Centro de Investigación Operativa

I-2004-26

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December 2004

ISSN 1576-7264

Depósito legal A-646-2000

TRABAJOS I+D

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Allocating Fixed Costs and Budgets by Using Data Envelopment Analysis

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Abstract

Fixed costs or common budgets often exist in numerous economic contexts which must be allocated among the entities making up an organization. Let's say this is the case of a university that has to share a certain amount of money to buy equipment (microscopes, software, etc.) among the different degree courses. The dilemma facing the manager in charge is how to allocate the budget fairly among the entities involved.

This problem has already been taken on previously, from the perspective of Data Envelopment Analysis (DEA). Two conceptually different approaches, one having as reference the work of Cook and Kress [8], and the other that of Beasley [3], are until now presented as the sole alternatives in equitable cost allocation using DEA. Within this context we will present a new model with the same objectives as the previous ones, leading to allocations possessing good properties and easy calculations. Our sharing rule will be compared to existing ones, and the differences between them examined using an example taken from literature. Finally, the validity of the new method will be checked by solving an actual allocation problem in a university extracted from Pulido et al. [15].

Key words: Data Envelopment Analysis (DEA); sharing rule; Fixed cost and budget allocation; Efficiency.

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[2]Financial support from the Generalitat Valenciana through project GV04B-678 and from the Ministerio de Educación y Ciencia through project MTM2004-07473 are gratefully acknowledged.

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1 Introduction: Data Envelopment Analysis

Data Envelopment Analysis (DEA), introduced by Charnes et al. [5], is a technique that permits the measurement of relative efficiency of entities, like for example non-profit organizations possessing multiple outputs and inputs, that cannot, however, be handled by the establishment of some prices (costs) or other previously known weights. In recent years the validity of this approximation has been proven in a wide variety of applications: maintenance activities of U.S. Air Force bases, applications by the police forces of England and Wales, as well as the evaluation of branch banks in countries like Cyprus and Canada. These types of applications extend to evaluations of city, region and country management, these having great quantities of different types of inputs and outputs. In this context the units or entities evaluated are given the name Decision Making Units (DMUs).

The most common productivity measurement found in economic literature assumes the form of the ratio:

$$\frac{Output}{Input}.$$

When this measurement is used to evaluate worker or employee performance, it is interpreted as "output per worker hour" or "output per worker employed". It is obvious that the manager wants this ratio to be as large as possible. The inverse of this measurement (input/output) can also be evaluated, minimizing this ratio.

The question when confronted with situations of multiple outputs and inputs is how to express the output that appears in the ratio numerator. In the case of DEA, the overall output is obtained as a linear combination of weights and observed outputs from each DMU. The overall input as the denominator acts in the same way.

The weights utilized by DEA are interpreted as prices (costs) per output unit (input) produced (consumed). The DEA philosophy states that the only restriction required of these weights is that they be non-negative. This liberty is used for the calculation of the efficiency measurement for each DMU in a way that the ratio between the overall output and the overall input is maximized.

The efficiency measurement with DEA is calculated using mathematical programming models. Specifically, the model proposed by Charnes et al. [5], baptized as CCR, assumes constant return to scale, and will be that used throughout the work in its output orientation, in the form that appears next when DMU 0 is evaluated:

$$
\begin{array}{ll}
Min & \phi_0 = \frac{\sum_i v_i x_{i0}}{\sum_r u_r y_{r0}} \\
s.t.: & \\
 & \frac{\sum_i v_i x_{ij}}{\sum_r u_r y_{rj}} \geq 1, \quad \forall j \\
 & v_i, u_r \geq 0, \qquad \forall i, r
\end{array}
\tag{M.1}
$$

Here, it is assumed that each DMU j ($j = 1, 2, \ldots, n$) consumes a known amount x_{ij} (> 0) of each m inputs in the production of s outputs in the amounts y_{rj} (> 0). The weights are represented by v_i and u_r for the inputs and outputs, respectively.

If ϕ_0 takes the value of 1 in (M.1), then we say that DMU 0 is efficient. In any other case it is classified as being an inefficient unit.

The model generalization (M.1) to another type of return to scale and orientation (input orientation) can be simply realized (see Banker et al [1]).

The non-linear problem brought up by Charnes et al. [5] to obtain a relative efficiency score is habitually replaced by an equivalent linear program, (M.2).

$$
\begin{array}{ll}
Min & \phi_0 = \sum_i v_i x_{i0} \\
s.t.: & \\
 & \sum_r u_r y_{r0} = 1 \\
 & -\sum_i v_i x_{ij} + \sum_r u_r y_{rj} \leq 0, \quad \forall j \\
 & v_i, u_r \geq 0, \qquad \forall i, r
\end{array}
\tag{M.2}
$$

For more details about linear fractional programming problems and linear programming problems see Charnes and Cooper [4].

Even though at first it may appear that a possible meeting point between DEA is not found, one technique to estimate efficiency measurements and the definition of a sharing rule will be shown in the next section, of how a series of work appearing in the literature has managed to establish links between both areas.

The remainder of the paper is organized as follows. Section 2 shows the existing literature linking DEA with distribution rules. The main achievements of each author are detailed. Section 3 develops a new budget allocation model. Section 4 shows the good properties of the model. Section 5 compares the new solution with those existing in the literature until now. Section 6 takes on the new approximation application using an actual empirical example. The conclusions are in section 7.

2 Allocation of a common fixed cost or budget using DEA: precedents

The attention in this section is directed upon existing literature in which DEA and optimal budget allocation are mixed together. Four recently published articles form the group of work dedicated to these tasks: [3], [8], [9] and [11].

As far as we are aware, the seminal work relating the DEA technique with cost allocation is that of Cook and Kress [8]. A model is presented based on the idea that observed outputs and inputs for each one of the DMUs have already been conditioned by the existence of a common cost such as a fixed overhead. The treatment given in this case to the cost or budget to be shared is that of non-discretionary input (see Banker and Moore [2]). This fact causes that any allocation of costs that does not alter the value of the efficiency measure is equitable (invariance condition). What is obtained in the one-dimensional case, as a consequence, is that the solution is unique (pareto-minimality condition), and depends exclusively upon the observed inputs. The solution, however, becomes complicated in general cases, multiple inputs against multiple outputs, not obtaining a unique allocation. Finally, a consistent complex procedure is developed, giving priority to certain DMUs by way of a series of steps that does permit the arrival at a unique solution. This procedure is based on the use of conditions of cone-ratio type constraints (see Charles et al. [6]).

Jahanshahloo et al. [11] work on the model [8] with the objective of computationally simplifying such a solution. They demonstrate that in general cases it is not necessary to solve a linear programming problem, but instead is enough to make use of a simple formula exclusively dependent on observed inputs. Finally, the authors stress the fact of the non-uniqueness of the allocation with use of a numeric example.

In [3] Beasley presents an alternative model to that of the Cook and Kress [8] approximation. It is fundamentally based on a reinterpretation of the DEA philosophy. The traditional DEA technique says that each DMU selects in an independent manner the weights that maximize its individual efficiency. Beasley demonstrates that an equally valid interpretation indicates that the weights are simultaneously selected by all of the DMUs, maximizing the average efficiency. To complete his model it will be necessary to take equal weights for each DMU and introduce the budget or cost as a new input variable. The total sum of these new variables will coincide with the budget. For the resolution of this non-linear model turn to the definition of a complex algorithm.

Finally, in the paper of Cook and Zhu [9], they work on the Cook and Kress [8] pareto-minimality condition that does not permit the cost allocation only among inefficient DMUs. The procedure needs to classify the DMUs from the resolution of various linear programming models as either efficient or inefficient, and finally solve a system of linear equations. The allocation is not unique and can produce for some DMUs a cost or profit of zero.

In the following section an alternative model is introduced to those existing, stressing the differences with the remaining approximations.

3 A new DEA approximation to the allocation of a fixed cost or budget

The question that motivated this work was to create an alternative method to those introduced in [8] and [3], that would overcome the problems presented by these: computation difficulty, unsatisfaction of good properties, etc. The approximation proposed, as will be later shown, satisfies a full series of good properties demanded in the allocation of a budget or fixed cost. The differences between our model and the previous ones will be clarified in this and the next section and in a definitive manner with a numeric example.

Evidently the principles supporting our approximation are based on the DEA technique, and as a consequence, in the resolution of a linear programming problem. Specifically we will make use of a philosophy that completely restricts the freedom of the weights associated with inputs and outputs, obligating that these coincide for each and every one of the evaluated DMUs. This philosophy is known by the name of "Common Set of Weighs" (CSW). In Roll et al. [16] and Roll and Golany [17] various seminal DEA models based on CSW are found. Lozano and Villa [14] is one of the most recent works moving in the same direction.

The proposed model is as follows:

$$\begin{array}{lll} Min & \sum_j \sum_i v_i x_{ij} & \\ s.t.: & & \\ & \sum_r u_r y_{rj} \geq f_j, & \forall j \\ & -\sum_i v_i x_{ij} + \sum_r u_r y_{rj} \leq 0, & \forall j \\ & \sum_j f_j = F & \\ & v_i, u_r, f_j \geq 0, & \forall i, r \end{array} \qquad \text{(M.3)}$$

where F (> 0) corresponds with the budget allocated among the distinct entities or DMUs, and f_j represents the assigned quantity of the DMU j in the allocation. (M.3) is made up of $m + s + n$ variables and $2n + 1$ constraints.

To be able to interpret in economic terms what (M.3) is doing, we will need to define the notional benefits and costs concepts. Remember that the weights utilized by DEA are interpreted as prices (costs) per output unit (input) produced (consumed). Consequently $\sum_r u_r y_{rj}$ is interpreted as the notional benefits of the DMU j. $\sum_i v_i x_{ij}$ can be similarly interpreted as the notional costs. Like this, (M.3) attempts to find a common set of weights (system of prices and costs) for all of the DMUs, and an allocation vector of the budget $(f_1, \ldots, f_n)$ that covers the

costs of the common expense source. At the same time the allocation is obligated to sum the total budget assigned by the manager of the organization.

The substantial difference between the approximation presented here and those of [8] and [3] is associated with how the common cost in the model is introduced. Cook and Kress [8] use it as a new non-discretionary input of the DMU. Beasley [3] introduces a new variable in the model on the input side, and gives it a treatment similar to these, except for the fact of not assigning it weight in the model. In our case we do not contemplate the allocation as a new input. Very much the opposite, we modify the normalization condition of (M.2), demanding that each DMU covers costs, and as a consequence is not considered loss-making.

4 Properties of the new approximation

The series of results that will be shown later will permit us to make an idea about the improvements that the new model has represented with respect to prior solutions.

The first proposition indicates how to calculate the allocation having the optimal weights of (M.3).

Proposition 1 *Let* $(\{v_i^*\}, \{u_r^*\}, \{f_j^*\},)$ *be an optimal solution of (M.3), then*

$$\sum_r u_r^* y_{rj} = f_j^* \quad \forall j.$$

Proof. The dual problem of (M.3) is

$$\begin{array}{lll} Max & F\omega & \\ s.t.: & & \\ & \omega - \phi_j \leq 0, & \forall j \\ & \sum_j \lambda_j x_{ij} \leq \sum_j x_{ij}, & \forall i \\ & \sum_j \phi_j y_{rj} - \sum_j \lambda_j y_{rj}, \leq 0 & \forall r \\ & \phi_j, \lambda_j \geq 0, & \forall j \\ & \phi_j, \lambda_j \geq 0, & \forall j \\ & \omega \in R & \end{array} \tag{M.4}$$

$(\{\overline{\phi}_j\}, \{\overline{\lambda}_j\}, \overline{\omega})$ so that $\overline{\phi}_j = 1 \ \forall j$, $\overline{\lambda}_j = 1 \ \forall j$ and $\overline{\omega} = 1$, is the solution of (M.4). As a consequence $\omega^* > 0$. Therefore necessarily $\phi_j^* > 0$ due the model constraints. Now applying the Complementary Theorem of linear programming the proof is complete. ■

As a consequence of the above result, once (M.3) is solved, the allocation of each DMU depends exclusively upon the observed outputs. In this sense it would seem that our model is similar to that proposed by Cook and Kress [8] and to the multidimensional treatment of Jahanshaloo et al. [11], by depending upon a single type of variables. There exists, however, a substantial difference. If with models [8] and [11] one of the outputs is varied while the inputs remain constant, the same allocation will be obtained. In our case, if any of the inputs are varied while the outputs remain constant, the allocation will be different from that initially. With this (M.3) is shown utilizing the inputs just as much as it does the outputs for the final calculation of the solution. This fact will be illustrated in section 5 as an example.

In [3] the following property is stated: "*If two DMUs have identical input profiles the fixed cost allocated to the DMU with higher output must exceed the fixed cost allocated to the DMU with a lower ouput*". This condition is not satisfied in model [8] but it is in [3][1], and is verified for the solution that (M.3) produces.

Proposition 2 *Let* $(x_1^1, \ldots, x_m^1; y_1^1, \ldots, y_s^1)$ *and* $(x_1^2, \ldots, x_m^2; y_1^2, \ldots, y_s^2)$ *be the input and ouput vectors associated with two DMUs, DMU 1 and DMU 2, so that* $x_i^1 = x_i^2 \ \forall i$ *and* $y_r^1 \leq y_r^2 \ \forall r$, *then* $f_j^{1*} \leq f_j^{2*} \ \forall j$.

The proof is immediate as a consequence of proposition 1. Note how the above proposition is intimately related with the dominance concept in the efficiency and productivity analysis (see Koopmans [13]).

Continuing along on the search for good properties, allocations of a budget in which a DMU remains without receiving any quantity would be refused by the manager of the organization and by the very own DMUs. Unfortunately, the Cook and Zhu model [9] presents this disadvantage. (M.3) on the other hand, does adequately verify such a property.

Proposition 3 *Let* $(\{v_i^*\}, \{u_r^*\}, \{f_j^*\},)$ *be the optimal solution of (M.3), then* $f_j^* > 0 \ \forall j$.

Proof. If there exists some $k = 1, \ldots, n$ so that $f_k^* = 0$ then by proposition 1 we have $u_1^* = u_2^* = \ldots = u_s^* = 0$. However, this would imply that $f_j^* = 0 \ \forall j$, contradicting the fact that $\sum_j f_j^* = F > 0$. ■

Two other desirable properties are the units invariance and translation invariance of (M.3). This theme appears profoundly developed in the work of Knox Lovell and Pastor [12] about classical DEA models. Despite this question frequently being examined in DEA literature, it has not been studied in any of the work mentioned in section 2. Referring to (M.3), it will be shown satisfying the units invariance property being lightly transformed, while it does not support in an acceptable manner a translation of the inputs and outputs variables.

[1] Although the result does not formally appear in Beasley [3].

Proposition 4 *If in (M.3) both the inputs and outputs are multiplied by $\alpha > 0$, then the same allocation is obtained as would be with the original data.*

Proof. Given an optimal solution of (M.3), with the original data, the same with the weights divided by α, is at the same time an optimal solution of the modified model. ■

The translation property will be shown with the data from the example of annex 1 with F = 100, as not being verified by the solution of (M.3).

Variables	Original Data	"× 2"	"+ 1"
v_1	7.58	3.79	6.25
u_1	10.43	5.21	8.33
u_2	0.47	0.24	0
f_1	24.17	24.17	25
f_2	53.08	53.08	50
f_3	22.75	22.75	25

The columns of the preceding table denote from left to right: the different variables of (M.3); the optimal solution with the original data; the solution with the data multiplied by 2; and the solution with the data plus one unit. The same allocation is obtained using both the data multiplied by 2 and using the original data from the example. DMU 1 is allocated 24.17 units, DMU 2 obtains 53.08, and DMU 3, 22.75. When considering the translation, it can be seen in the last column that the optimal solution has been modified.

Cook and Kress [8] present the solution in the one-dimensional case, one input one output. Such an allocation depends exclusively upon the inputs observed for each DMU. If (M.3) is simplified to adjust it to this context we will equally obtain an explicit formulation of the solution. In our case the allocation will depend upon the outputs.

Proposition 5 *If $m = s = 1$, the only optimal solution of (M.3) is*

$$u^* = F/\sum_j y_j,$$

$$v^* = max\{\frac{y_k}{x_k} : k = 1, \ldots, n\}u^*,$$

$$f_j^* = y_j u^*.$$

The proof is immediate.

In Jahanshahloo [11] an explicit formulation dependent upon inputs was obtained that generalized the one-dimensional solution of Cook and Kress [8]. Just the opposite, such a generalization with the multidimensional case is not possible to carry it out with our model.

In DEA literature the allocation of fixed costs and budgets one of the most important properties is that of uniqueness. Speaking about this, Beasley [3] says: "*In terms of practical applicability we regard it as important that any approach for fixed cost allocation produces unique value*". In Cook and Kress [8] the search emphasizes the uniqueness of the solutions. Just as with a model like the other the uniqueness is not assured. What about the model introduced in this work? The solution will be partially satisfactory. Even though in general the answer is negative, we will show that like in practice the allocation that (M.3) produces is unique.

The following result will be the cause of the empirical uniqueness of the (M.3) solution.

Proposition 6 *(M.3) is equivalent in optimal solutions to*

$$\begin{array}{lll} Min & \sum_i v_i \sum_j x_{ij} & \\ s.t.: & & \\ & \sum_r u_r \sum_j y_{rj} = F, & \\ & -\sum_i v_i x_{ij} + \sum_r u_r y_{rj} \leq 0, & \forall j \\ & v_i, u_r \geq 0, & \forall i, r \end{array} \tag{M.5}$$

Proof. Let $(\{v_i^*\}, \{u_r^*\}, \{f_j^*\})$ be an optimal solution of (M.3), then $(\{v_i^*\}, \{u_r^*\})$ is a feasible solution of (M.5). If it was not optimal, then a feasible solution of (M.5) would exist, $(\{\overline{v}_i\}, \{\overline{u}_r\})$, such that $\sum_i \overline{v}_i \sum_j x_{ij} < \sum_i v_i^* \sum_j x_{ij}$. However, $(\{\overline{v}_i\}, \{\overline{u}_r\}, \overline{f}_j\})$ with $\overline{f}_j = \sum_r \overline{u}_r y_{rj}$ verifies the constraints of (M.3). Therefore, a contradiction is obtained with the optimality of $(\{v_i^*\}, \{u_r^*\}, \{f_j^*\})$. The converse is proved in a similar manner. ■

(M.5) corresponds with the CCR[2] model (M.2) when the aggregate DMU is evaluated. This is to say, $(\sum_j x_{1j}, \ldots, \sum_j x_{mj}; \sum_j y_{1j}, \ldots, \sum_j y_{sj})$. Such a DMU represents the operation levels of the organization as an overall entity.

On the other hand, it is well known in DEA literature that inefficient units habitually present a unique solution for the model (M.2) (see Cooper et al. [10]). As a consequence, it would be enough to prove that the aggregate DMU is inefficient to finish displaying the frequent uniqueness of our solution.

Proposition 7 *Except in the case that the n DMUs are on the same efficient face of the production frontier,* $(\sum_j x_{1j}, \ldots, \sum_j x_{mj}; \sum_j y_{1j}, \ldots, \sum_j y_{sj})$ *is inefficient in the Pareto-Koopmans sense.*

[2]Except for the normalization constant detail, where instead of being 1 it has a value of F. Notwithstanding, this fact does not modify the DMU classifications as being either efficient or inefficient.

Proof. Suppose that the DMU k $(k = 1, \ldots, n)$ is inefficient. Then there exists a DMU p $(p = 1, \ldots, n)$ so that $x_{ip} \leq x_{ik} \quad \forall i$, $y_{rp} \geq y_{rk} \quad \forall r$ and, without loss of generality, exists an output $l = 1, \ldots, s$ with $y_{lp} > y_{lk}$. Now the aggregate DMU is a conic combination of all of the DMUs, and as such

$$\sum_j x_{ij} \geq \sum_{j \neq k} x_{ij} + x_{ip}, \quad \forall i,$$

$$\sum_j y_{rj} \leq \sum_{j \neq k} y_{rj} + y_{rp}, \quad \forall r \neq l,$$

$$\sum_j y_{lj} < \sum_{j \neq k} y_{lj} + y_{lp}.$$

This is to say, the aggregate DMU is dominated by a DMU of the production set.

Suppose that the n DMUs are efficient. Then, some of the efficient units belong to different efficient faces. As a consequence the aggregate DMU, as a conic combination of these DMUs, must be inefficient. ■

The above result indicates in a way that is simple to calculate (M.3) making use of the DEA software. For that it is enough solving (M.2) evaluating the aggregate DMU, and calculating the payments like this:

$$f_j^* = F \sum_r u_r^* y_{rj}$$

where u_r^* are the optimal weights associated with the outputs of (M.2).

In this latest sense, our approximation improves the proposed and developed non-linear model by Beasley [3] with regards to the computational complexity.

5 Numeric example with different DEA allocations

We will show how our general model works in the division of a common budget using the same data that appears in [8]. This data bank is found in annex 2. From a total of 12 DMUs the desired allocation comes from a budget made up of 100 €. The example fits within the general case, as it is formed with 3 inputs and 2 outputs.

The information appearing in the table, from left to right, is the following: the DMU index; the Cook and Kress solution [8]; the solution based on the Jahanshahloo et al. [11] model; the

Beasley [3] solution; the Cook and Zhu [9] solution; the solution of the model (M.3); and, lastly, the (M.3) solution applied with the data from annex 2 in which the input from one of the DMUs has been modified.

DMU	Cook and Kress	Jahanshahloo	Beasley	Cook and Zhu	(M.3)	(M.3)*
1	14.52	8.22	6.78	11.22	7.86	7.74
2	6.74	6.86	7.21	0	8.06	8.29
3	9.32	9.50	6.83	16.95	8.17	8.48
4	5.60	6.32	8.47	0	7.92	8
5	5.79	6.67	7.08	0	7.84	8.39
6	8.15	8.39	10.06	15.43	10.08	9.69
7	8.86	11.73	5.09	0	7.60	8.07
8	6.26	6.49	7.74	0	8.46	8.81
9	7.31	7.29	15.11	17.62	9.36	8.83
10	10.08	10.62	10.08	21.15	9.27	8.72
11	7.31	7.29	1.58	17.62	3.10	2.94
12	10.08	10.62	13.97	0	12.28	12.04

In the Cook and Kress [8], Jahanshahloo et al. [11], and Cook and Zhu [9] allocations, the original efficiency scores (invariance condition) are unmodified. The solution proposed by Beasley [3], has the average efficiency maximized. In the allocation obtained by applying the approximation (M.3), the efficiency of an aggregate DMU, representing all existing DMUs, is maximized and at the same time assures that each DMU is covering the costs of the source of common expenses.

The solutions appearing in the table are different, evidence of the existing discrepancies between the supposed allocations in each one of the approximations.

It can be confirmed that in both the Beasley [3] proposed solution, as well as that calculated by (M.3) are unique. In our case it is enough verifying that each of the reduced costs of the non-basic variables in the solution were strictly positive. The same cannot be said of the allocations consequent from the invariance condition.

Cook and Kress [8] allocate to DMUs 10 and 12, and to DMUs 9 and 11, the same exact quantity. This is due the sharing rule depending entirely upon the inputs. As the cited DMUs possess the same input levels, the model allocates them the same quantity. Contrasting this, in (M.3) there corresponds a greater quantity to DMU 12, because DMU 12 dominates DMU 10. The same relationship is given in the allocation of 9 and 11. This is an empirical illustration of proposition 2.

The Cook and Zhu [9] solution is unacceptable because it allocates nothing to some of the DMUs.

In the last column of the table, (M.3)*, the solution appears making use of (M.3) but by applying

a modification of the data. In particular, the input 1 from DMU 1 was changed from 350 to 1. As can be observed the solution obtained, that in addition whose results are unique, does not coincide with the calculation using (M.3) with the original data. We show like this how our solution is influenced as much by inputs as is by outputs, and this contradicts with how proposition 1 could be erroneously interpreted.

6 Budget allocation techniques for university management

In Pulido et al. [15], the problematic equitable allocation of money among the different degree courses is taken on for the acquisition of new laboratory equipment in the University Miguel Hernandez of Elche.

In this university the monetary distribution for the purchase of new equipment (software, microscopes, video recorders, and so on) for different degree courses received much criticism from 1997-2000. To solve this problem, the university decided to create a committee in charge of analyzing and proposing new solutions.

To make allocation decisions the committee decided to establish an objective measure to evaluate each degree course. This objective measure was based on the same criteria used by the Valencian Government (Generalitat Valenciana) for financing the public university system.

Being known the objective measure and the demand (claim, denoted by c_j) realized by each degree course, the committee opted to use the equity principle for the allocation of the budget.

The variables used by the committee to tackle the problem were:

- The number of financed students (fs_j): It depends on the number of students registered for each subject of the degree course and the length (in hours) of these studies.
- The experimental level (el_j): In the University Miguel Hernandez of Elche, $el_j \in 0.1, 0.3, 0.5, 0.7$, depending on the kind of degree course under consideration. For example, the experimental level in Law is 0.1 and Telecommunications Engineering has a level of 0.7.
- The implantation level (im_j): This is an aggregated measure, calculated using the number of new groups (inside degree course j), the number of two-year-old groups, and those which have existed for more than two years.

The objective measure for each degree course j is calculated like this

$$r_j = fs_j \times el_j \times im_j \times (336.567) \times 0.1.$$

The equity principle proposes a distribution such that

$$\frac{\alpha_j}{\omega_j} = \ldots = \frac{\alpha_n}{\omega_n},$$

where α_j is the quantity that each degree course will receive and ω_j is the weight that will allow us to compare the payments between different agents. These weights were calculated by resorting to the following formula:

$$\omega_j = 0.2\frac{c_j}{\sum_k c_k} + 0.8\frac{r_j}{\sum_k r_k}.$$

The weights 0.2 and 0.8 were subjectively determined by the committee. Lastly, it is indicated that the fixed total amount of money to be distributed among the degree courses is 721214.53 €.

Our intention is to make use of the model introduced in this work within this context and determine the payments for each one of the degree courses. We will compare at the same time the result obtained with that ultimately applied by the committee.

One of the advantages of our method is that it does not need to define arbitrary weights for the calculation of the allocation. However, by the very nature of the DEA technique it is necessary to classify from the beginning of the study which variables are inputs and which are outputs. By wanting to make the solution proposed by the committee comparable with that associated with (M.3), fs_j, el_j and im_j were used as outputs, and c_j was used as the sole input[3].

Applying the equity principle in [15], a solution was obtained that some DMUs exceeded their corresponding claim. In this case, the committee opted to allocate such DMUs exactly the quantity demanded. Specifically speaking, this problem tells us that the budget is not totally shared among its programs. In order to avoid this in our model, the following n linear constraints were added to (M.3) that will guarantee the payment assigned to each program will not exceed its claim:

$$f_j \leq c_j \quad \forall j. \tag{M.6}$$

This is one of the numerous advantages of realizing an allocation by making use of a linear programming model. Constraints can be added to the allocation in a very simple fashion.

The claim, the solution applied by the committee and the allocation calculated by our approximation appear in a table format in annex 3. The total demanded quantity by the degree courses

[3]The experimental level and the implantation level could be used as non-discretionary variables. However, the optimal weights obtained by (M.3) are zero for these variables. Therefore this fact is irrelevant for our purpose.

ascended to 4078097.6 €. The total sum of the allocation realized by the committee is 717239.11 €, while the total from the solution of (M.3) + (M.6) is 721214.53 €. This exactly coincides with the budget to allocate between the degree courses.

The solutions provided by one method or another are clearly different. Make note, for example, that the DMUs 16 and 22 are the degree courses allotted a greater payment by both the committee solution and by (M.3) + (M.6), respectively. DMU 16 is to receive 72127.98 € when it solicited 530070.44 € in the committee solution. DMU 22 is to acquire 59975.19 € when its claim was 250845.44 € in our solution. However, degree course 22 soliciting less money than 16 presents greater outputs than does 16. This indicates that degree course 16 is presenting an exaggerated claim. In our model it receives a payment of 44821.64 €. As can be observed, the technique of demanding more than is necessary does not work well when using our approximation. Unfortunately, the solution that was applied is strongly sensitive to the size of the claim.

On the other hand, it was commented in [15] of how problematic the selection of arbitrary weights resulted in the allocation. The introduced approximation in this work is just the opposite, determining some objective and general weights for each one of the degree courses.

7 Conclusions

There are different contexts in which a quantity of money must be distributed among a group of agents. Classically this problem has not been dealt with from the efficiency and productivity analysis point of view. However, a production function contains the information about the relationship existing between inputs and outputs. Such information can be used when calculating the allocation of a budget. This line of thinking has recently been taken on in the literature.

In this work a new alternative approximation was formally introduced to those existing, and this complies several interesting properties: dominance, uniqueness, computational ease, units invariant, etc. In this sense it improves the methodologies proposed by other authors.

Finally, the new model was applied using data from an actual problem: allocation of a budget between the degree courses of a university. The solution obtained was compared with that actually applied. An entire series of advantages convert it into an attractive sharing rule in university management applications.

Acknowledgements

Our most sincere gratitude to professors M. Pulido, J. Sánchez-Soriano, and N. Llorca for their assistance in the materialization of section 6.

8 Annexes

8.1 Annex 1

DMU	X	Y1	Y2
1	9	2	7
2	7	5	2
3	3	2	4

8.2 Annex 2

The following data appear in Cook and Kress [8].

DMU	Input 1	Input 2	Input 3	Output 1	Output 2
1	350	39	9	67	751
2	298	26	8	73	611
3	422	31	7	75	584
4	281	16	9	70	665
5	301	16	6	75	445
6	360	29	17	83	1070
7	540	18	10	72	457
8	276	33	5	78	590
9	323	25	5	75	1074
10	444	64	6	74	1072
11	323	25	5	25	350
12	444	64	6	104	1199

8.3 Annex 3

Degree course	Claim	Committee solution	(M.3)+(M.6)
1	15720.66	5525.89	15720.66
2	25532.20	5396.65	25532.20
3	32960.44	26325.69	21361.03
4	13664.61	4010.00	11228.23
5	8173.76	4245.87	8173.76
6	3904.17	3904.17	3904.17
7	14869.04	8267.42	10954.37
8	289753.13	40688.95	83070.66
9	250962.13	43498.27	20995.88
10	126857.13	34592.78	20539.45
11	248338.50	36258.13	32132.83
12	227091.64	62390.01	45643.22
13	63069.72	7582.93	7302.92
14	15915.98	6154.03	15915.98
15	10059.72	4178.78	7302.92
16	530070.44	72127.98	44821.64
17	121229.15	43877.06	24008.33
18	233163.45	53338.87	30763.53
19	248008.45	56406.21	32497.97
20	169534.83	10612.70	6298.76
21	240404.84	42826.19	40348.61
22	250845.44	48431.50	59975.19
23	70752.96	9661.04	9767.65
24	140679.05	10796.30	5294.61
25	227684.44	28023.43	54498.01
26	234125.14	26210.73	48929.53
27	264726.58	21907.51	34232.42
Total	4078097.60	717239.11	721214.53

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