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# Anisotropic geometric diffusion in surface processing

## 1 Introduction

Geometric evolution problems for curves and surfaces and especially curvature flow problems are an exciting and already classical mathematical research field. They lead to interesting systems of nonlinear partial differential equations and allow the appropriate mathematical modelling of physical processes such as material interface propagation, fluid free boundary motion, crystal growth.

On the other hand, curves and surfaces are essential objects in computer aided geometric design and computer graphics. Here, issues are fairing, modelling, deformation, and motion. Recently, geometric evolution problems and variational approaches have entered this research field as well and have turned out to be powerful tools.

The aim of our work in the field of surface fairing and surface modelling is to modify "classical" curvature motion in a suitable way and apply it in computer graphics.

## 2 A general scheme

Consider an image $I : \Omega \to \mathbb{R}$. A well known approach to image processing consists in modifying the heat equation by introducing an anisotropic diffusion tensor $a(\nabla I)$. The design of the tensor $a$ is such that next to edges only tangential smoothing along the edges is allowed for a scale of images obtained by:

$$\partial_t I - \mathrm{div}(a(\nabla I)\nabla I) = 0$$

The most basic anisotropy $a$ is given by

$$a(\nabla I) = G(|\nabla I|) = \frac{1}{1 + (|\nabla I|/\lambda)^2}$$

for a parameter $\lambda$, the choice of which separates smooth image domains from edges.

This process can be interpreted and generalized as follows:

We consider a noisy initial curve or surface $\mathcal{M}_0$.

- A local classification is involved to figure out the expected curve or surface shape especially distinguishing between smooth areas, edges and corners.
- In time, a multiscale evolution is driven by forces derived from the current local classification.

Thereby a family of surfaces $\{\mathcal{M}(t)\}_{t\in\mathbb{R}_0^+}$ is generated, where the time $t$ serves as the scale parameter.

A general approach to fairing images and surfaces thus is to use a suitable classification and a corresponding anisotropic surface evolution. We use essentially two approaches. Either the classification is based on the evaluation of curvature or it is based on the analysis of local zero and first order surface moments. In both cases we obtain information about smoothness. Additionally in the case of edges, the edge direction may be computed. For more details to surface classification we refer to [4].

## 3 Surface Processing

In our cycle of classification and evolution, we now consider different diffusion processes corresponding to the above classifications. Both are suitable generalizations of the mean curvature flow given by:

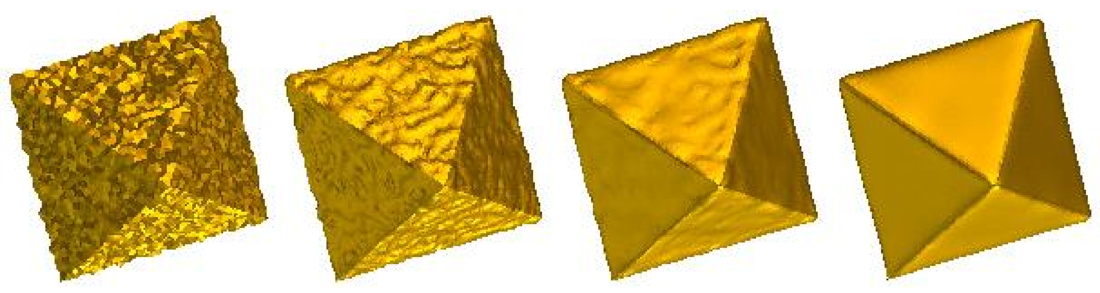
$$\partial_t x - \Delta_{\mathcal{M}} x = 0\,.$$

The first method is based on the definition of an anisotropic diffusion tensor which incorporates the local surface classification using an approximate curvature tensor $S_{\mathcal{T}_\epsilon\mathcal{M}}$. The diffusion equation is very similar to the image processing case. Additionally, we introduce a projection of the velocity onto normal direction which improves the grid regularity within the evolution. Our model is given by

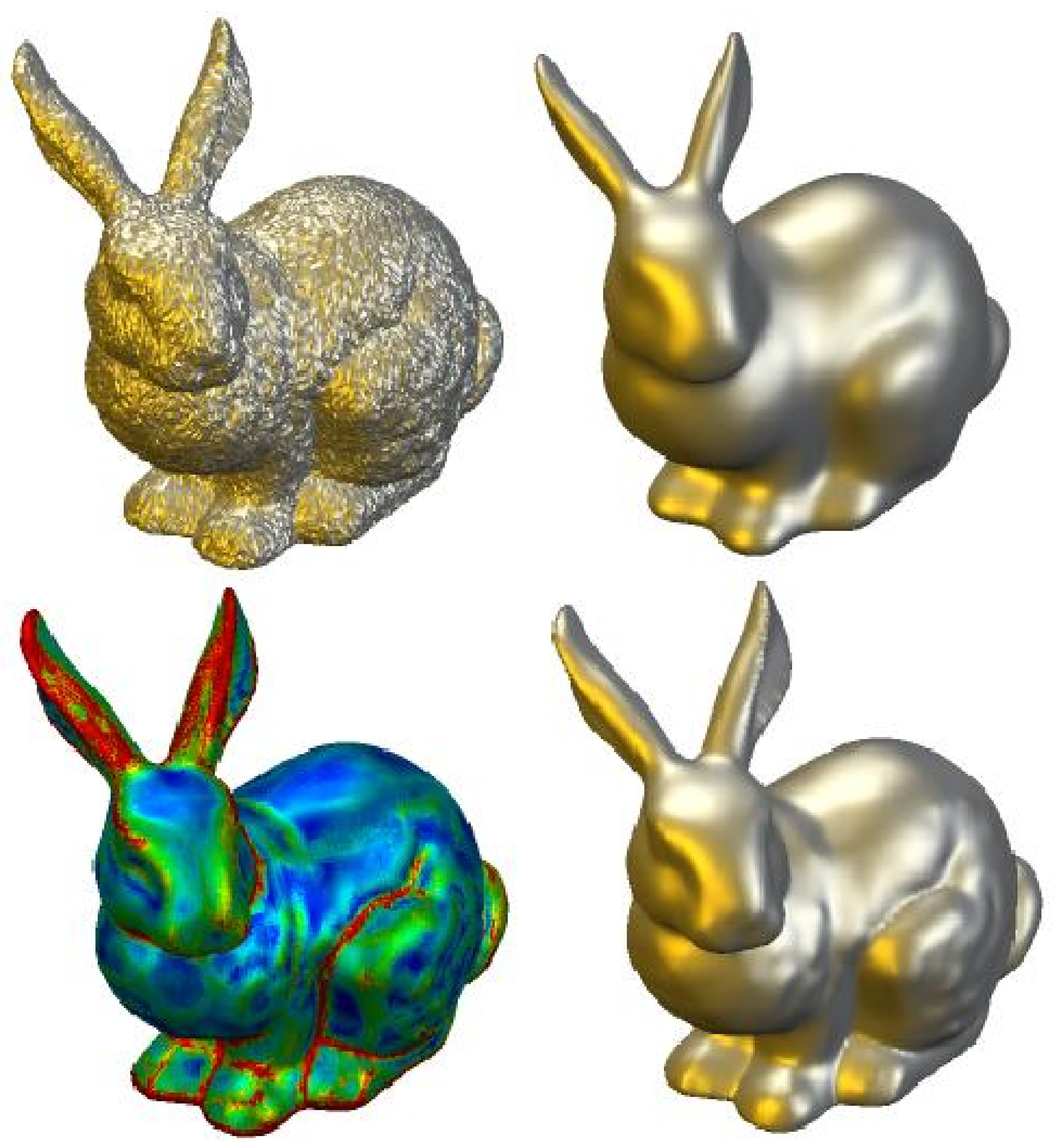
$$\partial_t x - (\mathrm{div}_{\mathcal{M}}(a(S_{\mathcal{T}_\epsilon\mathcal{M}})\nabla_{\mathcal{M}} x)\cdot n)\, n = 0\,,$$

where $n$ is the surface normal.

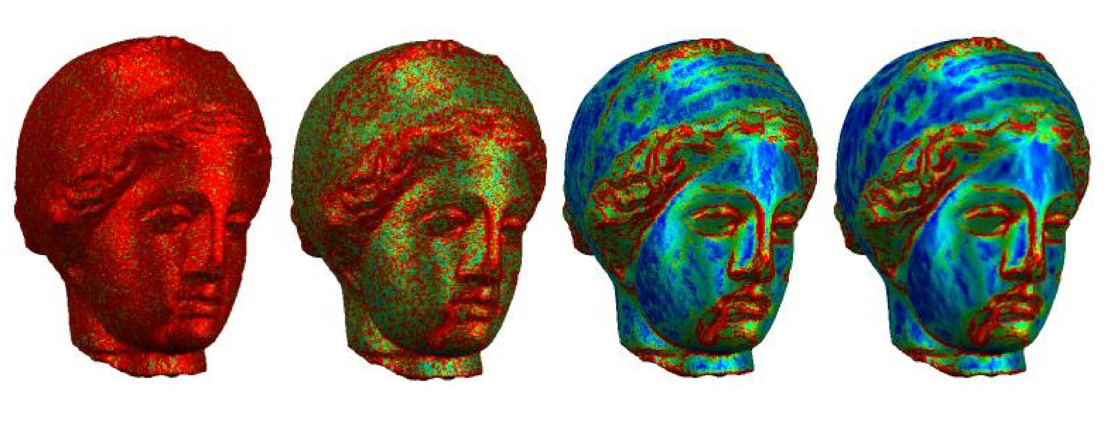
Below, there are shown different time-steps of this diffusion process starting with a noisy initial configuration.



Below, a noisy initial surface (top left) is evolved by discrete mean curvature flow (top right) and by the new anisotropic diffusion method (bottom right). Furthermore for the latter surface the dominant principal curvature - on which the diffusion tensor depends - is color coded (bottom left). The snapshots are taken at the same time-steps.



The next figure shows an initial surface (left) and three time-steps of the evolution of a venus head consisting of $268714$ triangles. The evolution times are $0.00005$, $0.0001$, and $0.0002$. Furthermore, the dominant principal curvature is color coded.



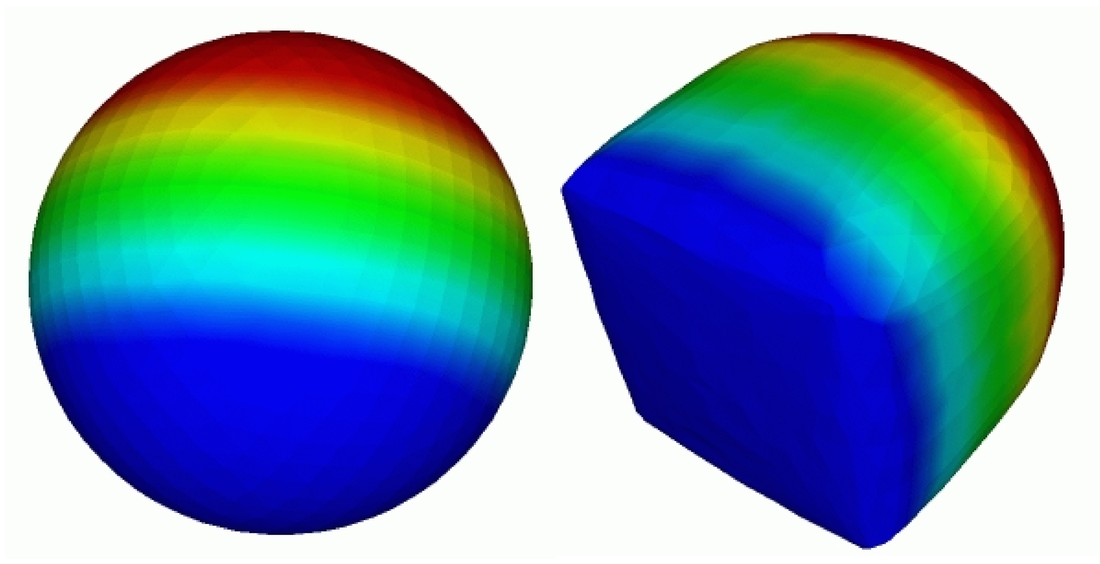
If we have additional color information on the surface, we can improve our classification. The above diffusion tensor may now depend on the geometry of the surface as well as on the color. Vice versa, we can use the geometry as an additional information if we want to smooth colors on a surface. The following figure shows, that the color information indeed is able to improve the quality of our smoothing method. On top, domains of a surface obtained by a laser scanning device combined with photography are depicted with and without color. This surface will be our initial configuration. In the second row, the result of an evolution process under the anisotropic diffusion without incorporating color information is shown, whereas in the third row we see the result of a combined diffusion for the same time-step.

Let us describe a different approach to image processing. One may consider the mean curvature evolution as the $L^2$-gradient flow for the area functional. From this perspective one may look for generalizations of the area functional that are better suited for surface fairing.

Indeed, taking into account a local moment analysis, we are able to define a position dependent anisotropic surface energy integrand leading to the energy:

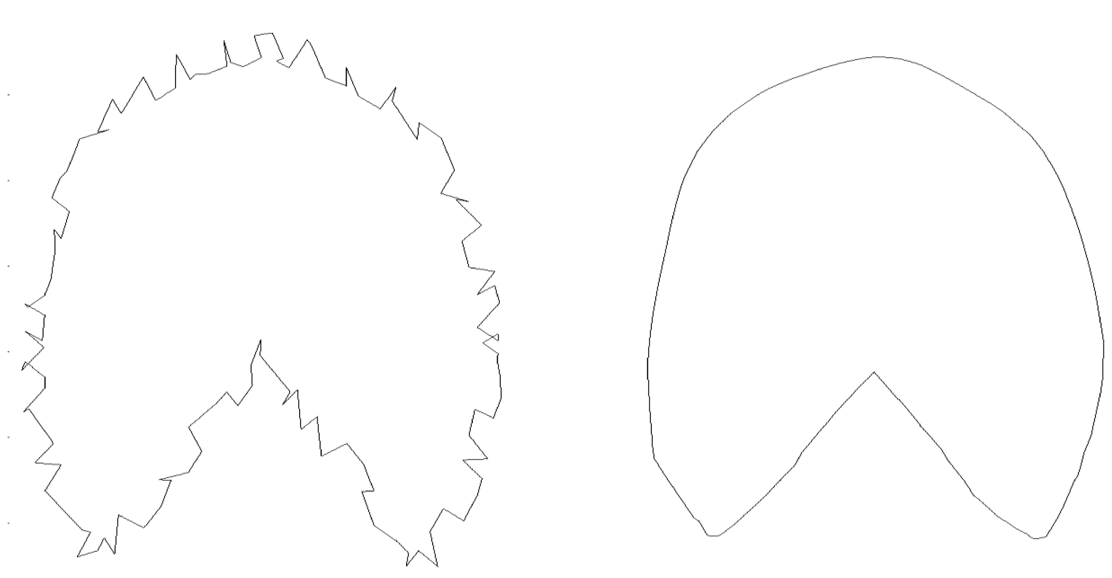
$$\int_{\mathcal{M}} \gamma(\xi, n(\xi))\, dA(\xi)\,.$$

In its second entry, the integrand is a convex function on the sphere. This means that we prescribe locally different convex shapes. As evolution process, again, we consider the corresponding $L^2$-gradient flow.



Above, on the left, an initial configuration is depicted and on the right the result of a gradient flow evolution. This figure shows that sharp edges and corners can be modelled with our method. Furthermore we are able to prescribe in different areas of the surface different (especially smooth and non-smooth) surface configurations.

We have implemented this method on curves. The figure below demonstrates, that using this approach, one is able to cope with very strong noise.



To our knowledge, the gradient flow approach is the first multiscale method, which not only retains features for longer times, but robustly enhances them.

## References

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[4] U. Clarenz, M. Rumpf, and A. Telea, *Robust feature detection and local classification for surfaces based on moment analysis*, (2003). submitted.