Stable Topology Control for Mobile Ad-Hoc Networks

João B. D. Cabrera, Ram Ramanathan, Carlos Gutiérrez, and Raman K. Mehra

***Abstract*— Topology control is the problem of adjusting the transmission parameters, chiefly power, of nodes in a Mobile Ad Hoc Network (MANET) to achieve a desired topology. Over the last several years, this problem has received much attention. Despite this work however, the stability of available techniques has not been studied. This paper presents the first control-theoretic investigation of topology control in MANETs. We take a simple representative fully distributed topology control algorithm called LINT and show that it is unstable under certain conditions. We then formulate LINT in a control-theoretic context, and derive a new mechanism called CLINT that is shown to be stable for a wide range of parameter variations. We compare the in-practice performance of LINT and CLINT using comprehensive simulations and show that CLINT provides a higher throughput.**

***Index Terms*— Mobile ad-hoc networks, power control, topology control, stability.**

I. Introduction

The topology of a Mobile Ad Hoc Network (MANET) is the set of nodes and communication links between nodes. The topology of a MANET depends both on uncontrollable factors such as mobility, terrain, fading, etc. and on controllable factors such as transmit power, antenna direction, processing gain, etc. Topology control is the adjustment of node parameters to get the desired topology, and maintain it in face of changes due to the uncontrollable factors. Topology control has been a subject of much research recently [1]. To the best of our knowledge however, none of the existing algorithms take notice of the fact that the parameter adjustment is a part of a control loop, involving a dynamical system. We show in this note that failure to do so may lead to unstable behavior of the resulting system. For the problem in question, we will also show a way to fix the problem by designing a feedback control system that takes into account the uncertainty in the dynamics of the topology control loop.

II. A Control-Theoretic Problem Formulation

A. LINT operation as a linear discrete time feedback loop

The Local Information No Topology (LINT) topology control algorithm works by changing the transmitting power in order to regulate the node degree (number of neighbors) of each node about a desired setpoint d_d. If p_c is the current transmission power in dB and d_c is the current node degree,

Manuscript received February 16, 2007. The associate editor coordinating the review of this letter and approving it for publication was Prof. Homayoun Yousefi'zadeh. This was work was supported by DARPA under contract W31P4Q-06-C-0068. Approved for Public Release. Distribution Unlimited. The authors are grateful to Lt. Col. James McCormick for his interest and suggestions.

J. B. D. Cabrera, C. Gutiérrez, and R. K. Mehra are with Scientific Systems Company, 500 West Cummings Park, Suite 3000, Woburn, MA 01801 USA (email: {cabrera, carlos, rkm}@ssci.com).

R. Ramanathan is with BBN Technologies, 10 Moulton Street, Cambridge, MA 02138 USA (email: ramanath@bbn.com).

Digital Object Identifier 10.1109/LCOMM.2007.070256.

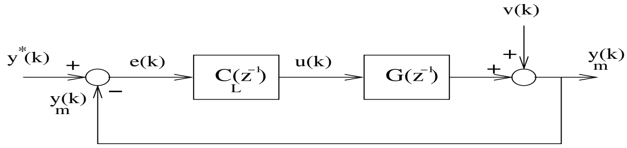

Fig. 1. The LINT feedback control loop.

it is shown in [2] that when $d_d \geq 1$ and $d_c \geq 1$ the desired transmission power p_d in dB to assure a node degree equal to d_d is given by:

$$p_d = p_c + 5\mathcal{E}\log_{10}\left(\frac{d_d}{d_c}\right) \tag{1}$$

where $\mathcal{E}$ is a constant usually between 2 and 5 depending on the propagation properties of the media [3]. Assume that power control is updated every Δ units of time, and define $u(k) = p_d$ as the desired transmission power in dB at time $k\Delta$. Clearly $u(k-1) = p_c$ is the transmission power in dB at time $(k-1)\Delta$. Define also $y_m(k) = \log_{10} d_c$ as the logarithm of the measured node degree at time $k\Delta$, and $y^*(k) = \log_{10} d_d$ as the logarithm of the desired node degree at time $k\Delta$. Assuming that $d_c \geq 1$ and $d_d \geq 1$ for all time, equation (1) becomes:

$$\begin{aligned} u(k) &= u(k-1) + 5\mathcal{E}e(k), \text{ where} \\ e(k) &= y^*(k) - y_m(k), \end{aligned}$$

which can be readily identified as Integral Control (eg. [4]). This is the first step in describing LINT operation as a linear discrete feedback control loop, as depicted in Figure 1. In Figure 1, $C_L(z)$ represents the controller, with input $e(k)$ and output $u(k)$. In terms of z-transforms, the LINT controller can be expressed as:

$$U(z^{-1}) = C_L(z^{-1})E(z^{-1}), \quad C_L(z^{-1}) = 5\mathcal{E}\frac{1}{1-z^{-1}}$$

where $U(z^{-1})$ and $E(z^{-1})$ are the z-transforms of $u(k)$ and $e(k)$ respectively, and $C_L(z^{-1})$ is the transfer function of the LINT controller. The output $u(k)$ is applied to the plant, denoted as $G(z^{-1})$. An additive disturbance term $v(k)$ is also included. Here, $G(z^{-1})$ and $v(k)$ are expected to represent the combined effects of: (1) the actuator dynamics associated with changing power, (2) the network dynamics, associated with node mobility, and (3) the sensor dynamics, related with how *actual* changes in the node degree gets reflected into the *measured* node degree at the node. What are $G(z^{-1})$ and $v(k)$ in Figure 1? It follows from equation (1) and developments in [2] that $p_c = b + 5\mathcal{E}\log_{10}(d_c)$, where b is a constant depending on the propagation properties of the media, the average node density and the radio characteristics. This gives $y_m(k) = \frac{1}{5\mathcal{E}}u(k-1) - \frac{b}{5\mathcal{E}}$ resulting in $G(z^{-1}) = G_1(z^{-1}) = \frac{1}{5\mathcal{E}}z^{-1}$, $v(k) = -\frac{b}{5\mathcal{E}}$, where $G_1(z^{-1})$ is the model of the plant used in LINT. Viewing b as a constant (but unknown) disturbance, the LINT controller is simply an integrator, which is known to provide for zero steady-rate error while tracking constant reference signals $y^*(k)$ and rejecting constant disturbances $v(k)$. If the actual plant $G(z^{-1})$ is indeed equal to $G_1(z^{-1})$ then the resulting open loop transfer function is $C_L(z^{-1})G_1(z^{-1}) = \frac{z^{-1}}{1-z^{-1}}$ providing a closed loop transfer function $\frac{C_L(z^{-1})G_1(z^{-1})}{1+C_L(z^{-1})G_1(z^{-1})} = z^{-1}$ which is stable, and results in $y_m(k) = y^*(k-1)$, i.e. the measured node degree will follow a constant desired node degree.

B. Instabilities in the LINT loop - Theory

One important element which was not considered in LINT design was the transport delay due to HELLO exchanges. Let δ denote the time interval between HELLO messages issued by a node, and N denote the number of HELLO messages taken into account in making a decision about the existence of a link[1]. The average delay between power changes in a node and its effect in the measured node degree is given by $N\delta$. Define $\rho = \lceil \frac{N\delta}{\Delta} \rceil$. It is clear that a more accurate model for the plant is $G_\rho(z^{-1}) = \frac{1}{5\mathcal{E}} z^{-\rho}$. When $\rho \geq 2$, the open loop transfer function formed by LINT and the plant becomes $C_L(z^{-1})G_\rho(z^{-1}) = \frac{z^{-\rho}}{1-z^{-1}}$ providing a closed loop transfer function $\frac{C_L(z^{-1})G_\rho(z^{-1})}{1+C_L(z^{-1})G_\rho(z^{-1})} = \frac{z^{-\rho}}{1-z^{-1}+z^{-\rho}}$, which has a characteristic polynomial in z given by $\lambda(z) = z^\rho - z^{\rho-1} + 1$. Recalling that the product of the roots of the polynomial $z^m + a_{m-1}z^{m-1} + \cdots + a_0$ is given by a_0, it follows that the product of the roots of $\lambda(z)$ is given by 1. This implies that for all $\rho \geq 2$ the polynomial $\lambda(z)$ will either have all roots in the border of the unit circle $|z| = 1$, or it will have at least one root strictly outside the unit circle. In either case $\lambda(z)$ will be an unstable polynomial [4], meaning that the LINT loop fails to provide adequate tracking. It means that if $N\delta > \Delta$ then the LINT algorithm will not work properly.

III. Controlled LINT (CLINT)

A. Instabilities in the LINT loop - Practice

Ideally, both Δ and δ must be decreased in response to increasing mobility. However, decreasing δ increases the protocol overhead acting in the system. In practice, one can assume a lower bound in δ below which the overhead caused by HELLO traffic becomes harmful to the operation of the network. There is no cost however in decreasing Δ in response to mobility, as long as it does not interfere with the updates of the routing tables. Decreasing N is not desirable, since it will increase the noise level acting on the system.

Given the above, one can easily imagine a scenario where in response to network mobility, the designer decreases Δ, but keeps δ constant to avoid overhead. As shown in the previous section this will result in instability. We show below that one can operate with lower Δ, quickly responding to mobility, without necessarily decreasing δ.

B. Designing CLINT: Closed loop pole placement

Figure 2 depicts the model used in designing CLINT (Controlled LINT). We utilize closed loop pole placement as in [5], page 146. We keep the LINT controller in the forward path, to assure tracking of constant reference inputs. The problem at hand is to design a compensator $C_C(z^{-1})$ to assure loop stability when $\rho \geq 2$. We now utilize the notation in [5], page 149, Figure 5.3.2 and define $\frac{B(z^{-1})}{A(z^{-1})} = C_L(z^{-1})G_\rho(z^{-1}) = \frac{z^{-\rho}}{1-z^{-1}}$, giving $B(z^{-1}) = z^{-\rho}$, $A(z^{-1}) = 1 - z^{-1}$. Define also $\frac{P(z^{-1})}{L(z^{-1})} = C_C(z^{-1})$ where $P(z^{-1})$ and $L(z^{-1})$ are polynomials in z^{-1} with degree $\rho - 1 : P(z^{-1}) = p_0 + p_1 z^{-1} + \cdots + p_{\rho-1} z^{-\rho+1}$, $L(z^{-1}) = \ell_0 + \ell_1 z^{-1} + \cdots + \ell_{\rho-1} z^{-\rho+1}$. In practice, the value of ρ may not be known, and/or the user may want the flexibility of varying Δ, δ and N. The poles of the resulting closed loop system will be roots in z of the polynomial $\Pi_\rho(z^{-1}) = L(z^{-1})(1 - z^{-1}) + z^{-\rho}P(z^{-1})$. To produce a fixed controller capable of stabilizing the closed-loop system for wide variations in ρ we proceed as following:

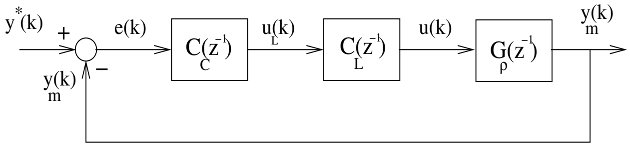

Fig. 2. The CLINT design model.

1) Pick a fixed value of $\rho = \rho^*$ and compute $P^*(z^{-1})$ and $L^*(z^{-1})$ so that $\Pi_{\rho^*}(z^{-1}) = L^*(z^{-1})(1 - z^{-1}) + z^{-\rho^*}P^*(z^{-1}) = A^*(z^{-1})$, where $A^*(z^{-1}) = (z^{-1} - 3)^{2\rho^*-1}$, i.e. we are assigning all closed loop poles to $z = \frac{1}{3}$ which is known to provide a good trade-off between overshoot, rise-time and settling time for discrete time systems ([5], p. 159).
2) For each value of $\rho \geq 2$ verify if the roots in z of the polynomial $\Pi_\rho(z^{-1}) = L^*(z^{-1})(1 - z^{-1}) + z^{-\rho}P^*(z^{-1})$ are all inside the unit circle or not. If they are, then the closed loop system formed by $P^*(z^{-1})$, $L^*(z^{-1})$, $B(z^{-1}) = z^{-\rho}$, and $A(z^{-1}) = 1 - z^{-1}$ is stable. Otherwise, it is unstable.

In our design, we picked $\rho^* = 3$, and verified that the corresponding $P^*(z^{-1})$ and $L^*(z^{-1})$ produced stable control loops for all values of ρ in the interval $1 \leq \rho \leq 9$. The resulting closed loop is unstable for $\rho \geq 10$. The equations governing the CLINT controller are as following:

$$
\begin{aligned}
u_L(k) &= \frac{1}{\ell_0^*}[-\ell_1^* u_L(k-1) - \ell_2^* u_L(k-2) \\
&+ p_0^* e(k) + p_1^* e(k-1) + p_2^* e(k-2)] \\
u(k) &= u(k-1) + 5\mathcal{E} u_L(k), \quad \text{where}
\end{aligned}
$$

$\ell_0^* = -243, \ell_1^* = 162, \ell_2^* = -108, p_0^* = -18, p_1^* = -15, p_2^* = 1.$

IV. Experimental Studies

We simulated a MANET with 50 nodes using the ns-2 simulation package. The details are as following. **Motion**: Nodes move for 1000 s in a 1000 $m \times 1000$ m square, changing direction and velocity (uniform distribution between 2 m/s and 6 m/s) randomly every 100 s; **Network layer**: Hazy-Sighted Link State Routing (HSLS); **MAC layer**: IEEE 802.11 MAC as provided in ns-2; **Maximum power**: The maximum power for each node correspond to 4.6 W (range of 500 m); **Desired node degree**: For both LINT and CLINT, the desired setpoint is $d_d = 8$. The LINT controller is only active

[1] As noted in [2], nodes are incorporated in or taken out from neighborhood tables based on a majority vote from N HELLO updates.

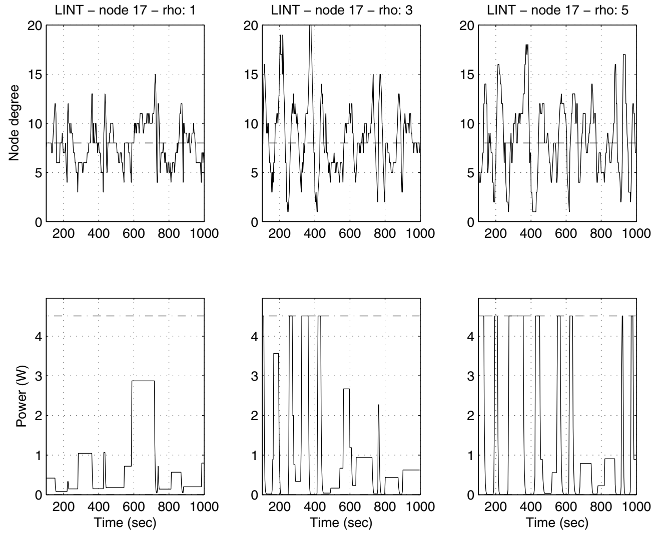

Fig. 3. LINT performance for varying ρ.

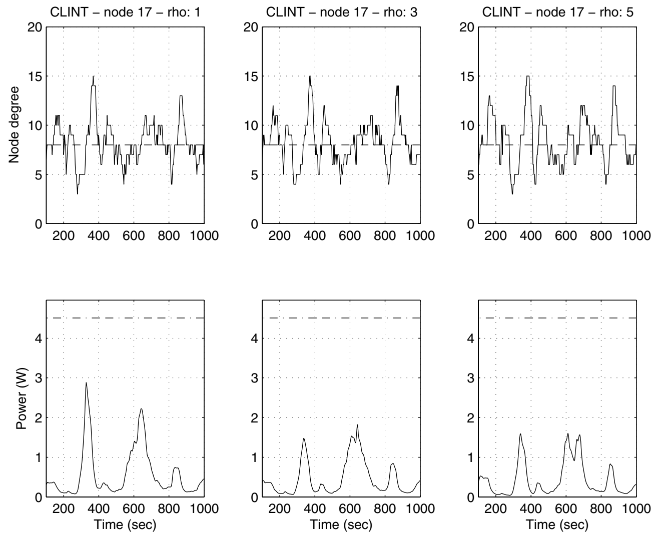

Fig. 4. CLINT performance for varying ρ.

when the measured node degree is below 5 or above 11[2]. To verify the effect of delay, we kept $\Delta = 3\ s$ constant, $N = 3$ constant, and considered three cases: $\delta = 1\ s,\ 3\ s$ and $5\ s$, corresponding respectively to $\rho =\ 1,\ 3,\ 5$. The evolution of the measured node degree and the transmitted power for one of the nodes is depicted in Figure 3 (LINT) and Figure 4 (CLINT). It is clear the performance of LINT degrades considerably with the increase in ρ, while the performance of CLINT is not affected. For $\rho = 3$ and $\rho = 5$ the power in LINT essentially oscillates between minimum power and maximum power, which is a clear evidence of instability. The CLINT loop in contrast remains stable.

Table I presents the Error RMS[3] for the regulated output $y_m(k)$ and the the overall average (among all nodes and across time) of transmitting power. The results clearly show the superiority of CLINT. To verify the effect of power control on data throughput, we ran another set of experiments in which data was sent between 5 node pairs. The data was UDP/CBR data at a rate of 4 KBps, with the packet size being 256 bytes. All sets contained data for LINT and CLINT, as well as a baseline run in which no power control was performed. In all cases, the throughput falls with increasing ρ. However, the CLINT system provides the same throughput as the baseline with much less usage of power.

[2]This was shown experimentally to improve LINT overall performance; it was also shown experimentally that CLINT behavior is not affected. This also explains the plateaus in LINT power profiles shown in Figure 3.

[3]Error RMS= $\sqrt{\frac{1}{n}\sum_{k=1}^{n}(y_m(k)-8)^2}$, where n is the number of data samples across all nodes.

TABLE I

ERROR RMS OF $y_m(k)$ AND AVERAGE POWER (WATTS).

Error RMS	$\rho = 1$	$\rho = 3$	$\rho = 5$
LINT	2.36	3.80	4.49
CLINT	2.40	2.85	3.20
Av. Power	$\rho = 1$	$\rho = 3$	$\rho = 5$
LINT	1.31	1.91	2.00
CLINT	1.20	1.20	1.20

TABLE II

THROUGHPUT WITH $\Delta = 3\ s$ AND VARYING δ.

Throughput	$\rho = 1$	$\rho = 3$	$\rho = 5$
Baseline (4.6 W)	0.71	0.36	0.27
LINT	0.66	0.23	0.19
CLINT	0.70	0.37	0.26

V. CONCLUDING REMARKS

The present note is a first step into the understanding of topology control algorithms from a control theoretic viewpoint. For global topology control schemes, where power decisions in one node depends on information on the other nodes, the problem becomes much more involved, as the topology control mechanism interacts with the dynamics of the routing algorithm being employed. An integrated design of routing and topology control, taking into account the overall dynamics of the resulting loop is a future research direction.

ACKNOWLEDGEMENT

This work was supported by DARPA contract W31P4Q-06-C-0068. Approved for Public Release. Distribution Unlimited. The authors are grateful to Lt. Col. James McCormick for his encouragement.

REFERENCES

[1] P. Santi, *Topology Control in Wireless Ad Hoc and Sensor Networks*. John Wiley & Sons, Ltd., 2005.

[2] R. Ramanathan and R. Rosales-Hain, "Topology control of multihop wireless networks using transmit power adjustment," in *Proc. IEEE INFOCOM*, Tel Aviv, Israel, March 2000.

[3] T. S. Rappaport, *Wireless Communications, Principles and Practice*. Prentice-Hall, 1996.

[4] B. C. Kuo, *Digital Control Systems*. Holt-Saunders International Editions, 1980.

[5] G. C. Goodwin and K. W. Sin, *Adaptive Filtering, Prediction and Control*. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1984.