UM-P-80/34

A PHENOMENOLOGICAL NUCLEON-NUCLEON INTERACTION

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ABSTRACT

A phenomenological NN interaction which fits the deuteron properties and the elastic scattering data, is presented. The only parameters are the π, ρ and ω masses and one overall strength.

1. INTRODUCTION

Many NN interactions have been derived, either purely phenomenologically or from field and dispersion theoretical approaches. The former vary greatly in their fit of the elastic scattering data as not all were derived principally to satisfy this criterion. The latter type are more realistic in that they use coupling constants derivable from other considerations, but need additional parameters to fit the data, especially the scattering lengths and deuteron properties. The need for additional parameters is due partly to the fact that the coupling constants are actually functions of scalar variables of the problem, but is also a manifestation of more fundamental problems such as the lack of any real criterion for choosing which mesons to include.

The following interaction, specifically a neutron-proton interaction, is presented for its relative simplicity and good fit of the data. The interaction should be regarded as phonomenological because although there is an underlying model, it has not been developed sufficiently to be presented at this stage. Contributions from one pion, two pion, ρ and ω mesons are included. The functional form (Petris 1971, Banks 1974) is essentially a Gaussian momentum dependence divided by the usual Yukawa-producing denominator, and effectively limits each meson contribution to a particular energy range. The different meson contributions are weighted analytically in the functional form. The operators used are partly novel and appear to be dictated by the deuteron properties and phase shifts.

The interaction is presented in sections 2 and 3, and the fit to the data in section 4. The application of this interaction in nuclear structure calculations is discussed in section 5.

2. THE INTERACTION

The interaction has the form

$$\langle \underline{k}' | V | \underline{k} \rangle = \frac{\hbar c}{M} \frac{G}{\pi^2} \sum_{i,\underline{A},\underline{B},\ldots} (\underline{A}'.\underline{B}')^{\dagger} \; f_j/\omega_i^2 \; (\underline{A}.\underline{B}) \qquad (2.1)$$

where M is twice the reduced nucleon mass and m_i the masses of the π^0, $2\pi^{\pm}$, ρ and ω mesons in fm^{-1}; G is the overall strength in fm^{-1}; $\underline{A},\underline{B},\ldots$ are operators discussed below; $\omega_i^2 = [(\underline{k}-\underline{k}')^2 + m_i^2]$ is the usual denominator, with $\underline{k}$ and $\underline{k}'$ denoting the initial and final center-of-mass momenta; and there are two form factors f_j

$$f_1 \; (k^2,k'^2,M,m) = \frac{M}{m} \exp\left[- \frac{M}{m} - \frac{(k^2+k'^2)}{2m^2}\right] \qquad (2.2)$$

and

$$f_2 \; (k^2,k'^2,M,m) = \frac{(k^2+k'^2)}{2m^2} \; f_1 \; (k^2,k'^2,M,m). \qquad (2.2b)$$

The interaction has the simple partial-wave form for each state $\alpha \equiv$ LSJT

$$V_{\alpha\alpha}(k,k') = G\left[V_{\pi^0}^{\alpha\alpha} \; F_L(m_{\pi^0}) + V_{2\pi^{\pm}}^{\alpha\alpha} \; F_L''(2m_{\pi^{\pm}}) + V_{\rho}^{\alpha\alpha} \; F_L''(m_{\rho}) + V_{\omega}^{\alpha\alpha} \; F_L''(m_{\omega})\right] \qquad (2.3)$$

where $F_L(m)$ and $F_L''(m)$ are given by

$$F_L(m) = \frac{1}{kk'} \; Q_L\left(\frac{k^2+k'^2+m^2}{2kk'}\right) \; f_1 \; (k^2,k'^2,M,m) \qquad (2.4a)$$

$$F_L''(m) = \frac{1}{kk'} \; Q_L\left(\frac{k^2+k'^2+m^2}{2kk'}\right) \; f_2 \; (k^2,k'^2,M,m) \qquad (2.4b)$$

and the $V_i^{\alpha\alpha}$ are the matrix elements for state α of the operator expressions $\sum(\underline{A}'.\underline{B}')^{\dagger}\underline{A}.\underline{B}$ discussed in section 3. There are no

$Q_{L\pm1}$ terms because the tensor operators defined below, have identically zero diagonal ($\alpha = \alpha'$) matrix elements. The off-diagonal matrix elements ($\alpha \neq \alpha'$, $L = L' \pm 2$) are given by

$$V^{TE}_{\alpha\alpha'}(k,k') = G[V^{\alpha\alpha'}_{\pi^0} F_J''(m_{\pi^0}) + V^{\alpha\alpha'}_{2\pi^\pm} F_J''(2m_{\pi^\pm}) + V^{\alpha\alpha'}_{\rho} F_J''(m_\rho) + V^{\alpha\alpha'}_{\omega} F_J''(m_\omega)] \tag{2.5}$$

It is required phenomenologically that the "non-tensor" part of the π^0 term have the form $F_L(m_{\pi^0})$, while the tensor part and all the other meson contributions have the form $F_L''(m)$.

3. ORIGIN OF THE $V^{\alpha\alpha}_i$

When the factor $\frac{M}{m} e^{-M/m}$ was included in the functional form, it was found that the strengths of the various meson terms for each state conformed to definite patterns. On factorizing out a common strength, a set of $V^{\alpha\alpha}_i$ very similar to the ones on Table 1, were obtained (Petris 1978). In order to reproduce the scattering lengths and deuteron properties it was found that the mass parameter required for the first term was that of the π^0 and that for the second term was twice the mass of the π^+ or π^-. No other combination of masses gives a simultaneous fit to the deuteron binding energy and triplet scattering length. In addition $V^{\alpha\alpha}_{\pi^0}$ and $V^{\alpha\alpha}_{2\pi^\pm}$ for the $^3S_1$ state have the remarkable values of -6.000 and -2.250 which are exactly 1½ times the corresponding ones for the $^1S_0$ state. Because of these and many other $V^{\alpha\alpha}_i$ which appear to be rigidly fixed by the data, it was decided to search for an appropriate set of operators to characterise them.

It was found eventually, that for the π^0 and $2\pi^\pm$ terms the $V^{\alpha\alpha}_i$ could be obtained from the matrix elements of the scalar products of operators such as $\hat{\underline{k}}(\hat{\underline{k}}')$, $\hat{\underline{L}} = \frac{\underline{L}}{\sqrt{L(L+1)}}$ *, $\hat{\underline{k}}x\hat{\underline{L}}(\hat{\underline{k}}'x\hat{\underline{L}})$, $\hat{\underline{J}}, \hat{\underline{S}}, \hat{\underline{S}}x\hat{\underline{L}}$ and $\hat{\underline{J}}x\hat{\underline{L}}$.

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* $\underline{A} = 0$ if $|\underline{A}| = 0$

Since $\hat{k},\hat{L},\hat{k}x\hat{L}$ and $\hat{k}',\hat{L},\hat{k}'x\hat{L}$ are orthogonal sets associated with the initial and final states, the number of non-vanishing scalar products is quite small and can be divided into three main types. These are the scalar products of $\hat{k}$ ($\hat{k}'$) and $\hat{k}x\hat{L}(\hat{k}'x\hat{L})$ with other operators producing the tensor terms, the scalar products that produce the important kronecker δ terms, for example,

$$\hat{S}.\hat{S}(\hat{k}.\hat{k} - \hat{L}.\hat{L}) = \delta_{S1}\,\delta_{L0} = \delta^1_{L0} \tag{3.1a}$$

$$\hat{J}.\hat{J}(\hat{J}.\hat{J} - \hat{S}.\hat{S}) = \delta_{S0}(1 - \delta_{J0}) = \delta^0_{J>0} \tag{3.1b}$$

$$\hat{J}.\hat{L}\;\hat{S}.\hat{S} - 2\,\hat{J}.\hat{S}\;\hat{L}.\hat{S} = \delta_{S1}\,\delta_{LJ} = \delta^1_{LJ} \tag{3.1c}$$

$$(\hat{S}.\hat{S})^2 - [(\hat{L}.\hat{S})^2 + (\hat{J}.\hat{S})^2] = (1 - \frac{1}{J(J+1)})\;\;\delta^1_{LJ} \tag{3.1d}$$

while the remaining ones reduce to small powers of $\hat{L}.\hat{S}$, $\hat{J}.\hat{S}$ and $\hat{J}.\hat{L}$, for example,

$$\hat{S}.\hat{S}x\hat{L} = i\,\hat{S}.\hat{L}/\sqrt{2} \tag{3.2a}$$

$$\hat{S}x\hat{L}\;.\;\hat{S}x\hat{L} = \delta_{S1} - (\hat{L}.\hat{S}) \tag{3.2b}$$

The operators required to characterise $V^{\alpha\alpha}_{\rho}$ and $V^{\alpha\alpha}_{\omega}$ were found to be essentially L^2, $L.S$ and $(L.S)^2$ operators commonly used in NN interactions.

The following operator expressions V_i were constructed to produce the $V^{\alpha\alpha'}_i$ in Table 1,

$$V_{\pi 0} = \tau_1.\tau_2[-4\delta_{J0} + \delta^0_{J>0} + 2\{\delta^1_{LJ} + (\hat{J}.\hat{S})^2 - (\hat{L}.\hat{S})^2\} + \hat{S}_{12}] \tag{3.3a}$$

$$V_{2\pi\pm} = -\frac{3}{4} + \frac{1}{4}\,\delta^0_{J>0} - \hat{L}.\hat{S} + \tau_1.\tau_2[-\frac{3}{4} + 3\delta^1_{LJ} + (\hat{J}.\hat{S} + \hat{L}.\hat{S})\;(\hat{J}.\hat{S} - \hat{L}.\hat{S} + \tfrac{1}{4}) + 2\hat{K}_{12}] \tag{3.3b}$$

$$V_\rho = \tau_1.\tau_2 \left[-\frac{1}{3}\delta_{J>0} + \delta^1_{L>0} + \underline{J}x\underline{L}.\underline{J}x\underline{L} - (\underline{L}.\underline{S})^2 + S_{12}\right]^* \quad (3.3c)$$

$$V_\omega = \delta^0_{J0} + \delta_{L\,0} - 4\underline{L}.\underline{S} - S_{12} \quad (3.3d)$$

where the $\tau_1.\tau_2$ arises for isovector contributions, the $2\pi^{\pm}$ being a mixture of isoscalar and isovector contributions, and the tensor operators are given by

$$\hat{S}_{12} = (\hat{\underline{k}}'.\hat{\underline{S}})^{\dagger}\hat{\underline{k}}.\hat{\underline{S}} + (\hat{\underline{S}}.\hat{\underline{k}}')^{\dagger}\hat{\underline{S}}.\hat{\underline{k}} - (\hat{\underline{S}}x\hat{\underline{L}}.\hat{\underline{k}}')^{\dagger}\hat{\underline{S}}x\hat{\underline{L}}.\hat{\underline{k}} - (\hat{\underline{k}}'.\hat{\underline{S}}x\hat{\underline{L}})^{\dagger}\hat{\underline{k}}.\hat{\underline{S}}x\hat{\underline{L}} \quad (3.4a)$$

$$\hat{K}_{12} = (\hat{\underline{S}}x\hat{\underline{L}}.\hat{\underline{k}}'x\hat{\underline{L}})^{\dagger}\hat{\underline{S}}x\hat{\underline{L}}.\hat{\underline{k}}x\hat{\underline{L}} + (\hat{\underline{k}}'x\hat{\underline{L}}.\hat{\underline{S}}x\hat{\underline{L}})^{\dagger}\hat{\underline{k}}x\hat{\underline{L}}.\hat{\underline{S}}x\hat{\underline{L}}$$

$$- (\hat{\underline{S}}.\hat{\underline{k}}'x\hat{\underline{L}})^{\dagger}\hat{\underline{S}}.\hat{\underline{k}}x\hat{\underline{L}} - (\hat{\underline{k}}'x\hat{\underline{L}}.\hat{\underline{S}})^{\dagger}\hat{\underline{k}}x\hat{\underline{L}}.\hat{\underline{S}} \quad (3.4b)$$

$$S_{12} = \underline{S}^2\hat{S}_{12} = 2\hat{S}_{12} \quad (3.4c)$$

The diagonal matrix elements ($\alpha = \alpha'$) of $\hat{S}_{12}$ and $\hat{K}_{12}$, as already mentioned, are identically zero, while the off-diagonal ones are given by

$$\langle LSJT|\hat{S}_{12}|(L\pm2)SJT\rangle = \tfrac{1}{2}\left[3\sqrt{J(J+1)} + \sqrt{(J-1)(J+2)}\right]/(2J+1) \quad (3.5a)$$

$$\langle LSJT|\hat{K}_{12}|(L\pm2)SJT\rangle = \tfrac{1}{2}\left[2\sqrt{(J-1)(J+2)} + (J-1)(J+2)/\sqrt{J(J+1)}\right]/(2J+1) \quad (3.5b)$$

The form of the tensor terms, $(\underline{A}.\hat{\underline{k}}')^{\dagger}\underline{A}.\hat{\underline{k}}$ and $(\underline{A}.\hat{\underline{k}}'x\underline{L})^{\dagger}\underline{A}.\hat{\underline{k}}x\underline{L}$, indicates that one scalar product is associated with the initial state and the other with the final state. It is possible that all the terms in equations (3.3) have this form, i.e. $(\underline{A}_f.\underline{B}_f)^{\dagger}\underline{A}_i.\underline{B}_i$. The $\underline{J}x\underline{L}.\underline{J}x\underline{L}$ term in V_ρ would then have an origin from terms such as $\underline{J}x\hat{\underline{L}}.\underline{J}x\hat{\underline{L}}\ \underline{L}.\underline{L}$, and the $\underline{L}.\underline{S}$ in V_ω from terms such as $\underline{L}.\hat{\underline{L}}\ \hat{\underline{L}}.\underline{S}$ and $\underline{L}.\hat{\underline{S}}\ \hat{\underline{S}}.\underline{S}$. The contention that all the V_i expressions, or ones very similar, can be produced from all admissible scalar products of this type for a specific subset of operators, is being investigated.

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* $\underline{J}x\underline{L}.\underline{J}x\underline{L} = \left[\underline{L}^2(\underline{S}^2-1) - (\underline{L}.\underline{S})^2\right]\delta_{J>0}$

4. FIT OF THE DATA

The matrix inversion technique (Haftel and Tabakin 1970) for solving the Lippmann-Schwinger equation (LSE), was used to obtain the R-matrix elements for the interaction. These give the Blatt-Biedernheim phase shifts and coupling parameters from which are obtained the Nuclear Bar ones. The latter are presented in figures 1 to 6 together with the most commonly accepted phase shift analysis of MacGregor et al. (MacGregor et al. 1969), at laboratory energies E_{LAB} = 25, 50, 95, 142, 210, 330 and 425 MeV. There are a few phase shifts in this analysis, such as the $^1P_1$ at 50 MeV, the $^3S_1$ at 330 MeV and some of the ϵ_1 values, which appear to be unacceptable. The ϵ_1 as predicted by this interaction are in better agreement with a more recent analysis of Signell and Holdeman (Signell and Holdeman 1971). The overall fit to the phase shifts is remarkable nevertheless, with a χ^2/(data points) of 3.

The contribution to the phase shift from the various terms is mainly from the π^0 term below 30 MeV, the $2\pi^{\pm}$ term dominates from 50 to 250 MeV, and the ρ and ω contributions are important above 200 MeV and completely determine the phase shift behaviour above 350 MeV. These three types of contributions form ideal "building blocks" which combine in the proportions given by the $V_i^{\alpha\alpha}$ to reproduce the phase shifts of all the states. There are two states, the $^3P_1$ and $^3F_4$, for which the fit is improved by a better choice of $(V_\rho^{\alpha\alpha}+V_\omega^{\alpha\alpha})$. To improve the operator expressions to obtain a better fit for these two states, is both difficult and unwarranted at this stage. Both examples illustrate the importance of the ρ and ω contributions above 200 MeV. The operator expressions for π^0 and $2\pi^{\pm}$ cannot be greatly improved upon.

The scattering lengths and deuteron properties are presented in Table 2. It is these properties which require the first contribution

to come from a π^0 and the second from $2\pi^{\pm}$. The deuteron properties also serve to rigidly fix the $V_i^{\alpha\alpha'}$ corresponding to the coupling parameter ϵ_1. These are zero for $V_{2\pi^{\pm}}^{\alpha\alpha'}$ and simple multiples of $\sqrt{2} = \sqrt{J(J+1)}$ for the other mesons. Together with the other ϵ_J, this virtually determines $\hat{S}_{12}$ and $\hat{K}_{12}$ to have the above forms. The feature that both $\hat{S}_{12}$ and $\hat{K}_{12}$ make no contribution to the diagonal potential matrix elements was not contrived and seems to blend well with the other features of the interaction. Although this is a soft-core interaction, unlike other potentials in this class which predict too small a D-state probability for the deuteron, it predicts a P_D of 5%. The overall strength G was found to have the same value of 20.2 fm^{-1} for both S=0 and S=1 states. It is interesting to note that this value of G is close to $2(2M+m_\pi) = 20.4$ fm^{-1}. Unfortunately because of the $2\pi^{\pm}$ term and also the nature of the functional form one cannot make comparisons with coupling constants of meson exchange theories.

5. APPLICATION OF INTERACTION IN NUCLEAR STRUCTURE AND NUCLEAR MATTER

The reason for presenting this interaction before it can be put on a more rigorous field theoretic basis, is that the results of preliminary nuclear structure calculations (Petris 1978) were very promising. The reduced matrix elements for this interaction are given by

$$\langle n\ {}^{2S+1}L_J||V||n'\ {}^{2S+1}L'_J\rangle =$$

$$\frac{2}{\pi}\frac{\hbar c}{M}\iint \phi_{nL}(\sqrt{2}bk)V_{\alpha\alpha'}(k,k')\phi_{n'L'}(\sqrt{2}bk')\,k^2k'^2dkdk' \qquad (5.1)$$

where $\frac{\hbar c}{M} = 41.5$ MeV fm^2, b is the oscillator length parameter and the $\phi_{nL}(x)$ are given by

$$\phi_{nL}(x) = \sqrt{\frac{2(\sqrt{2}b)^3\Gamma(n+1)}{\Gamma(n+L+3/2)}}\, i^{2n-L}\, x^L\, e^{-\frac{x^2}{2}}\, L_n^{L+\frac{1}{2}}(x) \qquad (5.2)$$

where $L_n^{L+\frac{1}{2}}$ are Laquerre polynomials. The $n = n' = 0$ matrix elements for the $^1S_0$ and $^3S_1$ states for $b = 1.7$ are respectively -5.5 and -8.3 MeV. These are similar to those of other popular potentials. The matrix elements for $L \geq 1$, however, decrease in magnitude much more rapidly than those of any other interaction. This difference is transmitted to the jj coupled matrix elements that are not dominated by $^1S_0$ or $^3S_1$ contributions, thereby affecting appreciably at least some of the nuclear structure properties in any calculation.

The resultsof a preliminary Hartree-Fock calculation on O^{16} are presented in Table 3. This calculation was preliminary in the sense that a small dimensionality of 3 had to be used, and the center-of-mass correction was included approximately by modifying the kinetic energy term by the factor (N-1)/N, where N is the number of nucleons in the level. Nevertheless the results are remarkable for an interaction which is in accordance with the scattering data. Simple particle-hole calculations for the giant dipole regions of C^{12} and O^{16} also produced spectra with the main levels close to the observed ones. It is hoped that more rigorous calculations, especially on O^{16} and Ca^{40}, will be carried out to determine just how suitable this interaction is for nuclear structure calculations.

A simple perturbation nuclear matter calculation produced a binding energy per particle of 15 MeV at $k_F = 1.4$ fm^{-1}. Saturation was not obtained, however, and a more rigorous G-matrix calculation is required to see if the $^3S_1$ contribution levels off above $k_F = 1.4$ fm^{-1} as it does for all potentials that produce saturation (Haftel and Tabakin 1970).

6. SUMMARY

The interaction that has been presented is very simple in comparison to contemporary ones. All of its features - such as the functional form, the operators and the parameters - were derived from fitting the deuteron properties and the more accurate of the elastic scattering data. Any variation of the functional form, the parameters or the operator expressions, quickly destroys the delicate equilibrium which reproduces the deuteron properties so accurately. It may appear that one can easily construct alternative operator expressions to characterise the $V_i^{\alpha\alpha}$, this however is not true. In view of the relative simplicity of the interaction and the specificity and effectiveness of its features, an explanation of its origin is being sought from a more fundamental underlying model of particle interactions. For the present this interaction is proposed as phonomenological and it is hoped that in the meantime its usefulness in nuclear structure calculations will be tested more rigorously.

ACKNOWLEDGEMENTS

This work was carried out at the invitation of Dr. K. Amos. Without his support and advice this work would not have reached its present state. The author wishes to express his gratitude to Dr. K. Amos and also to Professor B. Spicer and Professor B. McKellar for their critical readings of this paper, and Dr. E. Muirhead for extending the facilities of the department.

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Signell, P. and Holdeman Jr., P. 1971 Phys. Rev. Letters 27 1393.

FIGURE CAPTION:

Figures 1 - 6 The phase shifts and coupling parameters for the interaction as a function of laboratory energy. The dots and error bars denote the phase shifts of MacGregor et al. The open circles in figure 3 denote the ϵ_1 of Signell and Holdeman. The dashed lines in figure 4 and 6 correspond to alternative values of V_ρ and V_ω as given in Table 1.

TABLE 1: $V_i^{\alpha\alpha}$ CORRESPONDING TO THE V_i OF EQN.3.3

STATE $^{2S+1}L_J$	$V_{\pi^0}^{\alpha\alpha}$	$V_{2\pi^\pm}^{\alpha\alpha}$	$V_\rho^{\alpha\alpha}$	$V_\omega^{\alpha\alpha}$
$^1S_0$	-4.000	-1.500	0.000	1.000
$^1D_2$	-1.0	-1.25	-6.3	0
$^1G_4$	-1	-1.25	-20	0
$^1P_1$	3.0	1.75	7	0
$^1F_3$	3	1.75	37	0
$^1H_5$	3	1.75	91	0
$^3S_1$	-6.000	-2.250	1.000	0.000
ϵ_1	-2.12	0.0	-4.24	-1.41
$^3D_1$	3	4.9	34	13
$^3D_2$	-6	-7.1	-14	5
$^3D_3$	-2	-1.1	4	-7
ϵ_3	-3	-4	-6	-2
$^3G_3$	2	4	88	21
$^3G_4$	-6	-7	-56	5
$^3G_5$	-1	-1	34	-15
$^3P_0$	-6.0	-1.75	-3	9
$^3P_1$	2	2.0	0.7	5 (10)*
$^3P_2$	1	-1.1	0.7	-3
ϵ_2	1	1.1	1.9	-1.9
$^3F_2$	-1	-1.3	-19	17
$^3F_3$	2	1.7	11	5
$^3F_4$	1	-1.5 (-1.0)*	-5	-11 (-5)*
ϵ_4	1	1.4	2 (8)*	-2
$^3H_4$	0	-1.3	-41	25
$^3H_5$	2	1.7	29	5
$^3H_6$	0	-1.6	-19	-19

* Values of the $V_i^{\alpha\alpha}$ which are in better agreement with the phase shifts of MacGregor et al, are represented by the dashed lines in figures 1 to 6. The number of significant figures reflects the accuracy to which the $V_i^{\alpha\alpha}$ are required to fit the scattering lengths, deuteron properties and phase shifts.

TABLE 2: LOW EVERGY RESULTS FOR N-P SYSTEM

QUANTITY	G(fm^{-1})	EXP.VALUE	INTERACTION	METHOD OF CALCULATION
a_s(np)fm	20.17 (20.18)	-23.7±.1	-23.72 (-23.88)	LSE
$a_t$fm	20.20	5.39±.01	5.390	LSE
$r_t$fm	20.20	1.73±.02	1.72	EFFECTIVE RANGE APPROXIMATION
DEUTERON ($^3S_1$-$^3D_1$)				
E_B MeV	20.20	-2.2245	-2.220	LSE (DETERMINANT METHOD)
P_D	20.20	4-8%	5.0%	
Q fm^2	20.20	.278 .008	.28	$Q = \frac{\varepsilon_1}{\sqrt{2k^2}}$
$^1S_0$ VIRTUAL STATE (MeV)	20.17	.066	.05-.10	LSE (DETERMINANT METHOD)

TABLE 3: HARTREE-FOCK RESULTS FOR O^{16}

	Single-particle levels without & with CMC		EXP. (Neutron levels)
$Os_{1/2}$	-44	-50	~ -50
$Op_{3/2}$	-25	-28	-22
$Op_{1/2}$	-16	-22	-16
$1s_{1/2}$	0	-2	-2
BE	80	100	128
$Op_{1/2}$-$Op_{3/2}$	9	6	6

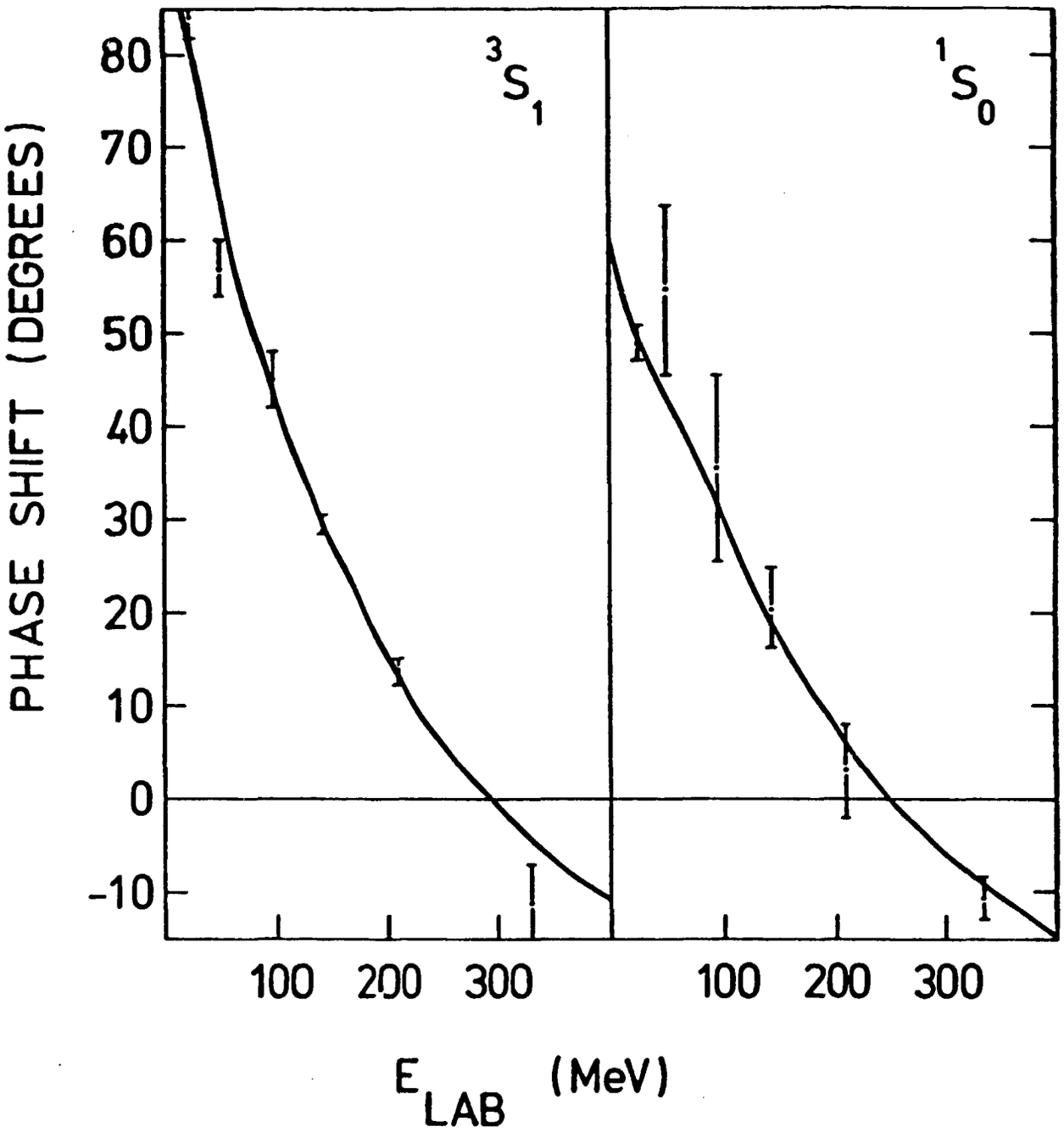
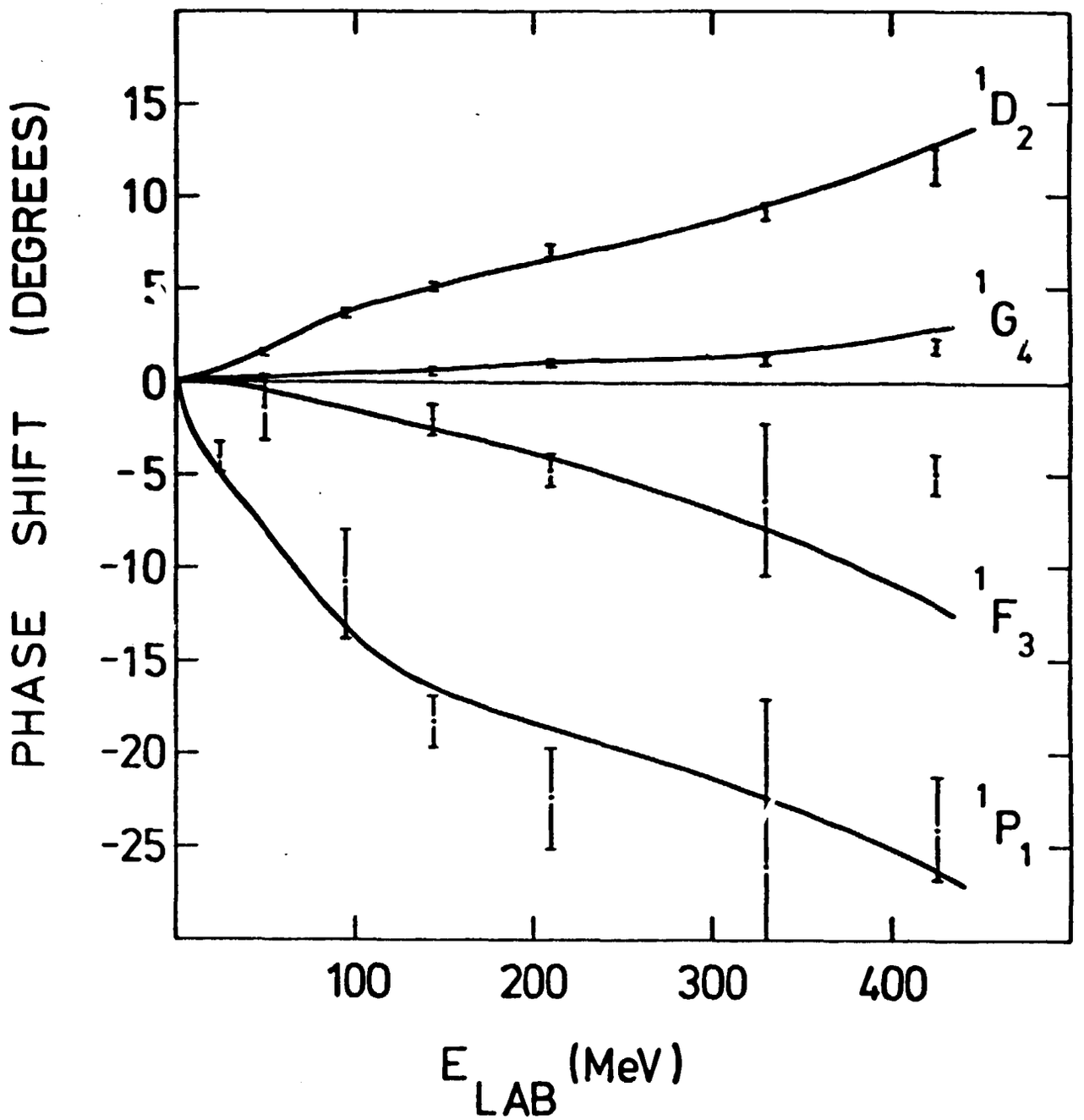

PHASE SHIFT (DEGREES)
80
70
60
50
40
30
20
10
0
-10
$^3S_1$
$^1S_0$
100
200
300
100
200
300
E_{LAB} (MeV)

Fig.1.


PHASE SHIFT (DEGREES)
15
10
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-25
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400
E LAB (MeV)
1D2
1G4
1F3
1P1

Fig. 2.

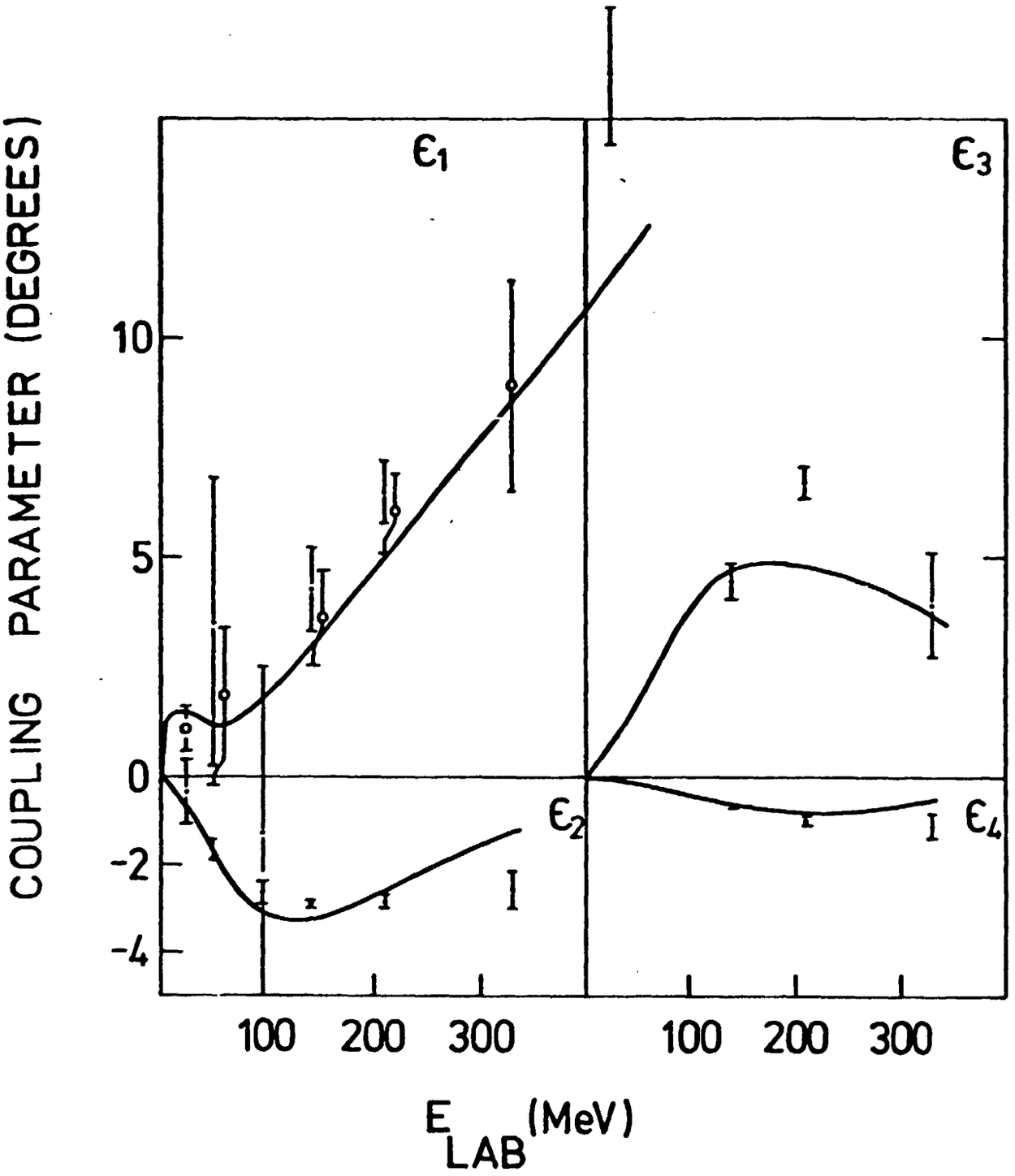
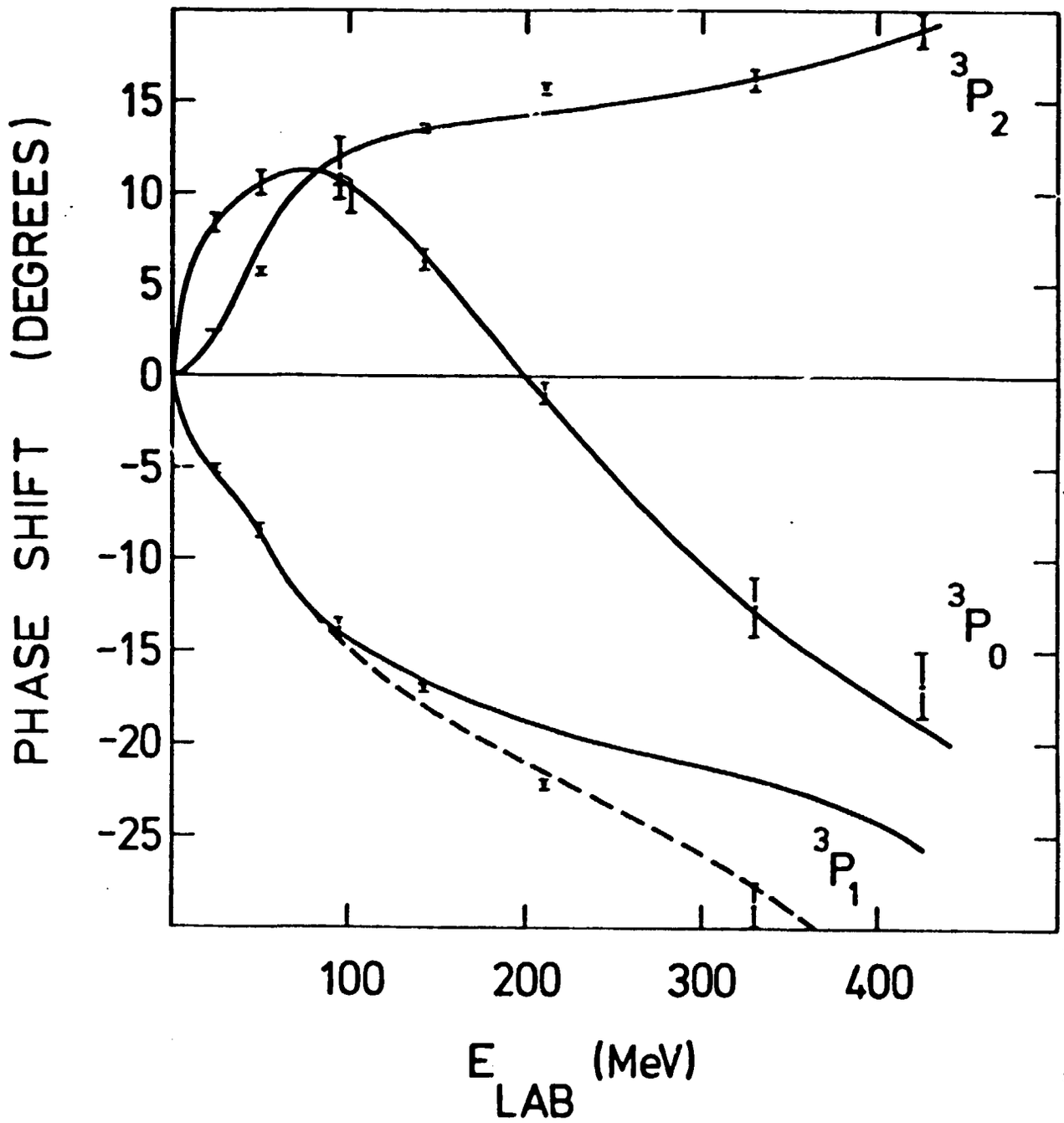

COUPLING PARAMETER (DEGREES)
ϵ_1
ϵ_3
ϵ_2
ϵ_4
10
5
0
-2
-4
100
200
300
100
200
300
E_{LAB} (MeV)

Fig. 3.


PHASE SHIFT (DEGREES)
15
10
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-20
-25
100
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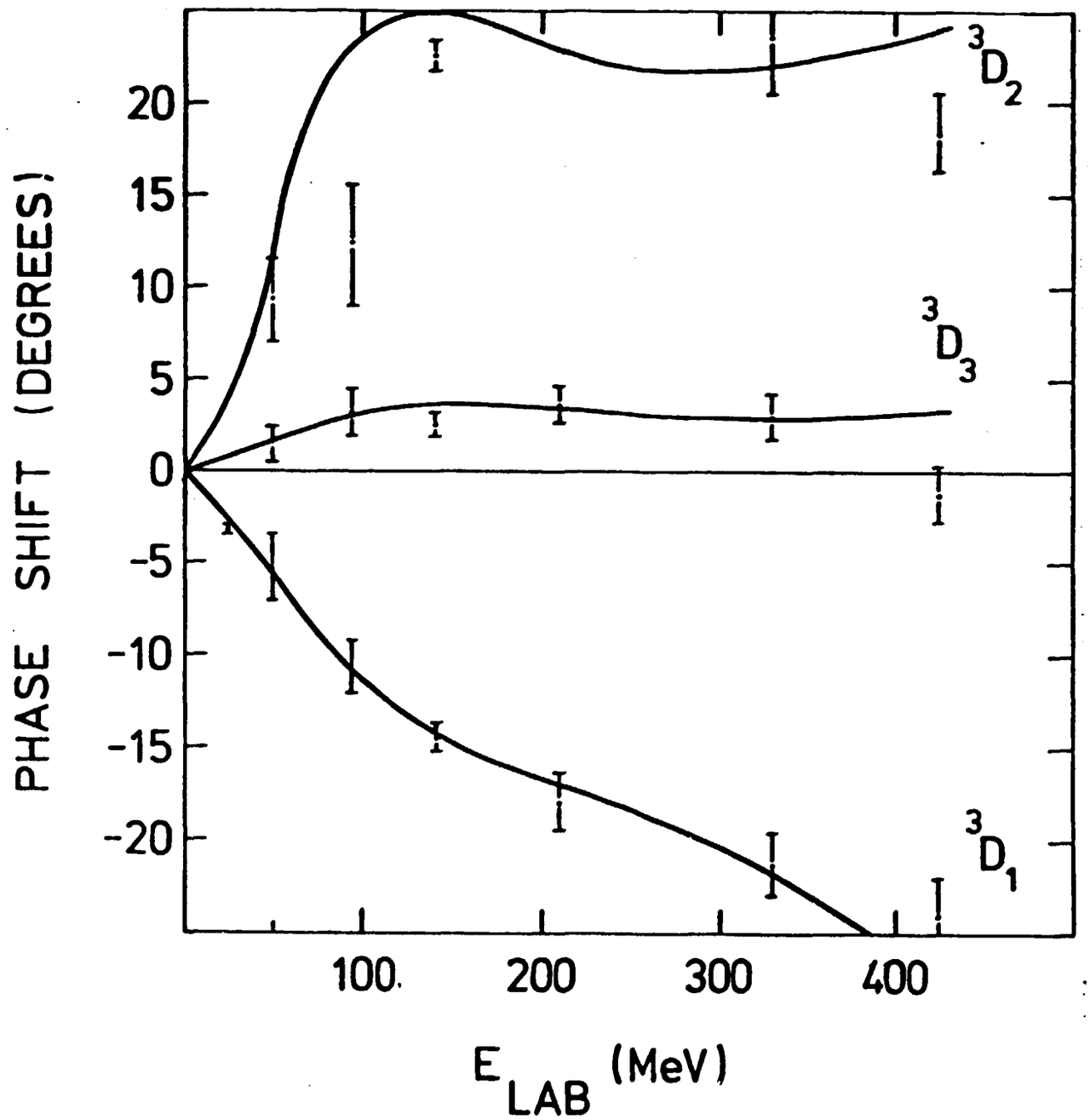
400
E LAB (MeV)
3P2
3P0
3P1

Fig. 4.


PHASE SHIFT (DEGREES)
20
15
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-10
-15
-20
100
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300
400
E LAB (MeV)
3D2
3D3
3D1

Fig. 5.

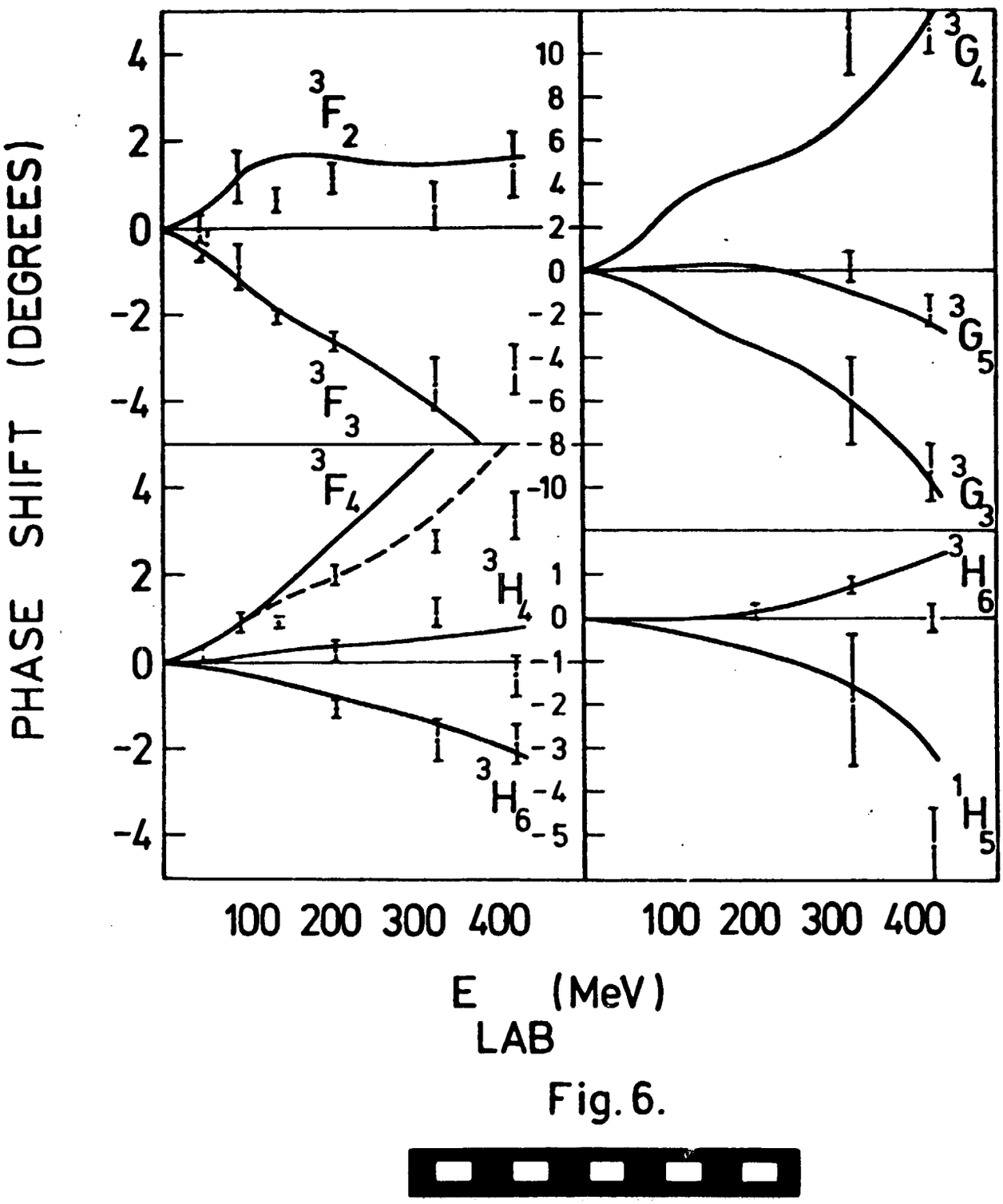
PHASE SHIFT (DEGREES)
$^3F_2$
$^3F_3$
$^3F_4$
$^3H_4$
$^3H_6$
$^3G_4$
$^3G_5$
$^3G_3$
$^3H_6$
$^1H_5$
100 200 300 400
100 200 300 400
E (MeV)
LAB

Fig. 6.