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# ON THE INTERSECTION FORMS OF CLOSED 4-MANIFOLDS

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*Abstract*

Given a closed 4-manifold $M$, let $M^*$ be the simply-connected 4-manifold obtained from M by killing the fundamental group. We study the relation between the intersection forms $\lambda_M$ and $\lambda_{M^*}$. Finally some topological consequences and examples are described.

## 1. Introduction.

Let $M^4$ be a closed connected orientable (PL) 4-manifold with fundamental group $\Pi_1$ .

Denote by $\lambda_M$ the intersection form of $M$

$$\lambda_M : FH_2(M) \times FH_2(M) \longrightarrow Z$$

where $FH_2(M) = H_2(M; Z)/\text{torsion}$ (see for example [5], [10]).

Let $M^*$ be the simply-connected closed 4-manifold obtained from $M$ by killing the fundamental group $\Pi_1$ (see [6]).

Our purpose is to study what relation links $\lambda_M$ to $\lambda_{M^*}$ . Then we obtain some topological consequences about $M^*$ . Finally we give some examples which illustrate the results.

## 2. Main results.

Let $[\alpha]$ be a generator of $\Pi_1$. Since $M$ is orientable, we can extend $\alpha : S^1 \longrightarrow M$ to an embedding $\psi : S^1 \times D^3 \longrightarrow M$.

Recall that there are two ways to extend $\alpha$ since $\Pi_1(SO(3)) \cong Z_2$.

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Denote by $M' = M \backslash \psi(S^1 \times \overset{o}{D}{}^3) \cup D^2 \times S^2$ the closed 4-manifold obtained from $M$ by surgery on $\psi$ .

Since $\Pi_1(M') \cong \Pi_1(M)/[\alpha]$, iterated surgeries on generators of $\Pi_1(M)$ give a simply-connected closed 4-manifold $M^*$ .

**Problem.** Study the relations between $\lambda_M, \lambda_{M'}$ and $\lambda_M, \lambda_{M^*}$ respectively.

First we have the following

**Proposition 1.** *If $\Pi_1(M)$ has no elements of finite order, then $\lambda_{M^*}$ is isomorphic over the integers to $\lambda_M$ .*

The proof is given for example in [**1**] .

Therefore from now on we will consider manifolds with $\Pi_1(M)$ finite.

**Proposition 2.** *If $[\alpha]$ has finite order, then*

$$\lambda_{M'} \cong \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix} \cong \begin{cases} \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & a \quad even \\ \lambda_M \oplus \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & a \quad odd \end{cases}$$

*for some integer $a \in Z$.*

In any case $\lambda_{M'}$ is indefinite. For these forms there is the following well-known classification:

$$\begin{aligned} &1) \quad \lambda_{M'} \quad \text{even} \Longrightarrow \quad \lambda_{M'} \cong pE_8 \oplus q \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &2) \quad \lambda_{M'} \quad \text{odd} \Longrightarrow \quad \lambda_{M'} \cong p(1) \oplus q(-1) \end{aligned}$$

for some non negative integers $p, q \in Z$ .

Furthermore, S. K. Donaldson (see [**2**]) proved the following

**Theorem 3.** *Let $M^4$ be a closed connected orientable 4-manifold with arbitrary fundamental group. If $\lambda_M$ is definite, then $\lambda_M$ is isomorphic over the integers to either $(1) \oplus \cdots \oplus (1)$ or $(-1) \oplus \cdots \oplus (-1)$.*

The parity of $\lambda_M$ is related to the second Stiefel-Whitney class

$w_2(M) \in H^2(M; Z_2)$ as follows. Using the universal coefficient sequence

$$0 \longrightarrow Ext(H_1(M); Z_2) \longrightarrow H^2(M; Z_2) \longrightarrow Hom(H_2(M); Z_2) \longrightarrow 0,$$

it is easily proved that $\lambda_M$ is even if and only if $w_2(M) \in Ext(H_1(M); Z_2)$.

In particular, if $H_1(M)$ has no 2-torsion, then $\lambda_M$ is even if and only if $w_2(M) = 0$.

Thus proposition 2 implies the following

**Proposition 4.** *If $w_2(M) \neq 0$ , then*

$$\lambda_{M^*} \cong \lambda_M \oplus p\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cong r(1) \oplus s(-1)$$

*for some non negative integers $p, r, s \in Z$ .*

*Further, $M^*$ is homeomorphic to the connected sum $r(CP^2)\#s(-CP^2)$, being $CP^2$ the projective complex plane.*

Now we can also apply theorem 2 of [**2**] to obtain the following consequence of proposition 2.

**Corollary 5.** *Let $M^4$ be a closed connected orientable spin 4-manifold with fundamental group $\Pi_1(M) \cong Z_m$ .*

*If $\lambda_M$ has a positive part of rank 1, then $M^*$ is homeomorphic to either $2(CP^2)\#(2-\sigma(M))(-CP^2)$ or $2(S^2 \times S^2)$.*

*In the last case, $\lambda_M \cong \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Here $\sigma(M)$ denotes the signature of $M$.*

*Proof:* By proposition 2, we have either $\lambda_{M^*} \cong \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or $\lambda_{M^*} \cong \lambda_M \oplus \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, hence $\lambda_{M^*}$ has a positive part of rank 2.

In the first case, $\lambda_{M^*}$ is even. Since $H_1(M^*) \cong 0$ has no 2-torsion, theorem 2 of [**2**] implies that

$$\lambda_{M^*} \cong \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cong \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

hence $\lambda_M \cong \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (see [**7**], [**9**]) and $M^* \underset{\mathrm{TOP}}{\cong} 2(S^2 \times S^2)$ as required.

In the second case, $\lambda_{M^*} \cong 2(1) \oplus (2-\sigma(M))(-1)$, hence $M^* \underset{\mathrm{TOP}}{\cong} 2(CP^2)\#(2-\sigma(M))(-CP^2)$. ■

**3. Examples.**

3.1) Let $K = \{z_0^4 + z_1^4 + z_2^4 + z_3^4 = 0\} \subset CP^3$ be the Kummer surface and let $T : CP^3 \longrightarrow CP^3$ be the fixed point free involution defined by

$$T(z_0, z_1, z_2, z_3) = (\bar{z}_1, -\bar{z}_0, \bar{z}_3, -\bar{z}_2).$$

Since $T(K) = K$, we can consider the orbit space $M = K/T$, called the Habegger manifold (see [**4**]).

It is known that $\Pi_1(M) \cong Z_2$ and the intersection form

$$\lambda_M \cong (-E_8) \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

is even with a positive part of rank 1.

Since $w_2(M) \neq 0$, proposition 2 gives

$$\lambda_{M^*} \cong (-E_8) \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cong 10(-1) \oplus 2(1),$$

hence $M^* \underset{\mathrm{TOP}}{\cong} 10(-CP^2)\#2(CP^2)$ by the Freedman classification (see [**3**]).

We also recall that C. Okonek (see [**8**]) has shown that all homotopy Enriques surfaces are homeomorphic to the Habegger manifold.

3.2) Let $M^4 = S(\eta \oplus \eta \oplus \eta)$ be the sphere bundle of $\eta \oplus \eta \oplus \eta$, where $\eta \longrightarrow RP^2$ is the canonical bundle over the real projective 2-space.

Then we have $\lambda_M \cong 0$, $w_2(M) \neq 0$ and $\Pi_1(M) \cong Z_2$, hence

$$\lambda_{M^*} \cong \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $M^* \underset{\mathrm{TOP}}{\cong} CP^2\#(-CP^2) \underset{\mathrm{TOP}}{\cong} S^2 \underset{\sim}{\times} S^2$.

3.3) Let $M^4 = S(\eta \oplus \epsilon^2)$ be the sphere bundle of $\eta \oplus \epsilon^2$, where $\epsilon^2 = \epsilon^1 \oplus \epsilon^1 \longrightarrow RP^2$ is the 2-dimensional trivial bundle over $RP^2$.

Then we have $\lambda_M \cong 0$, $w_2(M) = 0$ and $\Pi_1(M) \cong Z_2$.

It is very easy to see that

$$H^2(M; Z_2) \underset{\text{iso}}{\xrightarrow{i^*}} H^2(M_0; Z_2) \underset{\text{iso}}{\xleftarrow{i'^*}} H^2(M^*; Z_2)$$

where $M_0 = M \backslash \psi(S^1 \times \overset{\circ}{D}{}^3)$, $\psi : S^1 \times D^3 \longrightarrow M$ represents the generator of $\Pi_1(M)$ and $i : M_0 \longrightarrow M$, $i' : M_0 \longrightarrow M^*$ are the natural inclusions.

Thus $w_2(M^*) = 0$, hence $\lambda_{M^*} \cong \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is even and $M^* \underset{\mathrm{TOP}}{\cong} S^2 \times S^2$.

## 4. Proofs.

*Proof of proposition 2:* For convenience we assume that $\Pi_1(M) \cong Z_m, m > 0$, with generator $[\alpha] = [\psi|_{S^1 \times 0}]$. For the general case, see remark 1 below.

We set $M_0 = M \backslash \psi(S^1 \times \mathring{D}^3)$ and consider the cobordism

$$W = M \times I \cup_\psi D^2 \times D^3 \quad (I = [0,1])$$

between $M$ and $M' = M_0 \cup D^2 \times S^2$.

Obviously the pairs $(W, M), (W, M')$ are homology equivalent to $(D^2 \times D^3, S^1 \times D^3)$ and $(D^2 \times D^3, D^2 \times S^2)$ respectively.

We have the following diagram

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & H_3(M, M_0) \cong Z \arrow[r, "\partial_*"] \arrow[d, "\text{iso}"'] & H_2(M_0) \arrow[r, "i_*"] \arrow[d, "i'_*"] & H_2(M) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & H_3(W, M') \cong Z \arrow[r, "\partial'_*"] & H_2(M') \arrow[r] \arrow[d, "k_*"] \arrow[dr, "j_*"] & H_2(W) \arrow[r] \arrow[d] & 0 \\
 & & Z \cong H_2(M', M_0) \arrow[r, "\text{iso}"'] \arrow[d] & H_2(W, M) & \\
 & & H_1(M_0) \cong H_1(M) \cong Z_m \arrow[d] & & \\
 & & 0 & &
\end{tikzcd}$$

where $i, i', j, k$ are inclusions.

Obviously $H_2(M')$ is a free group of rank $rkH_2(M) + 2$ and $H_2(M_0)$ is free of rank $rkH_2(M) + 1$ since it injects into $H_2(M')$.

Here we often identify an element of $H_2(M_0)$ with its image under $i'_*$.

Now we have

$$\lambda_M(i_*(u), i_*(v)) = \lambda_{M'}(i'_*(u), i'_*(v))$$

for every $u, v \in H_2(M_0)$.

Let $e \in H_2(M_0)$ be a primitive element such that $i_*(e)$ generates the subgroup $\mathrm{Tor}H_2(M) \cong Z_m$ and suppose that $f \in H_2(M')$ maps to the integer $m \in Z \cong H_2(M', M_0)$. Similarly $f$ is chosen to be primitive. Furthermore, denote by $V$ the span of $\{e, f\}$ in $H_2(M')$.

**Lemma 6.** *With the above notation, we have*

$$\lambda_{M'}|_V \cong \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix}$$

*where* $\lambda_{M'}(f,f) = a \in Z$.

*Proof:* From the diagram, it follows that

$$\lambda_{M'}(\partial'_*(x), y) = \lambda_W(x, j_*(y)) \tag{1}$$

for every $x \in H_3(W, M')$ and $y \in H_2(M')$.

Note that

$$\partial'_*[D^2 \times D^3, D^2 \times S^2] = m i'_*(e) = me$$

and

$$j_*(f) = m[D^2 \times D^3, S^1 \times D^3],$$

where $[\,,\,]$ denotes the fundamental class.

Thus relation (1) implies

$$\begin{aligned}\lambda_{M'}(me, f) &= \lambda_{M'}(\partial'_*[D^2 \times D^3, D^2 \times S^2], f)\\ &= \lambda_W([D^2 \times D^3, D^2 \times S^2], j_*(f))\\ &= m\lambda_W([D^2 \times D^3, D^2 \times S^2], [D^2 \times D^3, S^1 \times D^3]) = m,\end{aligned}$$

hence $\lambda_{M'}(e, f) = 1$ as required.

Furthermore, we have

$$\begin{aligned}m^2\lambda_{M'}(e,e) &= \lambda_{M'}(me, me)\\ &= \lambda_{M'}(\partial'_*[D^2 \times D^3, D^2 \times S^2], \partial'_*[D^2 \times D^3, D^2 \times S^2])\\ &= \lambda_W([D^2 \times D^3, D^2 \times S^2], j_* \circ \partial'_*[D^2 \times D^3, D^2 \times S^2]) = 0\end{aligned}$$

since $j_* \circ \partial'_* = 0$ by the exactness. Thus $\lambda_{M'}(e,e) = 0$ and the proof of Lemma 6 is completed. ■

**Lemma 7.** *Let* $V^\perp \subset H_2(M')$ *be the orthogonal complement of* $V$. *Then* $V^\perp \subset H_2(M_0)$ *and the restriction*

$$i_*|_{V^\perp} : V^\perp \longrightarrow FH_2(M)$$

*is an isomorphism.*

*Proof:* To prove that $V^\perp \subset H_2(M_0)$, we have to show that for every $y \in H_2(M')$ with

$$\lambda_{M'}(y, e) = \lambda_{M'}(y, f) = 0,$$

then $y \in H_2(M_0)$ , i. e. $j_*(y) = 0$ .

Suppose, on the contrary, $j_*(y) \neq 0$, i. e. $j_*(y) = q[D^2 \times D^3, S^1 \times D^3]$ for some integer $q \neq 0$. Then we have

$$\begin{aligned}\lambda_{M'}(me, y) &= \lambda_{M'}(\partial'_*[D^2 \times D^3, D^2 \times S^2], y)\\ &= \lambda_W([D^2 \times D^3, D^2 \times S^2], j_*(y))\\ &= q\lambda_W([D^2 \times D^3, D^2 \times S^2], [D^2 \times D^3, S^1 \times D^3]) = q \neq 0,\end{aligned}$$

hence $\lambda_{M'}(e, y) \neq 0$, which is a contradiction.

To prove that $i_*|_{V^\perp}$ is mono, let $x \in V^\perp$ be an element such that $i_*(x) \in \mathrm{Tor} H_2(M) \cong Z_m$.

Then we have $i_*(x) = hi_*(e)$ for some integer $h$, and so $i_*(x - he) = 0$.

By the exactness, it follows that

$$\partial'_*(h'[D^2 \times D^3, D^2 \times S^2]) = i'_*(x - he),$$

hence $mh'e = x - he, h, h' \in Z$.

But we have (use (1))

(2)
$$\begin{aligned}\lambda_{M'}(\partial'_*(h'[D^2 \times D^3, D^2 \times S^2]), f) &= \lambda_W(h'[D^2 \times D^3, D^2 \times S^2], j_*(f))\\ &= \lambda_W(h'[D^2 \times D^3, D^2 \times S^2], m[D^2 \times D^3, S^1 \times D^3]) = mh'\end{aligned}$$

and

(3)
$$\begin{aligned}\lambda_{M'}(\partial'_*(h'[D^2 \times D^3, D^2 \times S^2]), f) &= \lambda_{M'}(i'_*(x - he), f)\\ &= \lambda_{M'}(x - he, f)\\ &= \lambda_{M'}(x, f) - h\lambda_{M'}(e, f) = -h.\end{aligned}$$

Comparing relations (2) and (3) gives $mh' = -h$, hence $mh'e = x - he$ implies that $x = 0$ as required.

To prove that $i_*|_{V^\perp}$ is epi, let $z \in H_2(M)$ and let $u \in H_2(M_0)$ be an element such that $i_*(u) = z$.

We consider the element $u' = u - \lambda_{M'}(u, f)e \in H_2(M_0)$.

Then we have

$$\begin{aligned}\lambda_{M'}(me, u') &= \lambda_{M'}(\partial'_*[D^2 \times D^3, D^2 \times S^2], u')\\ &= \lambda_W([D^2 \times D^3, D^2 \times S^2], j_* \circ i'_*(u')) = 0\end{aligned}$$

since $j_* \circ i'_* = 0$; therefore $\lambda_{M'}(u', e) = 0$.

Furthermore

$$\begin{aligned}\lambda_{M'}(u', f) &= \lambda_{M'}(u - \lambda_{M'}(u, f)e, f)\\ &= \lambda_{M'}(u, f) - \lambda_{M'}(u, f) = 0,\end{aligned}$$

i. e. $u' \equiv i'_*(u') \in V^\perp$.

Finally

$$\begin{aligned}i_*(u') &= i_*(u) - \lambda_{M'}(u, f) i_*(e)\\ &= i_*(u) = z \bmod \mathrm{Tor} H_2(M).\end{aligned}$$

This completes the proof. ■

By Lemmas 6 and 7, we have the result

$$\lambda_{M'} \cong \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix}.$$

*Proof of Proposition 4:*

Suppose now $w_2(M) \neq 0$. Because $(M, M_0)$ and $(M', M_0)$ are homology equivalent to $(S^1 \times D^3, S^1 \times S^2)$ and $(D^2 \times S^2, S^1 \times S^2)$ respectively, we have also the diagram

$$\begin{tikzcd} 0 & & \\ H^2(M_0; Z_2) \arrow[u] & H^2(M; Z_2) \arrow[l, "i^*"'] & 0 \arrow[l] \\ H^2(M'; Z_2) \arrow[u, "i'^*"'] & & \end{tikzcd}$$

which implies

$$i^*(w_2(M)) = w_2(M_0) = i'^*(w_2(M')). \tag{4}$$

Since $i^*$ is injective, relation (4) and $w_2(M) \neq 0$ give $w_2(M') \neq 0$, hence $\lambda_{M'}$ is odd. ■

**Remark 1.** The proof of proposition 2 can be easily generalized to manifolds with arbitrary fundamental groups. Indeed, this follows from Lemma 8 below.

Suppose now $M$ a closed connected orientable (PL) 4-manifold with fundamental group $\Pi_1$.

Let

$$\psi_1, \psi_2, \ldots, \psi_p : S^1 \times D^3 \longrightarrow M$$

be disjoint embeddings which kill $\Pi_1$.

Setting

$$M_0 = M \setminus \bigcup_{j=1}^{p} \psi_j(S^1 \times \overset{0}{D}{}^3)$$

and

$$M^* = M_0 \cup \bigcup_{j=1}^{p} (D^2 \times S^2),$$

we have

**Lemma 8.**

(1) $H_1(M_0) \cong H_1(M)$, $H_3(M_0) \cong \bigoplus_{p-1} Z$

(2) $H_2(M_0)$ *is a direct summand of the free group* $H_2(M^*)$

(3)

$$0 \longrightarrow H_2(M_0) \longrightarrow H_2(M^*) \longrightarrow H_2(M^*, M_0) \cong \bigoplus_p Z \longrightarrow$$
$$\longrightarrow H_1(M_0) \cong H_1(M) \longrightarrow 0$$

(4)

$$0 \longrightarrow H_3(M) \longrightarrow H_3(M, M_0) \cong \bigoplus_p Z \longrightarrow H_2(M_0) \longrightarrow H_2(M) \longrightarrow 0$$

(5)

$$H_2(M) \cong H_2(M_0) \cong H_2(M^*) \Longleftrightarrow$$
$$\Longleftrightarrow H_1(M) \cong H_3(M) \cong H_3(M, M_0) \cong \bigoplus_p Z.$$

The proof is straightforward.

Now we indicate how Lemma 8 yields Proposition 2 in the general case.

Suppose $\Pi_1(M)$ finitely generated by elements of finite orders, hence

$H_1(M) = Z_{m_1} \oplus \cdots \oplus Z_{m_p}$. Since $H_3(M) \simeq H^1(M) \simeq FH_1(M) \simeq 0$, by Lemma 8 we have the same diagram as at the beginning of section 4 with

$$H_3(M, M_0) \simeq H_3(W, M^*) \simeq \oplus_p Z,$$
$$H_2(M^*, M_0) \simeq H_2(W, M) \simeq \oplus_p Z$$

and

$$H_1(M) \simeq H_1(M_0) \simeq Z_{m_1} \oplus \cdots \oplus Z_{m_p}.$$

Observe that $H_2(M^*)$ is a free group of rank $rkH_2(M) + 2p$ and $H_2(M_0)$ is free of rank $rkH_2(M) + p$.

Now we can choose primitive elements

$$e_1, e_2, \ldots, e_p \in H_2(M_0) \text{ and } f_1, f_2, \ldots, f_p \in H_2(M^*)$$

such that $i_*(e_h)$ generates the subgroup $Z_{m_h} \subset TorH_2(M)$ and $f_h$ maps to the integer $m_h$ which belongs to the corresponding $h^{st}$ factor of $H_2(M^*, M_0)$, for $h = 1, 2, \ldots, p$.

Now we apply the previous results by replacing $V$ with the span $V_h$ of $\{e_h, f_h\}$. As a consequence we also obtain

$$\lambda_{M^*} = \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & a_1 \end{pmatrix} \oplus \cdots \oplus \begin{pmatrix} 0 & 1 \\ 1 & a_p \end{pmatrix}.$$

**Remark 2.** Let $M$ be a closed connected orientable spin 4-manifold with $\Pi_1(M)$ finite.

Let $\psi : S^1 \times D^3 \longrightarrow M$ be an embedding which represents a generator $[\alpha] \in \Pi_1(M)$.

Then

$$\lambda_{M'} \cong \lambda_M \oplus \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix}$$

by proposition 2 and $a$ defines a map

$$\tilde{a} : \widetilde{\Pi_1(M)} \longrightarrow Z_2$$

where $\widetilde{\Pi_1(M)}$ is a certain extension of $\Pi_1(M)$ by $Z_2$ which takes care not only of $[\alpha]$ but also of its extension $\psi$ (for details see [**10**, p. 44]). What type of invariant is $\tilde{a}$ ? : the examples show that $\tilde{a}$ is not trivial.

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