

THE INFLUENCE OF LANGUAGE ON PROBLEM SOLVING IN MATHEMATICS

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1. INTRODUCTION

Consider the following two problems:

- P1. Peter went to the shop to buy 5 packets of biscuits. If each packet contains 8 biscuits, how many biscuits did Peter buy?
- P2. Eight teams enter for a hockey tournament. Each team will play every other team once. How many matches must be played?

Problem no. 1. can be regarded as a common type of textbook problem with the characteristic that it can be solved by the direct application of one or more previously learned algorithms. The basic task for the pupil is to identify which operations or algorithms are appropriate for solving the problem. These problems allow children to work with the acquired skills and knowledge of the operations in situations which are surrogates or replacements for their real-world counterparts.

Problem no. 2. is an example of a 'process problem' which requires the use of strategies or some non-algorithmic approach. This type of problem stresses the process of obtaining the solution rather than the solution itself.

Success in solving this type of problem does not depend on the application of specific mathematical concepts, formulas, or algorithms; but rather on the use of one or more strategies.

The term 'problem solving' is generally reserved for the second type of problem and is described as the process of investigation that occurs when a person confronts a situation for which a routine response is inappropriate.

In this address, my main concern is about the influence of language on the pupil's ability to solve problems, whether of the first or the second type. I shall therefore concern myself with the influence of language on solving problems (not only of the non-routine type).

2. SOLVING PROBLEMS

The ability to solve problems is one of the most important objectives in the study of mathematics, and as Knifong and Burton (1985:13) put it:

"... learning to solve word problems prepares students to use mathematics in the real world. Teaching children to think logically about word problems is at the core of the professional responsibility of mathematics educators".

Many teachers, however, do not feel confident in teaching verbal problems and because many children find these problems one of the most difficult challenges in mathematics, negative attitudes to this aspect of teaching mathematics have developed in both teachers and pupils. One must, however, identify the causes for these difficulties and help teachers and pupils to surmount their difficulties.

Solving problems has four facets:

1. Understanding the problem
2. Devising a plan, i.e. selecting mathematical notions that may lead to a solution of the problem.

3. Carrying out the plan and using the mathematical notion to find the solution.
4. Examining the feasibility of the solution. Although researchers have found several reasons for children's mistakes in solving problems, e.g. errors in reasoning, ignorance of mathematical principles, rules or processes, and insufficient mastery of computational skills, the main reasons seem to centre around the facet of solving problems, i.e. understanding the problem.

3. UNDERSTANDING THE PROBLEM

The inability of children to understand the problem can be attributed to -

- reading difficulties,
- lack of understanding of the social context of the problem, and
- lack of conceptual understanding of the situation.

3.1 Reading difficulties

Shuard and Rothery (1984:1) defines reading as

"...the whole process by which a pupil examines the written word and the pictorial material and obtains its meaning. In mathematics, for us, reading is 'getting the meaning from the page'.

Reading and language processing are crucial abilities that influence problem solving behaviour but often, 'getting the meaning from the verbal problem' in mathematics, is a difficulty because of the differences between reading Ordinary English (OE) and Mathematical English (ME). Some of the differences, noted by J.C. Barnett et al. (1980:97-99), are given below.

1. Mathematical word problems are more compact and conceptually dense than ordinary prose. An ordinary paragraph of prose usually contains one major idea, but mathematical word problems often squeeze several important ideas into a single sentence.

2. The writing style found in word problems is usually different from that used in most other types of prose. Word problems usually contain relatively short thought units that are closely related to each other.
3. Often the meaning of a word in a mathematical problem is entirely different from the meaning of that same word in ordinary prose, e.g. the words difference, operation, power, base, etc.
4. Normal reading patterns are often ineffective for word problems. Symbols and numerals in word problems can cause breaks in the pupils chain of thought. The reader may then focus on the numbers in the problem statement and become distracted from the relationships implied by verbs and nouns.

In trying to deal with childrens' reading difficulties, researchers have suggested several guidelines:

1. An emphasis on isolated word cues, e.g. "If the problem asks how many left, you subtract", is misleading for attention is directed away from the recognition of the relationships inherent in the problem that may be crucial to its solution.
2. Use a tape recorder to aid poor readers.
3. Present some problems in separate sentences rather than in the usual paragraph format.
4. Have pupils write their own problems, formulating them for given conditions.
5. Use problems without numbers so that the reader may concentrate on the relationships implied by the verbs and nouns and not become distracted by numbers.
6. Activities stressing certain reading skills, such as selecting main ideas, making inferences, constructing sequences may improve problem-solving achievement.
7. Specific instruction on quantitative vocabulary, such as "times as many", "more" and "sweets per bag" may be helpful for some pupils.

The Cockcroft Report (1982), however, warns in paragraph 311:

"The policy of trying to avoid reading difficulties by preparing work cards in

which the use of language is minimised or avoided altogether should not be adopted".

3.2 Lack of understanding of the social context of the problem

Consider the following problems:

- P1. A man invests his money as follows:
bonus bonds R5 900; savings bank R2 386; building society R1 890; shares R580. How much money did he invest?
- P2. An astronaut completes his orbit of 43 200 km round the earth every 1,5h. Calculate his speed in metres per second.

A pupil to whom terms such as bonus bonds, shares, astronaut, have no meaning within his social context is at an immediate disadvantage and has little hope of understanding the problem, let alone solving it.

3.3 Lack of conceptual understanding of the situation

In an article by Ana Quintero (1986:34), she refers to studies by Carpenter, Hiebert and Moser in which it is pointed out that the main difficulty children have with word problems is understanding the situation described in the problem and not in determining the correct operation to apply. She suggests that in teaching word problems, more emphasis should be placed on helping children understand the situation described in the problem. To do this, however, teachers must be aware of the different situations that can be modelled by an operation as well as the different concepts needed to understand each situation. Consider the following three problems which can all be solved by 5×3 , yet the situations described exemplify three different types of structures found in multiplication word problems. (Quintero, 1985:36).

- P1. John has five marbles.
Mary has three times as many marbles as John has.

How many marbles does Mary have?

P2. The store sells five candies per bag.

Joan bought three bags of candies.

How many candies did she buy?

P3. Peter has five pairs of pants and three skirts.

How many outfits can he make?

If children have only experienced multiplication as repeated addition and not any of the other structures, the chances are that they would not have the conceptual understanding of the above situations to recognise the process of multiplication.

C.S. Thompson and A.D. Hendrickson (1986:21) report similar problems with children taught addition only as "putting together" and subtraction only as "taking away". Problems with a different inherent structure, e.g.

"Jean has two pencils. She has three fewer pencils than Bill. How many pencils does Bill have?"

are then difficult to solve, even though pupils can read the problem and fully understand the social context.

They conclude (1986:25):

"...we have learned that children can become good solvers of verbal problems. What they need is an instructional program that proceeds from the concrete to the symbolic and the opportunity to encounter the various problem situations that occur in real life".

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