The Motion of a Railway Axle

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The authors investigated the motion of a single axle and pair of wheels by means of a model, paying particular attention to the action of the flanges and to their influence on the development of "hunting". In justification of the use of a single axle, evidence is produced that, owing to the various clearances, the axles of a bogie are in fact largely free to follow their individual paths. The effects of unsprung mass, wear of tyres and rails, flange clearance, lateral stiffness of the track, and other variables were studied. It is shown from considerations of dynamical similarity that the speed at which oscillation of a given violence occurs varies approximately inversely as the square root of the unsprung mass, and that a reduction of this mass would therefore be beneficial. Possible changes in tyre profile are discussed.

Introduction. The lateral oscillation of railway vehicles has been the subject of many investigations, but the influence on the motion of the impacts between the flanges and the rails does not seem to have been adequately studied. The authors have investigated the effect of these impacts on the running of a single free axle, by means of analysis and model experiments. The results are considered with particular reference to bogie passenger vehicles.

The use of a single axle needs justification, since it is natural to assume that for lateral‡ and yawing‡ movements the axles and the frame of a bogie are one rigid body. There is plenty of evidence that this assumption is incorrect, and that the plays of the axleboxes in their guides, together with the end plays of the journals in their brasses, are sufficient to allow each axle to perform lateral and yawing oscillations almost as if it were independent.

It is well known that on straight and perfect track, and in the absence of action by the flanges, a single axle wanders in a sine-curve of wavelength $\lambda = 2\pi\sqrt{(rb/T)}$, where r is the tread radius of the wheels, $2b$ the gauge, and T the tangent of the coning angle of the treads. This will be called the "natural wavelength"; it is about 60 feet for an axle of a normal bogie with treads coned at 1 in 20, and may fall to about 30 feet when the tyres are badly worn. It can also be shown that a "rigid" four-wheeler, with no axlebox or journal plays, wanders in a sine-curve whose wavelength, for a normally proportioned bogie, is more than twice the natural wavelength of its axles (Carter 1916)§. This has been confirmed by experiments with models (Davies 1939). With few exceptions, the wavelengths recorded in full-size experiments are compatible with the natural wavelength of the individual axles, and not with the much longer wavelength of a rigid four-wheeler (Davies 1939, Newberry 1945). This seems conclusive.

The problem is also investigated mathematically in Appendix I, p. 433, on the assumption of ideal track, and the result is shown in Fig. 1. Provided that the relation connecting the wavelength of the axle motions and the phase angle between them lies in the unshaded part of the graph, the first interference with the natural motions comes from the flanges, and not from the axlebox guides. It will be seen that the available range of phase angle is 215 deg. with wavelengths of 60 feet, and 60 deg. with wavelengths of 30 feet. One would expect that the motions would somehow adjust their relationship to lie within this free range, and in fact it can be shown analytically, and confirmed by model experiment, that they do so (Davies 1939). Heumann (1937) seems to have reached these conclusions.

The motion of a real axle is of course much affected by the imperfections of the track, which constantly bring the flanges, and probably also the axlebox guides, into play. Mauzin (1933) found that high or low spots in one rail had a pronounced effect on the lateral oscillation. None the less, full-size records suggest that the natural motion of a single axle may well be the fundamental factor; it is hard to overlook the general agreement in wavelength. At any rate, there seems to be ample justification for attacking the problem in its simplest form—the motion of a single axle, particularly when the primary object is to study flange-rail impacts.

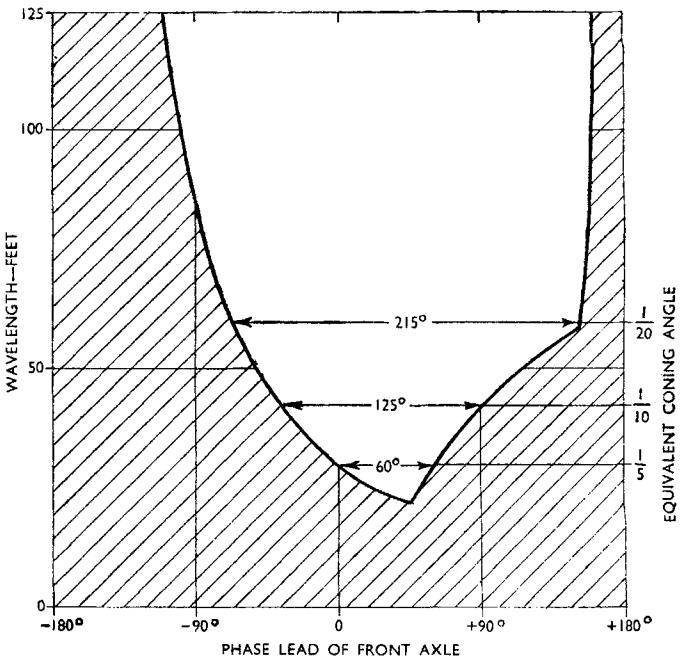

Fig. 1. Limitations imposed by the Axlebox and Bearing Plays on the Natural Motions of the Individual Axles of a Bogie

Tyre and Rail Profiles. Fig. 2*a* shows the profile of a new tyre on a new rail, the rail being shown in the two extreme positions relative to the wheel which are permitted by the flange clearance of $\frac{3}{4}$ inch. The corresponding points of tread-rail contact are A and B, and the effective tread radii are r_a and r_b. Clearly, when one wheel of a pair is running on radius r_a

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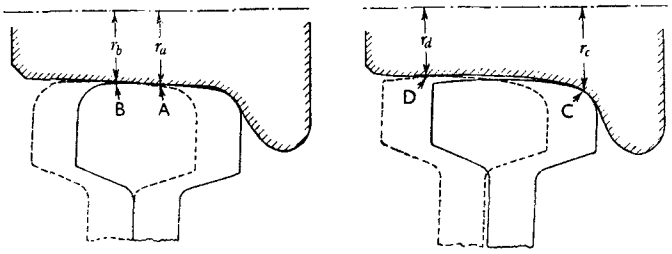
‡ For brevity, "fore-and-aft" and "longitudinal" are used to mean parallel to the rails, "lateral" to mean parallel to the sleepers, and "yaw" to mean angular movement about a vertical axis.

§ An alphabetical list of references is given in Appendix III, p. 434.

the other is running on radius r_b, and the maximum difference between the effective tread radii is $(r_a - r_b) = \frac{3}{4}/20 = 0{\cdot}038$ inch, assuming a coning angle of 1 in 20.

Fig. 2*b* shows a worn tyre on a worn rail; the profiles were selected from a number supplied by the London, Midland and Scottish Railway Company, and seem to be representative. Whereas with new profiles the position of the point of contact



a New profiles. *b* Worn profiles.

Fig. 2. Cross-sections of Full-size Tyres and Rail Heads

on the *rail* hardly moved, with these worn profiles, owing to the concavity of the tread, it moves nearly 2 inches between its extreme positions C and D. The point of contact on the *tread* therefore moves more than $2\frac{1}{2}$ inches laterally, and the maximum difference between the effective tread radii $(r_c - r_d)$, is much greater than $(r_a - r_b)$. This seems to be the most important effect of wear, that it increases the maximum difference between the effective radii of a pair of wheels, and so increases the centering action and reduces the natural wavelength; it is well known that worn wheels oscillate with shorter wavelengths than unworn. Clearly, even for a given pair of wheels, the positions of C and D, and therefore the value of $(r_c - r_d)$, depend largely on the rail profile; this suggests one reason why hunting is spasmodic.

The figures show another effect of wear. Whereas with new profiles there is two-point contact between wheel and rail while a flange is in action, with worn profiles, owing to the increased radius at the root of the flange, there is commonly single-point contact. The actual point of contact depends on the lateral flange force, and as speed increases, and the motion becomes more violent, the tendency is for the point to move farther down the flange; that is $(r_c - r_d)$ increases with speed.

The Action of the Flanges. Some useful conclusions can be drawn without either mathematics or experiment. Consider an axle rolling "east" at a speed V, and let the flange of the "north"

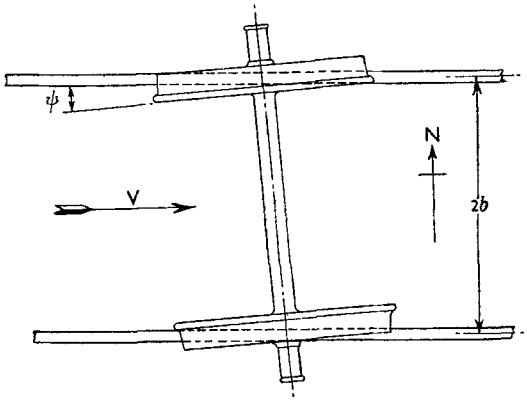


Fig. 3. An Axle during Flange Contact

wheel be in contact with the rail (Fig. 3). Let the angle between the plane of the wheel and the rail be ψ, and let its value when the flange first hit the rail have been Ψ, hereafter called the "angle of incidence"; the corresponding angle when the flange loses contact with the rail will be called the "angle of reflection".

If the angle of reflection at any flange impact is less than the angle of incidence, flange action is tending to damp out the lateral oscillation. If, however, the angle of reflection is equal to, or greater than, the angle of incidence, flange action is sustaining, or even increasing the lateral oscillation. It follows that, for hunting to persist, the axle must yaw during the period of flange contact so that the angle of incidence is destroyed and a definite angle of reflection is built up; this is true whether flange impact is due to natural oscillation or to track imperfections. The angle of yaw during flange contact depends on the rate of yaw in radians per unit distance travelled and on the distance for which flange contact lasts.

The rate of yaw is, by simple geometry, $h/2br$, where h is the difference between the effective radii of the wheels. The effective radius of the south wheel is its least tread radius (r_b or r_d of Fig. 2); that of the north wheel is greater than its greatest tread radius, since this wheel is running partly on its flange in the case of two-point contact, or on the root of its flange if there is single-point contact (Fig. 2). Thus h may well exceed $\frac{1}{8}$ inch.

In the absence of inertia forces, lateral friction at the treads maintains flange contact while ψ is positive, and causes separation when ψ becomes negative. Flange contact therefore ceases when the axle is square with the track; pure rolling is then resumed at the treads and the north wheel leaves the rail tangentially. So at low speed the flanges steady the motion, because, whatever the angle of incidence, the angle of reflection is zero.

At high speed the lateral momentum may be sufficient to maintain flange contact after ψ has become negative, and thus allow an angle of reflection to be built up. When the flange hits the rail the lateral momentum is $MV\Psi$, where M is the mass of the wheels and axle. This momentum is not instantly destroyed, but is absorbed and then in part reversed by the elasticity of the rail, which acts as a lateral spring. It is convenient to define the stiffness of this spring by the frequency N with which the mass M would oscillate if attached to it; this does not imply any idea of resonant oscillation. Of course the rail is not a perfect spring; its deflexion under lateral load is the result of its own bending and twisting, of the compression of the keys, and no doubt of sleeper and ballast movements. The value of N must vary from place to place, but is probably between 10 and 20 periods per second; lower values imply improbably large displacements, while higher values are incompatible with the known stiffness of the rail in bending.

When a heavy mass charges into a light spring the combined system performs half an oscillation before separating. So, neglecting lateral friction at the treads in comparison with the inertia force, flange contact would last for $1/2N$ seconds, and the angle of yaw would then be $(V/2N) \times (h/2br)$ radians. On inserting in this expression likely figures for worn wheels at high speeds, an angle of the order of 0·01 radians is obtained, which is comparable with the maximum angle of yaw geometrically possible with standard flange and axlebox clearances.

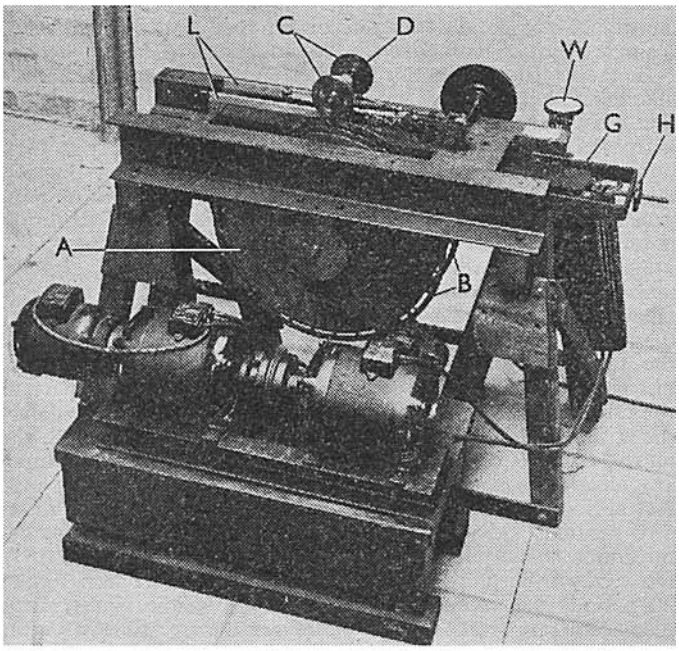
The Model. The model (Fig. 4) was one-tenth full-size, the wheel diameter being 4 inches. For the sake of compactness the rails were circular rings, mounted on a drum whose spindle turned in fixed bearings. The wheels rested on the rails, and the wheel-and-axle unit had every degree of freedom except that of fore-and-aft translation. The arrangement was equivalent to keeping the axle stationary, and moving the track under it.

The drum A was driven from a Ward-Leonard set, the tachometer W indicating the speed. The rail rings B had an outer diameter of $23\frac{3}{4}$ inches, a width of $\frac{7}{32}$ inch, and a depth of $\frac{9}{16}$ inch; they were attached to the drum by short spokes of spring steel, spaced, to scale, at sleeper pitch. The axle unit C ran in ball bearings in the block D. A downward extension of D formed the journal of a self-aligning bearing, whose housing E was mounted on the table F. The sprung load was applied vertically to the bottom of the extension of D by a cord which passed round a pulley to the spring balance G and the hand-wheel H. The table F was attached to the frame by a parallel motion consisting of two arms L, hinged at their ends by crossed steel strips to posts on the frame and on the table; the axle therefore had lateral freedom. The bearing in E allowed the axle to yaw; also, being self-aligning, it equalized the wheel loads.

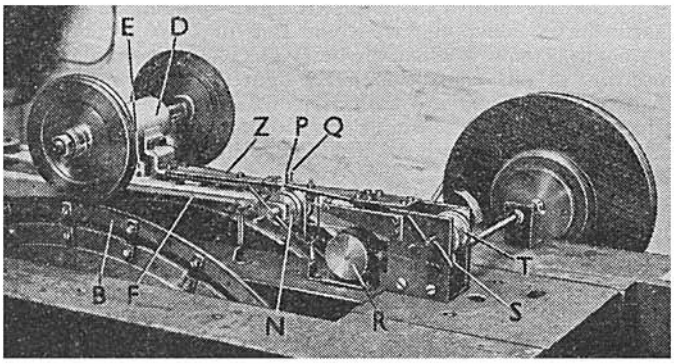
The table F carried the roller N, over which passed a strip

of uncoated film; the film thus translated with the wheel-and-axle unit, but did not yaw. A fixed style P recorded the lateral translation of the axle. A style Q, mounted on an arm Z attached to block D, traversed the film as the axle yawed, thus tracing a curve whose ordinate gave the angle of yaw. The film was drawn from the magazine R by the roller S, which was driven from the drum. A single control lowered the idle roller T, and thus set the film in motion by pressing it against the continuously revolving roller S, and also raised the roller N to bring the film into contact with the styles.

Four pairs of wheels were used in the main tests. Owing to an oversight in proportioning the model, the standard coning of



a General view.



b Axle unit and recording gear.

Fig. 4. Model Used in the Experiments

1 in 20 could not be used, because the natural wavelength was then so close to the circumference of the rails that the oscillation of the axle unit was drawn into step with the revolutions of the drum. The "new" wheels were therefore made with treads coned at 1 in 15. They were provided with three interchangeable sets of flanges, having coning angles of 50 deg., 60 deg., and 70 deg.

The reproduction of worn profiles one-tenth full size was difficult and expensive, and only a few could be made with the resources available. The first pair of worn tyres was ground so hollow that the effect of reversed coning was produced, and the axle ran hard over to either side. Two other pairs of worn tyre profiles, W_1 and W_2, and two of rail profiles, R_1 and R_2, were made; they are shown in Fig. 5. One important effect of wear, the increase in the maximum difference between the effective radii, can be reproduced by the simple expedient of increasing the coning angle; and the authors felt that the use of this expedient might give the best indication of the general behaviour of worn wheels, as distinct from the idiosyncrasies of particular profiles. Accordingly, a pair of wheels with treads coned at 1 in 5 was provided. With all wheels the flange clearance could be varied by inserting washers between the hubs and the shoulders of the axle.

The effective mass M and the principal radii of gyration of the axle unit could be varied independently by fixing various combinations of weights to the wheels and to the block D. Table 1 gives the data for the arrangements used; k is the radius of gyration about the axis of the axle, and K about a perpendicular through its mid-point. The dimensionless ratios

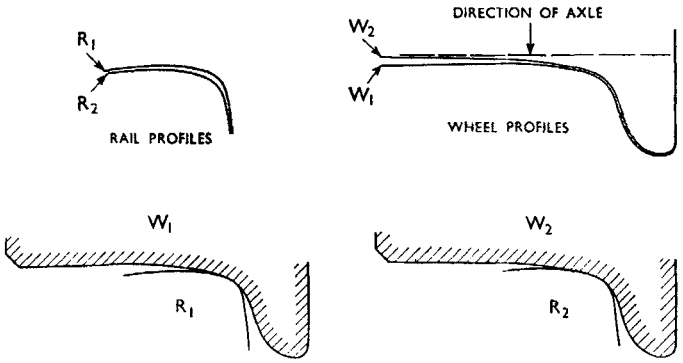


Fig. 5. Worn Profiles used on Model

k/r and K/b are used for comparison with full-size conditions. The value of N for each value of M was found by direct measurement.

TABLE 1. DATA FOR VARIOUS ARRANGEMENTS OF APPARATUS DURING TESTS

Arrangement	Mass M, lb.	Ratio k/r	Ratio K/b	Frequency N, periods per sec.	Remarks
(*a*)	4·0	0·53	0·86	34·4	Without added weights
(*b*)	7·0	0·48	0·85	26·5	With various added weights
(*c*)	6·0	0·43	0·75	28·3	
(*d*)	6·0	0·60	1·04	28·3	
Full size	3,000*	0·7	1·11	15?	Axle of a typical bogie

* Reduced to scale, $3{,}000 \times (1/10)^3 = 3$ lb.

The variables in the experiments were: the dynamical properties of the axle unit, specified by arrangement (*a*), (*b*), (*c*), or (*d*) of Table 1; the sprung load, specified by the ratio n of total to unsprung load; the profiles; the coefficient of friction between the wheels and the rails; the flange clearance p; and the speed V. It is shown later, using an estimated value for the lateral stiffness of full-size track, that arrangement (*d*) with $n = 2$ should give dynamical similarity with full-size conditions at a speed scale of one-fifth; this combination was therefore extensively used.

The coefficient of friction was measured, in what seemed to be the only practicable way, by applying a known vertical load to the axle, locking the wheels against rotation, and measuring the torque required to start the rail drum turning. The nominal coefficient thus obtained will be called μ; its value is recorded amongst the data for each test, but it is important to realize its limitations. It is a measure of the coefficient of longitudinal static friction at the treads. But in actual running there is sliding, superimposed on rolling, and with transverse and longitudinal

components; further, there is no reason to assume that the coefficient at the flange is the same as at the tread.

With clean dry surfaces the value of μ was about 0·3 when the wheels were new, and increased to about 0·6 when they were fully run in. It could, of course, be reduced by lubrication. It often changed rapidly and erratically.

Inability to measure the true coefficient was a major experimental difficulty. First, it threw doubt on comparisons of records taken at different times, or with different wheels; and, in particular, made it impossible to repeat any test with the certainty that conditions were as before. Second, even in a single test, there was the danger that unsuspected changes in the coefficient might mask the effect of changes made in whatever other variable was being investigated. Whenever possible, all the records required for a particular comparison were taken quickly, with measurements of μ before and after; and if μ had not changed significantly in the interval, it is reasonable to assume that the true coefficient had not done so either. This technique probably overcame the second difficulty, but it was limited to comparisons which did not involve a change of wheels.

The following were the principal differences between model and full-size conditions:—

(i) The model consisted of a single axle carrying its appropriate sprung load. This point has already been discussed.

(ii) The model track had to be considerably truer than ordinary full-size track; a series of kinks repeating at regular intervals of one revolution of the drum would have been even more unrealistic. In studying individual flange impacts this is irrelevant, but in long records it exaggerates the part played by the natural oscillation, or at least shows it in an unnaturally pure form.

(iii) Unlike real rails, the model rails were almost perfect springs; deflexion was proportional to the lateral force, and damping was negligible. This would affect the details of the motion, but hardly its character.

(iv) The model rails were rings instead of straight lines. It is hard to see how this could seriously affect the motion; the diameter of the rings was six times that of the wheels.

The Records. The films were projected in an optical enlarger for inspection and tracing. Some records of the path, made by the style P, are reproduced in Figs. 6, 7, and 8; their slopes are magnified about 125 times because of the difference between the longitudinal and transverse scales. No records made by the style Q are reproduced; their ordinates are a measure of the yawing of the axle, and much information was obtained from them, but they convey little to visual inspection because the eye cannot readily interpret an ordinate as an angle. Two sorts of record were made: (i) short records, each covering a single flange-rail contact at a controlled angle of incidence; (ii) long records, covering many impacts, with the axle oscillating in the manner, and with the angles of incidence, dictated by the speed and the other variables.

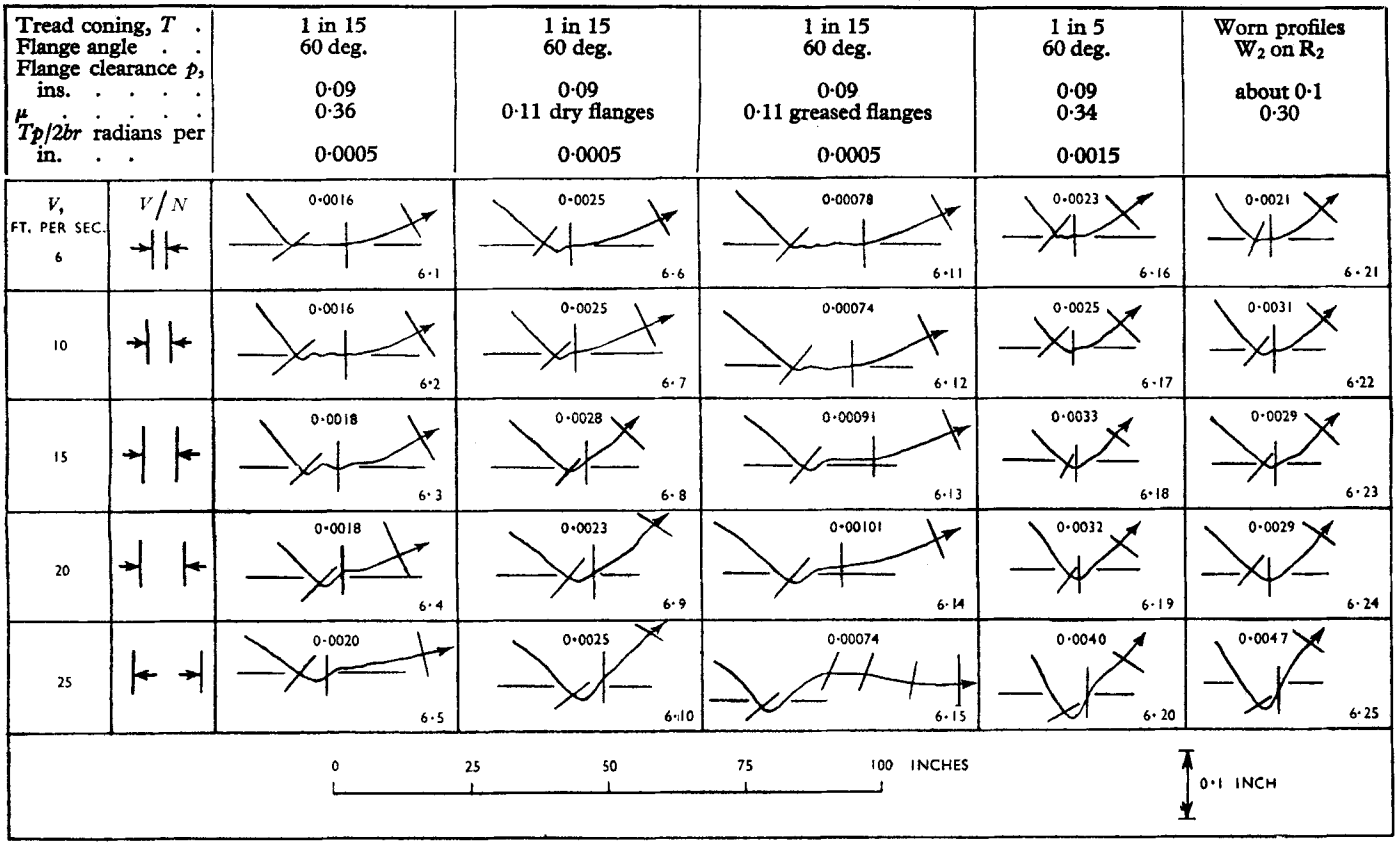


Fig. 6. Records of Flange Contact

Arrangement (*d*): $n = 2$.

Note.—The figure at the top of each panel is the actual maximum rate of yaw, in radians per inch, obtained from the maximum slope of the yaw record.

Representative short records are shown in Fig. 6. The strokes across the path indicate the direction of the axle as deduced from the yaw records; the broken line marks what would be the boundary of the path if the rails were rigid. An attempt was made to keep the angle of incidence constant, but it was difficult to control. The speed and the space-wavelength V/N for a complete lateral oscillation on one rail as a spring are given on the left of each row. The profiles, flange clearance and value of μ are given at the head of each column. With treads of coning T the difference between the tread radii during flange contact is Tp, and if there were no longitudinal slip at the treads the corresponding rate of yaw would be $Tp/2br$; this quantity is given at the head of the first four columns. The figure at the top of each panel is the actual maximum rate of yaw, as obtained from the maximum slope of the yaw record.

It will be seen that at low speed the flange hugs the rail until the axle is square with the track, and then runs off tangentially; the rail is not much displaced, but slight oscillation of wavelength about V/N is sometimes noticeable. The actual rate of yaw is increased by wear, by increasing the coning angle, and by increasing the coefficient of friction at the flange relative to that at the tread. All these effects therefore reduce the length of flange contact; this is because they increase the difference between the effective radii of the wheels. The effect of increasing the angle of incidence without changing the other variables is not shown in Fig. 6; it increases the length of flange contact less than might be expected, because the increased flange pressure increases the rate of yaw.

As the speed increases, the deflexion of the rail becomes more and more pronounced, while the oscillation is reduced to a single half-cycle owing to the increase in the length V/N. Variations in the deflexion at a given speed are due to differences in the angles of incidence.

The actual rate of yaw always exceeds $Tp/2br$; clearly, the difference between the effective radii is greater than Tp, both because the deflexion of the rail increases the effective value of p, and because one wheel is running partly on its flange. In general, the rate of yaw increases with the speed because of the increased flange pressure and rail deflexion; but at the higher speeds with a low coefficient of friction the rate falls off. The explanation probably is that, whereas a given rate of yaw per unit distance travelled implies an angular acceleration of yaw proportional to the square of the speed, the angular acceleration is in fact limited by the frictional forces available, and these forces are small because lateral slip is occurring simultaneously. The deceleration of yaw is similarly limited, and the relation between the orientation of the axle and its path at the higher speeds shows that the axle continues to skid round, and to skid laterally, after flange contact has ceased.

Apart from the effects of skidding, which are important, the records agree well with the theory outlined earlier. In particular, the records for the 1 in 5 coning are very similar to those for the worn profiles.

Figs. 7 and 8 show portions of long records, for speeds ranging from the lowest at which there was sustained oscillation up to the highest which seemed safe.

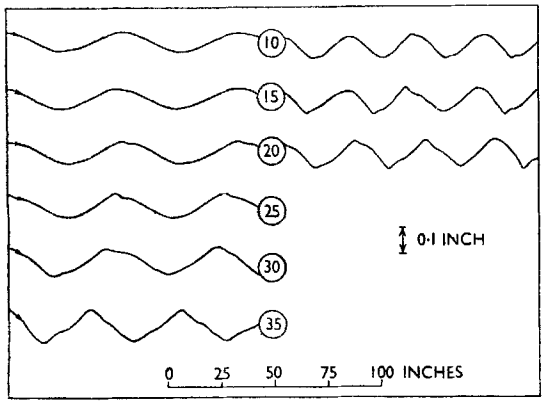
With 1 in 15 coning, the path at 10 ft. per sec. is entirely governed by the treads, with no sign of flange contact. As the speed rises the amplitude of the motion builds up, owing to creep, and at 20 ft. per sec. the left-hand flange is in action, but there is no appreciable angle of reflection. The right-hand flange does not come into action until 30 ft. per sec.; this pair of wheels had a bias to the left. At 30 ft. per sec. angles of reflection are built up, but they are not as large as the angles of incidence. At 35 ft. per sec. the angles of incidence and reflection are about equal, the path is a sharp zig-zag from rail to rail, and the wavelength is consequently shortened; the deflexion of the rails considerably increases the amplitude of the motion.

The records for the 1 in 5 coning show a shorter wavelength, in agreement with theory. The motion passes through much the same stages as the speed increases, but at any given speed it is much more violent; for instance, the motion at 20 ft. per sec. is very similar to that of the 1 in 15 coning at 35 ft. per sec. Earlier tests with 1 in 10 coning gave results intermediate between 1 in 5 and 1 in 15.

Fig. 8 shows records for three combinations of the worn wheel and rail profiles of Fig. 5. The wavelengths are greater than with 1 in 5 coning, but considerably less than with 1 in 15. Each combination gave a distinctive path, the same tyre profile running differently on different rail profiles; some of the records in the middle are quite unlike those on their right and left, although they have one profile in common with each. This confirms that the running of worn wheels depends more on how the wheel profiles fit the rails than on their precise shape; the records for the best-fitting profiles, W_2 on R_2, show the most violent motion.

Hundreds of long records were taken, and some means of comparing them, more definite and convenient than visual inspection, was necessary. Two criteria were chosen, and were plotted against speed from the records of each test. The mean wavelength of the motion was an obvious criterion, for the shorter the wavelength the rougher the motion. The second criterion will be called the "mean yaw"; it is the average value of the angle between successive extreme orientations of the axle, two such extreme orientations occurring, of course, in each wavelength; it was obtained from the mean amplitude of the yaw record.

Dynamical Similarity. It is shown in Appendix II, p. 433, that if two free axles are geometrically similar they will, in theory, move in similar paths, when the following three quantities have the same values for both: the coefficient of friction, and the quantities N^2L/n, and V/NL, where L, any representative linear dimension, is a measure of the scale. An important practical conclusion follows. Suppose that, for a given axle on given track, some mass could be transferred from "unsprung" to "sprung", reducing the unsprung mass from M to M/q, and



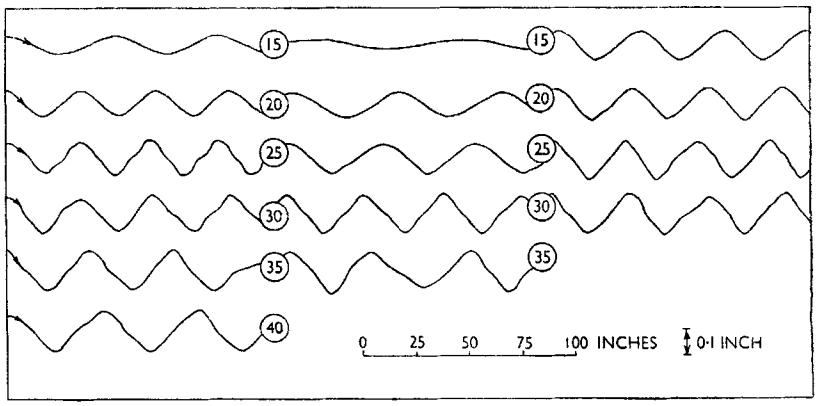
Treads, 1 in 15. Flanges, 60 deg. Flange clearance, 0·09 in. μ about 0·28.

Treads, 1 in 5. Flanges, 60 deg. Flange clearance, 0·09 in. μ about 0·20.

Fig. 7. Axle Paths with Coned Treads

Arrangement (d): $n = 2$.

The circled figures give the speeds in feet per second.



W_1 on R_1 W_1 on R_2. W_2 on R_2.

Fig. 8. Axle Paths with Worn Profiles

Arrangement (d): $n = 2$; μ about 0·27; flange clearance about 0·1 inch.

The circled figures give the speeds in feet per second.

leaving all else unchanged. Then n would be increased to nq and, the stiffness of the track being unchanged, N would be increased to $N\sqrt{q}$; thus both n and N^2 would be increased in the ratio q and, L being constant, N^2L/n would be unchanged. Then the speed for any definite path would be increased in the same ratio as N, that is, $\sqrt{q}$ times. Thus, if half the unsprung mass could be sprung, the speed at which oscillation of a definite violence occurred would be increased by a factor of $\sqrt{2}$. The argument does not assume straight and perfect track or natural oscillation under the influence of the coning.

This conclusion seemed so important that some of the first experiments were directed to verifying it. Arrangements (*a*) and (*b*), with the same wheels, are almost geometrically similar, and L is the same for both. So, using the actual measured values of N from Table 1, p. 428, N^2L/n will be the same if $n_b = (N_b/N_a)^2\ n_a = (26{\cdot}5/34{\cdot}4)^2\ n_a = 0{\cdot}592\ n_a$. Convenient settings of the spring balance were $n_a = 4$ and $n_b = 2{\cdot}37$. If the tests are made in quick succession, the coefficient of friction should be the same for both arrangements, and they should then give identical records at corresponding speeds given by $V_a/V_b = N_a/N_b = 1{\cdot}3$. The test was therefore to compare the speed-wavelength and speed-yaw curves, the actual speeds of (*b*) being scaled up 30 per cent before plotting. This test was made under various conditions; and Fig. 9 shows a typical result. The evidence seems convincing; the 30 per cent speed correction brings the (*b*) points on to the (*a*) curve in a most satisfactory way.

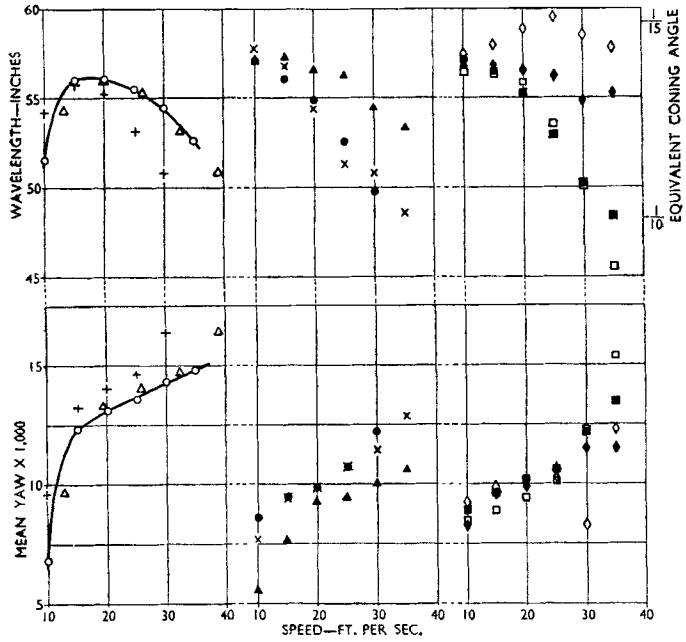


Fig. 9 Fig. 10 Fig. 11

Fig. 9. Check on Dynamical Similarity

Treads, 1 in 15; flanges, 60 deg.; flange clearance, 0·12 inch; μ about 0·47.

○ Arrangement (*a*): $N = 34{\cdot}4$; $n = 4$.
\+ ,, (*b*): $N = 26{\cdot}5$; $n = 2{\cdot}37$.
△ As for +, but recorded speeds scaled up in ratio 34·4/26·5.

Fig. 10. The Effect of N and n

Treads, 1 in 15; flanges, 60 deg.; flange clearance, 0·09 inch; μ about 0·48.

● Arrangement (*b*): $N = 26{\cdot}5$; $n = 2$.
× ,, (*a*): $N = 34{\cdot}4$; $n = 2$.
▲ ,, (*a*): $N = 34{\cdot}4$; $n = 4$.

Fig. 11. The Effect of μ

Treads, 1 in 15; flanges, 60 deg.; arrangement (*d*), $n = 2$; flange clearance, 0·09 inch.

■ μ about 0·48.
□ μ about 0·30.
◇ As □, but flanges greased.
◆ about 0·12.

Effect of N and n. Since the running was improved by increasing both N and n in a certain relationship, it was natural to investigate the effect of increasing them separately. This test too was made under various conditions, and Fig. 10 shows typical results. Evidently, increasing N alone makes no difference, outside the limits of experimental error; whereas increasing n alone makes a distinct improvement, since it increases the wavelength and decreases the mean yaw at a given speed. It seems, therefore, that a reduction in the unsprung mass is beneficial because it increases n; and that an increase in N obtained by using stiffer track would do little good.

Effect of the Coefficient of Friction. This has already been mentioned in the remarks on the short records. Fig. 11 shows the speed-wavelength and speed-yaw relations for various values of μ, the nominal coefficient of friction. It will be seen that at low speeds the value of μ makes little difference, but that at high speeds a low value, and particularly greasing the flanges, improves the running. Path records showed that with $\mu = 0{\cdot}12$ the motion at high speed was rather irregular; with a low coefficient of friction at the treads some random sliding about is to be expected. Liberal oiling of both rails produced the curious effect that at times the wheels could be made to run hard over to either side at will.

When one rail was lubricated and the other dry, the axle had a pronounced bias towards the dry rail. A possible explanation is that the frictional couple at the axle journals must be overcome by the moment of the backward forces at the wheel treads. The force at the dry rail tends to be greater than that at the lubricated rail, and there is thus a couple about a vertical axis tending to turn the axle towards the dry rail. This explanation is supported by indirect evidence; in a test with one wheel free on the axle, the fixed wheel ran hard against its rail.

Effect of Principal Radii of Gyration. Arrangements (*c*) and (*d*) differ greatly in their radii of gyration, but in no other way. Comparative tests under various conditions showed no significant difference between their behaviour.

Effect of Flange Angle. Comparative tests were made with 50 deg., 60 deg., and 70 deg. flanges, in combination with both 1 in 15 and 1 in 5 coned treads. The speed-wavelength and speed-yaw relations obtained show that the flange angle, within the range tested, had no consistent or significant effect on the motion; this confirms the impression gained during the tests. Flange climbing was often the factor which limited the safe speed, but it was not appreciably more pronounced with one flange angle than with another.

Effect of Wear. The axle paths with the three combinations of worn profiles, W_1 on R_1, W_1 on R_2, and W_2 on R_2, have already been considered. Fig. 12* shows the speed-wavelength and speed-yaw relations for the same combinations. It will be seen that, judged by these criteria, W_2 on R_2 gave the most violent oscillation, and W_1 on R_2 the least violent. On comparing Fig. 12 with Fig. 13, which gives representative results for 1 in 15 coned wheels at about the same coefficient of friction,

* In Figs. 12, 13, and 14 the ordinates are plotted to a much smaller scale than in Figs. 9, 10, and 11.

the differences are striking. As the speed rises, the wavelength for all three combinations of worn profiles at first falls and then rises again, whereas with the 1 in 15 coned wheels it generally falls at an increasing rate throughout the range.

For the worn combinations, with one exception, as the speed rises the yaw at first increases and then diminishes, whereas with the 1 in 15 coned wheels it generally increases at an increasing rate throughout the range. These effects are not really peculiar to worn profiles; at high speeds, and with a low coefficient of friction, they occurred with 1 in 5 coned wheels. They probably imply that the angular acceleration of yaw is being limited by the frictional forces, and it is likely that they would have occurred with 1 in 15 coned wheels, had sufficiently high speeds been attainable.

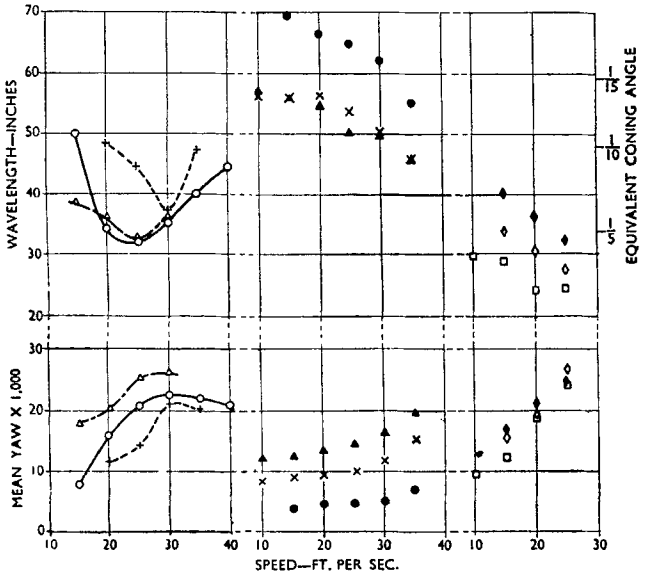


Fig. 12 Fig. 13 Fig. 14

Fig. 12. The Effect of Wear

Arrangement (*d*): $n = 2$; flange clearance about 0·1 inch; μ about 0·26.

○ W_1 on R_1.
\+ W_1 on R_2.
△ W_2 on R_2.

Fig. 13. The Effect of Flange Clearance with Coned Treads

Treads, 1 in 15; flanges 60 deg.; arrangement (*d*): $n = 2$; μ about 0·3.

● Flange clearance 0·06 inch.
× Flange clearance 0·09 inch.
▲ Flange clearance 0·12 inch.

Fig. 14. The Effect of Flange Clearance with Worn Profiles (W_2 on R_2)

Arrangement (*d*): $n = 2$; μ about 0·34.

□ Flange clearance 0·06 inch.
◇ Flange clearance 0·09 inch.
♦ Flange clearance 0·12 inch.

Effect of Flange Clearance. Fig. 13 shows the speed-wavelength and speed-yaw relations for 1 in 15 coned wheels with flange clearances corresponding to 0·6, 0·9, and 1·2 inches full size, a range which should cover practical conditions. It will be seen that increasing the flange clearance increases the yaw, presumably because, with coned wheels, it increases the maximum difference between the radii. The long wavelengths recorded with the smallest flange clearance are artificial; the imperfections of the track were then relatively appreciable, and their regular repetition tended to draw the oscillation into step with the revolutions of the drum. When the flange clearance was reduced still further, the model ran very steadily, showing little tendency to oscillate, even within the limits still permitted, but following closely the slight imperfections in the lateral alignment of the track. It is likely that, under full-size conditions also, the smaller the flange clearance the less violent the oscillation but the greater the influence of track imperfections on the motion.

Fig. 14 shows the speed-wavelength and speed-yaw relations for worn wheels of profile W_2 on rails of profile R_2, with the same flange clearances as in Fig. 13. Increasing the flange clearance still increases the yaw, but much less than with the coned wheels. The wavelength, on the other hand, increases markedly with each increase of flange clearance, which is the opposite of the effect with coned wheels. The explanation probably is that with a small clearance the worn wheels run entirely on the inner and steeply coned portions of their profiles. No other combination of worn profiles was tested for the effect of flange clearance, and too much importance must not therefore be attached to these results.

Conclusions. Apart from confirming well-known results, the investigation suggests the following conclusions.

(1) Other conditions being the same, the speed at which oscillation of any given type occurs varies approximately inversely as the square root of the unsprung mass of the axle unit. For this reason, and also to reduce maintenance costs on the track in general and on the rail joints in particular, the reduction of unsprung weights seems very desirable.

(2) A change in the lateral stiffness of the track has little effect.

(3) With new wheels, the violence of the oscillation increases markedly with the flange clearance. It might be worth trying the effect of reducing the flange clearance of a full-size vehicle. The flange angle, within practical limits, has little effect on the character of the motion.

(4) Some improvement in the running was produced by greasing the flanges. The effect of using a flange-greaser on straight track could easily be investigated.

(5) Not enough worn profiles were tested to justify more than tentative conclusions, but apparently it is the closeness of fit between wheel and rail profiles, rather than the precise shape of the wheel profiles, which governs the violence of the oscillation; this explains why, in practice, hunting is spasmodic. Increasing the coning angle of the treads approximately reproduces the effect of wear.

Possible Changes in Tyre Profile. Since wear can only occur where there is contact, it must tend to produce, and then maintain, tyre and rail profiles such that the whole width of the rail surface is used. Further, since the tyre is wider than the rail, the tyre profile becomes concave and the rail profile remains convex. Owing to the lateral motion of the axle, such profiles cannot fit exactly in any one position, but they will tend to be such that, as the axle traverses across the track, the contact areas traverse across the width of the rail surfaces in the opposite direction, as shown in Fig. 2*b*.

If the proportions of new, part-worn, and badly worn tyres in operation do not vary much from time to time, it may be expected that as a rail wears down it soon reaches a definite profile (generated by the "average tyre"), which profile it thereafter retains. This profile will not be constant from place to place, because the conditions of wear will be affected by curves, small errors of gauge, etc. (similarly, one would expect the minor imperfections of the vehicles to cause their wheels to wear to slightly different profiles). Any change in the coning angle of all new tyres must lead to a corresponding change in the profile of worn rails, that is, of almost all rails in service. The trouble is that wheels and rails inevitably wear so that their profiles nearly fit, and that, starting from any practicable profiles, this leads to a large difference between the effective radii during flange contact.

This effect of wear cannot be avoided, but it might possibly be delayed. For instance, it has been suggested that the present 1 in 20 coning on all four wheels of a bogie might be replaced by 1 in 10 coning on diagonally opposite wheels, and cylindrical treads on the other two. Such an arrangement would be just as effective in accommodating the motion of the axles to curves in the track; and even if it were universally adopted, the combined effect of the two different tyre profiles should leave the worn rail profile much as at present, while the amount of wear

needed to bring either tyre profile to a near fit with the rail profile would be increased. A wheel-pair consisting of one cylindrical wheel and one coned at 1 in 7·5 was tested on the model. Its motion was generally similar to that of wheels coned at 1 in 15, but was rather more violent. However, the tests were not very extensive, and the differences in the motion may well have been due to differences in the effective coefficient of friction.

Alternatively, a small groove might be turned at the root of the flange of the present 1 in 20 profile, so as to remove in advance the offending part of the worn profile. But there may be practical objections to this, and in any case the effect on the worn rail profile would have to be carefully investigated.

Acknowledgements. The experimental work was carried out in the Engineering Laboratory of the University of Cambridge, with help and financial support from the London, Midland and Scottish Railway Company. The authors wish to express their thanks to that Company, and also to the Cambridge Instrument Company for advice about recording on celluloid.

APPENDIX I

AXLEBOX AND BEARING PLAYS

Let the yawing play of each axle relative to the bogie frame be $\pm\beta$; it is due to the fore-and-aft play of the axleboxes in their guides. Let the lateral play of each axle relative to the frame be $\pm d$; it is made up of the lateral play of the axleboxes in their guides and the end play of the journals in their brasses. Let the wheelbase be $2c$, and let $d/c = \gamma$.

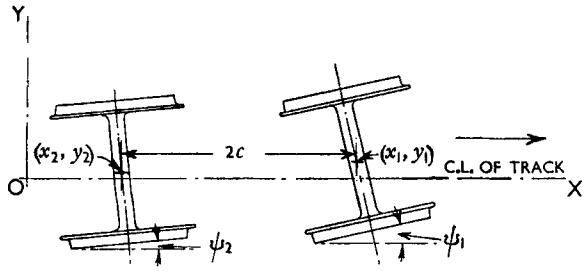


Fig. 15. Diagram of the Axles of a Bogie

Assume that in their mid-positions the axles are parallel and in track. In Fig. 15 the centre of the front axle is at (x_1, y_1) and the front wheels make an angle ψ_1 with OX, the centre-line of the track; the corresponding quantities for the back axle are x_2, y_2, and ψ_2. The track is assumed to be straight and perfect, and y_1, y_2, ψ_1, and ψ_2 are all small. Then the lateral displacements of the axles are subject to the conditions

$$|(y_1-2c\psi_1)-y_2|<2c\beta+2d$$

that is $$|y_1-y_2-2c\psi_1|<2c(\beta+\gamma) \quad . \; . \; . \; . \quad (1a)$$

and $$|y_1-y_2-2c\psi_2|<2c(\beta+\gamma) \quad . \; . \; . \; . \quad (1b)$$

The angular displacements are subject to the condition

$$|\psi_1-\psi_2|<2\beta \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

If the axles are rolling freely, within the limits imposed by the plays and without interference from the flanges, they may be assumed to move in sine-curves of the same wavelength λ. Then, expressing both motions in terms of x_1,

$$\left.\begin{aligned} y_1 &= A_1 \sin\frac{2\pi x_1}{\lambda} \\ y_2 &= A_2 \sin\left(\frac{2\pi x_1}{\lambda}-B\right) \\ \psi_1 &= \frac{dy_1}{dx_1} = A_1\frac{2\pi}{\lambda}\cos\frac{2\pi x_1}{\lambda} \\ \psi_2 &= \frac{dy_2}{dx_1} = A_2\frac{2\pi}{\lambda}\cos\left(\frac{2\pi x_1}{\lambda}-B\right) \end{aligned}\right\} \quad . \; . \; . \quad (3)$$

where A_1 and A_2 are the amplitudes of the motions, and B is the angle by which the motion of the back axle lags in phase behind that of the front axle.

To investigate the conditions under which the flanges, and not the β and γ plays, limit the sinoidal motion of individual axles, assume that each axle is moving with the maximum amplitude allowed by the flanges, i.e. put $A_1 = A_2 = A =$(half the flange clearance). Then, substituting from (3) in (1a) and putting $A/2c = \alpha$,

$$\left|\sin\frac{2\pi x_1}{\lambda}-\sin\left(\frac{2\pi x_1}{\lambda}-B\right)-\frac{4\pi c}{\lambda}\cos\frac{2\pi x_1}{\lambda}\right|<\frac{\beta+\gamma}{\alpha} \quad . \; . \quad (4)$$

The left-hand side of (4) gives the vector diagram in Fig.16. For no interference by the β and γ plays throughout the cycle, the dotted vector PQ must be less than $(\beta+\gamma)/\alpha$, and the point Q must therefore lie within a circle of centre P and radius $(\beta+\gamma)/\alpha$; whence, by trigonometry from Fig. 16,

$$\sin B+\sqrt{\left(\frac{\beta+\gamma}{\alpha}\right)^2-(1-\cos B)^2}>\frac{4\pi c}{\lambda}$$

$$>\sin B-\sqrt{\left(\frac{\beta+\gamma}{\alpha}\right)^2-(1-\cos B)^2} \quad . \; . \; . \quad (5)$$

Condition (1b) gives a diagram which is a mirror image of Fig. 16, and therefore repeats condition (5).

Substituting from (3) in (2) we get

$$\left|\cos\frac{2\pi x_1}{\lambda}-\cos\left(\frac{2\pi x_1}{\lambda}-B\right)\right|<\frac{\lambda\beta}{2\pi c\alpha}$$

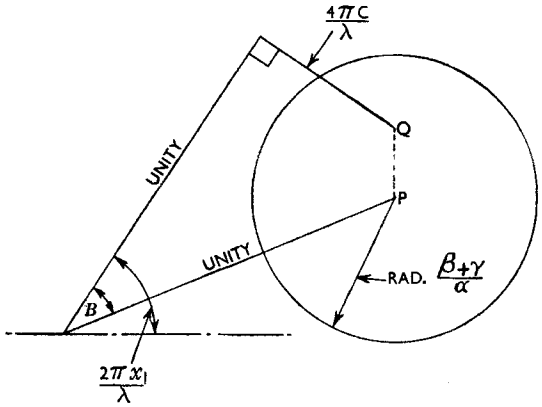


Fig. 16. Diagram of Vectors

which gives, as another condition for no interference by the β plays throughout the cycle,

$$\lambda>\frac{4\pi c\alpha}{\beta}\sin\frac{B}{2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Conditions (5) and (6) are the necessary and sufficient conditions for no interference by the β and γ plays with the sinoidal motions of individual axles moving on straight and perfect track with the maximum amplitude allowed by the flanges, assuming that within the positive constraints imposed by those plays the frame moves to conform with the motions of the axles.

Representative dimensions seem to be: wheelbase, 108 inches; flange clearance, $\frac{3}{4}$ inch; fore-and-aft and lateral plays of axleboxes in their guides, each $\pm\frac{1}{8}$ inch; lateral distance between axlebox guides, 76 inches; end play of journals in brasses, $\pm\frac{1}{16}$ inch. These give $\alpha = 0{\cdot}0035$, $\beta = 0{\cdot}0033$, $\gamma = 0{\cdot}0035$; and conditions (5) and (6) then give the curves in Fig. 1.

APPENDIX II

DYNAMICAL SIMILARITY

The paths of two free axles, (1) and (2), will be similar if the conditions of geometrical and dynamical similarity are fulfilled.

For geometrical similarity the relevant dimensions—gauge, wheel radius, tyre and rail profiles, principal radii of gyration, and position of the centre of gravity—must be to scale; this

scale is L_1/L_2, where L_1 and L_2 are corresponding dimensions. The masses will be proportional to L^3.

For dynamical similarity all horizontal forces must be in the same ratio, the ratio of the mass-accelerations. Since the radii of curvature of similar paths are proportional to L, the mass-accelerations are proportional to MV^2/L, that is to L^2V^2, where M is the mass of the wheels and axle, and V is the speed.

The horizontal forces are the lateral reactions of the rails, and the frictional forces. Unit deflexion of a rail corresponds to a force of $4\pi^2N^2M$, which is proportional to N^2L^3, where N is the frequency with which M would oscillate laterally on one rail as a spring; but for similar paths rail deflexion is proportional to L, so the force is proportional to N^2L^4. The frictional forces are proportional to the lateral reactions of the rails, to the axle load, and to the coefficient of friction. Let the axle load be nMg, which is proportional to nL^3, where n is the ratio of total to unsprung load.

Then the conditions for dynamical similarity are that the coefficients of friction should be equal, and that

$$n_1L_1^3/n_2L_2^3 = N_1^2L_1^4/N_2^2L_2^4 = L_1^2V_1^2/L_2^2V_2^2.$$

The first equation requires that N^2L/n should have the same values for axles (1) and (2). The second equation states that the paths will then be similar at corresponding speeds given by $V_1/V_2 = N_1L_1/N_2L_2$.

APPENDIX III

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Communications

Mr. W. E. GELSON, M.Sc. (Eng.), M.I.Mech.E., wrote that idealization was of use both in analytical and experimental work in railway mechanics, but important results were likely to be overlooked if it were carried too far.

The rear axle or axles of a truck, whether in a locomotive bogie, in a rigid wheelbase, or in coaching or goods stock, usually oscillated with greater violence than leading axles. That was an experimentally observed fact, attributable to the effect of the spring-borne mass and to wheel contact well into the root radius of the flange—a phenomenon made possible by the trammelling effect of the axlebox guides.

In most wheel-axle applications in railway practice, the superimposed sprung weight was applied in two planes eccentric to the track centre-line. One of its effects appeared in oscillation—as a couple, often of considerable amount, alternating at the frequency of the wheel-axle oscillation in the vertical plane transverse to the track.

Had the model been arranged to test a four-wheeled truck, supporting a mass through springs spaced to scale at appropriate journal centres, the results would, in his own opinion, have been of more practical value.

The effect of lateral track stiffness and sleeper pitch had been shown by full-size experiment to be important, and that might have been more clearly brought out in the authors' records, had the model been more representative of actual practice.

The authors' proposal to experiment with reduced flange clearances would, if carried out, produce considerably improved running. Reduced clearance, accompanied by a necessary corresponding reduction in distance between tyres (to avoid damage to crossing noses) had been found by test in India to reduce lateral forces appreciably. Thicker flange profiles had therefore been specified for carrying wheels of all Indian passenger locomotives commissioned since the 1939–45 war. That feature was incorporated in the design of passenger locomotives some years ago on one of the French railways, he believed.

Some caution seemed desirable when experimenting with flange grease. While the effect was to minimize the danger of derailment by climbing, an unstable vehicle, such as a 0–6–0 locomotive travelling at 60 m.p.h. or more on poorly maintained track, would be expected to have an increased tendency to derail by sliding on the lubricated surface.

Mr. W. S. GRAFF-BAKER, B.Sc. (Eng.), M.I.Mech.E., wrote that the problem was of practical importance. The aspects of the problem which had been studied were only part of a broader one, and the value of the paper would probably have been enhanced had it been extended to include the effect of curvature of the track upon the motion of the axle. He urged the authors to go forward with their investigations, so ably carried out.

The paper dealt with the problem as it affected "trailing" wheels and axles, and factors introduced by a motored drive, as in electric traction, had been disregarded.

Railway engineers had for some time been aware of the significance of the unsprung weight, and a good deal of work had been and was being directed towards its reduction. Major changes in the design of the wheel and axle assembly seemed to be indicated if an improvement in that respect was to be effected. The axle unit dimensions were largely governed by strength considerations and any substantial reduction in the weight of the components, using the conventional materials, was unlikely. The authors stated that, at the speeds tested, an increase in the lateral stiffness of the track did not increase the wavelength nor reduce the mean yaw of the axle unit. Railway mechanical engineers had assumed that a lateral impact between the tyre and rail developed certain energy, part of which was expended in deflecting the portion of the axle between the wheels; the deflexion might be expected to have a damping effect and to improve the running of the car, and the authors' opinion on this point would be of interest.

The results of the experiments, which in the interests of simplification had been carried out on a single-axle unit, whilst indicating the character of the motion, might not be applicable to a service axle, as the enclosure of the wheel and axle unit in a bogie frame introduced constraining forces which might mask the fundamental wave-form. A design of bogie frame which could be readily distorted from a rectangle into a parallelogram by the forces from the leading wheel and axle would, it was suggested, substantially modify the natural motion of the trailing axle. In addition, depending upon the design of the bogie frame, the constraining forces introduced might prevent an increase in the yaw of the axle unit except such as might be due to the clearances in the axlebox suspensions. In the case of axles provided with roller-bearing axleboxes, the fore-and-aft clearances were usually much smaller than those assumed in the calculations in Appendix I of the paper.

The modifications in tyre profile suggested as a means of improving the tyre performance, and motion of the wheel and axle units, were simple, but difficulties might arise in some workshops in integrating them with existing shop practices. Some amendment in the design of the conventional bogie brake-

work might be required if the proposed modification in tyre profiles were adopted.

The authors' conclusion that a low coefficient of friction between tyre tread and rail was advantageous, in so far as improved running resulted, was in accord with experience on London Transport railways. Certain types of their stock, comprising a substantial proportion of the whole, were fitted with non-metallic brake blocks, the use of which polished the tyre tread surface and reduced the coefficient of friction between tyre tread and rail. The improved finish thus gained reacted very favourably upon both tyre and rail performance and reduced considerably discomfort to passengers, due to noise.

The lubrication of rails and the extension of the use of flange greasers to straight tracks would be a practice requiring checking to ensure that braking efficiency of the rolling stock were not impaired.

Mr. H. G. IVATT, M.I.Mech.E., wrote that the paper dealt with the behaviour of a single axle running on straight track under practically ideal conditions. The results threw new light upon the academic end of the problem of railway dynamics, but it was difficult to correlate them with the very involved dynamics of the steam locomotive as it ran along the actual track.

Investigations were being carried out, on the London Midland Region of British Railways, on the lateral forces produced by the locomotive in service, and electrical measuring equipment had been fitted to various classes of locomotives to record the forces transmitted by the axleboxes to the frame. Those were not necessarily the same as the forces at the tyre flanges, however.

Owing to the many variables—those of the track such as alignment and curvature, and those of the engine, such as disturbing forces due to the driving loads, tendencies to hunt and roll, and individual aberrations of the locomotive as a vehicle—the natural tendencies of the individual wheels were liable to be swamped. A further complication arose from the interaction between the frame and the axleboxes, when, for example, the frame was constraining a particular pair of wheels to pass round a curve—instead of the wheels guiding the locomotive.

At present the investigations had enabled comparisons to be made of the magnitude of the lateral forces arising in different locomotives, when running over the same stretch of track, namely: Derby–Trent and Derby–Manchester. It might be possible, by comparison of the magnitude of the lateral forces, to decide which classes required modification. By relatively small alterations to the locomotive, a redistribution of the forces could probably be effected, and the results assessed from further tests. That was perhaps a hit-and-miss method and more information was required of the dynamics underlying the characteristics of different classes of locomotives.

The investigations were continuing, and it was hoped that it would eventually be possible to separate many of the variables, and then to determine to what extent the theories of Ubelacker, and the results of the experiments conducted by the present authors could be substantiated, and correlated with actual locomotive conditions.

Dr. F. C. JOHANSEN, M.I.Mech.E., wrote that this latest study of the detailed motion of a railway axle fitted with coned wheels combined with Dr. Davies's previous work to form the most adequate investigation of the subject published. More particularly, the theory now advanced to explain bogie hunting was of practical importance, since the bad riding of railway vehicles due to hunting bogies tended to counteract the advantages of high speeds of travel, and any means of raising the critical hunting speed well above the maximum speed permitted by other considerations of railway operation, was well worth knowing. In this connexion the dimensional analysis which the authors had made was a most instructive feature of the paper.

The conclusion they drew regarding the advantage to be gained by reducing "the unsprung mass of the axle unit" presumably referred only to the wheel-axle assemblies and was perhaps rather curiously worded since in conventional designs those assemblies were necessarily unsprung. Although, moreover, the clearances between axle and bogie frame were such that the wavelength of transverse oscillation conformed more closely to that appropriate to a single-wheel pair than to that theoretically appropriate to two pairs connected without clearances by a rigid bogie, it was an observed fact that the frame of a four-wheeled bogie did pursue a sinuous course as it travelled, and Newberry (1945) ascribed the double wave effects, recorded in several of his full-scale tests, to interaction, through the bogie frame, of one axle assembly on the other. He found that such effects were more prevalent at higher than at lower speeds, which suggested that they might be of considerable importance if the wheels were hunting. Would it, therefore, be more valuable to reduce the mass of the bogie—or perhaps more accurately its moment of inertia about its centre pin—or to increase it, in order to raise the hunting speed? Again, if interaction did take place between the axles, would it be advantageous to increase the bogie wheel-base; and what would be the effect of the third, central, axle in a six-wheeled bogie?

The authors' explanation, in conclusion (5), of the spasmodic nature of hunting might well account for the fact that hunting was sometimes observed to occur on a length of newly railed track. There was a good deal of evidence to show that, for the wheel and rail profiles usual on most British railways, the radius of the running surface of the rail was reduced by wear to a value probably somewhat smaller than that of worn tyre treads. The latter were therefore a closer fit on new rails than on worn ones, and hunting might suddenly start when a train ran onto a length of new rails.

The ultimate profile of rails during probably the greater part of their useful life was commonly due at least as much to low-speed passenger and freight trains as to the high-speed trains in which bogie hunting was a serious practical problem. While, therefore, it was true in principle that any general change in the coning of all new tyres would eventually lead to wear such that rails and treads tended to fit one another and so conduce to hunting, there nevertheless seemed some advantage to be gained by reducing the coning on wheels of fast passenger stock to such a value as experience showed would give the optimum compromise between riding and wearing qualities. By such means not only would the critical hunting speed of new wheels be raised well above the operating maximum, but the improved riding, by itself reducing the rate of tyre-tread wear, might be expected markedly to increase the mileage run before hunting could occur at actual service speeds. After all, since the critical hunting speed diminished in general with increased wear of treads, the most that any anti-hunting measure could do was to delay the time when the amount of tread wear needed to cause hunting had taken place; and a reduction of tread coning, provided it proved to be an effectual measure, was likely to be cheaper and easier to introduce and maintain than reduction of unsprung mass, reduction of rail gauge, or rail lubrication.

Mr. J. C. LOACH, M.Sc., A.M.I.Mech.E., confirmed that, as a rail wore, the top of the head soon reached a definite profile which it afterwards retained. If tyre profiles were maintained more nearly to the new profile than was the case at present, however, the normal profile of the rail-head would differ from its present shape. With existing standards of maintenance of tyres, the rail-head on main lines used by steam trains wore to a radius of 9–10 inches within a year or two of being laid, in situations where other disturbing features were not present.

The observation that some bogies did not ride well over new track led to the deduction that tyres at some time during their life had a profile which, being partly worn, nearly fitted the new rails having the 12-inch radius on the running surface and so bogie-hunting on newly laid track occurred whilst smooth running was usual at other places. Consequently the designed radius of head of new flat-bottom rails had been modified recently to 9 inches with the result that bogies rode as well on new track as elsewhere.

Investigators agreed that bogie-hunting was much more severe when the profiles of tread of tyre and head of rail nearly fitted, and he agreed with the authors that a universal change in coning would lead to a change in the profile of worn rails; presumably a flatter coning of tyre would give a rail head with a larger radius. In his paper, Newberry deduced from his work that tyre-tread wear was reduced by flattening the coning and, in his reply to communications, he stated that flatly-coned tyres would accordingly have a much longer period of service before riding

became bad. But he himself wondered whether the increase would be as much as Newberry suggested if the rail head normally wore flatter, and so the tyre and rail profiles nearly fitted after less tyre wear had occurred. That element in the problem was worthy of early attention.

Dr. R. D. DAVIES and Mr. A. F. COOK, in reply, thanked the contributors for their helpful criticisms and suggestions. Several contributors, they wrote, had referred to the limitations of experiments with a single free axle. Despite many investigations in the past, the difficult problem of hunting was still a long way from being solved, and they had felt that it would only yield to patient step-by-step investigations. In conducting their experiments they had had in mind the simple case of a trailing axle of a coach bogie, which, with the clearances usually provided, had very considerable freedom. They did not suggest that their results had much bearing on the running of locomotives, and they regretted that they had not said so specifically in their paper. They had long contemplated extending their investigation to a model of a four-wheeled bogie, and Mr. Cook had already designed such a model which had adjustable clearances and was arranged for loading through a bolster in the conventional way.

The difficulty and cost of reducing the unsprung weight (the mass of the wheels, axle, and axleboxes) had been referred to, and were obviously great, but so were the advantages to be gained. The running would probably be much improved, with consequent decrease in flange and tread wear; and track maintenance, particularly at rail joints, would certainly be reduced. With proper diffidence, they made the following suggestions for the consideration of more practical engineers. Even a small reduction in the distance between axlebox centres would reduce the bending moment in the axle, and so make a lighter axle possible; it might perhaps be achieved by using dished wheel-centres and, or alternatively, short roller bearings, as was sometimes done in tramway practice. Light alloy wheel-centres and axleboxes, tubular axles of alloy steel, and thinner tyres (in anticipation of reduced wear) were obvious suggestions.

Mr. Graff-Baker had referred to curved track. They had suffered much from that in their experiments. Any slight difference between the diameters of the "rail rings" of the model was equivalent to a curvature of the track, and gave the axle a bias towards the larger ring (the outside of the curve). Fig. 7 showed this effect; apart from the heavier flange impacts on the "outer" rail the character of the motion did not seem to be affected.

Driving or braking the axles of a bogie, if sufficiently violent, must surely eliminate all axlebox clearances by holding the axleboxes hard against their front or rear guides, and so practically turn the bogie into a "rigid four-wheeler".

Some of the energy used in bending the axle was certainly lost, but it was a small fraction of that lost at each impact. The oscillation of an axle was quite different in character from that of a pendulum or a mass on a spring. Under certain conditions, by drawing energy from the forward motion, it persisted despite a large loss at every impact.

They were very interested to hear of Mr. Ivatt's experiments on lateral forces, and looked forward to seeing the results published.

They agreed that the axles acted on, and interacted through, the bogie frame. Mr. Cook's model might supply the answers to some of Dr. Johansen's questions.

Dr. Johansen had referred to the use of reduced coning on the wheels of fast passenger stock. The ultimate effect of any change in tyre profile on the regime of tyre-rail wear was very difficult to predict. It was possible to benefit certain privileged vehicles by fitting them with special tyre profiles; but the ultimate effect might be slightly to modify the worn rail profile to the disadvantage of other vehicles. The result would depend on the number of the privileged. On lines carrying heavy goods-traffic, it might be practicable to give all passenger stock special tyre profiles.

The improved running on new rails rolled with heads of 9-inch radius was interesting, and supported one of the authors' conclusions.