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THE SEQUENCE SPACE $E_n^q(M,p,s)$ AND N_k- LACUNARY STATISTICAL CONVERGENCE

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ABSTRACT. In this paper we define the sequence space $E_n^q(M,p,s)$ by using an Orlicz function and we study various properties and obtain some inclusion relations involving this space. We give some relations between N_k- lacunary statistical convergence and strongly N_k- lacunary convergence.

1. INTRODUCTION AND PRELIMINARIES

Let $\sum_{k=0}^{\infty} a_k$ be an infinite series with sequence of partial sums (s_k) and $q>0$ any real number. The Euler means (E,q) of the sequences (s_n) are defined to be

$$E_n^q=\frac{1}{(1+q)^n}\sum_{v=0}^{n}\binom{n}{v}q^{n-v}s_v.$$

The series $\sum_{n=0}^{\infty} a_n$ is said to be summable (E,q) to the number s if

$$E_n^q=\frac{1}{(1+q)^n}\sum_{v=0}^{n}\binom{n}{v}q^{n-v}s_v\to s \quad \text{as} \quad n\to\infty,$$

and is said to be absolutely summable (E,q) or summable $|E,q|$, if

$$\sum_k |E_k^q-E_{k-1}^q|<\infty.$$

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Let $a = (a_n)$ be a sequence of scalars, for $k \geq 1$ we will denote by $N_n(a) = E_n^q - E_{n-1}^q$, where E_n^q is defined as above. After applications of Abel's transform we have

$$N_n(a) = -\frac{1}{(1+q)^{n-1}} \sum_{k=0}^{n-2} a_{k+1} A_k + \frac{s_{n-1} A_{n-1}}{(1+q)^{n-1}} + \frac{s_n}{(1+q)^n} - \frac{q^{n-1}}{(1+q)^n} s_0,$$

where $A_k = \sum_{i=0}^{k} \left[\frac{q}{1+q}\binom{n}{i} - \binom{n-1}{i}\right] q^{n-i-1}$.

Note that for any sequences $a = (a_n)$, $b = (b_n)$ and scalar λ, we have:

$N_n(a+b) = N_n(a) + N_n(b)$ and $N_n(\lambda a) = \lambda N_n(a)$.

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, nondecreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. It is well known that if M is a convex function and $M(0) = 0$; then $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 < \lambda \leq 1$. Two Orlicz functions M_1 and M_2 are said to be equivalent if there are positive constants α and β, and x_0 such that $M_1(\alpha x) \leq M_2(x) \leq M_1(\beta x)$ for all x with $0 \leq x \leq x_0$. (see [15, 16]).

Lindentrauss and Tzafirir [17] used the idea of Orlicz function and defined the following sequence space

$$l_M = \left\{ x = (x_i) \in w : \sum_{i=1}^{\infty} M\left(\frac{|x_i|}{\varrho}\right) < \infty, \varrho > 0 \right\}$$

which is called an Orlicz sequence space. The space l_M is a Banach space with the norm

$$||x|| = \inf \left\{ \varrho > 0 : \sum_{i=1}^{\infty} M\left(\frac{|x_i|}{\varrho}\right) \leq 1 \right\}.$$

The space l_M is closely related to the space l_p which is an Orlicz sequence space with, $M(x) = x^p, 1 \leq p < \infty$.

In the later stage different Orlicz sequence spaces were introduced and studied by Altın et al. [1], Altun and Bilgin [2], Bhardwaj and Singh [3], Braha [7], Et et al. [9], Mursaleen et al. [18] and many others.

2. Main results

Let M be an Orlicz function and $p = (p_k)$ be a sequence of positive real numbers. We define the following sequence space:

$$E_n^q(M, p, s) = \left\{ a = (a_k) : \sum_{k=1}^{\infty} k^{-s} \left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{p_k} < \infty, s \geq 0, \text{ for some } \varrho > 0 \right\}.$$

If we take $s = 0$, then we have

$$E_n^q(M, p) = \left\{ a = (a_k) : \sum_{k=1}^{\infty} \left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{p_k} < \infty, \text{ for some } \varrho > 0 \right\},$$

if $p_k = 1$ for all $k \in \mathbb{N}$, then we have

$$E_n^q(M,s)=\left\{a=(a_k):\sum_{k=1}^{\infty}k^{-s}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]<\infty, s\geq 0,\ \text{for some } \varrho>0\right\}.$$

In what follows we will prove the structure properties of the spaces defined above.

Theorem 2.1 Let the sequence (p_k) be bounded. Then the spaces $E_n^q(M,s)$, $E_n^q(M,p)$ and $E_n^q(M,p,s)$ are linear spaces over the field $\mathbb{C}$ of complex numbers.

Proof. Clear.

Theorem 2.2 Let $p=(p_i)$ be a bounded sequence of strictly positive real numbers. Then the sequence space $E_n^q(M,p,s)$ is a paranormed (need not total paranorm) space with

$$g(a)=\inf\left\{\rho^{p_n/H}:\left(\sum_{k=1}^{\infty}k^{-s}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{p_k}\right)^{\frac{1}{H}}\leq 1,\ n=1,2,\dots\right\}$$

where $H=\max(1,\sup p_k)$.

Proof. Clearly $g(a)=g(-a)$ and $g(a+b)\leq g(a)+g(b)$. Since $M(0)=0$, we get $\inf\left\{\rho^{r_n/H}\right\}=0$ for $a=0$.

Finally, we prove that scalar multiplication is continuous. Let λ be any number. Since

$$g(\lambda a)=\inf\left\{\rho^{p_n/H}:\sum_{k=1}^{\infty}k^{-s}\left[M\left(\frac{|\lambda\ N_k(a)|}{\varrho}\right)\right]^{p_k}\leq 1,\ n=1,2,\dots\right\},$$

we may write

$$g(\lambda a)=\inf\left\{(\lambda s)^{p_n/H}:\left(\sum_{k=1}^{\infty}k^{-s}\left[M\left(\frac{|N_k(a)|}{s}\right)\right]^{p_k}\right)^{\frac{1}{H}}\leq 1,\ n=1,2,\dots\right\},$$

where $s=\varrho/|\lambda|$. Since $|\lambda|^{p_k}\leq\max\left(1,|\lambda|^H\right)$, then $|\lambda|^{p_k/H}\leq\left(\max\left(1,|\lambda|^H\right)\right)^{\frac{1}{H}}$. Hence

$$g(\lambda a)\leq\left(\max\left(1,|\lambda|^H\right)\right)^{\frac{1}{H}}\cdot$$

$$\inf\left\{(s)^{p_n/H}:\left(\sum_{k=1}^{\infty}k^{-s}\left[M\left(\frac{|N_k(a)|}{s}\right)\right]^{p_k}\right)^{\frac{1}{H}}\leq 1,\ n=1,2,\dots\right\}$$

and therefore $g(a)$ converges to zero when $g(a)$ converges to zero in $E_n^q(M,p,s)$. Now suppose that $\lambda_n\to 0$ as $n\to\infty$ and a in $E_n^q(M,p,s)$. For arbitrary $\varepsilon>0$, let n_0 be a positive integer such that

$$\sum_{k=n_0+1}^{\infty}k^{-s}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{p_k}<\frac{\varepsilon}{2}$$

for some $\varrho > 0$. This implies that

$$\left(\sum_{k=n_0+1}^{\infty} k^{-s}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{p_k}\right)^{\frac{1}{H}} \leq \frac{\varepsilon}{2}.$$

Let $0 < |\lambda| < 1$, then using convexity of M we get

$$\sum_{k=n_0+1}^{\infty} k^{-s}\left[M\left(\frac{|\lambda N_k(a)|}{\varrho}\right)\right]^{p_k} < \sum_{k=n_0+1}^{\infty} k^{-s}\left[|\lambda|\frac{|N_k(a)|}{\varrho}\right]^{p_k} < \left(\frac{\varepsilon}{2}\right)^{H}.$$

Since M is continuous everywhere in $[0,\infty)$, then

$$f(t) = \sum_{k=1}^{n_0} k^{-s}\left[M\left(\frac{|tN_k(a)|}{\rho}\right)\right]^{p_k}$$

is continuous at 0. So there is $0 < \delta < 1$ such that $|f(t)| < \frac{\varepsilon}{2}$ for $0 < t < \delta$. Let K be such that $|\lambda_n| < \delta$ for $n > K$, then for $n > K$ we have

$$\left(\sum_{k=1}^{n_0} k^{-s}\left[M\left(\frac{|\lambda_n N_k(a)|}{\rho}\right)\right]^{p_k}\right)^{\frac{1}{H}} < \frac{\varepsilon}{2}.$$

Thus

$$\left(\sum_{k=1}^{\infty} k^{-s}\left[M\left(\frac{|\lambda_n N_k(a)|}{\rho}\right)\right]^{p_k}\right)^{\frac{1}{H}} < \varepsilon, \text{ for } n > K.$$

Hence $g(\lambda a) \to 0$, as $\lambda \to 0$. This completes the proof of the Theorem.

Theorem 2.3 Let M_1 and M_2 be two Orlicz functions and s, s_1, s_2 be non-negative real numbers, then we have
i) $E_n^q(M_1,p,s) \cap E_n^q(M_2,p,s) \subseteq E_n^q(M_1+M_2,p,s)$,
ii) If $s_1 \leq s_2$, then $E_n^q(M_1,p,s_1) \subseteq E_n^q(M_1,p,s_2)$.
iii) If M_1 and M_2 are equivalent, then $E_n^q(M_1,p,s) = E_n^q(M_2,p,s)$.

Proof. Clear.

Corollary 2.4 Let $s > 1$ and M be Orlicz function.
i) $E_n^q(M,p) \subset E_n^q(M,p,s)$
ii) $E_n^q(M) \subset E_n^q(M,s)$

Theorem 2.5 Suppose that $0 < u_k \leq p_k < \infty$ for each k. Then $E_n^q(M,u) \subseteq E_n^q(M,p)$.

Proof. Let $a \in E_n^q(M,u,s)$. Then there exists some $\varrho > 0$ such that

$$\sum_{k=1}^{\infty}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{u_k} < \infty.$$

This implies that $M\left(\frac{|N_i(a)|}{\varrho}\right) \leq 1$ for sufficiently large values of i, say $i \geq k_0$ for some fixed $k_0 \in \mathbb{N}$. Since M is non-decreasing, we get

$$\sum_{k\geq k_0}^{\infty}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{p_k} \leq \sum_{k\geq k_0}^{\infty}\left[M\left(\frac{|N_k(a)|}{\varrho}\right)\right]^{u_k} < \infty.$$

Hence $a \in E_n^q(M,p)$.

The following results are consequence of the above result.

Corollary 2.6 (i) If $0 < p_k \leq 1$ for each k, then $E_n^q(M,p) \subseteq E_n^q(M)$.
(ii) If $p_k \geq 1$ for all k, then $E_n^q(M) \subseteq E_n^q(M,p)$.

3. Results on Statistical Convergence

The idea of statistical convergence was given by Zygmund [26] in the first edition of his monograph published in Warsaw in 1935.The concept of statistical convergence was introduced by Steinhaus [25] and Fast [11] and later reintroduced by Schoenberg [24] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory, measure theory, trigonometric series, turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Connor [4], Çolak [5], Fridy [13], Et et al. [10], Mursaleen [19], Rath and Tripathy [20], Salat [21], Savaş [22] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

Let $\theta = (k_r)$ be the sequence of positive integers such that $k_0 = 0$, $0 < k_r < k_{r+1}$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. Then θ is called a lacunary sequence. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio k_r/k_{r-1} will be denoted by q_r. Lacunary sequences have been studied by ([3],[6],[8],[12],[14],[23]).

In this section we give some results about N_k-lacunary statistical convergence and give some relations between the set of N_k-lacunary statistical convergence sequences and other spaces which defined with respect to an Orlicz function.

Definition 3.1 Let $\theta = (k_r)$ be a lacunary sequence, then the sequence $a = (a_k)$ is said to be N_k-lacunary statistically convergent to the number ℓ provided that for every $\varepsilon > 0$,

$$\lim_r \frac{1}{h_r}\left|\{k \in I_r : |N_k(a) - \ell| \geq \varepsilon\}\right| = 0.$$

In this case we write $[N_k, S]_\theta - \lim a = \ell$ or $a_k \to \ell\,([N_k, S]_\theta)$.

Definition 3.2 Let $\theta=(k_r)$ be a lacunary sequence, M be an Orlicz function and $p=(p_k)$ be any sequence of strictly positive real numbers. A sequence $a=(a_k)$ is said to be strongly N_k-lacunary convergent to the number ℓ with respect to the Orlicz function M, provided that

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k}=0.$$

The set of all strongly N_k-lacunary convergent sequences to the number ℓ with respect to the Orlicz function M is denoted by $[N_k,w,M,p]_\theta$. In this case we write $a_k\to\ell([N_k,w,M,p]_\theta)$. In the special case $M(x)=x$, $p_k=p_0$ for all $k\in\mathbb{N}$ we shall write $[N_k,w]_\theta$ instead of $[N_k,w,M,p]_\theta$.

Theorem 3.3 Let $\theta=(k_r)$ be a lacunary sequence.

i) If a sequence $a=(a_k)$ is strongly N_k-lacunary convergent to ℓ, then it is N_k-lacunary statistically convergent to ℓ.

ii) If a bounded sequence $a=(a_k)$ is N_k-lacunary statistically convergent to ℓ, then it is strongly N_k-lacunary convergent to ℓ.

Proof. (i) Let $\varepsilon>0$ and $a_k\to\ell\left([N_k,w]_\theta\right)$. Then we can write

$$\sum_{k\in I_r}|N_k(a)-\ell|^{p_0}\geq\sum_{\substack{k\in I_r\\|N_k(a)-\ell|\geq\varepsilon}}|N_k(a)-\ell|^{p_0}\geq\varepsilon^{p_0}\left|\{k\in I_r:|N_k(a)-\ell|\geq\varepsilon\}\right|.$$

Hence $a_k\to\ell\left([N_k,S]_\theta\right)$.

ii) Suppose that $a_k\to\ell\left([N_k,S]_\theta\right)$ and let $a\in\ell_\infty$. Let $\varepsilon>0$ be given and select N_ε such that

$$\frac{1}{h_r}\left|\left\{k\in I_r:|N_k(a)-\ell|\geq\left(\frac{\varepsilon}{2}\right)^{\frac{1}{p_0}}\right\}\right|\leq\frac{\varepsilon}{2K^{p_0}}$$

for all $r>N_\varepsilon$ and set $L_r=\left\{k\in I_r:|N_k(a)-\ell|\geq\left(\frac{\varepsilon}{2}\right)^{\frac{1}{p_0}}\right\}$, where $K=\sup_k|a_k|<\infty$. Now for all $r>N_\varepsilon$ we have

$$\begin{aligned}\frac{1}{h_r}\sum_{k\in I_r}|N_k(a)-\ell|^{p_0} &= \frac{1}{h_r}\sum_{\substack{k\in I_r\\k\in L_r}}|N_k(a)-\ell|^{p_0}+\frac{1}{h_r}\sum_{\substack{k\in I_r\\k\notin L_r}}|N_k(a)-\ell|^{p_0}\\ &\leq \frac{1}{h_r}\left(\frac{h_r\varepsilon}{2K^{p_0}}\right)K^{p_0}+\frac{\varepsilon}{2h_r}h_r=\varepsilon.\end{aligned}$$

Thus $(a_k)\in[N_k,w]_\theta$. This completes the proof.

Theorem 3.4 For any lacunary sequence θ, if $\liminf_{r\to\infty} q_r>1$, then $[N_k,S]\subset[N_k,S]_\theta$.

Proof. If $\liminf_{r\to\infty} q_r>1$, then there exists a $\delta>0$ such that $1+\delta\leq q_r$ for sufficiently large r. Since $h_r=k_r-k_{r-1}$, we have $\frac{k_r}{h_r}\leq\frac{1+\delta}{\delta}$. Let $a_k\to\ell\left([N_k,S]\right)$.

Then for every $\varepsilon > 0$ we have

$$\begin{aligned}\frac{1}{k_r}\left|\{k \le k_r : |N_k(a)-\ell| \ge \varepsilon\}\right| &\ge \frac{1}{k_r}\left|\{k \in I_r : |N_k(a)-\ell| \ge \varepsilon\}\right| \\ &\ge \frac{\delta}{1+\delta}\frac{1}{h_r}\left|\{k \in I_r : |N_k(a)-\ell| \ge \varepsilon\}\right|.\end{aligned}$$

Hence $[N_k, S] \subset [N_k, S]_\theta$.

Theorem 3.5 For any lacunary sequence θ, if $\limsup_r q_r < \infty$, then $[N_k, S]_\theta \subset [N_k, S]$.

Proof. Suppose that $\limsup_r q_r < \infty$. Then there exists a $\beta > 0$ such that $q_r < \beta$ for all r. Let $a_k \to \ell\left([N_k, S]_\theta\right)$, and set $E_r = \left|\{k \in I_r : |N_k(a)-\ell| \ge \varepsilon\}\right|$. Then there exists an $r_0 \in \mathbb{N}$ such that $\frac{E_r}{h_r} < \varepsilon$ for all $r > r_0$. Let $K = \max\{E_r : 1 \le r \le r_0\}$ and choose n such that $k_{r-1} < n \le k_r$, then we have

$$\begin{aligned}\frac{1}{n}\left|\{k \le n : |N_k(a)-\ell| \ge \varepsilon\}\right| &\le \frac{1}{k_{r-1}}\left|\{k \le k_r : |N_k(a)-\ell| \ge \varepsilon\}\right| \\ &\le \frac{1}{k_{r-1}}\left\{E_1 + E_2 + \ldots + E_{r_0} + E_{(r_0+1)} + \ldots + E_r\right\} \\ &\le \frac{K}{k_{r-1}}r_0 + \frac{1}{k_{r-1}}\left\{\frac{E_{(r_0+1)}}{h_{r_0+1}}h_{r_0+1} + \ldots + \frac{E_r}{h_r}h_r\right\} \\ &\le \frac{K}{k_{r-1}}r_0 + \frac{1}{k_{r-1}}\left(\sup_{r>r_0}\frac{E_r}{h_r}\right)\left\{h_{r_0+1} + \ldots + h_r\right\} \\ &\le \frac{K}{k_{r-1}}r_0 + \varepsilon\frac{k_r - k_{r_0}}{k_{r-1}} \\ &\le \frac{K}{k_{r-1}}r_0 + \varepsilon q_r \\ &\le \frac{K}{k_{r-1}}r_0 + \varepsilon\beta.\end{aligned}$$

This completes the proof.

Theorem 3.6 Let $\theta = (k_r)$ be a lacunary sequence with $1 < \liminf_r q_r \le \limsup_r q_r < \infty$, then we have $[N_k, S]_\theta = [N_k, S]$.

Proof. The proof follows from Theorem 3.4 and Theorem 3.5.

Theorem 3.7 Let $\theta = (k_r)$ be a lacunary sequence, M be an Orlicz function and $0 < h = \inf_k p_k \le p_k \le \sup_k p_k = H$. Then $[N_k, w, M, p]_\theta \subset [N_k, S]_\theta$.

Proof. Let $a \in [N_k, w, M, p]_\theta$. Then there exists a number $\rho > 0$ such that

$$\frac{1}{h_r}\sum_{k \in I_r}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k} \to 0, \text{ as } r \to \infty.$$

Then given $\varepsilon > 0$ we have

$$\begin{aligned}\frac{1}{h_r}\sum_{k\in I_r}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k} &\geq \frac{1}{h_r}\sum_{\substack{k\in I_r\\ |N_k(a)-\ell|\geq\varepsilon}}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k}\\ &\geq \frac{1}{h_r}\sum_{\substack{k\in I_r\\ |N_k(a)-\ell|\geq\varepsilon}}\left[M\left(\varepsilon_1\right)\right]^{p_k}, \text{ where } \varepsilon/\rho=\varepsilon_1\\ &\geq \frac{1}{h_r}\sum\min\left\{\left[M\left(\varepsilon_1\right)\right]^{h},\left[M\left(\varepsilon_1\right)\right]^{H}\right\}\end{aligned}$$

$$\geq \frac{1}{h_r}\left|\left\{k\in I_r:|N_k(a)-\ell|\geq\varepsilon\right\}\right|\cdot\min\left\{\left[M\left(\varepsilon_1\right)\right]^{h},\left[M\left(\varepsilon_1\right)\right]^{H}\right\}.$$

Hence $x\in[N_k,S]_\theta$.

Theorem 3.8 Let $\theta=(k_r)$ be a lacunary sequence, M be an Orlicz function and $a=(a_k)$ be bounded sequence, then $[N_k,S]_\theta\subset[N_k,w,M,p]_\theta$.

Proof. Suppose that $a\in\ell_\infty$ and $a_k\to\ell\left([N_k,S]_\theta\right)$. Since $a\in\ell_\infty$, there is a constant $T>0$ such that $|N_k(a)-\ell|\leq T$. Given $\varepsilon>0$ we have

$$\begin{aligned}\frac{1}{h_r}\sum_{k\in I_r}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k} &= \frac{1}{h_r}\sum_{\substack{k\in I_r\\ |N_k(a)-\ell|\geq\varepsilon}}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k}\\ &+\frac{1}{h_r}\sum_{\substack{k\in I_r\\ |N_k(a)-\ell|<\varepsilon}}\left[M\left(\frac{|N_k(a)-\ell|}{\rho}\right)\right]^{p_k}\\ &\leq \frac{1}{h_r}\sum_{\substack{k\in I_r\\ |N_k(a)-\ell|\geq\varepsilon}}\max\left\{\left[M\left(\frac{T}{\rho}\right)\right]^{h},\left[M\left(\frac{T}{\rho}\right)\right]^{H}\right\}+\frac{1}{h_r}\sum_{\substack{k\in I_r\\ |N_k(a)-\ell|<\varepsilon}}\left[M\left(\frac{\varepsilon}{\rho}\right)\right]^{p_k}\\ &\leq \max\left\{\left[M\left(K\right)\right]^{h},\left[M\left(K\right)\right]^{H}\right\}\frac{1}{h_r}\left|\left\{k\in I_r:|N_k(a)-\ell|\geq\varepsilon\right\}\right|\\ &+\max\left\{\left[M\left(\varepsilon_1\right)\right]^{h},\left[M\left(\varepsilon_1\right)\right]^{H}\right\},\quad \frac{T}{\rho}=K,\ \frac{\varepsilon}{\rho}=\varepsilon_1.\end{aligned}$$

Hence $a\in[N_k,w,M,p]_\theta$.

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