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Intrinsically knotted and 3-linked Graph

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Abstract. Intrinsically linked graphs are very important spatial graphs. We say that is intrinsically knotted and 3-linked graph if every spatial embedding of this graph contains nontrivial knot and a non-split 3-component link. This paper exhibit a new intrinsically knotted and 3-linked graph. This paper is devoted to the operation of the $\Delta - Y$ exchanges preserve intrinsically knotted and 3-linked graph.

Introduction

The spatial graph theory is the natural generalization of the knotted topology theory, as an active branch of the topology. The study of the Intrinsically linked and intrinsically knotted graphs began In 1983, when Conway and Gordon[1] showed that every embedding of K_6 (the complete graph on six vertices) contains a non-trivial link, and every embedding of K_7 contains a non-trivial knot. Since the existence of such a non-trivial link or knot depends only on the graph and not on the particular embedding of the graph in S^3, we say that K_6 is intrinsically linked graph and K_7 is intrinsically knotted graph. Sachs defined the Petersen family as the set of graphs which can be obtained from K_6 by a finite sequence of moves which either replace a triangle by Y, or Y by a triangle. A vertex splitting of a vertex v in a graph G is achieved by replacing v with two vertices v' and v'', adding the edge $\overline{v'v''}$ and connecting a subset of the edges that were incident to v to v' and the rest of the edges that were incident to v to v''. Sachs conjectured that this family was the complete set of minor minimal intrinsically linked graphs. (Recall that a graph H is said to be a minor of a graph G if H can be obtained from G by deleting and/or contracting a finite number of edges. A graph G is said to be minor minimal with respect to a property if G has the property but no minor of G has the property[2,3].) A natural generalization of an intrinsically linked graph is an intrinsically 3- linked graphs, a graph is intrinsically 3-linked if there exists a non-split 3-component link in every spatial embedding. It is important to found the every spatial embedding of this graph contains nontrivial knot and a non-split 3-component link. That is the intrinsically knotted and 3-linked graph. And show that the operation of the $\Delta - Y$ exchanges preserve intrinsically knotted and 3-linked graph.

The new intrinsically knotted and 3-linked graph G^*

Firstly, constructing the graph G^*, we will use two of the graph $K_{3,3,1}$. This graph is one of the Petersen graphs and hence is minor minimal with respect to being intrinsically 2-linked. The graph G^* can be constructed as follows. We start with two copies of $K_{3,3,1}$, the vertices of $K_{3,3,1}$ are labeled by A, v_1^1, v_2^1, v_3^1, v_1^2, v_2^2, v_3^2, the vertices of another copy $K_{3,3,1}$ are labeled by B, v_4^1, v_5^1, v_6^1, v_4^2, v_5^2, v_6^2. We create the graph W by adding an edge between the pair of vertices A and B, and adding an edge between the pair of vertices v_1^2 and v_4^1, between the pair of vertices v_2^2 and v_5^1, between the pair of vertices v_3^2 and v_6^1, between the pair of vertices v_1^2 and v_6^1, the G^* is illustrated in Fig. 1. The paper obtain the following results.

Theorem1. Let G^* denote the graph illustrated in Fig. 1. Then G^* is new intrinsically knotted and 3-linked graph.

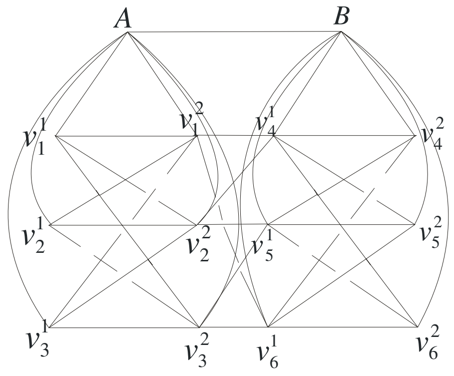

Fig. 1 The graph G^*

Proof: consider an arbitrarily embedding of G^*, G^* is constructed by the two copies of $K_{3,3,1}$, because $K_{3,3,1}$ is one of the Petersen graphs, so there will be linked cycles. Through the analysis of the $K_{3,3,1}$, the two linked cycles are 3-cycle and 4-cycle, the vertices A and B must be contained in each 3-cycles, one of the linked 3-cycles containing the vertex A, we will denote this cycle C_1, another linked 3-cycles will contain the vertex B, we will denote this cycle C_2. The C_1 can pass through at one vertices in { v_1^1, v_2^1, v_3^1 }, another vertices in { v_1^2, v_2^2, v_3^2 }, the other linked cycle C_2 can pass through at one vertices in { v_4^1, v_5^1, v_6^1 }, and another vertices in { v_4^2, v_5^2, v_6^2 }. the 4-cycle linked C_1 pass through at two vertices in { v_1^1, v_2^1, v_3^1 }, other two vertices in { v_1^2, v_2^2, v_3^2 }, we will denote this cycle C_3, the 4-cycle linked C_2 pass through at two vertices in { v_4^1, v_5^1, v_6^1 }, other two vertices in { v_4^2, v_5^2, v_6^2 }, we will denote this cycle C_4. Without loss of generality, suppose C_1 use the vertices { A, v_1^1, v_2^2 }, C_3 use the vertices { $v_2^1, v_3^1, v_1^2, v_3^2$ }, then C_2 use the vertices { B, v_4^1, v_6^2 }, C_4 use the vertices { $v_5^1, v_6^1, v_4^2, v_5^2$ }. Let c be a path in G^* that connects A to B, and let d be a path in G^* that connects v_2^2 to v_4^1, consider the cycle C^* is formed by joining together the paths A - v_1^1 - v_2^2 - d - v_4^1 - v_6^2 - B - c, because $lk(C_3, C^*) + lk(C_3, C_1) = lk(C_3, C^* + C_1)$. If $lk(C_3, C^* + C_1) = 1$, then $lk(C_1, C_3) = 1$ and $lk(C_2, C_4) = 1$, then there is a non-split 3-component link in that embedding. Otherwise $lk(C_3, C^*) = 1$ and we can again conclude that the embedded graph has a 3-component link.

Next, we will prove the embedding of graph G^* contains nontrivial knot, because of graph H (see Fig. 2) is the the minor of G^*, and J.Foisy[3] has proof the important result, the graph H is a newly recognized intrinsically knotted graphs, so the every spatial embedding of this graph H contains nontrivial knot. So we get every spatial embedding of graph G^* contains nontrivial knot. Overall, every spatial embedding of this graph G^* contains nontrivial knot and a non-split 3-component link. Theorem1 is proven as well.

Because of the $K_{3,3,1}$ is Petersen graph, Garry Bowlin[4] showed that "let G be a graph from the Petersen family as sub-graphs, if there are edges between the two sub-graphs such that the edges form a 6-cycle with vertices that alternate between the two sub-graphs, then G is Intrinsically 3-linked graph". But is it minor minimal Intrinsically 3-linked graph form this method, the graph

G^* in Theorem1 answer the question as well, the answer is no. because the graph G^* is minor of the graph G that is constructed from two Petersen graph $K_{3,3,1}$ and the edges form a 6-cycle with vertices that alternate between the two graphs.

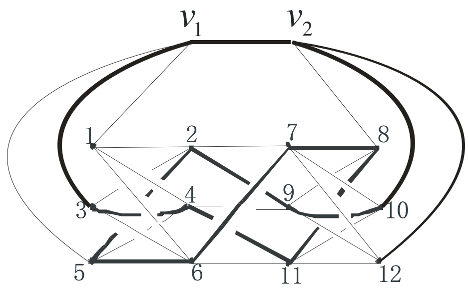

Fig. 2 Graph H [3]

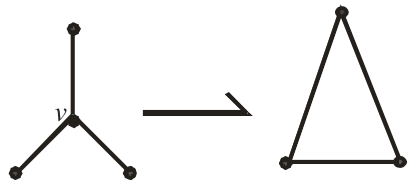

Fig. 3 "$\Delta - Y$"exchange [1]

The operation of the $\Delta - Y$ exchanges

Now, we show that the $\Delta - Y$ exchanges preserve intrinsically knotted and 3-linked graph. **Let** $\Delta - Y$ exchanges illustrated in Fig.3.

Theorem2. The operation of the $\Delta - Y$ exchanges preserve intrinsically knotted and 3-linked graph.

Proof: Fristly, using the Dijkstra's algorithm to compute shortest paths from a starting vertex to each other vertex. We will compute shortest distances in G_Δ .

A starting vertex is chosen, from which all distances will be computed. Each vertex in the graph G_Δ is assigned a distance value, initially for all but the starting vertex. Then the vertex with the minimum distance is examined, and any vertices adjacent to the current vertex have their distances updated to the current vertex's distance plus the edge between the current and the adjacent vertex in $H - e_{v_1 v_2}$ .Then to get the path v_1-v_2 to $v_2^1 - v_5^2$ of $\Delta \to Y$,between a vertex v_i^1 (i=1,…,6) and the starting verte v_i^2 (i=1,…,6), follow the back edges from A_i (i=1,2) back to the start. The path from the start to B (i=1,2) is the reverse of this.

Edge[ ] Dijkstra(Graph G_Δ, Vertex A and B):

Construct an array D[ ] with one cell for each vertex in G_Δ ;

Construct an array of edges E[ ] with one cell for each vertex in G_Y ;

Set D[v_1-v_2] = 0 where s is the starting vertex;

Set D[$v_1^1 - v_4^2$] = (really big) for all vertices u except the starting vertex;

Construct a min priority queue P containing each node n, with D[n] as its priority;

for each vertex z still in P adjacent to v do:

v = P. remove MinElement();

for each vertex z still in P adjacent to v do:

if D[v] + w(v, z) < D[z] then:

D[z] = D[v] + w(v, z);

E[z] = edge(v, z);

Reorder P according to the new priorities D[ ];

fi;

od;

od

return E;

So the The operation of the $\Delta \to Y$ preserve intrinsically knotted and 3-linked graph.

Conclusion

The main results of this paper are that: (1) Exhibit a new intrinsically knotted and 3-linked graph. (2) Using the Dijkstra's algorithm, show that the $\Delta - Y$ exchanges preserve intrinsically knotted and 3-linked graph.

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