{\mu}$	-0.8010 (0.1367)
	$\hat{\sigma}$	2.7023 (0.1551)	$\hat{\sigma}$	1.5642 (0.0926)
	$\hat{\gamma}$	-0.2097 (0.0317)	$\hat{\gamma}$	-0.1638 (0.0424)
GPD	$\hat{\sigma}$	0.5691 (0.1876)	$\hat{\sigma}$	0.5107 (0.1781)
	$\hat{\gamma}$	0.0927 (0.2480)	$\hat{\gamma}$	0.1165 (0.2363)

The standard deviation estimates as shown in the blanket are relative low which implies that my block size choice is also appropriate for the parameter estimation of GEV, and they are good responsibility for GPD also.

The parameter estimation of the copulas and realted dependence function

Table 2 and III are summary estimations for the parametric models discussed in Section III. Not only the value of the objective function (20) is given, but also an AIC goodness-of-fit measure,

$$AIC = -2L + 2\frac{n_p}{n}, \quad (26)$$

where L has been defined in (23) and (24), and n_p is the number of parameters.

Table 2 Summary of dependence function estimation for Bgev.

Model	Parameters	AIC
log	$\hat{r} = 0.9990$	1377.486
alog	$\hat{\theta} = 0.6900,\ \hat{\phi} = 0.6799, \hat{r} = 1.0000$	1371.383
hr	$\hat{r} = 0.2005$	1376.220
neglog	$\hat{r} = \mathbf{0.0530}$	**1366.106**
aneglog	$\hat{\theta} = 0.1105,\ \hat{\phi} = 0.0013, \hat{r} = 0.1507$	1371.288
bilog	$\hat{\alpha} = \mathbf{0.9477},\ \hat{\beta} = \mathbf{0.9984}$	**1369.625**
negbilog	$\hat{\alpha} = \mathbf{10.8941},\ \hat{\beta} = \mathbf{9.5691}$	**1368.200**
ct	$\hat{\alpha} = 0.0019,\ \hat{\beta} = 0.1222$	1372.330
amix	$\hat{\alpha} = 0.0007,\ \hat{\beta} = 0.0774$	1378.964

From Table 2, the three best fitting models for BGEV areneglog, negbilog and bilog model, which their AIC are1366.106, 1368.200 and 1369.625, respectively. Next, tail probability, dependence functions and tail dependence are calculated from parameters of these models.

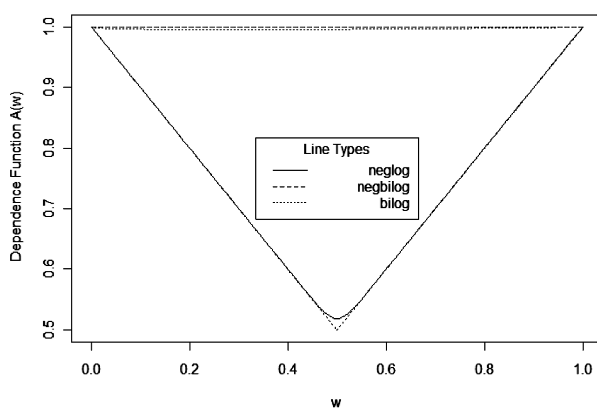

Figure 1 Parametric estimations of the dependence function $A(w)$ for BGEV. In Fig. 1, the three bestfitting; neglog, negbilog and bilog,estimations of the dependence function are represented. It can be seen that the "neglog" modelprovidesbest results from the represent of dependence function.

Table 3 Summary of dependence function estimation for BGPD.

Model	Parameters	AIC
log	$\hat{r} = \mathbf{0.9994}$	**495.0349**
alog	$\hat{\theta} = 0.7040,\ \hat{\phi} = 0.6947, \hat{r} =$ 0.9998	497.6283
hr	$\hat{r} = \mathbf{0.2034}$	**493.5208**
neglog	$\hat{r} = \mathbf{0.0524}$	**493.5208**
aneglog	$\hat{\theta} = 0.0884,\ \hat{\phi} = 0.0014, \hat{r} =$ 0.3538	498.2955
bilog	$\hat{\alpha} = 0.9449,\ \hat{\beta} = 0.9981$	496.3350
negbilog	$\hat{\alpha} = 15.8405,\ \hat{\beta} = 15.7989$	495.5213
ct	$\hat{\alpha} = 0.0017,\ \hat{\beta} = 0.1515$	496.4902
amix	$\hat{\alpha} = 0.6046,\ \hat{\beta} = -0.2013$	503.3025

From Table 3, the three best fitting models for BGPD arehr, neglog and log.In Fig. 2, the three best fitting estimations of the dependence function are represented. It can be seen that the "hr" and "neglog" models provide similar results and it seems all of them are able to represent the dependence function.

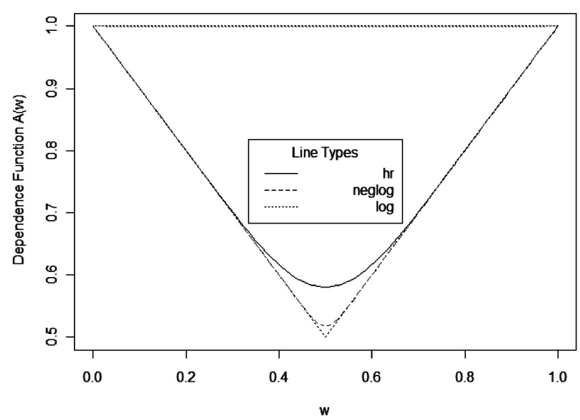

Figure 2 Parametric estimations of the dependence function $A(w)$ for BGPD.

Tail probability and Tail dependence

Table 4 gives the estimates of tail probability which is exceeding over 95th and 99thquantile under difference levels for BGEV and BGPD. These results showthat the probability of simultaneously exceeding respective quantile of two extremal return data is quite low and they are very similar.The $\hat{A}(1/2)$ presents the information of tail dependence between the variables, results of all model shows $\hat{A}(1/2) \sim 0.5$ that means variables are strong dependence (see the value of $\hat{\lambda}_u$).

Table 4 Tail probability , dependence function and tail dependence under different models for bgev and bgpd

Model	Tail Probability exceeding p^th^ quantiles		$\hat{A}(1/2)$	$\hat{\lambda}_u$
	95^{th}	99^{th}		
BGEV				
neglog	0.11830	0.06851	**0.5180**	0.9639
BGPD				
hr	0.10026	0.020053	**0.5794**	0.8412
neglog	0.10026	0.020053	**0.5990**	0.8020

According to $\hat{\lambda}_u$ in section IV which claim the upper tails of variables are independent, $\hat{\lambda}_u \geq 0.$

Conclusions

The combination, in this article, between extreme value theory and extreme value copulas to make analysis on the extremely dependence for the selected data obtained from the foreign exchange market. The estimation of the GEV distribution for monthly maxima and the GPD for daily maxima using MLEapproach. Then the calibration of copula functions to recover the tail probability distribution and tail dependence properties by comparing different model of extreme value copula. The result shows that EV copula whichare selected, are all suitable copulas that have the desired property to measure tail probability and tail dependence of empirical financial management variables.

Finally, the application of multivariate EVT to the field of financial management and related fields are currently quite an active research topic and it provides a lot of opportunities for exploration.

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