Frequency-Domain Equalization for OFDM over Doubly-Selective Channels

Imad Barhumi[†], Geert Leus[‡] and Marc Moonen
K.U.Leuven-ESAT, Kasteelpark Arenberg 10, 3001 Leuven, Belgium
Email: {imad.barhumi,geert.leus,marc.moonen}@esat.kuleuven.ac.be

***Abstract*— In this paper we propose a new equalization technique for multi-carrier systems over time- and frequency-selective channels. Due to the time-variations of the channel, the orthogonality between subcarriers is destroyed. A 1-tap time-varying filter is used to combat the time-variations of the channel. However, we show that by transferring the time-domain operation to the frequency domain we can approach the performance of the purely frequency-selective channel. An important feature of the proposed technique is that no bandwidth expansion or incorporating redundancy is required.**

I. Introduction

THE wireless communication industry has experienced rapid growth in recent years, and digital cellular systems are currently being designed to provide high data rates at high speeds. High data rates give rise to intersymbol interference (ISI) due to multipath fading. Such an ISI channel is called frequency-selective. On the other hand, due to mobility and/or carrier frequency offsets the received signal is subject to frequency shifts (Doppler shifts). The Doppler effect in conjunction with ISI gives rise to a so-called doubly-selective channel (frequency- and time-selective). In this paper, we present a new equalization technique for multicarrier transmission over such a challenging channel.

Orthogonal Frequency Division Multiplexing (OFDM) has attracted a lot of attention, due to its simple implementation and robustness against frequency-selective channels. However, the channel variations over an OFDM block destroy the orthogonality between the subcarriers resulting into inter-carrier interference (ICI). In this paper, we propose a new ICI-mitigation technique that works on a per-tone basis.

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Different approaches for reducing ICI have been proposed including frequency-domain equalization and or time-domain windowing. In [1] the authors propose matched-filter, least-square (LS) and minimum mean-square error (MMSE) receivers that incorporate all subcarriers. A low complexity MMSE receivers incorporating the dominant adjacent subcarriers is presented in [2]. A frequency-domain ICI mitigation technique is proposed in [3] for multiple-input multiple-output (MIMO) OFDM over doubly-selective channels. A time-domain windowing (linear pre-processing) to restrict ICI support in conjunction with iterative MMSE estimation is presented in [4]. An ICI self-cancellation scheme is proposed in [5] where redundancy is added to enable self-cancellation which implies a substantial reduction in bandwidth efficiency. A good review of these different schemes is presented in [6], where also the author proposes a new higher order self-cancellation scheme.

Previously, a time-invariant (TI) FIR time-domain equalizer (TEQ) is used to shorten a purely frequency-selective channel when its delay spread is larger than the cyclic extension. In this paper, the channel has a delay spread that fits within the cyclic extension, but on the other hand has a high Doppler spread. Hence, in a dual fashion, a 1-tap time-varying (TV) TEQ is applied at the receiver. The purpose of this 1-tap TV TEQ is to convert the doubly-selective channel into a purely frequency-selective channel. Then, an FFT is applied on the equalized samples followed by a 1-tap frequency-domain equalizer (FEQ) to recover the frequency-domain symbols. Similar to the case of a purely frequency-selective channel where the TIV FIR TEQ is transfered to the frequency-domain resulting into a so-called per-tone equalizer (PTEQ) that treats each tone separately [7], we now transfer the 1-tap TV TEQ to the frequency-domain resulting in a new frequency-domain equalization structure. This new structure enhances the performance at the cost of slight increase in complexity.

This paper is organized as follows. The system model is described in section II. The proposed equalization technique is introduced in section III. In section IV, we show through computer simulations the performance of the proposed equalizer. Finally, our conclusions are drawn in section V.

Notations: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We denote

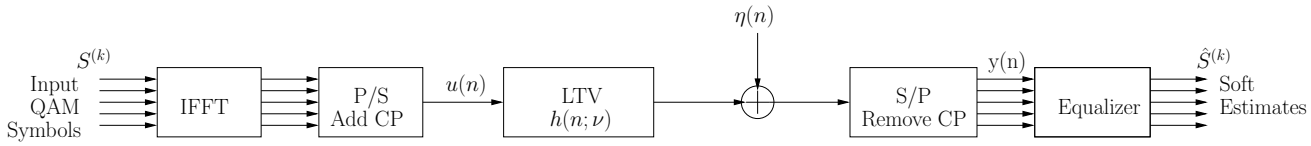

Fig. 1. System Model

the 1- and 2-dimensional Kronecker delta as δ_n and $\delta_{n,m}$, respectively. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. Finally, diag$\{\mathbf{x}\}$ denotes the diagonal matrix with $\mathbf{x}$ on the diagonal.

II. System Model

We assume a single-input single-output (SISO) system (see Figure 1), but the results can be easily extended to a single-input multiple-output (SIMO) or a multiple-input multiple-output (MIMO) system. At the transmitter, the incoming bit sequence is parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed into a time-domain transmitted sequence using an N-point IFFT. To avoid inter-block interference (IBI), a cyclic extension of length ν equals to or larger than the channel order L is inserted at the head of each block. The time-domain blocks are then serially transmitted over a multipath fading channel. The channel is assumed to be linear time-varying (LTV). Focusing only on the baseband-equivalent description and assuming symbol rate sampling, the received sequence can be written as:

$$y(n) = \sum_{\theta=-\infty}^{\infty} h(n;\theta)u(n-\theta) + \eta(n) \tag{1}$$

where $u(n)$ is the time-domain transmitted sequence, $\eta(n)$ is the additive noise, and $h(n;\theta)$ is the baseband equivalent of the doubly-selective (time- and frequency-selective) channel, which includes the physical channel as well as the transmit and receive filters. Suppose $S^{(k)}$ is the QAM symbol transmitted on the kth tone, $u(n)$ can be written as:

$$u(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S^{(k)} e^{j2\pi nk/N} \quad n \in \{-\nu, \cdots, N-1\}$$

Note that this description includes the transmission of a CP of length ν.

In this paper we use the basis expansion model (BEM) to approximate the doubly-selective channel [8], [9], [10], [11], which has been shown to accurately model realistic channels. In this BEM, the channel is modeled as a TV FIR filter, where each tap is expressed as a superposition of complex exponential basis functions with frequencies on a DFT grid. Before we proceed to define the BEM channel, let us first make the following assumptions:

a1) the channel delay spread is bounded by τ_{max}
a2) the channel Doppler spread is bounded by f_{max}

Under assumptions a1) and a2), it is possible to accurately model the double-selective channel $h(n;\theta)$ for $n \in \{0, \cdots, N-1\}$ as:

$$h(n;\theta) = \sum_{l=0}^{L} \delta_{\theta-l} \sum_{q=-Q/2}^{Q/2} h_{q,l} e^{j2\pi qn/K} \tag{2}$$

where $h_{q,l}$ is the TIV coefficient of the lth tap and qth basis function of the channel, which is kept invariant over a period of NT seconds (T is the sampling time). Q is the number of time-varying basis functions and L is the channel order. K here determines the BEM frequency-resolution, and is assumed to be larger than the number of carriers, i.e., $K \geq N$. The parameters Q and L should satisfy the following conditions:

c1) $LT \geq \tau_{max}$
c2) $Q/(2KT) \approx f_{max}$

Substitute (2) in (1) we obtain the following input/output relation:

$$y(n) = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} e^{j2\pi qn/K} u(n-l) + \eta(n) \tag{3}$$

In the following we will describe an equalization technique.

III. Equalization Technique

At the receiver, the CP is removed and the resulting input sequence is filtered by means of 1-tap TV (purely time-selective) filter denoted by $g(n;0)$. The purpose of the 1-tap filter is toconvert the doubly-selective channel into a purely frequency-selective channel. This allows us to use a 1-tap FEQ applied at the ooutput of the FFT to recover the transmitted QAM symbols. Since we approximate the doubly-selective channel using the BEM, we also find it convenient to model the 1-tap TV TEQ $g(n;0)$ using the BEM with Q' time-varying basis functions. We design the equalizer such that to have the same frequency-resolution as the channel. Hence, an estimate of the frequency-domain symbol at the kth tone can be obtained as:

$$\begin{aligned}\hat{S}^{(k)} &= \frac{1}{d^{(k)}\sqrt{N}} \sum_{n=0}^{N-1} \sum_{q'=-Q'/2}^{Q'/2} g_{q'} e^{j2\pi nq'/K} y(n) e^{-j2\pi nk/N} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \sum_{q'=-Q'/2}^{Q'/2} w_{q'}^{(k)} y(n) e^{-j2\pi n(k/N - q'/K)}\end{aligned} \tag{4}$$

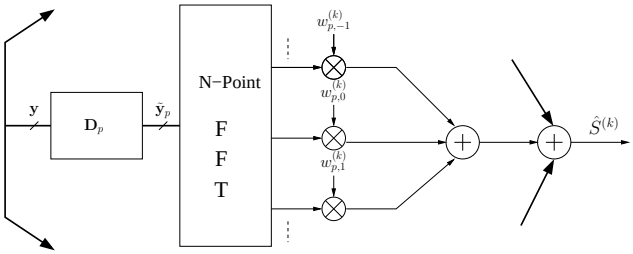

Fig. 2. Receiver structure with the PTEQ

where $w_{q'}^{(k)} = g_{q'}/d^{(k)}$ and $d^{(k)}$ is the 1-tap FEQ coefficient at the kth tone. From (4) we can see that it is possible to transfer the TEQ operations to the frequency-domain. This results into a per-tone equalizer (PTEQ), which allows us to optimize the equalizer coefficients $w_{q'}^{(k)}$ for each tone k separately, without taking into account the specific relation between $w_{q'}^{(k)}$, $g_{q'}$, and $d^{(k)}$. Let us now discuss how we can implement and design such an equalizer. In general K can be different than N. In this paper, we will assume that K is an integer multiple of the block size N i.e. $K = PN$, where P is an integer greater than or equal to 1. For this reason P-FFTs are applied to different phase-shifted versions of the received block as will be clear later.

Define $\mathbb{Q} = \{-Q'/2 \cdots, Q'/2\}$, $\mathbb{Q}_p = \{q \in \mathbb{Q} \mid |q| \bmod P = p\}$ for $p = 0, \cdots, P-1$. Hence, we can write (4) as:

$$\begin{aligned}\hat{S}^{(k)} &= \frac{1}{\sqrt{N}} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} w_{p,l_p}^{(k)} \sum_{n=0}^{N-1} \underbrace{e^{j2\pi\alpha_p n/N} y(n)}_{\tilde{y}_p(n)} \\ &\quad \times e^{-j2\pi n(k-l_p)/N} \\ &= \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} w_{p,l_p}^{(k)} \, \tilde{Y}_p^{(k-l_p)} \end{aligned} \tag{5}$$

where $l_p = \lfloor q_p/P \rfloor$, $\alpha_p = p/P$, and $w_{p,l_p}^{(k)} = w_{q_p}^{(k)}$ is the coefficient operating on the $(k-l_p)$th tone of the pth FFT to compute the symbol transmitted on the kth tone. $\tilde{Y}_p^{(k-l_p)}$ is the frequency resposnse of $\tilde{y}_p(n)$ on the $(k-l_p)$th tone. From (5), the receiver structure can be realized as depicted in Figure 2.

For each tone, we can find the MMSE-based PTEQ coefficients by minimizing the following cost function

$$\begin{aligned}\min_{w_{p,l_p}^{(k)}} J(w_{p,l_p}^{(k)}) &= \min_{w_{p,l_p}^{(k)}} \mathcal{E}\|\hat{S}^{(k)} - S^{(k)}\|^2 \\ &= \min_{w_{p,l_p}^{(k)}} \mathcal{E}\|\mathbf{w}^{(k)T}\mathbf{F}^{(k)}\mathbf{y} - S^{(k)}\|^2 \end{aligned} \tag{6}$$

where $\mathbf{w}^{(k)} = [\mathbf{w}_0^{(k)T}, \cdots, \mathbf{w}_{P-1}^{(k)T}]^T$, $\mathbf{w}_{p-1}^{(k)} = [\cdots, w_{p,-1}^{(k)}, w_{p,0}^{(k)}, w_{p,1}^{(k)}, \cdots]^T$ and $\mathbf{y} = [y(0), \cdots, y(N-1)]^T$. $\mathbf{F}^{(k)} = [\mathbf{F}_0^{(k)T}, \cdots, \mathbf{F}_{P-1}^{(k)T}]^T$ where,

$$\mathbf{F}_p^{(k)} = \begin{bmatrix} \vdots \\ \mathcal{F}^{(k-1)}\mathbf{D}_p \\ \mathcal{F}^{(k)}\mathbf{D}_p \\ \mathcal{F}^{(k+1)}\mathbf{D}_p \\ \vdots \end{bmatrix}$$

where $\mathcal{F}^{(m)}$ is the mth row of the unitary DFT matrix $\mathcal{F}$ and $\mathbf{D}_p = \text{diag}\{[1, e^{j2\pi\alpha_p/N}, \cdots, e^{j2\pi\alpha_p(N-1)/N}]\}$.

Solving for $\mathbf{w}^{(k)}$ in (6) we obtain:

$$\begin{aligned}\mathbf{w}^{(k)T} &= \mathbf{e}^{(k)T}\mathbf{R}_s\mathcal{F}\mathbf{H}^H\mathbf{F}^{(k)H} \\ &\quad \times \left(\mathbf{F}^{(k)}\mathbf{H}\mathcal{F}^H\mathbf{R}_s\mathcal{F}\mathbf{H}^H\mathbf{F}^{(k)H} + \mathbf{F}^{(k)}\mathbf{R}_\eta\mathbf{F}^{(k)H}\right)^{-1} \end{aligned} \tag{7}$$

where $\mathbf{e}^{(k)}$ is the $N \times 1$ unit vector with a 1 in position $k+1$ and $\mathbf{R}_s$ and $\mathbf{R}_\eta$ are the data and the noise covariance matrices respectively. $\mathbf{H}$ is the TV channel matrix given by:

$$\mathbf{H} = \sum_{q=-Q/2}^{Q/2} \mathbf{D}_q\mathbf{H}_q \tag{8}$$

where $\mathbf{H}_q$ is a circulant matrix with $[h_{q,0}, \cdots, h_{q,L}, \mathbf{0}_{N-L-1\times 1}]^T$ as its first column and $\mathbf{D}_q = \text{diag}\{[1, e^{j2\pi q/K}, \ldots, e^{j2\pi q(N-1)/K}]\}$. For white source and white noise with variances σ_s^2 and σ_η^2 respectively (i.e. $\mathbf{R}_s = \sigma_s^2\mathbf{I}_N$ and $\mathbf{R}_\eta = \sigma_\eta^2\mathbf{I}_N$) (7) reduce to:

$$\begin{aligned}\mathbf{w}^{(k)T} &= \mathcal{F}^{(k)}\mathbf{H}^H\mathbf{F}^{(k)H} \\ &\quad \times \left(\mathbf{F}^{(k)}\mathbf{H}\mathbf{H}^H\mathbf{F}^{(k)H} + \frac{\sigma_\eta^2}{\sigma_s^2}\mathbf{F}^{(k)}\mathbf{F}^{(k)H}\right)^{-1} \end{aligned} \tag{9}$$

A. Unifying Frame Work

In this section we show that our frame work unifies and extends earlier proposed techniques for both LTI and LTV channels.

1) Linear Time-Invariant (LTI) ($Q = 0$, $Q' = 0$, $P = 1$): the solution in (7) boils down to the MMSE [12]:

$$w^{(k)} = \left(H^{(k)*}H^{(k)} + \frac{\sigma_\eta^2}{\sigma_s^2}\right)^{-1} H^{(k)*} \tag{10}$$

where $H^{(k)}$ is the frequency response of the LTI channel on the kth tone. σ_η^2 and σ_s^2 are the power spectral density of the noise and transmitted sequence respectively.

2) Linear Time-Varying (LTV) channels with $P = 1$: the solution in (7) boils down to the MMSE solution obtained in [2], where only one FFT is applied and Q' adjacent subcarriers are used to cancel interference.

B. Complexity

The complexity of the proposed equalizer is of two types:

1) Design complexity: is the complexity associated with computing the PTEQ coefficients.
2) Show time complexity: is the complexity associated with estimating a block of N symbols.

In Table I, we show the design and run-time complexities associated with the proposed technique.

TABLE I

COMPLEXITY OF PTEQ

	Design	Run-Time
Add	$(2Q'+L+4)N^2+$ $2(Q'+1)^2N+\mathcal{O}(Q'+1)^3$	$Q'N$
Multiply	$(2Q'+L+3)N^2+$ $2(Q'+1)^2N+\mathcal{O}(Q'+1)^3$	$(Q'+1)N$
FFT's		P-FFTs*

* each FFT requires $\mathcal{O}(N\cdot\log_2(N))$ multiplications and $\mathcal{O}(N\cdot\log_2(N))$ additions.

Note that the complexity table shown above is not true for the case of LTI channels by setting $Q'=0$, since the circulant property of the channel can be exploited to perform low complexity matrix multiplication.

IV. Simulation Results

In this section we show some simulation results for the proosed ICI mitigation technique. We consider a SISO system, the channel is assumed to be doubly-selective of order $L=6$ and a maximum Doppler frequency $f_d=100Hz$. The channel taps are simulated as i.i.d., correlated in time with a correlation function according to Jakes' model $\mathcal{E}\{h(n_1;l_1)h^*(n_2;l_2)\}=\sigma_h^2J_0(2\pi f_dT(n_1-n_2))\delta(l_1-l_2)$, where J_0 is the zeroth- order Bessel function of the first kind and σ_h^2 denotes the variance of the channel. We consider $N=128$ subcarriers, and a cyclic prefix of length $\nu=6$. The sampling time is $T=50\mu sec$, the total OFDM symbol duration is $6.7msec$. QPSK signaling is assumed. We define the $SNR=\sigma_h^2(L+1)E_s/\sigma_\eta^2$, where E_s is the QPSK symbol power.

We use the BEM to approximate the channel, with $Q=2$ TV basis functions. The BEM coefficients of the approximated channel are used to design the MMSE equalizer. The BEM resolution is determined by $K=PN$ where $P=1,2$. The 1-tap TV TEQ is modeled using the BEM with $Q'=10$. The proposed equalizer is used to equalize the true Jakes' channel. The performance is measured in terms of BER vs. SNR. Figure 3 shows the BER curve of the proposed equalizer for different BEM resolutions ($P=1,2$). As a benchmark we show the case of LTI channel, we see that the proposed equalizer approaches the performance of the LTI channel. In the same figure we also show the matched-filter bound (see [2], [13]) for the LTV channels. Finally, we measure the

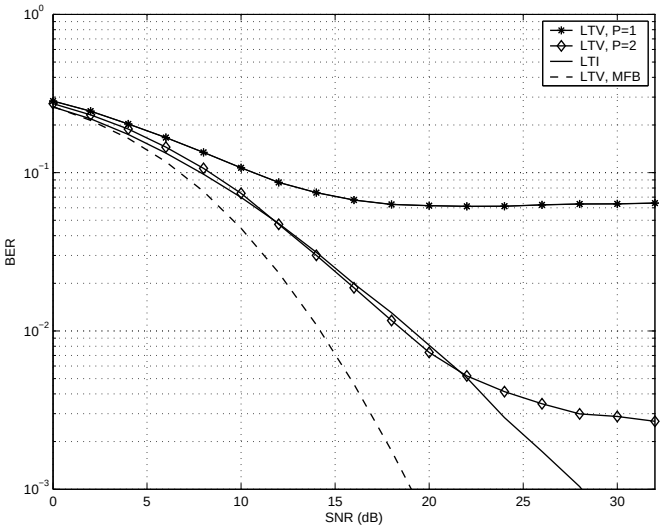

Fig. 3. BER vs. SNR for $P=1,2$

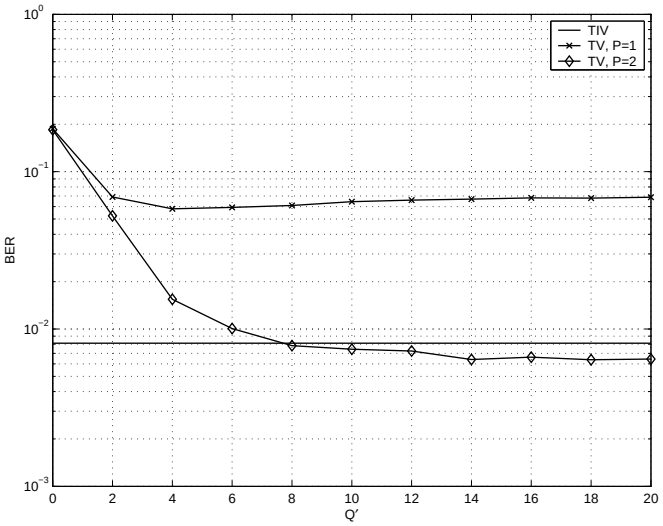

Fig. 4. BER vs. Q' for $P=1,2$

performance of the proposed equalizer as a function of Q' at a fixed SNR of 20 dB. As shown in figure 4, a significant gain can be obtained by increasing Q' up to a certain threshold value ($Q'=2$ for $P=1$ and $Q'=8$ for $P=2$) after which almost no gain is obtained.

V. Conclusions

In this paper, we have proposed a new ICI mitigation technique for OFDM over doubly-selective channels. The proposed technique uses adjacent subcarriers to mitigate ICI on specific carrier. By increasing the BEM resolution beyond the size of the FFT, we can approach the performance of OFDM over purely frequency-selective channels. Except for the additional number of FFTs, the design and run-time complexity are comparable to the low complexity MMSE receiver proposed in literature.

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