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A fast algorithm of linear canonical transformation for radar signal processing system

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Abstract. Linear canonical transformation is a new signal processing tools developing in recent years. As a unified multi-parameter linear integral transform, linear canonical transformation has its unique advantages when dealing with non-stationary signal. However, from the existing literatures, the basic theoretical system is not perfect, some of the theories associated with signal processing needs to be further established or strengthened, the research of linear canonical transformation has important theoretical significance and practical significance, but linear canonical transformation needs a lot of calculation, it is not like Fourier transform, fractional Fourier transform, Fresnel transform and scale operator, they have already been widely used in various fields of expertise, in order to reduce the amount of calculation, this paper puts forward a fast algorithm which uses duality theorem of linear canonical transformation to reduce the amount of calculation, it can quickly complete the operation when we use linear canonical transformation to process the signal during radar signal processing, the time for normal algorithm is 5s, the fast algorithm needs only 0.2s.

Introduction

For a long time, Fourier transform[1] is a powerful tool for dealing with stationary signal, there is no signal processing can better than it, however, Fourier transform has an obvious limitation, because of its degrees of freedom and the overall transformation parameters makes it unable to describe the properties of partial change characteristics in time and frequency signal, therefore, in the treatment of non-stationary signals, the Fourier transform is not work, with the deepening of the signal processing method, fractional transform was raised, because of its good time-frequency characteristics of rotating, so it has obtained the widespread application in the field of non-stationary signal analysis, but there is only one parameter in fractional transformation , for the unsteady signal it is still not enough, with the introduction of more parameters, fractional transformation can further promote to linear canonical transformation[2](LCT),linear canonical transformation has more strong flexibility and processing capacity. But it has been a lack of scientific physical interpretation and an effective fast algorithm. Based on the above problems, this paper study the fast algorithm of the linear canonical transformation, puts forward a fast algorithm for linear canonical transformation, the fast algorithm can quickly calculate the linear canonical transformation and generality[3].

The discrete algorithm of LCT

The definition of linear canonical transformation[4]is as follows:

$$F_{(a,b,c,d)}(u) = L_{(a,b,c,d)}(f(t)) =$$
$$\begin{cases} \sqrt{\dfrac{1}{j2\pi b}} e^{\frac{jd}{2b}u^2} \int_{-\infty}^{+\infty} e^{-\frac{j}{b}ut} e^{\frac{ja}{2b}t^2} f(t)dt & b \neq 0 \\ \sqrt{d} e^{\frac{jcd}{2}u^2} f(d) & b = 0 \end{cases} \qquad ad - bc = 1 \tag{1}$$

First of all the input function $f(t)$ and the output function $F_{(a,b,c,d)}(u)$ with interval Δt, Δu for sampling is as bellows:

$$f(n) = f(n\Delta t),\quad n = -N, -N+1, \ldots, N,$$

$$F_{(a,b,c,d)}(m) = F_{(a,b,c,d)}(m\Delta\mu),\ m = -M, -M+1, \ldots, M, \tag{2}$$

We can get the expression of discrete canonical transformation:

$$F_{(a,b,c,d)}(m) = A\sum_{n=-N}^{N}\exp(-j\frac{2\pi nm}{2M+1}\bullet\frac{b}{|b|})\bullet\exp(j\frac{a}{2b}n^2\Delta t^2)f(n) \tag{3}$$

Where, $A = \sqrt{\dfrac{1}{2M+1}}\exp(j\dfrac{d}{2b}m^2\Delta u^2)$ (4)

The fast algorithm of linear canonical transformation

The expression of discrete canonical transformation[5] also can be writen as follow:

$$F_{(a,b,c,d)}(m) = \sum_{n=-N}^{N}E_{(a,b,c,d)}(m,n),\ m\in(-M,M), n\in(-N,N) \tag{5}$$

$$E_{(a,b,c,d)}(m,n) = \sqrt{\frac{1}{2M+1}}\exp(-j\frac{2\pi nm}{2M+1}\bullet\frac{b}{|b|})\exp[j(\frac{a}{2b}n^2\Delta t^2 + \frac{d}{2b}m^2\Delta u^2)]f(n),\ m\in(-M,M), n\in(-N,N) \tag{6}$$

We can see that, $n\in(-N,N)$, $m\in(-M,M)$ is symmetrical about $n=0$, $m=0$ so if we can get the relationship between $E_{(a,b,c,d)}(m,n)$, $m\in(-M,M), n\in(1,N)$ and $E_{(a,b,c,d)}(m,-n)$, $m\in(-M,M), n\in(1,N)$ the relationship between: $E_{(a,b,c,d)}(m,n)$, $m\in(1,M), n\in(-N,N)$ and $E_{(a,b,c,d)}(-m,n)$, $m\in(1,M), n\in(-N,N)$ We can simplify the calculation. Through the analysis of (6), we can get:

$$E_{(a,b,c,d)}(m,-n) = E_{(a,b,c,d)}(m,n)\exp(j\frac{4\pi nm}{2M+1}\bullet\frac{b}{|b|})f(-n)/f(n),\quad m\in(-M,M), n\in(1,N) \tag{7}$$

$$E_{(a,b,c,d)}(-m,n) = E_{(a,b,c,d)}(m,n)\exp(j\frac{4\pi nm}{2M+1}\bullet\frac{b}{|b|}),\ m\in(1,M), n\in(-N,N) \tag{8}$$

Especially when $n=0$, we can get:

$$E_{(a,b,c,d)}(m,0) = \sqrt{\frac{1}{2M+1}}\exp(j\frac{d}{2b}m^2\Delta u^2)f(0),\quad m\in(-M,M) \tag{9}$$

When $m=0$ we can get:

$$E_{(a,b,c,d)}(0,n) = \sqrt{\frac{1}{2M+1}}\exp(j\frac{a}{2b}n^2\Delta t^2)f(n) \tag{10}$$

Thus it is concluded that the calculation of linear canonical transformation[6] steps are as follows:

1)According to (6) we can get the value of $E_{(a,b,c,d)}(m,n)$, $m\in(1,M), n\in(1,N)$.

2)According to (7) and (9) we can get the value of $E_{(a,b,c,d)}(m,n)$, $m\in(1,M), n\in(-N,1)$ and the value of $E_{(a,b,c,d)}(m,0)$, $m\in(-M,M)$.

3)According to (8) and (10) we can get the value of $E_{(a,b,c,d)}(m,n)$, $m\in(-M,1), n\in(-N,N)$.

and the value of $E_{(a,b,c,d)}(0,n)$ $n\in(-N,N)$ .then we can get $F_{(a,b,c,d)}(m)$, $m\in(-M,M)$.

The results of simulation

In this paper $a=\sqrt{3}/2, d=\sqrt{3}/2, b=1/2, c=-1/2, M=60, \Delta t=0.3s$, $\Delta u=1s$,the values of N is 1:100 the step of the simulation is 10,the result of simulation is shown as figure 1, the ratio between the fast algorithm and the original algorithm with the changing of sampling values are as figure 2:

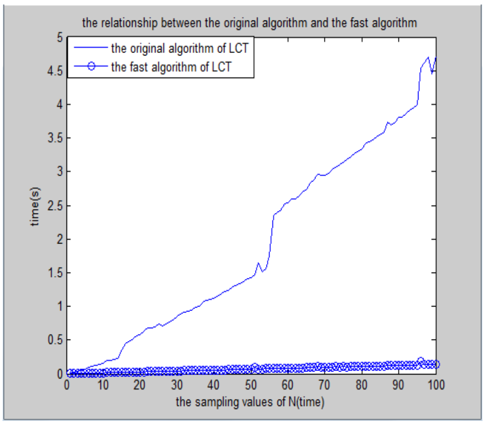

Fig.1 The relationship between the fast algorithm and the original algorithm

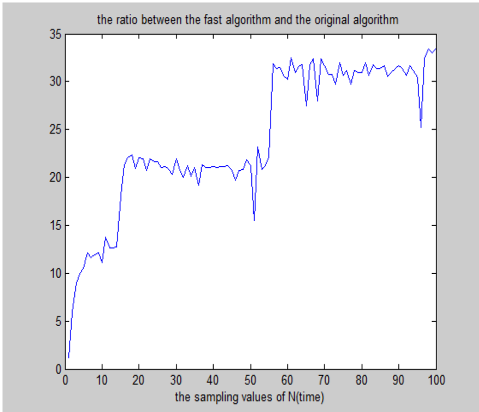

Fig.2 The ratio between the fast algorithm and the original algorithm

This figures show that the rapid algorithm can reduce the amount of calculation, especially when a larger amount of data operation was exist, when the normal time needed is 5 seconds, fast algorithm only needs 0.2 seconds.

As the figure shows, the ratio of speed with the increasing of number in operations is becoming more and more better. Especially when handling large amounts of data,it can quickly calculate the data, $\Delta t = 0.1 \sim 1$,he step of the simulation is 0.1,the result of simulation is shown as figure 3,the ratio between the fast algorithm and the original algorithm with the changing of sampling values are as figure 4:

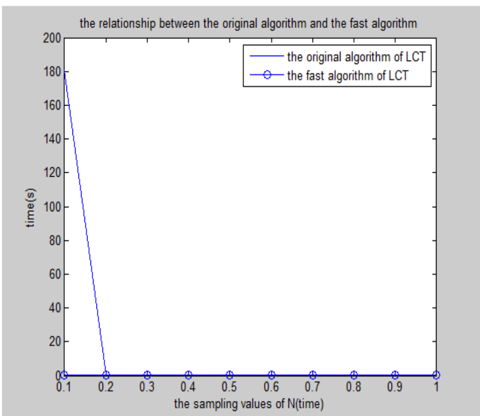

Fig.3 The relationship between the fast algorithm and the original algorithm

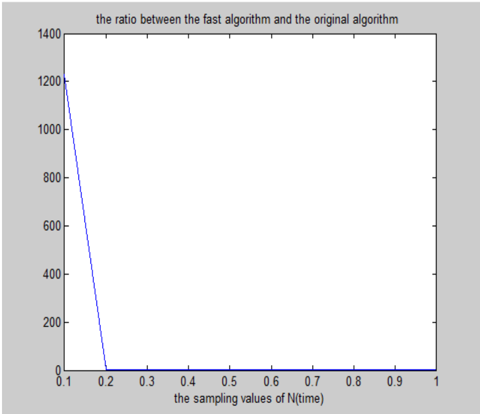

Fig.4 The ratio between the fast algorithm and the original algorithm

Conclusion

Based on the decomposition of discrete canonical transformation, the use of various parts in the discrete canonical transformation and duality theorem to reduce the amount of calculation, this method in this paper can quickly realize discrete linear canonical transformation, it can save time and this method has universality, it can make discrete canonical transformation more specific and more explicit.

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