

Outage Detection in Power Distribution Networks with Optimally-Deployed Power Flow Sensors

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Abstract—An outage detection framework for power distribution networks is proposed. The framework combines the use of optimally deployed real-time power flow sensors and that of load estimates via Advanced Metering Infrastructure (AMI) or load forecasting mechanisms. The distribution network is modeled as a tree network. It is shown that the outage detection problem over the entire network can be decoupled into detection within subtrees, where within each subtree only the sensors at its root and on its boundary are used. Outage detection is then formulated as a hypothesis testing problem, for which a maximum a-posteriori probability (MAP) detector is applied. Employing the maximum misdetection probability P_e^{\max} as the detection performance metric, the problem of finding a set of a minimum number of sensors that keeps P_e^{\max} below any given probability target is formulated as a combinatorial optimization. Efficient algorithms are proposed that find the globally optimal solutions for this problem, first for line networks, and then for tree networks. Using these algorithms, optimal three-way trade-offs between the number of sensors, the load estimate accuracy, and the outage detection performance are characterized for line and tree networks using the IEEE 123 node test feeder system.

I. INTRODUCTION

Outage detection and management has been a long-standing problem in power distribution networks. As society becomes more dependent on electric power, the economic and societal costs due to loss of loads from distribution outages have been increasingly severe. Outages are mainly caused by permanent short circuit faults in the distribution system. When a short circuit fault occurs, protective devices close to the fault will automatically isolate the faulted area. The loads downstream of the protective devices will be in outage. We employ the term *outage detection* to denote the task of finding the status of the protective devices, and the term *fault detection* to denote that of finding the faults that caused the resulting outage situation.

Many methods for outage and fault detection based on artificial intelligence have been developed. Outage detection is often performed prior to fault detection and can greatly improve the accuracy of fault diagnosis. For outage detection, fuzzy set approaches have been proposed based on customer calls and human inspection [1], and based on real-time measurement with a single sensor at the substation [2]. In networks

where supervisory control and data acquisition (SCADA) systems are available, a subset of the protective devices' status can be obtained via direct monitoring. When two-way communications from the operator and the smart meters are available, AMI polling has been proposed to enhance outage detection [3]. There have also been knowledge based systems that combine different kinds of information (customer calls, SCADA, AMI polling) [4]. For fault detection, using only a single digital transient recording device at the substation, fault location and diagnosis systems have been developed based on fault distance computation using impedance information in the distribution system [5]. Using only the outage detection results, i.e., the status of the protective relays, expert systems have been applied to locate the underlying faults [6]. Incorporating voltage measurements in the distribution system with the outage detection results, fault detection methods based on knowledge based systems have been proposed [7]. Fault detection that uses during fault voltage-sag measurements and matching has been proposed in [8], [9]. Fault diagnosis based on fuzzy systems and neural networks have also been proposed that can resolve multiple fault detection decisions [10].

Nonetheless, current practice of outage and fault detection does not provide real-time detection decisions. In addition, as the existing outage and fault detection methods based on artificial intelligence do not provide any analytical metric on how well the algorithms perform, while their performance can be evaluated numerically, it is in general hard to examine the *optimality* of the algorithms. Moreover, because of this lack of an analytical metric, while some of the existing approaches depend on near real-time sensing (e.g. SCADA), they do not provide guidance on where to deploy the limited sensing resources within the distribution system.

In this paper, we focus on *real-time outage detection* based on *optimally deployed power flow sensors* within the distribution system and *load estimates* via AMI or load forecasting mechanisms. The proposed sensing and feedback framework exploits the combination of *real-time sensing and feedback* from a limited number of power flow sensors and the *infrequent load update* from AMI or forecasting mechanisms. We develop a probabilistic model of the outage detection problem, and formulate outage detection as a hypothesis testing problem. This formulation not only allows the development of optimal detectors, but also enables an analytical metric of

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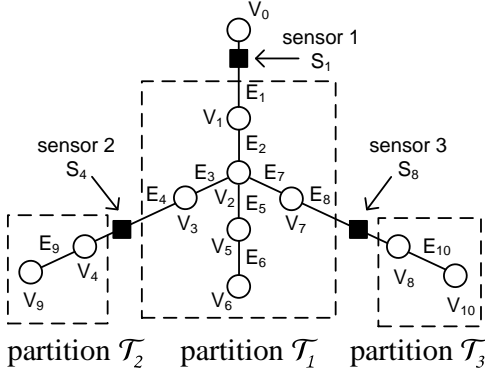


Fig. 1. Diagram for a distribution network with a tree structure.

the overall detection performance. Based on this performance metric, we propose efficient algorithms that determine the *globally optimal locations* for deployment of real-time sensors, and characterize the *optimal trade-off* between the number of sensors to use and the optimal detection performance.

II. SYSTEM MODEL

We consider a power distribution network that has a tree structure. The power is supplied from the feeder at the root, and is drawn by all the downstream loads. We consider an outage to be line tripping when it leads to loss of the corresponding downstream loads. An outage can be caused by any type of fault event that triggers protection devices to isolate the fault. We investigate the optimal design and performance of automatic outage detection systems, with the use of the following two types of measurements:

- Load pseudo-measurements from intermittently collected AMI data, or load forecasts. These are considered noisy observations of the true load at any given time.
- Real-time measurements of the power flows on a fraction of the lines obtained using accurate sensors placed on the selected lines. They are modeled as noiseless sensors, since the errors of these measurements are negligible compared to those of the load pseudo-measurements.

Topology of the Distribution System: We index the buses in the distribution network by V_0, V_1, \dots, V_N , with bus V_0 denoting the root of the tree. We index by E_n ($n \geq 1$) the line that connects bus V_n and its parent node. We denote by $\mathcal{T}(n)$ the subtree with V_n as the root node.

Outage Hypothesis: Outages are modeled as disconnected edges, corresponding to the lines tripped by the protective devices. For example, when we consider single line outages in a tree with N edges, there exist $N+1$ hypotheses to consider: H_i that denotes the trip of edge E_i , as well as H_{N+1} that denotes the non-outage situation.

Load Model: Each node V_n ($n \geq 1$) in the graph has a consumption load L_n . The pseudo-measurement of this load is \hat{L}_n . We denote the pseudo-measurement error of load L_n by $\epsilon_n = L_n - \hat{L}_n$. We consider the loads to have single phase real power. The developed methods can be generalized to loads of complex three phase values. We assume that the

errors are mutually independent random variables that follow $\epsilon_n \sim N(0, \sigma_n^2)$. Therefore, $L_i \sim N(\hat{L}_i, \sigma_i^2)$. We denote the vectors of true loads, load pseudo-measurements, and the error covariance matrix by \mathbf{L} , $\hat{\mathbf{L}}$ and $\mathbf{\Sigma}$, respectively. Thus, $\mathbf{L} \sim N(\hat{\mathbf{L}}, \mathbf{\Sigma})$, and $\mathbf{\Sigma}$ is a diagonal matrix due to the independence of errors.

Sensor Model: For any line E_n , we denote by S_n the power flow on it to all active downstream loads. The measured flow depends on the network topology, outage situation and the true loads. For example, under outage H_i on E_i ,

$$S_n = \sum_{V_j \in \mathcal{T}(n) \setminus \mathcal{T}(i)} L_j, \quad \forall n \geq 1. \quad (1)$$

III. PROBLEM FORMULATION

In this section, we formulate the problems of detection of outages and optimization of power flow sensor locations.

A. Tree Partitions and Decoupling Principle

Suppose there are M power flow sensors at lines $\mathcal{M} \triangleq \{p(1), \dots, p(M)\}$. Define $p(0) = 0$. Accordingly, the M sensors partition the tree network into $M+1$ subtrees \mathcal{T}_m , $m = 0, 1, \dots, M$:

$$\mathcal{T}_m \triangleq \mathcal{T}(p(m)) \setminus \bigcup_{\substack{m' \neq m, \\ V_{p(m')} \in \mathcal{T}(p(m))}} \mathcal{T}(p(m')), \quad (2)$$

An illustrative example is depicted in Figure 1, where the sensor placement $p(1) = 1$, $p(2) = 4$ and $p(3) = 8$ forms partitions \mathcal{T}_1 , \mathcal{T}_2 and \mathcal{T}_3 . We now state the following *decoupling principle* in using the sensor measurements for detecting outages in different partitions of the tree: the *optimal* detection of outage hypotheses within a partition \mathcal{T}_m can be performed using only the sensor measurement at the *root* of \mathcal{T}_m and those on the *boundary* of \mathcal{T}_m . For example, in Figure 1, the optimal detection decision in partition \mathcal{T}_1 is made using sensors $S_{p(1)}$, $S_{p(2)}$ and $S_{p(3)}$, while the optimal decisions in \mathcal{T}_2 and \mathcal{T}_3 are made using only $S_{p(2)}$ and $S_{p(3)}$ respectively.

B. Outage Detection within a Partition

An outage detection decision within partition \mathcal{T}_m is made using a MAP detector. For an outage H_k on $E_k \in \mathcal{T}_m$, we compute from the sensor measurements the sum of all the *remaining active loads* within \mathcal{T}_m , defined as the *effective measurement* of sensor $p(m)$:

$$\Delta S_{p(m)} \triangleq S_{p(m)} - \sum_{E_{p(m')} \in \mathcal{T}_m} S_{p(m')}. \quad (3)$$

In other words, $\Delta S_{p(m)}$ is the difference of the incoming and outgoing power within partition \mathcal{T}_m . With $\mathbf{L} \sim N(\hat{\mathbf{L}}, \mathbf{\Sigma})$, under H_k , $S_{p(m)}$ is a random variable with a conditional probability density function (pdf) of $N(\mu_k, \tau_k^2)$, where

$$\mu_k = \sum_{V_i \in \mathcal{T}_m \setminus \mathcal{T}(k)} \hat{L}_i, \quad \tau_k^2 = \sum_{V_i \in \mathcal{T}_m \setminus \mathcal{T}(k)} \sigma_i^2. \quad (4)$$

Given a set of outage hypotheses, MAP detection can then be performed based on $\{\mu_k\}$ and $\{\tau_k^2\}$. We evaluate the

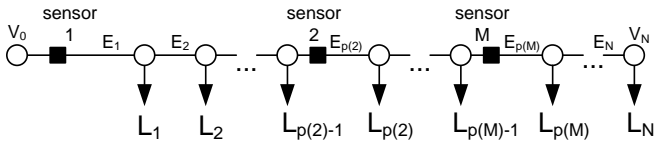


Fig. 2. A line network with N loads and M power flow sensors.

performance of the MAP detector based on the hypothesis misdetection probability, $\Pr(\hat{H} \neq H_k | H_k \text{ is true})$, and use the *maximum misdetection probability* as the detection performance metric, denoted by $P_e^{\max}(\hat{\mathbf{L}}, \Sigma, \mathcal{M})$.

C. Minimum Number of Sensors and Their Locations

For any given probability target P_e , we find the *minimum number* of sensors that keep the maximum misdetection probability below P_e . This is formulated as:

$$\begin{aligned} \min M & \\ \text{s.t. } \exists \mathcal{M}, |\mathcal{M}| \leq M, \mathcal{M} \subseteq \{1, \dots, N\}, & \\ P_e^{\max}(\hat{\mathbf{L}}, \Sigma, \mathcal{M}) \leq P_e. & \end{aligned} \quad (5)$$

We see that (5) is a combinatorial optimization over \mathcal{M} which is in general hard to solve. In the following sections, we will show that (5) can be solved very efficiently.

IV. OUTAGE DETECTION AND OPTIMAL SENSOR LOCATIONS IN LINE NETWORKS

In this section, we consider a special case of tree networks, namely, the line networks (cf. Figure 2).

A. Outage Detection

Line Partitioning: With M power flow sensors at lines $p(1) < \dots < p(M)$, partition \mathcal{T}_m is the segment from $V_{p(m)}$ to $E_{p(m+1)}$: $\mathcal{T}_m \triangleq \mathcal{T}(p(m)) \setminus \mathcal{T}(p(m+1))$, with \mathcal{T}_M as the segment from $V_{p(M)}$ to V_N .

Sensor Measurements and MAP Detector: Consider a single line outage H_k at $E_k \in \mathcal{T}_{m^*}$ for some m^* . From the decoupling principle, only the sensor at the root of \mathcal{T}_{m^*} , namely, $S_{p(m^*)} = \sum_{i=p(m^*)}^{k-1} L_i$ should be used. In this case, $\Delta S_{p(m^*)} = S_{p(m^*)}$. For each H_k , $\Delta S_{p(m^*)} | H_k \sim N(\mu_k, \tau_k^2)$, where $\mu_k = \sum_{i=p(m^*)}^{k-1} L_i$ and $\tau_k^2 = \sum_{i=p(m^*)}^{k-1} \sigma_i^2$.

B. Minimum Number of Sensors and Their Locations

Given a probability target P_e , we propose Algorithm 1 (cf. Table I) that solves the global optimum of (5). The main idea of the algorithm is as follows. We start from the root where a sensor is placed at E_1 . If P_e^{\max} does not fall below P_e with the 1st sensor only, the 2nd sensor is needed. We then move the 2nd sensor forward starting from E_2 to E_3, E_4, \dots , and compute P_e^{\max} on segment \mathcal{T}_1 . We choose line $p(2)$ to place the 2nd sensor, such that

- P_e^{\max} on segment \mathcal{T}_1 is below P_e , whereas
- if the 2nd sensor is instead placed at $E_{p(2)+1}$, P_e^{\max} on segment \mathcal{T}_1 will exceed P_e .

Similarly, if the m^{th} sensor is needed, we place it *as far from the root as possible* while satisfying P_e on segment \mathcal{T}_{m-1} .

TABLE I
ALGORITHM 1

Find a minimum set of sensor locations that satisfy P_e for line networks

Place the 1st sensor at E_1 , i.e., $p(1) = 1$.

Initialize $m = 2, p(m) = 2$.

Repeat

for segment \mathcal{T}_{m-1} ,

If $P_e^{\max}(\hat{\mathbf{L}}_{\mathcal{T}_{m-1}}, \Sigma_{\mathcal{T}_{m-1}}) \leq P_e$,

If $p(m) = N + 1$,

return $\mathcal{M} = \{p(1) \dots, p(m-1)\}$,
($m-1$ sensors are sufficient.)

Else $p(m) \leftarrow p(m) + 1$.

Else,

$p(m) \leftarrow p(m) - 1$,

$m \leftarrow m + 1$,

$p(m) = p(m-1) + 1$.

TABLE II
EXAMPLE OF HYPOTHESIS SET REDUCTION

Possible sets of line outages, $S_1 > 0$.

	$S_8 = 0$	$S_8 > 0$
$S_4 = 0$	$\{E_2\}$	$\{E_3\}, \{E_4\}$
$S_4 > 0$	$\{E_7\}, \{E_8\}$	$\{E_5\}, \{E_6\}, \emptyset$

V. OUTAGE DETECTION AND OPTIMAL SENSOR LOCATIONS IN TREE NETWORKS

Here, we first describe hypothesis set reduction in trees, and then develop the optimal sensor location algorithm for arbitrary tree networks.

A. Hypothesis Set Reduction

With M power flow sensors at lines $p(1) < \dots < p(M)$, define $p(0) = 0$. We observe that, $S_n = 0$ if and only if there is a line outage in the unique path from E_n to the root V_0 . Therefore, in any partition \mathcal{T}_m , the set of valid hypotheses can be greatly reduced using the knowledge of whether sensor m and the downstream sensors on the boundary of \mathcal{T}_m see zero or non-zero measurements.

We now give an illustrative example of the hypothesis set reduction for the tree network in Figure 1. Consider outage detection in partition \mathcal{T}_1 using S_1, S_4 and S_8 . If $S_1 = 0$, there is an outage on line 1 for sure. If $S_1 > 0$, for different combinations of S_4 and S_8 , we summarize all the possible sets of outage hypotheses in Table II.

B. Minimum Number of Sensors and Their Locations

Generalizing the intuition from Section IV-B, we first define an *efficient set of sensor locations*. This efficiency captures the fact that all the sensors are *as far as possible from the leaves of the tree*, while still maintaining $P_e^{\max} \leq P_e$:

Definition 1: Given a target P_e , a set of sensor locations $\mathcal{M} = \{p(1), \dots, p(M)\}$ with $p(1) = 1$ is efficient if and only if:

- 1) $P_e^{\max}(\hat{\mathbf{L}}, \Sigma, \mathcal{M}) \leq P_e$.

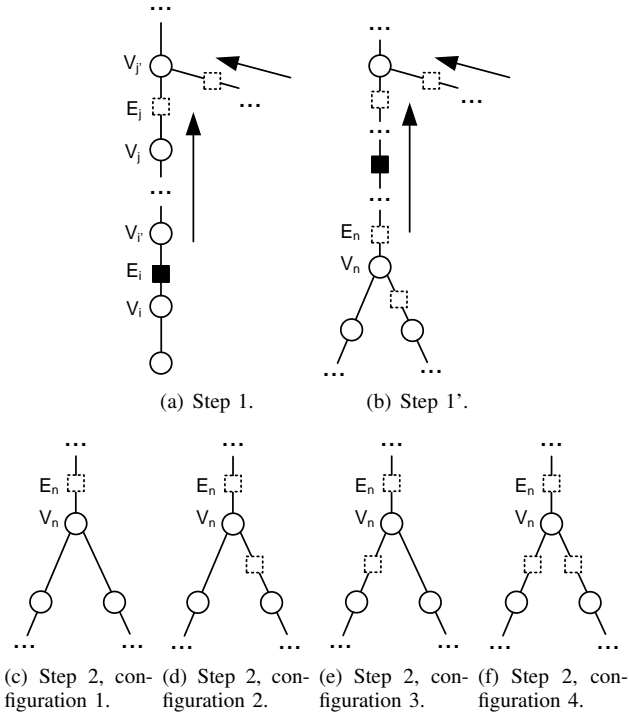


Fig. 3. Algorithm 2, solid squares denote placed sensors, and dashed open squares denote undecided candidate sensors.

- 2) $\forall m = 1, \dots, M$, with a sensor at $E_{p(m)}$, within subtree $\mathcal{T}(p(m))$, all sets of sensor locations of $|\{m' | V_{p(m')} \in \mathcal{T}(p(m))\}| - 1$ sensors yield $P_e^{\max} > P_e$.
- 3) $\forall m = 1, \dots, M$, with a sensor at $E_{p'(m)}$ where $V_{p'(m)}$ is the *parent* node of $V_{p(m)}$, within subtree $\mathcal{T}(p'(m))$, all sets of sensor locations of $|\{m' | V_{p'(m')} \in \mathcal{T}(p'(m))\}| - 1$ sensors yield $P_e^{\max} > P_e$.

By definition, we immediately observe that an efficient set of sensor locations, if it exists, solves (5).

Next, we propose the following algorithm that always finds an efficient set of sensor locations for any given P_e . We allocate sensors in a bottom-up manner with candidate sensors starting at all the leaves of the tree. We then move the sensors up as much as possible, for which the operational details can be described as a combination of two steps:

Algorithm 2, find an efficient set of sensor locations for a given P_e in tree networks:

1) Each candidate sensor moves up along its branch until one of the following three scenarios happens:

- It moves to E_i where V_i 's parent node $V_{i'}$ only has one child V_i , and P_e^{\max} within $\mathcal{T}(i)$ is below P_e , whereas P_e^{\max} within $\mathcal{T}(i')$ would exceed P_e had the candidate sensor moved up to $V_{i'}$. In this case, we place the sensor at E_i , and start a new candidate sensor moving up from $E_{i'}$. Then, we repeat step 1.
- It moves to E_j , where V_j 's parent node $V_{j'}$ is a *joint node* where multiple branches meet. In this case, we pause the current candidate sensor at E_j , and go to step 2).

- It moves to the root of the entire tree, i.e., E_1 . In this case, we place the candidate sensor at E_1 , and stop.

An illustrative example is depicted in Figure 3(a).

2) At a joint node V_n , with all of its current downstream candidate sensors moved up to the lines connecting to V_n 's children, we find the *minimum* number of sensors required to keep P_e^{\max} within $\mathcal{T}(n)$ below P_e , assuming that there is a sensor at E_n . This is done by enumerating all placement configurations of the candidate sensors immediately below V_n . An illustrative example where two branches join at V_n is depicted in Figure 3(c)-3(f).

3) After deciding the minimum required number of sensors below a joint node V_n , we start a new candidate sensor moving up from E_n , and repeat step 1).

An illustrative example is depicted in Figure 3(b), where the minimum required number of sensors below the joint node V_n is one, achieved by the configuration of Figure 3(d).

As Algorithm 2 always finds an efficient set of sensor locations, it solves for the global optimum of (5).

VI. SIMULATION

We simulate Algorithm 1 and 2 in line and tree networks each with 123 nodes. The IEEE 123 node test feeder system [11] is used for the tree topology, with a unique switch configuration to guarantee a fully connected tree. The true loads L_i are drawn from a uniform distribution $U[5, 10]$ KW, while the pseudo-measurement error standard deviation σ_i is proportional to the true load L_i : $\sigma_i = \kappa \times L_i$. The typical values of κ seen in load forecast mechanisms are 10 – 30%.

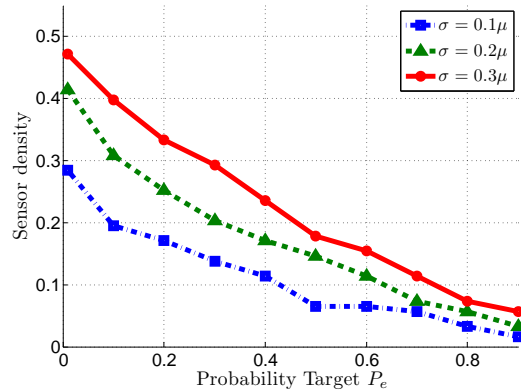


Fig. 4. Minimum number of sensors required for line networks.

The optimal trade-off curves between the number of sensors M and P_e^{\max} for line and tree networks are solved via Algorithm 1 and 2, and are depicted in Figure 4 and 5. P_e values of 1%, 10%, 20%, ..., and 90% are evaluated. We evaluated the trade-offs for different pseudo-measurement accuracies κ , namely 10%, 20% and 30%. For $\kappa = 30\%$, we see that roughly half of the lines must be monitored if $P_e^{\max} < 1\%$ is required, for both line and tree topologies. By improving κ to 10%, only about a third of the lines need to be monitored.

Discussion: On average the line network requires 40% fewer sensors than the tree network. When the sensor density is

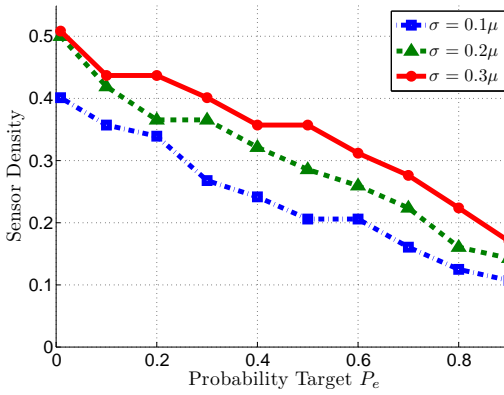


Fig. 5. Minimum number of sensors required for tree networks.

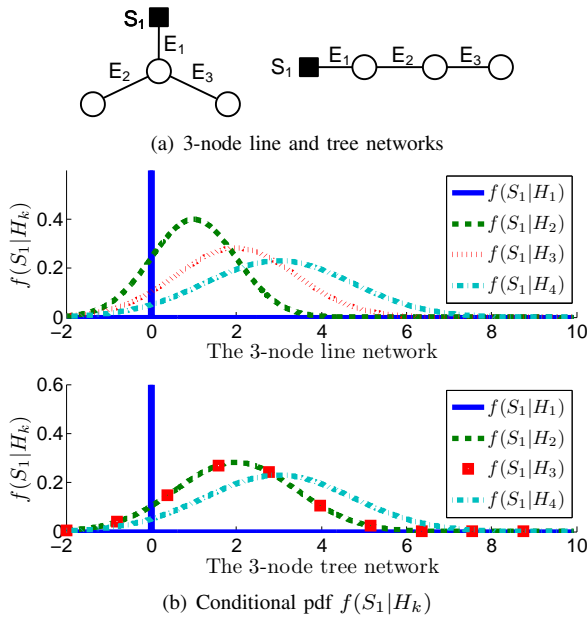


Fig. 6. Conditional pdf in 3-node line and tree networks.

higher, this difference is smaller. There are various factors that contribute to this difference. To illustrate these factors, we examine example 3-node line and tree networks (cf. Figure 6(a)). Considering single line outages, the line network has the following possible flow measurements, $S|H_1 = 0$, $S|H_2 = L_1$, $S|H_3 = L_1 + L_2$, $S|H_4 = L_1 + L_2 + L_3$. Likewise the tree network has $S|H_1 = 0$, $S|H_2 = L_1 + L_2$, $S|H_3 = L_1 + L_3$, $S|H_4 = L_1 + L_2 + L_3$. Assuming $L_i = 1$ and $\sigma_i^2 = 1, \forall i$, the conditional pdf of the flow measurement under each hypothesis is plotted in 6(b). Clearly, in the line network, all the hypotheses are distinguishable, whereas in the tree network H_2 and H_3 are not distinguishable. Algorithm 2 will then place a sensor at E_2 or E_3 in the tree network. The difference between line and tree networks narrows as P_e decreases for the following reasons. For tree networks, as Algorithm 2 proceeds to upper nodes within the tree, the leaves of the intermediate partitions are all terminated by downstream sensors, and the *hypothesis set reduction* (cf. Section V-A) plays an important role. For example, if the 3-node tree was

a partition terminated with 2 sensors below V_2 and V_3 , all the hypotheses will be fully distinguishable with zero misdetection probability. Indeed, we observe that there are large partitions having close to zero misdetection probability when P_e is small. Depending on the values of P_e and κ , one of the above two factors dominates the optimization of sensor locations.

VII. CONCLUSIONS

We have proposed outage detection in power distribution systems using optimally deployed real-time power flow sensors combined with load estimates from intermittent AMI data or load forecasts. We have shown a decoupling principle that allows detection of outages within a subtree using only the sensor measurements at its root and on its boundary. Accordingly, we have formulated the outage detection problem as one-dimensional hypothesis testing, and have applied the MAP detector. Employing the maximum misdetection probability P_e^{\max} as the outage detection metric, we have studied the problem of finding a set of a minimum number of sensors that keep P_e^{\max} below any given target. We have proposed efficient algorithms that find the globally optimal solution of this problem in line and tree networks respectively. Using these algorithms, we have evaluated the optimal trade-offs between the number of sensors, the pseudo-measurement (i.e., load estimate) accuracy, and the outage detection performance in line and tree networks. We have observed that on average 40% fewer sensors are needed in line networks than in tree networks to satisfy the same target P_e^{\max} .

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