Risk and Return in Bond, Currency and Equity Markets

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Abstract

We develop a general equilibrium long-run risks model that can simultaneously account for key asset price puzzles in bond, currency and equity markets. Specifically, we show that the model can explain the predictability of returns and violations of the expectations hypothesis in bond and foreign exchange markets. It also accounts for the levels and volatilities of bond yields and exchange rates, and the well-known risk premium and volatility puzzles in equity markets. The model matches the observed consumption and inflation dynamics. Using domestic and foreign consumption and asset markets data we provide robust empirical support for our models predictions. We argue that key economic channels featured in the long-run risks model — long-run growth fluctuations and time-varying uncertainty, along with a preference for early resolution of uncertainty — provide a coherent framework to simultaneously explain a rich array of asset market puzzles.

1 Introduction

An extensive list of financial puzzles commonly includes the level and volatilities of the nominal yields, failure of the expectations hypothesis in bond and foreign exchange markets (see Fama, 1984; Campbell and Shiller, 1991), exchange rate volatility, and the risk premium and stock-price volatility puzzles in equity markets. Absence of arbitrage opportunities implies that concerns about the same fundamental sources of risks should simultaneously account for prices of assets in all financial markets. In this paper we present a unified long-run risks framework that provides an integrated explanation of bond, currency, and equity market anomalies.

Specifically, we rely on the long-run risks model of Bansal and Yaron (2004), who largely focus on the equity premium and risk free rate puzzles. The key ingredients of the model is a small and persistent low-frequency component in consumption growth rate and a time-varying volatility of consumption growth. To price claims in nominal terms, we also model the inflation process for each country. The preferences of the representative agent are characterized by a non-expected recursive utility considered by Kreps and Porteus (1978), in a convenient parametrization of Epstein and Zin (1989). These preferences allow for a separation between risk aversion and intertemporal elasticity of substitution (IES) of investors; in our specification, agents have preference for early resolution of uncertainty, which increases the risk compensation for long-run growth and uncertainty risks.

In bond markets, an increase in the differential between the long rate and the short rate forecasts higher expected returns. Indeed, empirical evidence in US and foreign countries typically finds a significant drop in long rates following periods of high yield spreads. This result contradicts the expectations model of the yield curve, where an increase in the long-short spread would forecast a rise in long yields. Campbell and Shiller (1991) show that these violations become more dramatic for longer maturity bonds. More recent work in Cochrane and Piazzesi (2005) shows that a single bond factor constructed from a linear combination of three to five forward rates can sharply forecast future bond returns. We will refer to this as the bond predictability puzzle. In currency markets, Fama (1984), Backus, Foresi, and Telmer (2001) and Bansal (1997) show that uncovered interest rate parity is violated. The interest rate differential across countries forecasts future exchange rate changes — in particular, a rise in the domestic nominal rate forecasts an appreciation of the domestic currency. Fama (1984) shows that this forward-premium puzzle implies that the variability of the risk premium in currency markets is larger than that of the expected changes in foreign exchange rates. More recent work by Alexius (2001) and Chinn and Meredith (2004) show that these violations become less pronounced at longer maturities; in our calibrations, slopes in the foreign exchange projections become less negative with horizon, and the nominal one turns positive within 2 years. What economic mechanisms can account for all these puzzles? We show that preference for early resolution of uncertainty and the time-varying volatility channel in the long-run risks model are critical to account for these puzzles. That is, time-variation in economic uncertainty and expected growth generates enough persistence and variability in the risk premia in bond and currency markets to rationalize the violations of the expectations hypotheses in the data.

While the main focus of this paper is on bond and currency markets anomalies, we show that the model is quantitatively consistent with a richer set of key asset market facts. Using numerical calibrations, we show that the model can account for a high volatility of exchange rates — a dimension also explored in Colacito and Croce (2005), and for the market premium and asset-price and volatility puzzles in equity markets, as in Bansal and Yaron (2004). Further, we show that the co-movements of consumption volatilities with the bond and currency prices across countries are consistent with theoretical predictions of our model. We find in the data that the volatility of domestic consumption growth correlates negatively with the dollar price of foreign currencies and forward premia, and positively with the expected returns on foreign bonds for all the countries in our analysis. This evidence provides a considerable support for economic channels highlighted in this paper.

To the best of our knowledge, this is the first paper to provide a quantitative, unified general equilibrium framework that can simultaneously address the (1) failure of expectations hypothesis and level and volatility of yields in bond markets, (2) violation of the uncovered interest rate parity condition and foreign exchange volatility puzzle in currency markets, (3) risk premium and risk-free rate puzzles in equity markets. Eraker (2006), Piazzesi and Schneider (2005) use the long-run risks setup to analyze the unconditional moments of real and nominal term structure, but do not deal with predictability and violations of expectations hypothesis issues in bond markets. Colacito and Croce

(2005) use the long-run risks model to reconcile the exchange rate volatility with a low consumption correlation across countries. In the context of habit formation model of Campbell and Cochrane (1999), Wachter (2006) provides a consumption-based explanation for the violations of expectations hypothesis, while Verdelhan (2005) addresses the failure of the uncovered interest rate parity condition. Alvarez, Atkeson, and Kehoe (2006) set up a general equilibrium monetary model which generates time-varying risk premium through the endogenous asset market segmentation. They show that limited and changing participation of agents in financial markets can account for forward premium anomaly at short and long horizons; however, the authors do not explore the implications for equity markets. Lustig and Verdelhan (2007) construct portfolios of foreign currency returns sorted on the basis of foreign interest rates and test whether their consumption-based model can account for the expected return on these portfolios.

Earlier works include the general-equilibrium analysis of bond risk premium and violations of expectations hypothesis by Backus, Gregory, and Zin (1989), the term structure relations in a two-good setup featuring nonseparable preferences in Dunn and Singleton (1986), structural models of foreign exchange rates of Bansal, Gallant, Hussey, and Tauchen (1995), Bekaert (1996) and Backus, Gregory, and Telmer (1993).[1]

The rest of the paper is organized as follows. In the next section we document the violations of the expectations hypothesis in bond and currency markets. In Section 3 we setup the long-run risks model. We present the solution to the model and discuss its theoretical implications for bond, currency and equity markets in Section 4. Section 5 describes the data and calibration of the real and nominal economy and preference parameters. Model implications for bond, currency and equity markets at the calibrated parameter values are addressed in Section 6. Section 7 discusses the extension of the model to the two-volatility structure. Conclusion follows.

[1] Recent works in no-arbitrage and statistical literature include affine models of Dai and Singleton (2002), regime-switching models of Bansal and Zhou (2002), Bansal, Tauchen, and Zhou (2004) and Dai, Singleton, and Yang (2006), and macro-finance specifications of the term structure by Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2005), Rudebusch and Wu (2004) and Bikbov and Chernov (2006).

2 Predictability Puzzles and Evidence

A standard benchmark for the analysis of returns on bonds is provided by the expectations hypothesis. It states that in domestic bond markets, a high long-short yield spread today is offset by an anticipated loss on long maturity bonds in the future, and therefore should forecast an increase in the long rates. In foreign exchange context, low risk-free rates at home are compensated by the future appreciation of dollar and therefore should predict expected depreciation of the foreign currency. These conclusions formally obtain in structural models when the expected excess returns are constant, e.g., when investors are risk-neutral or economic uncertainty is constant.

As discussed below, none of these implications of the expectations hypothesis are supported by the data; in fact, the signs in predictability regressions are exactly the opposite. Reduced-form empirical projections imply that a high yield spread forecasts a drop in future long rates, and the regression coefficients become more negative with maturity. Likewise, low forward premium predicts appreciation of the foreign currency, though, the violations are less severe in longer horizon. Therefore, the forecasts of the change in future bond and currency prices based on expectations hypothesis or empirical projections will be radically different, both in terms of their magnitude and sign. The violations of these predictions in the data pose a challenge to the economic understanding of the asset markets and seriously question the constant (zero) expected excess return assumptions used to justify the expectations hypothesis model.

The economic principle of no-arbitrage across bond, currency and equity markets implies that the expected return in all these markets should be explained by common economic risk channels. In the context of long-run risks model, we show that these channels can successfully account for the predictability puzzles in bond and currency markets.

In the next two sub-sections we establish the notations and document the key empirical findings on predictability of domestic and foreign bond returns.

2.1 Bond Market Puzzles

Denote $y_{t,n}$ the yield on the real discount bond with n months to maturity. Then, we can write the excess log return on buying an n months bond at time t and selling it at time $t+m$ as an $n-m$ period bond as

$$rx_{t+m,n} = ny_{t,n} - (n-m)y_{t+m,n-m} - my_{t,m}. \tag{1}$$

Variables with a dollar superscript will refer to nominal quantities, such as nominal risk-free rate $y^{\$}_{t,1}$. To avoid clustering of superscripts, we lay out the discussion using real variables; same arguments apply for the nominal economy as well.

Under the expectations hypothesis, the expected excess bond returns are constant. This implies that the slope coefficient $\beta_{n,m}$ in bond regressions

$$y_{t+m,n-m} - y_{t,n} = const + \beta_{n,m}\frac{m}{n-m}(y_{t,n} - y_{t,m}) + error \tag{2}$$

should be equal to one at all maturities n and time steps m. Indeed, with rational expectations, the population value for the slope coefficient is given by

$$\beta_{n,m} = 1 - \frac{Cov(E_t rx_{t+m,n}, y_{t,n} - y_{t,m})}{mVar(y_{t,n} - y_{t,m})}. \tag{3}$$

If term-spread $y_{t,n} - y_{t,m}$ contains no information about expected excess bond returns $E_t rx_{t+m,n}$, e.g. expected excess returns are constant under the expectations hypothesis, then the slope is equal to unity. Alternatively, high long-short spread should predict a proportional decline in future bond prices, which eliminates the yield advantage to long-term bonds by expected capital loss.

In the data, however, the regression coefficients in bond projections (2) are negative and increasing in absolute value with horizon (Campbell and Shiller, 1991). In the second panel of Table 2, we tabulate the projection coefficients for nominal and inflation-adjusted bond yields in US and UK. Consistent with previous studies, virtually all of the slope coefficients are negative, and they are increasing in absolute value with maturity for all except UK nominal series. The standard errors of the estimates, however, are quite large. This empirical evidence suggests that, contrary to the predictions from the expectations hypothesis, high long-short yield spread forecasts an increase in future prices. That is, expected excess bond returns are time-varying and predictable by the term-spread with a positive sign. Dai and Singleton (2002) provide further discussion of the violations of the expectations hypothesis and bond return risk premia in context of affine models of the term structure.

Forward rate projections provide additional evidence for the time-variation in bond risk premia. We follow Cochrane and Piazzesi (2005) and regress the average of $m-$period excess returns on bonds of different maturities on the forward rates over equally spaced short, middle and long horizon. The fitted values $\widehat{rx}_{t,m}$ from these regressions are then used as a single bond factor in projections

$$rx_{t+m,n} = const + b_{m,n}\widehat{rx}_{t,m} + error. \tag{4}$$

Cochrane and Piazzesi (2005) show that the estimates $b_{m,n}$ are positive and increasing with horizon, and a single factor projection captures $20-30\%$ of the variation in bond returns. We demonstrate these results for US and UK bond markets in Table 4.

2.2 Currency Market Puzzles

Let s_t stand for a real spot exchange rate, in logs, per unit of foreign currency (dollars spot price of one pound), and denote by f_t^{FX} the logarithm of the foreign exchange forward rate, i.e. current dollar price of a contract to deliver one pound tomorrow. Superscript * will denote the corresponding variable in the second country, e.g. $y^*_{t,1}$ stands for the foreign risk-free rate. To avoid clustering of superscripts, we present the discussion in real terms.

A one-period excess dollar return in foreign bonds is given by

$$rx^{FX}_{t+1} = s_{t+1} - s_t + y^*_{t,1} - y_{t,1}. \tag{5}$$

This corresponds to an excess return on buying foreign currency today, investing the money into the foreign risk-free asset and converting the proceeds back using the spot rate next period.

Under the expectations hypothesis in currency markets, the excess returns are constant. Therefore, the slope coefficient in the projection

$$s_{t+1} - s_t = const + \beta^{UIP}(y_{t,1} - y^*_{t,1}) + error. \tag{6}$$

should be equal to one. Indeed, with rational expectations, the population value for the regression coefficient can be written as,

$$\beta^{UIP} = 1 + \frac{Cov(E_t rx^{FX}_{t+1}, y_{t,1} - y^*_{t,1})}{Var(y_{t,1} - y^*_{t,1})}. \tag{7}$$

Therefore, if the interest gap (forward premium) $y_{t,1} - y^*_{t,1}$ contains no information about the foreign bond risk premium $E_t rx^{FX}_{t+1}$, e.g. the latter is constant under the expectations hypothesis, the projection coefficient is unity. Alternatively, if the uncovered interest rate parity condition holds, high interest rate bearing countries are expected to experience a proportional depreciation of their currency.

Fama (1984), Hodrick (1987), Backus et al. (2001), Bansal and Dahlquist (2000) and many other studies show that at short maturities, the regression coefficient in foreign exchange projection (6) is negative and statistically significant. In the second panel in Table 3, we document these findings for UK, Germany and Japan in nominal and inflation-adjusted quantities for an investment horizon of 1 month. To focus on the return dimension, we subtract the forward premium from the both sides of (6). As investment horizon gets larger, the violations of the UIP condition in the data are less severe. As shown by Chinn and Meredith (2004) and Alexius (2001), at multiple year maturities the slope coefficient turns positive but remains below one. We confirm these findings for nominal bonds in US and UK for 2 to 5 years to maturity: in monthly regressions from January 1988 to December 2005, the slope coefficient is positive at 2 year horizon, 0.12 (0.98), and is very close to one for longer investment horizons. [2]

To get additional insight into the violations of expectations hypothesis in currency markets, Fama (1984) decomposes the difference between the forward and spot price of the currency into the risk premium part and the expected depreciation of the exchange

[2] Baillie and Bollerslev (2000) and Maynard and Phillips (2001) argue that the principal failure of the uncovered interest rate parity hypothesis could be statistical in nature, caused by the difference in persistence of exchange rates and forward premium. Maynard (2006) revisits the arguments using the robust econometric methods and finds that while the statistical theory could play the role in the anomaly, the successful resolution of the puzzle belongs to economics.

rate:

$$f_t^{FX} - s_t = (f_t^{FX} - E_t s_{t+1}) + (E_t s_{t+1} - s_t) \\ \equiv (-E_t rx_{t+1}^{FX}) + (E_t s_{t+1} - s_t). \tag{8}$$

As can be seen from the expression (7), to explain a negative slope in uncovered interest parity regressions, an asset pricing model should deliver a negative covariance between the (negative of) foreign bond risk premium, $-E_t rx_{t+1}^{FX}$, and expected depreciation of the currency, $E_t s_{t+1} - s_t$, and also a greater variance of the risk premium than that of the expected depreciation.

3 Long-Run Risks Model

3.1 Preferences and Real Economy

We consider a discrete-time real endowment economy developed in Bansal and Yaron (2004). The investors preferences over the uncertain consumption stream C_t can be described by the Kreps-Porteus, Epstein-Zin recursive utility function, (see Epstein and Zin, 1989; Kreps and Porteus, 1978):

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}. \tag{9}$$

The time discount factor is δ, $\gamma \geq 0$ is the risk-aversion parameter and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES). Parameter θ is defined $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Its sign is determined by the magnitudes of the risk aversion and the elasticity of substitution, so that if $\psi > 1$ and $\gamma > 1$, then θ will be negative. Note that when $\theta = 1$, that is, $\gamma = 1/\psi$, the above recursive preferences collapse to the standard case of expected utility. As is pointed out in Epstein and Zin (1989), in this case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk-version exceeds (is less than) the reciprocal of IES the agent prefers early (late) resolution of uncertainty of consumption path. Hence, these preferences allow for a departure on the agent's preference for the timing of the resolution of uncertainty. In the long-run risk model agents prefer early resolution of uncertainty of the consumption path.

As shown in Epstein and Zin (1989), the logarithm of the Intertemporal Marginal Rate of Substitution (IMRS) for these preferences is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{10}$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the log growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends each time period. This return is not observable in the data. It is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. Therefore, we assume exogenous process for consumption growth and use a standard asset pricing restriction

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1 \tag{11}$$

which holds for any continuous return $r_{t+1} = \log(R_{t+1})$, including the one on the wealth portfolio, to solve for the unobserved wealth-to-consumption ratio in the model.

Following Bansal and Yaron (2004), we assume that the real consumption growth contains a small and persistent long-run expected growth component and the time-varying volatility (economic uncertainty). The consumption dynamics is thus the following:

$$\begin{aligned} \Delta c_{t+1} &= \mu_g + x_t + \sigma_{gt} \eta_{t+1}, && (12) \\ x_{t+1} &= \rho x_t + \varphi_e \sigma_{gt} e_{t+1}, && (13) \\ \sigma^2_{g,t+1} &= \sigma^2_g + \nu_g (\sigma^2_{gt} - \sigma^2_g) + \sigma_{gw} w_{g,t+1}. && (14) \end{aligned}$$

The unconditional mean of the time-varying variance of consumption growth is σ^2_g. The variance of the long-run component in expected growth and consumption volatility is determined by φ_e and σ_{gw}, respectively. The parameters ρ and ν_g control the persistence of shocks to expected growth and consumption volatility. For analytical tractability, we assume that all the innovations are Gaussian and independent from each other. Hansen, Heaton, and Li (2006) consider a similar long-run risks model with predictable variation in expected growth, constant volatility and agent's preferences as in (9) with an intertemporal elasticity of substitution near one.

3.2 Nominal Economy

The economic channels in the real economy are sufficient to explain the violations of the expectations hypotheses and predictability of returns in bond and currency markets. However, most of the asset markets data is in nominal terms, and data on real bonds is not observable. To make our model-data closer to observed data, we pursue the simplest strategy of directly modeling inflation, which allows us to derive asset prices in nominal terms. A similar approach is pursued by Wachter (2006) and Piazzesi and Schneider (2005).

In particular, we assume that the inflation process follows

$$\pi_{t+1} \;=\; \bar{\pi}_t + \varphi_{\pi g}\sigma_{gt}\eta_{t+1} + \varphi_{\pi x}\varphi_e\sigma_{gt}e_{t+1} + \sigma_\pi\xi_{t+1}, \tag{15}$$

where the expected inflation, $\bar{\pi}_t \equiv E_t\pi_{t+1}$, is given by

$$\bar{\pi}_{t+1} \;=\; \mu_\pi + \alpha_\pi(\bar{\pi}_t - \mu_\pi) + \alpha_x x_t + \varphi_{zg}\sigma_{gt}\eta_{t+1} + \varphi_{zx}\varphi_e\sigma_{gt}e_{t+1} + \sigma_z\xi_{t+1}. \tag{16}$$

To maintain parsimony, we assume that the inflation shock ξ is homoscedastic and affects both the inflation rate and its conditional mean; extensions to time-varying volatility and separate shock structure are straightforward. Parameters $\varphi_{\pi g}, \varphi_{zg}$ and $\varphi_{\pi x}, \varphi_{zx}$ measure the sensitivity ("beta") of realized and expected inflation innovations to short and long-run consumption news. Thus, the conditional variance of realized and expected inflation is time-varying and proportional to that of consumption growth, σ_{gt}^2.

Our specification of the expected consumption and inflation growth rates is similar to that of Piazzesi and Schneider (2005). For parsimony, we assume that the inflation process has no effect on the real economy, while the real consumption growth, in particular, its expected growth, affects future expectations of inflation rate. We discuss the plausibility of this specification in the empirical section of the paper.

Given the real discount factor m_{t+1} and inflation process π_{t+1}, the logarithm of nominal pricing kernel is given by

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}. \tag{17}$$

3.3 Two-Country Setup

We extend our model to two-country setup—a similar specification is also used in Colacito and Croce (2005). In particular, we assume that the endowments are country specific, and the agents derive utility only from consumption of domestic goods. Financial markets are frictionless and open for domestic and foreign investment and equilibrium real and nominal exchange rates adjust exactly to offset all the net payoffs and preclude arbitrage. [3] This is similar to setup considered by Backus et al. (2001).

For simplicity, we impose complete symmetry and equate model and preference parameters across the two countries. Only the states and innovations are allowed to be country-specific; those for the foreign country are indexed by a superscript *. The correlation structure of the innovations is summarized by

$$\begin{aligned} Cov(e, e^*) &= \tau_e, \qquad Corr(\eta, \eta^*) = \tau_\eta, \\ Corr(w_g, w_g^*) &= \tau_{wg}, \quad Corr(\xi, \xi^*) = \tau_\xi. \end{aligned}$$

The discount factor used to price assets denominated in foreign currency is given by

$$m^*_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c^*_{t+1} + (\theta - 1) r^*_{c,t+1}, \tag{18}$$

where g^*_{t+1} is the log growth rate of foreign endowment growth, $r^*_{c,t+1}$ is the log return on foreign consumption portfolio, and δ, γ and ψ are the preference parameters of the representative agents at home and abroad.

4 Asset Markets

4.1 Real Marginal Rate of Substitution

The key ideas of the model rely on solutions which are derived using the standard log-linearization of returns. In particular, the log-linearized return on consumption claim is

[3] Refer to Colacito (2006) for a formal discussion on the existence of the equilibrium solution and the exchange rate in a limiting case when representative consumer's utility weight on foreign goods approaches zero.

given by

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{19}$$

where $z_t \equiv \log(P_t/C_t)$ is the log price-to-consumption ratio. Parameters κ_0 and κ_1 are approximating constants which are based on the endogenous average price-to-consumption ratio in the economy.

The approximate solution for the price-consumption ratio is linear in states, $z_t = A_0 + A_x x_t + A_{gs}(\sigma_{gt}^2 - \sigma_g^2)$. From the Euler condition (10) and the assumed dynamics of consumption growth, the solutions for A_x and A_{gs} satisfy

$$A_x = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}, \tag{20}$$

$$A_{gs} = \frac{1}{2}\frac{(1-\gamma)(1-\frac{1}{\psi})}{1-\kappa_1\nu_g}\left(1+\left[\frac{\varphi_e\kappa_1}{1-\kappa_1\rho}\right]^2\right). \tag{21}$$

It follows that A_x is positive if the IES, ψ, is greater than one. In this case the intertemporal substitution effect dominates the wealth effect. In response to higher expected growth, agents buy more assets, and consequently the wealth to consumption ratio rises. In the standard power utility model with risk aversion larger than one, the IES is less than one, and hence A_x is negative — a rise in expected growth potentially lowers asset valuations. That is, the wealth effect dominates the substitution effect.

Coefficient A_{gs} measures the sensitivity of price-consumption ratio to volatility fluctuations. If the IES and risk aversion are larger than one, then A_{gs} is negative. In this case a rise in consumption volatility lowers asset valuations and increases the risk premia on all assets. An increase in the permanence of volatility shocks, that is ν_g, magnifies the effects of volatility shocks on valuation ratios as changes in economic uncertainty are perceived by investors as being long lasting. Similarly, A_{gs} increases in absolute value with the persistence of expected consumption growth ρ, as the effects of volatility shocks are magnified as they feed in through the expected growth channel x_t.

Using the approximate solutions for the price-consumption ratio, we can provide an analytical expression for the marginal rate of substitution in (10). The log of the IMRS m_{t+1} can always be decomposed to its conditional mean and innovation. The former is affine in expected mean and variance of consumption growth and can be expressed as

$$E_t m_{t+1} \quad = \quad \mu_m - \frac{1}{\psi} x_t + \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[1 + \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right] (\sigma_{gt}^2 - \sigma_g^2) \tag{22}$$

for some constant μ_m.

The innovation in the IMRS is very important for thinking about risk compensation (risk premia) in various markets. Specifically, it is equal to

$$m_{t+1} - E_t m_{t+1} = -\lambda_\eta \sigma_{gt} \eta_{t+1} - \lambda_e \varphi_e \sigma_{gt} e_{t+1} - \lambda_{gw} \sigma_{gw} w_{g,t+1}. \tag{23}$$

Parameters λ_η, λ_e, and λ_{gw} are the market price of risks for the short-run, long-run, and volatility risks. The market prices of systematic risks, including the compensation for stochastic volatility risk in consumption, can be expressed in terms of the underlying preferences and parameters that govern the evolution of consumption growth:

$$\begin{aligned} \lambda_\eta &= \gamma \\ \lambda_e &= (\gamma - \frac{1}{\psi}) \left(\frac{\kappa_1}{1 - \kappa_1 \rho} \right) \\ \lambda_{gw} &= \frac{1}{2} (\gamma - \frac{1}{\psi})(1 - \gamma) \frac{\kappa_1}{1 - \kappa_1 \nu_g} \left(1 + \left[\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right]^2 \right) \end{aligned} \tag{24}$$

In the special case of power utility, $\theta = 1$ or more specifically, $\gamma = \frac{1}{\psi}$, the risk compensation parameters λ_e and λ_{gw} are zero, and the IMRS collapses to the standard power utility specification

$$m_{t+1}^{CRRA} = \log \delta - \gamma \Delta c_{t+1}. \tag{25}$$

With power utility there is no separate risk compensation for long-run growth rate risks and volatility risks — with generalized preferences both risks are priced. The pricing of long-run and volatility risks is an important feature of the long-run risks model. Specifically, at the calibrated parameter values, consumption volatility shocks, $w_{g,t+1}$, together with the innovations into the expected consumption, e_{t+1}, are the most important sources of risks in the economy, as measured by their contribution to the maximal Sharpe ratio in the economy (conditional variance of the discount factor).

The logarithm of nominal pricing kernel can be obtained from the condition (17).

In particular, nominal prices of immediate and long-run consumption risks depend on the inflation betas to short and long-run consumption news. Real and nominal discount factors in foreign country have analogous solutions.

4.2 Bond Returns

The equilibrium real and nominal yields are affine in the state variables:

$$
\begin{aligned}
y_{t,n} &= \frac{1}{n}\begin{bmatrix} B_{0,n} & B_{x,n} & B_{gs,n} \end{bmatrix}\begin{bmatrix} 1 & x_t & \sigma_{gt}^2-\sigma_g^2 \end{bmatrix}', && (26)\\
y_{t,n}^{\$} &= \frac{1}{n}\begin{bmatrix} B_{0,n}^{\$} & B_{x,n}^{\$} & B_{gs,n}^{\$} & B_{\pi,n}^{\$} \end{bmatrix}\begin{bmatrix} 1 & x_t & \sigma_{gt}^2-\sigma_g^2 & \bar{\pi}_t-\mu_\pi \end{bmatrix}', && (27)
\end{aligned}
$$

where the loadings satisfy the recursions given in Appendix B.

In particular, the solution to one-period real risk-free rate is given by,

$$
y_{t,1} = const + \frac{1}{\psi}x_t - \frac{1}{2}\left(\lambda_\eta^2 + \varphi_e^2\lambda_e^2 + (\frac{1}{\psi}-\gamma)(\gamma-1)\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right]\right)\sigma_t^2. \quad (28)
$$

For $\gamma > 1$ and $\psi > 1$, the bond loading on consumption volatility is negative, so that consumption uncertainty increases the demand for safe assets and thus drives the yields down. This is also true with power utility, when $\gamma = \frac{1}{\psi}$. However, with generalized preferences, the magnitude of this effect depends on both the risk aversion and the IES, and the persistence of the expected growth factor.

The bond risk premium is given by the negative of the covariation of bond returns with the discount factor, safe for the Jensen's inequality adjustment. For example, one period excess return on real bond with n months to maturity can be rewritten in the following form:

$$
\begin{aligned}
E_t(rx_{t+1,n}) + \frac{1}{2}Var_t(rx_{t+1,n}) &= -Cov_t(m_{t+1}, rx_{t+1,n})\\
&= -B_{gs,n-1}\lambda_{gw}\sigma_{gw}^2 - B_{x,n-1}\lambda_e\varphi_e^2\sigma_{gt}^2.
\end{aligned} \quad (29)
$$

Notably, the risk premium on holding period bond returns reflects the compensation for long-run and volatility risks — with power utility, these two sources of risks are not priced, as λ_{gw} and λ_e are all zero. When $\gamma > 1/\psi$, the price of long-run risks λ_e is positive. As the solution to $B_{x,n}$ is positive as well, the expected excess return on real bonds falls in response to a positive shock to consumption uncertainty.

Now the real term-spread is given by,

$$y_{t,n} - y_{t,1} = const + \left(\frac{1}{n}B_{x,n} - B_{x,1}\right) x_t + \left(\frac{1}{n}B_{gs,n} - B_{gs,1}\right) \sigma_{gt}^2.$$

When long yields are more sensitive to consumption variance than short yields, that is, $\frac{1}{n}B_{gs,n} < B_{gs,1}$, the long-short term-spread is going to decline in period of high consumption uncertainty. As bond risk premium is also low at these times, the real term-spread and bond risk premia are positively correlated, as required to explain the violations of the expectations hypothesis in the data. The actual magnitudes of the slopes coefficients in bond regressions (2) depend on the amount of persistence and variation in bond risk premium generated by the model. For instance, when the volatility of consumption growth is constant, the expected excess returns are constant as well, so the expectations hypothesis holds and the projection coefficients should be centered at unity.

A similar discussion of the bond risk premia holds in nominal terms. Market prices of risks and bond return sensitivities to sources of uncertainty should now account for inflation risks, so the actual responses of excess bond returns and slope of the nominal term structure to consumption volatility depends on the calibration of consumption and inflation rates. However, as long as the expected bond returns and the term-spread move in the same direction in response to consumption volatility shock, the slope coefficient in expectations hypothesis regressions is below one.

4.3 Expected Depreciation and the Forward premium

As discussed in Backus et al. (2001), the Euler equation implies that the change in exchange rate is equal to the difference between the logarithms of the discount factors in the two countries:

$$s_{t+1} - s_t = m_{t+1}^* - m_{t+1}. \tag{30}$$

Therefore, given the equilibrium solution to the pricing kernel, one-period expected depreciation rate of the domestic currency can be calculated in the following way:

$$E_t s_{t+1} - s_t = const + \frac{1}{\psi}(x_t - x_t^*) + \frac{(\gamma - \frac{1}{\psi})(\gamma - 1)}{2}\left[1 + \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho}\right)^2\right](\sigma_{gt}^2 - \sigma_{gt}^{*2}), \quad (31)$$

while the expression for the forward premium follows from equation (28):

$$\begin{aligned} y_{t,1} - y_{t,1}^* = const &+ \frac{1}{\psi}(x_t - x_t^*) \\ &- \frac{1}{2}\left(\lambda_\eta^2 + \varphi_e^2 \lambda_e^2 + (\frac{1}{\psi} - \gamma)(\gamma - 1)\left[1 + \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho}\right)^2\right]\right)(\sigma_{gt}^2 - \sigma_{gt}^{*2}). \end{aligned} \quad (32)$$

Therefore, the solution to expected excess return in foreign bond is linear in consumption volatilities at home and abroad:

$$\begin{aligned} E_t r x_{t+1}^{FX} &= E_t\left(s_{t+1} - s_t + y_{t,1}^* - y_{t,1}\right) \\ &= \frac{1}{2}\left(\lambda_\eta^2 + \varphi_e^2 \lambda_e^2\right)(\sigma_{gt}^2 - \sigma_{gt}^{*2}). \end{aligned} \quad (33)$$

As can be seen from the above expressions, in the full long-run risks specification with time-varying economic uncertainty and preference for early resolution of uncertainty, the expected excess return on foreign bonds unambiguously increases when the consumption uncertainty at home is high. Indeed, with a positive shock to domestic uncertainty, the equilibrium dollar price of the foreign currency s_t drops immediately, so that relative to the new level today, the foreign currency is expected to *appreciate* tomorrow (expression (31)), and the dollar return on investments abroad is expected to be high (expression (33)). At the same time, when $\gamma > 1$ and $\psi > 1$, increase in domestic consumption uncertainty lowers the yields on domestic bonds and the yield spread across countries (equation (32)). All together, in response to a positive shock to consumption uncertainty, the agents demand higher expected excess returns in foreign bonds, forecast appreciation of the foreign currency and at the same time push the yield on domestic risk-free assets down. This can qualitatively account for the violations of the uncovered interest rate parity condition in the data. The actual magnitude of the model-implied slope coefficients in foreign exchange projections depend on the calibration of preference and consumption growth parameters.

For example, if the consumption uncertainty σ_{gt}^2 is constant, the expected excess returns on foreign bonds are constant as well, so that the expectations hypothesis holds and the projection coefficient in foreign exchange regression should be centered at one. At the other extreme, when agent has power utility, the slope coefficient is given by

$$\beta_{UIP}^{CRRA} = \frac{Var(x_t - x_t^*)}{\gamma^2 Var(\sigma_{gt}^2 - \sigma_{gt}^{*2}) + Var(x_t - x_t^*)},$$

so that it less than one, but bigger than zero.

In terms of Fama (1984) conditions, while the power utility model can generate a negative covariation of the expected excess returns in foreign bonds and forward premium, so that the slope coefficient is below one, it fails to produce enough variation in the risk premium $E_t rx_{t+1}^{FX}$. In the long-run risks model, the risk premium is magnified by the compensation for the uncertainty in expected consumption growth, see expression (33). In particular, for the right persistence and variance of the long-run and volatility risks, we can match the empirical findings that the projection coefficients are negative at short horizon but become positive and closer to one at longer maturities.

The discussion for nominal variables is completely analogous, and is omitted for the interests of space.

4.4 Equity Returns

Following Bansal and Yaron (2004), we specify a dividend process

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_{gt} \eta_{d,t+1}.$$

The parameters $\phi > 1$ and $\varphi_d > 1$ determine the overall volatility of dividends, its persistence and correlation with consumption growth. The dividend innovation $\eta_{d,t+1}$ is allowed to be correlated with consumption news η_{t+1}. Note that consumption and dividends are not cointegrated in the above specification — Bansal, Gallant, and Tauchen (2007a) develop a specification that allows for cointegration between consumption and dividends.

Exploiting the Euler equation (11), the approximate solution for the log price-dividend ratio $z_{m,t}$ has the form $z_{m,t} = A_{0,m} + A_{x,m}x_t + A_{gs,m}\sigma^2_{gt}$, where the expressions for the loadings provide similar intuition as for the price-consumption ratio, and can be found in Bansal and Yaron (2004).

The equity premium in the presence of time-varying economic uncertainty is

$$E_t(r_{m,t+1} - r_{f,t}) = \beta_\eta \lambda_\eta \sigma^2_{gt} + \beta_e \varphi_e \lambda_e \sigma^2_{gt} + \beta_w \lambda_{gw} \sigma^2_{gw} - 0.5Var_t(r_{m,t+1}). \quad (34)$$

The first beta corresponds to the exposure to short-run risks, and the second to long-run risks. The last beta, $\beta_{m,w}$, captures the sensitivity of the asset return to volatility risks. The actual magnitudes of betas and their effects on the risk premium are discussed in detail in Bansal and Yaron (2004); however, it is important to note that unlike in reduced-form approaches, the sensitivity parameters in general equilibrium framework are completely pinned down by cash flow and preference parameters and therefore are endogenous to the model.

5 Data and Calibration

5.1 Real Economy

We choose four countries for our empirical analysis, such as United Kingdom, Germany, Japan and United States (domestic country). The financial data for the foreign countries is taken from from Datastream. This includes spot and forward rates, Euro-Currency Middle Rates of 1 month to maturity and returns on Morgan Stanley International Index for the period of January 1976 (July 1978 for Japan) to November 2005. Additional data on 1 to 5 year nominal discount bonds in US and UK come from CRSP and Bank of England, respectively. Market returns in US are calculated for a broad value-weighted portfolio from CRSP. The consumption and CPI measures for foreign countries are taken from the IMF's International Financial Statistics,[4] while the US consumption data come from BEA tables of real expenditures on non-durable goods and services.

The second column of Table 6 shows summary statistics for the US quarterly consumption series for the period from 1976Q2 to 2005Q2. The real consumption growth

[4] We thank John Campbell for providing us the dataset.

rate is mildly persistent, has an annualized volatility just below 1% and correlates positively with household expenditure rates in UK. To view the long-horizon properties of the series we compute autoregressive coefficients at different lags as well as the variance ratios which are themselves determined by the autocorrelations (see Cochrane (1988)). In the data the variance ratios first rise significantly and at about 3 years start to decline, while the autocorrelations fall uniformly with number of lags. The standard errors on these statistics, not surprisingly, are quite substantial. As the constructed series in other countries are more noisy proxies for the true consumption process of the agent, they are more volatile and less persistent than the US series. For the interests of space, we do not report their statistics in the paper.

The first panel in Table 1 tabulates summary statistics for excess market returns and inflation-adjusted rates across the four countries[5]. Notably, one-month interest rates are very persistent and vary from 1.9% in Japan to 3.61% in UK, while their standard deviations range between 2.1% and 3.1% for the sample period. For a post-war sample in US, the interest rate is equal to 0.8% with a standard deviation of 2.8%. Realized excess equity returns are several times more volatile and average $4.9\% - 5.5\%$ for UK, Germany and US (7.4% for a post-war US sample). High mean and variance of the market return relative to the interest rate are well-known puzzles in financial literature (see Mehra and Prescott, 1985).

The first panel of Table 3 reports summary statistics for the nominal and inflation-adjusted changes in foreign exchange rates across the countries. Consistent with previous findings, foreign exchange rates have a zero autoregressive coefficient (not reported), and the range for their volatilities is $10.7\% - 12.1\%$ both for nominal and inflation-adjusted series.

5.2 Nominal Economy

The second panel of Table 1 contains summary statistics for seasonally-adjusted monthly inflation rates and 1 month nominal yields across countries, while a more detailed description of the quarterly inflation in US is given in the second column of Table 7. The inflation rates are fairly persistent, as evidenced by the first and tenth-order autoregres-

[5]Inflation adjustment of interest rates is based on AR(2) filtered inflation, while realized inflation is used to adjust changes in foreign exchange rates.

sive coefficients of 0.64 and 0.36, respectively, in the US sample. In fact, the variance ratios rise substantially and start to decline only at 8.5 year horizon. The series co-move positively across countries, with a correlation of US and UK inflation rates of 0.73, and negatively with consumption growth rates, -0.21 being the correlation coefficient in US sample. The output for the other countries is similar, safe for a low predictability of inflation rates in Japan, and is omitted for the interest of space.

To capture the sensitivity of inflation news to consumption uncertainty, we set up and estimate a bivariate VAR(1) for these two series. We use the estimated model to compute analytically a $k-$period inflation beta defined as
$\widehat{Cov}_t(\frac{1}{k}\sum_{i=1}^{k}\pi_{t+i}, \frac{1}{k}\sum_{i=1}^{k}\Delta c_{t+i})/\widehat{Var}_t(\frac{1}{k}\sum_{i=1}^{k}\Delta c_{t+i})$.

A solid line in Figure 1 draws the inflation beta as a function of horizon. Consistent with negative correlation of consumption and inflation rates in the data, the inflation beta is negative and stabilizes at -1.3 at long horizons. Notably, if the conditional expectations of consumption growth and inflation rates were constant, the inflation beta computed using the unconditional moments in Tables 6 and 7 would amount to -0.44 at all maturities. This evidence suggests that the expected consumption and inflation rates are time-varying and negatively correlated.

5.3 Calibration of Consumption and Inflation

We calibrate the model outlined in (12) - (14) and (15) - (16) at monthly frequency and time-aggregate the output from monthly simulations to match the key aspects of the 1976Q2 - 2005Q2 sample of quarterly consumption growth and inflation rate in US. We use the solutions provided above to derive our model implications for the asset prices. In particular, we use the numerical method discussed in Bansal, Kiku, and Yaron (2007b). They develop a procedure to solve for the endogenous constants κ_0 and κ_1 in equation (19) associated with each return and document that the numerical solution to the model is accurate. We provide some details on this method in Appendix A.

The baseline calibration parameter values are reported in Table 5. Specifically, we set the persistence in the expected consumption growth ρ at 0.991. Our choice of φ_e and σ ensures that the model matches the moments of consumption growth in the data. In particular, the annualized volatility of monthly consumption growth is set to

1.1%, while the long-run risks volatility parameter is $\varphi_e = 0.055$. The persistence of the variance shocks is set at $\nu_g = 0.996$. We also calibrate a dividend dynamics and set the exposure of the corporate sector to long-run risks to $\phi = 1.5$, and choose $\varphi_d = 6$ to match a high volatility of dividend stream relative to consumption. The correlation of consumption and dividend news is set to $\tau_{gd} = 0.1$.

To capture the international dimensions of the data, we follow Colacito and Croce (2005) and set the correlation of long-run news τ_e to be nearly perfect, 0.999, and additionally impose a high correlation of the volatility news across the countries, $\tau_{gw} = 0.99$. In the extension of the model, we allow the volatilities of short-run and long-run consumption news to be different from each other, and calibrate the correlation between the short-run volatilities across countries to be zero, while the correlation of the long-run volatilities is set to one. This captures the intuition that in the long run, consumption processes across all the countries are nearly identical (the long-run means and volatilities are the same), while in the short-run, they can be quite different due to uncorrelated immediate consumption and short-run volatility news at home and abroad.

In Table 6 we report the calibration output of our model, which is based on 1,000 simulations of 360 months of consumption series aggregated to quarterly horizon. As in Bansal and Yaron (2004), the model implications for the volatility, persistence and multi-horizon properties of consumption growth rates are close to their empirical counterparts. Additionally, the model also delivers low correlation coefficient of consumption growth rates across countries, which matches well the historical estimates (0.24 in the model versus 0.25 in the US and UK data). To calibrate the dividend series, we first change the target frequency to annual horizon, due to strong seasonal patterns in quarterly dividend data. The model-implied dynamics of the dividend growth series matches the data very well, and is omitted for the interest of space. The median volatility of dividend growth rate is 6.96% in the model versus 6.38% in the data; the persistence is 0.24 versus 0.16, and the correlation of dividend and consumption growth rates is 0.26 both in the data and in the model (median value).

The inflation rate process is calibrated as follows. To maintain parsimony, we zero out inflation and expected inflation betas to immediate consumption news, $\varphi_{\pi g} = \varphi_{zg} = 0$, and set their sensitivity to long-run risks to be negative, $\varphi_{\pi x} = -2$ and $\varphi_{\pi z} = -1$. This is consistent with Piazzesi and Schneider (2005), who show that the negative correlation of inflation innovations with future long-horizon consumption growth helps explain the term structure of nominal bonds. We calibrate the parameters of the expected inflation to match the key properties of the data. In particular, the expected inflation loads negatively on the expected consumption growth, $\alpha_x = -0.35$, and its own autoregressive coefficient is α_π is 0.83. For simplicity, we set to zero the correlation of independent inflation shocks across countries and capture the co-movements in the series through covariation of long-run risks shocks.

Table 7 shows the calibration output for the inflation process, while Figure 1 depicts the model-implied inflation beta to consumption news based on the bivariate VAR(1) specification. The model quite successfully matches the univariate properties of the inflation series, as well as the joint behavior of inflation and consumption growth rate and the correlation of the inflation rates across countries.

5.4 Preference Parameters

We calibrate the subjective discount factor $\delta = 0.9987$. The risk-aversion coefficient is set at $\gamma = 8$. Mehra and Prescott (1985) and Bansal and Yaron (2004) do not entertain risk aversion values larger than 10. As in Bansal and Yaron (2004), we focus on an IES of 1.5 — an IES value larger than one is important for our quantitative results.

There is a debate about the magnitude of the IES. Hansen and Singleton (1982), Attanasio and Weber (1989), Guvenen (2001), Vissing-Jorgensen and Attanasio (2003) and Gruber (2006) estimate the IES over one. Hall (1988) and Campbell (1999) estimate the IES to be well below one. Bansal and Yaron (2004) argue that low estimates of IES are based on a model without time-varying volatility, which leads to a downward bias in estimation.

Bansal, Khatchatrian, and Yaron (2005) document that the asset valuations fall when consumption volatility is high; this is consistent only with $\psi > 1$. As we show in the next section of the paper, domestic consumption volatility also co-moves negatively with dollar prices of foreign currency and forward premia, and positively with expected returns on foreign bonds. This evidence is consistent with model predictions only when $\gamma > \frac{1}{\psi}$ and $\psi > 1$, which further motivates our calibration of preference parameters

6 Model Implications

6.1 Bond Markets

As shown in the first panel of Table 8, at the calibrated parameter values the model-implied term structure of nominal bond yields is upward sloping. The one-year nominal yield is 5.44%, and it increases to 6.22% at 5 years. The volatilities of the yields fall uniformly from 2.25% at 1 year to 1.91% at 5 year horizon. The population values for levels and volatilities of nominal yields are thus consistent with US historical estimates reported in Table 2. The term-structure of real rates is downward sloping: the model-implied real rate is 1.3% at one year horizon and 0.52% at 5 years. At the calibrated parameter values, the inflation risk premium is increasing with the maturity, which helps to reconcile the difference in the levels of the real and nominal yields and the the slopes of their term structures.

The second panel of Table 8 shows model-implied slope coefficients in bond projections (2). These regressions are done using the annual time step and bond maturities of 2 to 5 years, so they are directly comparable to the projections in the data reported in Table 2. The theoretical slope coefficients are all negative and decreasing with horizon: the nominal slopes fall from -0.17 to -0.34, while the real ones decrease from -0.63 to -0.69 for 2 and 5 year horizons, respectively. These values are consistent with the estimates based on historical data in US and UK bond markets in Table 2. In the model, shocks to the consumption variance move the expected excess bond returns and the term-spread in the same direction, which accounts for the violation of the expectations hypothesis and makes the slope coefficient in Campbell and Shiller (1991) regressions to be less than one. As in the data, these violations are more severe at longer horizons, as the model-implied slope coefficient in bond regressions increases in absolute value with horizon.

Recent evidence on the time-variation of bond risk premia comes from forward-rate projections considered in Cochrane and Piazzesi (2005). The preferred regression model of Cochrane and Piazzesi (2005) includes five forward rates, but we have to limit ourselves to three regressors to avoid perfect multicollinearity in the model. Indeed, with three states — expected consumption growth, expected inflation rate and consumption volatility — the three forward rates summarize all the information in the economy. In

Table 4, we compare the magnitudes of the coefficients and R^2 in common bond factor regressions (4) in the data and in the model. The loadings on a single bond factor are very similar across the countries: for US, they increase from 0.44 at 2 year horizon to 1.45 at 5 years. These values are well captured by the model: the slope coefficients increase from 0.39 to 1.58 for 2 and 5 year maturities, respectively. The R^2 in the data are in $20\% - 30\%$ range. The population R^2s are about $10-12\%$, however, the estimates in small samples (not reported) often reach magnitudes found by Cochrane and Piazzesi (2005).

6.2 Currency Markets

Table 9 shows that the nominal slope coefficient in foreign exchange projections is equal to -1.23 at one month horizon, and it increases to -0.71 at 1 year and to 3.19 at 5 year horizon. The real slope coefficient is about -7 at all the considered maturities. These findings are broadly consistent with Hollifield and Yaron (2003), who argue that risks from the real side of the economy are potentially important to capture the violations of the uncovered interest rate parity condition.

The value of the nominal projection coefficient at short horizon matches well the empirical estimates shown in Table 3. The model-implied nominal slope becomes positive at maturities above 2 years, so that the violations of the expectations hypothesis are less pronounced at longer maturities, which is consistent with the evidence reported in Chinn and Meredith (2004) and Alexius (2001). As also shown in Table 3, the model-implied volatility of the foreign exchange rates is 19.86%, both in real and nominal terms, which is somewhat higher than the usual estimates of $11-12\%$ in the data.

The data also supports broader predictions of the model for the foreign exchange markets. In Figures 2 - 4 we plot inflation-adjusted spot prices, forward premia and expected foreign bond returns against the difference in consumption volatility across countries. We follow Bansal et al. (2005) and construct consumption volatility measures non-parametrically as a 4.5 year sum of absolute residuals from AR(3) projections of consumption growth rates. Inflation adjustment for the interest rates is based on the fitted values from AR(2) model for the inflation rate. Consistent with the theoretical predictions in Section 4.3 for $\gamma > \frac{1}{\psi}$ and $\psi > 1$, consumption volatility differential co-moves negatively with dollar prices of foreign currency and forward premia, and positively with the expected dollar returns in foreign bonds. Indeed, the correlation coefficients for the spot exchange prices range between -0.1 and -0.5, and are equal to about -0.3 and 0.3 for the forward premia and expected excess returns, respectively, for all the countries in the sample.

The magnitudes of the preference parameters and in particular, the value of the IES relative to one, have the first-order implications for the dynamics of asset markets in the model. Indeed, if $\psi > 1$, a rise in consumption volatility lowers asset valuations (see equation (21)), while the opposite happens if the IES is less than one. This has a direct effect on the co-movements of the consumption variance with asset and currency valuations and forward premium in the economy. Ultimately, it determines the ability of the model to explain the violations of the expectations hypothesis. In Figure 5 we plot the model-implied nominal slope coefficient against the IES when all the other parameters are fixed at their benchmark values. When IES is less than one, the theoretical slope coefficient is positive, so calibrating the IES the at the value above one is important to explain the foreign exchange puzzle in the data.

6.3 Equity Markets

The third panel of Table 8 computes summary statistics for the real excess return on market portfolio and the real one-month interest rate. The model generates a sizable equity premium of 6.6%, which matches well the historical estimates in US and foreign countries reported in Table 1. The model-implied population value of the volatility of market returns is about 11.11%, which is broadly comparable to the historical estimates of $14\% - 15\%$ in US data (see Table 1).

In the model, one-month real interest rates average 1.5% and, unlike the stock market returns, are very persistent, with an autoregressive coefficient of 0.99. At the calibrated parameter values, the volatility of the real rates is 1.3% at one month horizon. These findings are consistent with the estimates reported in Table 1 and elsewhere in the literature.

To summarize, our model can account for the predictability of returns and failure of expectations hypotheses in bond and currency markets, nominal term structure, volatilities of yields and foreign exchange rates, and risk premium and risk-free rate puzzles in equity markets. In the next section we consider an extension of the model which can sharpen the quantitative results of the model.

7 Two Volatilities

7.1 Setup

Consider an extension of the baseline setup in which the volatilities of long-run and short-run consumption news are separate and driven by individual shocks,

$$\begin{aligned}
\Delta c_{t+1} &= \mu_g + x_t + \sigma_{gt}\eta_{t+1}, \\
x_{t+1} &= \rho x_t + \sigma_{xt} e_{t+1}, \\
\sigma^2_{g,t+1} &= \sigma^2_g + \nu_g(\sigma^2_{gt} - \sigma^2_g) + \sigma_{gw} w_{g,t+1}, \\
\sigma^2_{x,t+1} &= \sigma^2_x + \nu_x(\sigma^2_{xt} - \sigma^2_x) + \sigma_{xw} w_{x,t+1}.
\end{aligned}$$

The immediate news in consumption growth have a stochastic volatility σ^2_{gt}, while σ^2_{xt} is equal to the conditional variance of the low-run component x_t. This framework disentangles the low-frequency swings in consumption uncertainty from the short-run component in consumption volatility. Stock and Watson (2002) provide empirical evidence for the very-low frequency movements of several macroeconomic series including consumption growth.

In Appendix C we present the solution to the two-volatility model. Now, in addition to long-run risks x, expected inflation $\bar{\pi}_t$ and short-run variance σ^2_{gt}, asset prices also respond to movements in long-run volatility σ^2_{xt}.

7.2 Model Implications

Table 10 reports the adjustments to the baseline calibration values for the variance parameters and correlations of news across the countries. We keep the persistence of σ^2_{xt} at 0.996, and decrease the short-run volatility autoregressive coefficient to 0.85. While we increase the variance of consumption growth and the volatility of the long-run variance, the variance long-run risks was decreased to keep the predictability of consumption growth low. In international dimension, we increase the correlation of long-run risks shocks across the countries and shift all the co-movements of volatilities to low frequency. For the interest of space, we do not report the calibration output for consumption, inflation and dividend processes as they are virtually the same as the in baseline case.

In Table 11 we show the output for the nominal term structure and the violations of expectations hypothesis in the two-volatility setup. The first panel documents that the levels and variances of yields are very similar to the baseline calibration and are consistent with historical evidence. On the other hand, the violations of the expectations hypothesis in bond markets are more pronounced than in the baseline setup: for example, the slope coefficients in nominal bond projection are now equal to -0.26 and -0.45 at 2 and 5 year frequencies, respectively. The predictability of bond returns in single bond factor regressions also increases relative to the benchmark case: as shown in Table 4, the population R^2 increase to $11 - 13\%$, while the loadings on the single bond factor remain the same.

Table 12 shows that the real and nominal slope coefficients in foreign exchange regressions are -1.2 and -3.1, respectively, at 1 month horizon. They turn positive within 2 to 3 years and remain below one at the considered maturities (nominal one is equal to 0.86 and real one to 0.95 at 5 year horizon), which is consistent with historical evidence. The decrease in short-run consumption volatility also helps to match the volatility of foreign exchange rate, which now becomes 15.78% and is closer to the historical estimates of $11 - 12\%$ relative to the benchmark case.

Finally, the third panel of Table 11 shows that model generates the equity risk premium of 7.5% and the standard deviation of market returns of 12.11%, which is again closer to the estimates in the data. The mean of the real risk-free rate (1.32%) and its volatility (1.21%) are quite similar to the benchmark values. Therefore, the extension of the model to the two volatilities case can improve the quantitative predictions of the benchmark model.

Conclusion

We show that the long-run risks model (see Bansal and Yaron, 2004) can simultaneously account for a wide-range of asset price puzzles in bond, currency, and equity markets. The model features long-run growth fluctuations, time-varying consumption uncertainty and a preference for early resolution of uncertainty. The model implies that short-run, long-run, and volatility risks are priced in financial markets.

Using a specification that is similar to Bansal and Yaron (2004), we show that the model can account for the negative coefficients in expectations hypothesis and the foreign exchange projections (see Campbell and Shiller, 1991), bond predictability with forward rates (see Cochrane and Piazzesi, 2005), the exchange rate volatility, the equity premium and the low risk-free rate puzzles, the high volatility of the stock prices, and the upward sloping nominal term structure. The model also matches the salient properties of the consumption and inflation data within and across the countries.

The model captures the intuition that a positive shock to consumption volatility move the expected excess bond returns and the yield-spread in the same direction, which makes population slope coefficients in the expectations hypothesis regressions negative. At the same time, forward premium decreases and the domestic currency is expected to depreciate, which accounts for the violations of the uncovered interest rate parity condition. Model-implied slope coefficients in foreign exchange and expectations hypothesis are consistent with historical estimates. Using consumption and asset markets data, we provide empirical evidence to support the key economic channels highlighted in the paper. This evidence across financial markets implies that the long-run risks model provides a resolution for a rich array of asset pricing puzzles.

A Solution to κ_1

From the log-linearization of returns, we obtain that the mean price-to-consumption ratio $E(z_t) = A_0$ and a constant κ_0 are equal to

$$A_0 = \log \frac{\kappa_1}{1-\kappa_1}, \quad (35)$$

$$k_0 = -\log\left[(1-k_1)^{1-k_1} k_1^{k_1}\right]. \quad (36)$$

We can substitute the above expressions for A_0 and κ_0 into the solution for A_0 from the Euler equation, and after some algebra obtain

$$\log \kappa_1 = \log \delta + (1 - \frac{1}{\psi})\mu_g + A_{gs}(1 - \kappa_1 \nu_g)\sigma_g^2 + \frac{1}{2}\theta \kappa_1^2 A_{gs}^2 \sigma_{gw}^2.$$

To find the solution for κ_1 at the calibrated parameter values, we iterate on the equation above from an initial value δ. Similar expression can be derived for the approximating coefficients in the log-linearization of the return on the stock market portfolio. This approach closely follows Bansal et al. (2005).

B Solution to Bond Yields

The log prices of real and nominal discount bonds $q_{t,n}, q_{t,n}^{\$}$ satisfy the Euler conditions

$$e^{q_{t,n}} = E_t e^{m_{t+1} + q_{t+1,n-1}},$$
$$e^{q_{t,n}^{\$}} = E_t e^{m_{t+1}^{\$} + q_{t+1,n-1}^{\$}},$$

for $q_{t,0} = q_{t,0}^{\$} = 0$. Using the solutions to the real and nominal pricing kernels in (22) and (17), we obtain that the bond prices and therefore, bond yields, are affine in the states, as shown in expression (37) and (38), and the bond yield loadings satisfy

$$
\begin{aligned}
B_{x,n} &= \rho B_{x,n-1} + \frac{1}{\psi}, \\
B_{gs,n} &= \nu_g B_{gs,n-1} - \frac{(\frac{1}{\psi}-\gamma)(\gamma-1)}{2}\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right] - \frac{1}{2}\left(\lambda_\eta^2 + \varphi_e^2[\lambda_e + B_{x,n-1}]^2\right), \\
B_{0,n} &= B_{0,n-1} - \mu_m - \frac{1}{2}\left(\sigma_g^2[\lambda_\eta^2 + \varphi_e^2(\lambda_e + B_{x,n-1})^2] + \sigma_{gw}^2[\lambda_{gw} + B_{gs,n-1}]^2\right)
\end{aligned}
$$

for real bonds, and

$$
\begin{aligned}
B^{\$}_{x,n} &= \rho B^{\$}_{x,n-1} + \alpha_x B^{\$}_{\pi,n-1} + \frac{1}{\psi}, \\
B^{\$}_{\pi,n} &= \alpha_\pi B^{\$}_{\pi,n-1} + 1, \\
B^{\$}_{gs,n} &= \nu_g B^{\$}_{gs,n-1} - \frac{(\frac{1}{\psi}-\gamma)(\gamma-1)}{2}\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right] \\
&\quad - \frac{1}{2}\left([\lambda_\eta + \varphi_{\pi g} + \varphi_{zg}B^{\$}_{\pi,n-1}]^2 + \varphi_e^2[\lambda_e + \varphi_{\pi x} + \varphi_{zx}B^{\$}_{\pi,n-1} + B^{\$}_{x,n-1}]^2\right), \\
B^{\$}_{0,n} &= B^{\$}_{0,n-1} - \mu_m + \mu_\pi - \frac{1}{2}\left([\sigma_\pi + B^{\$}_{\pi,n-1}\sigma_z]^2 + \sigma_{gw}^2[\lambda_{gw} + B^{\$}_{gs,n-1}]^2\right. \\
&\quad \left. + \sigma_g^2[(\varphi_{\pi g} + \lambda_\eta + \varphi_{zg}B^{\$}_{\pi,n-1})^2 + \varphi_e^2(\varphi_{\pi x} + \lambda_e + B^{\$}_{x,n-1} + \varphi_{zx}B^{\$}_{\pi,n-1})^2]\right)
\end{aligned}
$$

for nominal ones.

C Two Volatilities Model

The equilibrium price-to-consumption ratio is affine in the expected consumption and inflation rates and the two volatilities:

$$v_t = \mu_v + A_x x_t + A_{gs}(\sigma_{gt}^2 - \sigma_g^2) + A_{xs}(\sigma_{xt}^2 - \sigma_x^2),$$

where the loadings satisfy

$$\begin{aligned}
A_x &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}, \\
A_{gs} &= \frac{1}{2}\frac{(1-\gamma)(1-\frac{1}{\psi})}{1-\kappa_1\nu_g}, \\
A_{xs} &= \frac{1}{2}\kappa_1^2\frac{(1-\gamma)(1-\frac{1}{\psi})}{(1-\kappa_1\nu_x)(1-\kappa_1\rho)^2}.
\end{aligned}$$

The log-linearization parameter κ_1 is given implicitly by

$$\log\kappa_1 = \log\delta + (1-\frac{1}{\psi})\mu_g + A_{gs}(1-\kappa_1\nu_g)\sigma_g^2 + A_{xs}(1-\kappa_1\nu_x)\sigma_x^2 + \frac{1}{2}\theta\kappa_1^2\left(A_{gs}^2\sigma_{gw}^2 + A_{xs}^2\sigma_{xw}^2\right).$$

The real discount rate now takes the following form:

$$\begin{aligned}
m_{t+1} &= \mu_m + m_x x_t + m_{gs}(\sigma_{gt}^2-\sigma_g^2) + m_{xs}(\sigma_{xt}^2-\sigma_x^2) \\
&\quad - \lambda_\eta\sigma_{gt}\eta_{t+1} - \lambda_e\sigma_{xt}e_{t+1} - \lambda_{gw}\sigma_{gw}w_{g,t+1} - \lambda_{xw}\sigma_{xw}w_{x,t+1},
\end{aligned}$$

for

$$\begin{aligned}
\mu_m &= \theta\log\delta - (\theta-1)\log\kappa_1 - \gamma\mu_g, \\
m_x &= -\frac{1}{\psi}, \\
m_{gs} &= \frac{1}{2}(\gamma-1)(\frac{1}{\psi}-\gamma), \\
m_{xs} &= \frac{1}{2}(\gamma-1)(\frac{1}{\psi}-\gamma)\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2,
\end{aligned}$$

and

$$\begin{aligned}
\lambda_\eta &= \gamma, \\
\lambda_e &= (\gamma-\frac{1}{\psi})\frac{\kappa_1}{1-\kappa_1\rho}, \\
\lambda_{gw} &= \frac{1}{2}(\gamma-\frac{1}{\psi})(1-\gamma)\frac{\kappa_1}{1-\kappa_1\nu_g}, \\
\lambda_{xw} &= \frac{1}{2}(\gamma-\frac{1}{\psi})(1-\gamma)\frac{\kappa_1}{1-\kappa_1\nu_x}\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2,
\end{aligned}$$

The equilibrium real and nominal bond prices are affine in the state variables:

$$q_{t,n} = -B_{0,n} - B_{x,n}x_t - B_{gs,n}(\sigma_{gt}^2 - \sigma_g^2) - B_{xs,n}(\sigma_{xt}^2 - \sigma_x^2), \tag{37}$$

$$q_{t,n}^{\$} = -B_{0,n}^{\$} - B_{x,n}^{\$}x_t - B_{gs,n}^{\$}(\sigma_{gt}^2 - \sigma_g^2) - B_{xs,n}^{\$}(\sigma_{xt}^2 - \sigma_x^2) - B_{\pi,n}^{\$}(\bar{\pi}_t - \mu_\pi), \tag{38}$$

where the loadings satisfy the recursions

$$\begin{aligned}
B_{x,n} &= \rho B_{x,n-1} - m_x, \\
B_{gs,n} &= \nu_g B_{gs,n-1} - (m_{gs} + \frac{1}{2}\lambda_\eta^2), \\
B_{xs,n} &= \nu_x B_{xs,n-1} - m_{xs} - \frac{1}{2}(\lambda_e + B_{x,n-1})^2, \\
B_{0,n} &= B_{0,n-1} - \mu_m - \frac{1}{2}\left(\lambda_\eta^2\sigma_g^2 + (\lambda_e + B_{x,n-1})^2\sigma_x^2\right. \\
&\quad \left. +(\lambda_{gw} + B_{gs,n-1})^2\sigma_{gw}^2 + (\lambda_{xw} + B_{xs,n-1})^2\sigma_{xw}^2\right).
\end{aligned}$$

and

$$\begin{aligned}
B_{x,n}^{\$} &= \rho B_{x,n-1}^{\$} + \alpha_x B_{\pi,n-1}^{\$} - m_x, \\
B_{\pi,n}^{\$} &= \alpha_\pi B_{\pi,n-1}^{\$} + 1, \\
B_{gs,n}^{\$} &= \nu_g B_{gs,n-1}^{\$} - m_{gs} - \frac{1}{2}\left(\varphi_{\pi g} + \lambda_\eta + \varphi_{zg}B_{\pi,n-1}^{\$}\right)^2, \\
B_{xs,n}^{\$} &= \nu_x B_{xs,n-1}^{\$} - m_{xs} - \frac{1}{2}\left(\varphi_{\pi x} + \lambda_e + B_{x,n-1}^{\$} + \varphi_{zx}B_{\pi,n-1}^{\$}\right)^2, \\
B_{0,n}^{\$} &= B_{0,n-1}^{\$} - \mu_m + \mu_\pi - \frac{1}{2}\left((\sigma_\pi + B_{\pi,n-1}^{\$}\sigma_z)^2 + (\lambda_{gw} + B_{gs,n-1}^{\$})^2\sigma_{gw}^2 + (\lambda_{xw} + B_{xs,n-1}^{\$})^2\sigma_{xw}^2\right. \\
&\quad \left. + (\varphi_{\pi g} + \lambda_\eta + \varphi_{zg}B_{\pi,n-1}^{\$})^2\sigma_g^2 + (\varphi_{\pi x} + \lambda_e + B_{x,n-1}^{\$} + \varphi_{zx}B_{\pi,n-1}^{\$})^2\sigma_x^2\right)
\end{aligned}$$

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D Tables and Figures

Table 1: **Descriptive Statistics Across Countries**

		UK	Germany	Japan	US	US47
Inflation-Adjusted	Mean	3.61	2.73	1.91	2.59	0.82
Interest Rate:	Std. Dev.	3.06	2.06	2.80	3.07	2.76
	AR(1)	0.74	0.90	0.91	0.84	0.78
Excess Market Return:	Mean	5.46	4.86	2.83	5.50	7.42
	Std. Dev.	17.30	20.78	18.56	14.74	14.60
	AR(1)	0.00	0.02	0.05	0.03	0.06
	Sharpe Ratio	0.32	0.23	0.15	0.37	0.51
Nominal	Mean	9.15	5.27	3.57	6.82	4.59
Interest Rate:	Std. Dev.	3.91	2.53	3.26	3.76	2.90
	AR(1)	0.97	0.99	0.98	0.98	0.97
Inflation Rate:	Mean	5.52	2.55	1.84	4.24	3.76
	Std. Dev.	1.58	0.72	1.24	1.02	1.20
	AR(1)	0.63	0.44	0.21	0.71	0.57

Descriptive statistics for interest rates and equity returns across countries. Inflation-adjusted interest rate corresponds to 1 month nominal interest rate adjusted for expected inflation. Excess market return is the return on Morgan Stanley International Index (CRSP portfolio for US) over the one month interest rate. Nominal interest rate is Euro-Currency Middle Rate of 1 month to maturity (CRSP risk-free rate for US47). Monthly observations from Jan 1976 (July 1978 for Japanese interest rate) to Nov 2005, and Feb 1947 to Nov 2005 for US47. Means and standard deviations are annualized.

Table 2: **Bond Market Evidence**

		1y	2y	3y	4y	5y
Nominal Yield:						
US	Mean	5.56	5.77	5.94	6.07	6.16
	Std. Dev.	2.91	2.86	2.79	2.75	2.72
UK	Mean	7.33	7.35	7.39	7.43	7.46
	Std. Dev.	2.82	2.57	2.45	2.40	2.37
EH Projection:						
US	Nominal		-0.70	-1.03	-1.41	-1.39
			(0.43)	(0.51)	(0.57)	(0.64)
	Infl Adjusted		-0.19	-0.16	-0.48	-0.43
			(0.23)	(0.43)	(0.51)	(0.57)
UK	Nominal		-0.14	-0.12	-0.10	-0.09
			(0.57)	(0.64)	(0.72)	(0.82)
	Infl Adjusted		0.26	-0.12	-0.31	-0.44
			(0.35)	(0.46)	(0.59)	(0.72)

Nominal term structure and tests of expectations hypothesis in US and UK bond markets. Monthly observations of 1-5 year yields on US and UK discount bonds for June 1952 to Dec 2005 and Jan 1985 to Dec 2005, respectively. EH Projection reports the slope coefficient $\beta_{n,m}$ in regression $y_{t+m,n-m} - y_{t,n} = const + \beta_{n,m}\frac{m}{n-m}(y_{t,n} - y_{t,m}) + error$, where time step m is set at 12 months and bond maturities n run from 2 to 5 years (see Campbell and Shiller, 1991). Standard errors are Newey-West adjusted with 10 lags.

Table 3: **Currency Market Evidence**

		UK	Germany	Japan
Foreign Exchange Rate:				
Nominal	Mean	-0.53	1.47	3.10
	Std. Dev.	10.69	11.18	12.09
Infl Adjusted	Mean	0.72	-0.25	0.65
	Std. Dev.	10.80	11.17	12.18
UIP Projection:				
Nominal	Slope	-1.72	-0.85	-2.83
		(0.95)	(0.77)	(0.66)
	R^2	0.04	0.02	0.06
Infl Adjusted	Slope	-1.33	-1.10	-1.50
		(0.89)	(0.74)	(0.76)
	R^2	0.04	0.02	0.03

Foreign exchange rate and tests of expectations hypothesis in currency markets. Monthly observations of changes in log spot foreign exchange rates from Jan 1976 to Nov 2005 for Germany and UK and from July 1978 to Nov 2005 for Japan. Nominal foreign exchange rates are adjusted by realized inflation. UIP Projection reports the slope coefficient β^{UIP} and R^2 in regression $s_{t+1} - s_t - y_{t,1} + y^*_{t,1} = const + (\beta^{UIP} - 1)(y_{t,1} - y^*_{t,1}) + error$, where $y_{t,1}$ and $y^*_{t,1}$ are foreign and US interest rate, respectively, and s_t is the foreign exchange rate. Standard errors are Newey-West adjusted with 10 lags.

Table 4: **Predictability of Bond Returns**

		2y	3y	4y	5y
Panel A: Data:					
US	slope	0.44	0.87	1.24	1.45
	R^2	0.20	0.23	0.24	0.22
UK	slope	0.47	0.88	1.20	1.45
	R^2	0.28	0.30	0.30	0.29
Panel B: Model:					
Baseline	Slope	0.39	0.82	1.21	1.58
	R^2	0.09	0.09	0.09	0.09
2 Vol	Slope	0.39	0.82	1.21	1.58
	R^2	0.11	0.12	0.12	0.13

Bond return predictability regressions report the slope coefficient $b^{\$}_{m,n}$ and R^2 in single latent factor regression $rx^{\$}_{t+m,n} = const + b^{\$}_{m,n}\widehat{rx}^{\$}_{t,m} + error$, where $rx^{\$}_{t+m,n}$ is an m-months excess return on $n-$period nominal bond, and $\widehat{rx}^{\$}_{t,m}$ corresponds to a single bond factor obtained from a first-stage projection of average bond returns on three forward rates (see Cochrane and Piazzesi, 2005). Panel A tabulates the estimation results in the US and UK bond markets, while Panel B is based on the average across 100 simulations of 300,000 months in baseline and two-volatility model.

Table 5: **Model Parameter Values**

Parameter		Value
Preference Parameters:		
Subjective discount factor	δ	0.9987
Intertemporal elasticity of substitution	ψ	1.5
Risk aversion coefficient	γ	8
Consumption Growth Parameters:		
Mean of consumption growth	μ_c	0.0016
Long-run risks persistence	ρ	0.991
Long-run risks volatility	φ_e	0.055
Volatility level	σ_g	0.0032
Volatility persistence	ν_g	0.996
Volatility of volatility	σ_{gw}	1.15e-06
Dividend Growth Parameters:		
Dividend leverage	ϕ	1.5
Volatility loading of dividend growths	φ_d	6.0
Correlation of consumption and dividend news	τ_{gd}	0.1
Inflation Parameters:		
Mean of inflation rate	μ_π	0.0032
Inflation leverage on long-run news	$\varphi_{\pi x}$	-2.0
Inflation shock volatility	σ_π	0.0035
Expected inflation AR coefficient	α_π	0.83
Expected inflation loading on long-run risks	α_x	-0.35
Expected inflation leverage on long-run news	φ_{zx}	-1.0
Expected inflation shock volatility	σ_z	4.0e-06
Cross-Country Parameters:		
Correlation of long-run news	τ_e	0.999
Correlation of short-run news	τ_η	0.0
Correlation of volatility news	τ_{gw}	0.99
Correlation of inflation news	τ_ξ	0.0

Calibrated parameter values for the baseline model. The model is calibrated at monthly frequency.

Table 6: **Consumption Growth Dynamics**

	Data		Model		
Variable	Estimate	S.E.	Median	95%	5%
$\sigma(\Delta c)$	0.78	(0.09)	1.38	2.03	0.96
AR(1)	0.36	(0.07)	0.38	0.61	0.18
AR(2)	0.20	(0.08)	0.21	0.47	-0.03
AR(5)	0.08	(0.08)	0.17	0.45	-0.08
VR(2)	1.36	(0.07)	1.38	1.59	1.18
VR(5)	2.15	(0.27)	2.03	2.97	1.32
VR(10)	3.01	(0.65)	2.78	5.06	1.31
$Corr(\Delta c, \Delta c^*)$	0.25	(0.09)	0.24	0.49	0.01

Calibration of consumption growth. Quarterly observations of US real consumption growth from 1976Q2 to 2005Q2. Cross-country correlation is computed for US and UK series. Standard errors are Newey-West corrected using 10 lags. Model output is based on 1000 simulations of 360 months aggregated to quarterly horizon.

Table 7: **Inflation Dynamics**

	Data		Model		
Variable	Estimate	S.E.	Median	95%	5%
$\sigma(\pi)$	1.62	(0.32)	1.68	2.70	1.19
AR(1)	0.64	(0.16)	0.70	0.88	0.40
AR(2)	0.67	(0.14)	0.60	0.84	0.22
AR(5)	0.57	(0.17)	0.48	0.79	0.13
AR(10)	0.36	(0.17)	0.32	0.69	-0.04
VR(2)	1.61	(0.17)	1.69	1.87	1.40
VR(5)	3.56	(0.60)	3.41	4.34	2.10
VR(10)	6.44	(1.81)	5.69	8.25	2.75
$Corr(\pi, \Delta c)$	-0.21	(0.14)	-0.39	-0.13	-0.63
$Corr(\pi, \pi^*)$	0.73	0.07	0.63	0.86	0.28

Calibration of inflation rate. Quarterly observations of US inflation rate from 1976Q2 to 2005Q2. Cross-country correlation is computed for US and UK. Standard errors are Newey-West corrected using 10 lags. Model output is based on 1000 simulations of 360 months aggregated to quarterly horizon.

Table 8: **Model Implications: Bond and Equity Markets**

	1m	1y	2y	3y	4y	5y
Nominal Term Structure:						
Mean	5.39	5.44	5.59	5.77	5.98	6.22
Std. Dev.	2.33	2.25	2.12	2.01	1.94	1.91
EH Projection:						
Nominal slope			-0.17	-0.30	-0.33	-0.34
Real slope			-0.63	-0.65	-0.67	-0.69

	Mean	Std. Dev.	AR(1)
Excess Market Return:	6.64	11.11	0.00
Real Risk Free Rate:	1.50	1.30	0.99

Model-implied population statistics for the nominal term structure, market excess return and real risk-free rate. EH Projection reports the slope coefficient $\beta_{n,m}$ in regression $y_{t+m,n-m} - y_{t,n} = const + \beta_{n,m}\frac{m}{n-m}(y_{t,n} - y_{t,m}) + error$, where time step m is set at 12 months and bond maturities n run from 2 to 5 years (see Campbell and Shiller, 1991). Slope coefficients in projections are based on the average of 100 simulations of 300,000 months of data.

Table 9: **Model Implications: Currency Markets**

	1m	3m	1y	2y	3y	4y	5y
UIP Projection:							
Nominal	-1.23	-1.18	-0.71	0.20	1.25	2.30	3.19
Real	-6.68	-6.79	-7.15	-7.35	-7.34	-7.21	-7.02

	Std. Dev.	AR(1)
Nominal FX Rate:	19.86	0.00
Real FX Rate:	19.80	0.00

Model-implied population statistics for changes in foreign exchange rates. UIP Projection reports the slope coefficient β^{UIP} in regression $s_{t+n} - s_t = const + \beta^{UIP}(ny_{t,n} - ny^*_{t,n}) + error$, where $y_{t,n}$ and $y^*_{t,n}$ are foreign and US yields on $n-$period bonds, respectively, and $s_{t+n} - s_t$ is the appreciation rate of the foreign currency over n months. Slope coefficients in projections are based on the average of 100 simulations of 300,000 months.

Table 10: **Two-Volatility Model Parameter Values**

Parameter		Value
Consumption Growth Parameters:		
Short-run volatility level	σ_g	0.004
Short-run volatility persistence	ν_g	0.85
Short-run volatility of volatility	σ_{gw}	1.15e-06
Long-run volatility level	σ_x	$0.04 \times \sigma_{g0}$
Long-run volatility persistence	ν_x	0.996
Long-run volatility of volatility	σ_{xw}	$0.06^2 \times \sigma_{gw}$
Cross-Country Parameters:		
Correlation of long-run news	τ_e	0.99995
Correlation of short-run news	τ_η	0.0
Correlation of short-run volatility news	τ_{gw}	0.00
Correlation of long-run volatility news	τ_{gw}	1.00

Calibrated parameter values for the two-volatility model. The model is calibrated at monthly frequency.

Table 11: **Two-Volatility Model: Bond and Equity Markets**

	1y	2y	3y	4y	5y
Nominal Term Structure:					
$E(y^\$)$	5.23	5.35	5.52	5.73	5.98
$\sigma(y^\$)$	2.04	1.92	1.84	1.82	1.84
EH Projection:					
Nominal slope		-0.26	-0.39	-0.43	-0.45
Real slope		-0.73	-0.76	-0.79	-0.82

	Mean	Std. Dev.	AR(1)
Excess Market Return:	7.51	12.11	0.00
Real Risk Free Rate:	1.32	1.21	0.99

Population statistics for the nominal term-structure based on the two-volatility model. EH projection reports the slope coefficient $\beta_{n,m}$ in regression $y_{t+m,n-m} - y_{t,n} = const + \beta_{n,m}\frac{m}{n-m}(y_{t,n} - y_{t,m}) + error$, where time step m is set at 12 months and bond maturities n run from 2 to 5 years (see Campbell and Shiller, 1991). Slope coefficients in projections are based on the average of 100 simulations of 300,000 months of data.

Table 12: **Two-Volatility Model: Currency Markets**

	1m	3m	1y	2y	3y	4y	5y
UIP Projection:							
Nominal	-1.18	-0.81	0.25	0.66	0.81	0.82	0.86
Real	-3.08	-2.79	-1.57	-0.52	0.34	0.73	0.95

	Std. Dev.	AR(1)
Nominal FX Rate:	15.77	0.00
Real FX Rate:	15.68	0.00

Population statistics for changes in foreign exchange rates and term structure of UIP projections in the two-volatility model. UIP Projection reports the slope coefficient β^{UIP} in regression $s_{t+n} - s_t = const + \beta^{UIP}(ny_{t,n} - ny^*_{t,n}) + error$, where $y_{t,n}$ and $y^*_{t,n}$ are foreign and US yields on $n-$period bonds, respectively, and $s_{t+n} - s_t$ is the appreciation rate of the foreign currency over n months. Slope coefficients are based on the average of 100 simulations of 300,000 months.

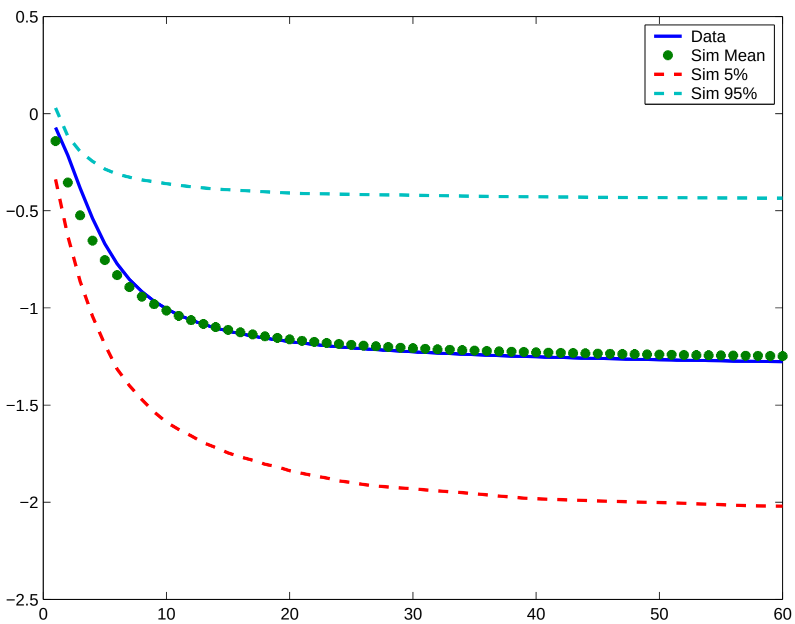
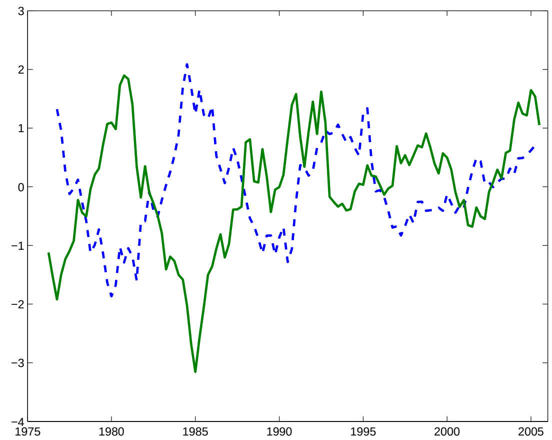
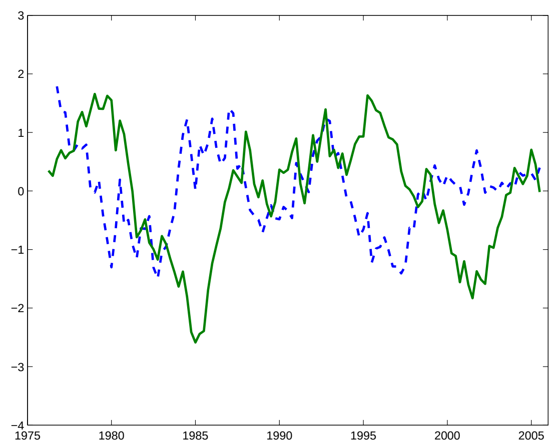


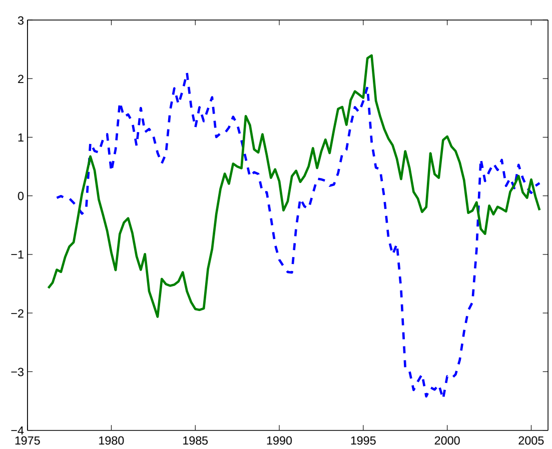
Figure 1: Inflation beta. Computations are based on VAR(1) fit of consumption growth and and inflation rate. Data are quarterly observations of US real consumption growth and inflation from 1976Q2 to 2005Q2. Model output is based on 1,000 simulations of 360 months aggregated to quarterly horizon.



(a) UK

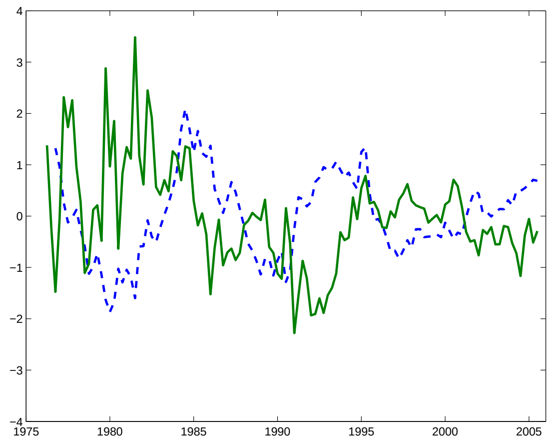


(b) Germany

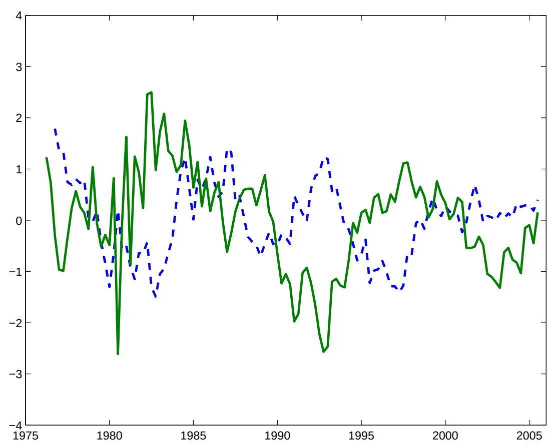


(c) Japan

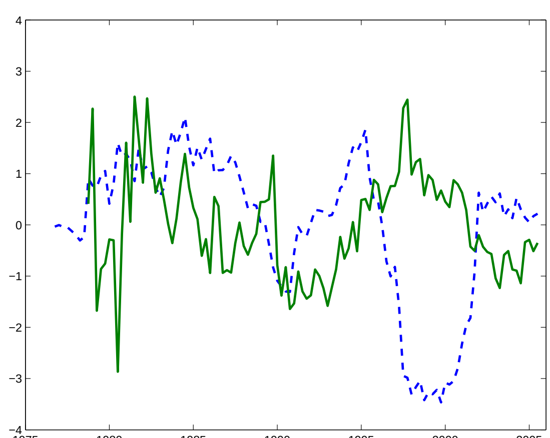
Figure 2: Real exchange rate (solid line) and domestic minus foreign consumption volatility (dashed line).



(a) UK

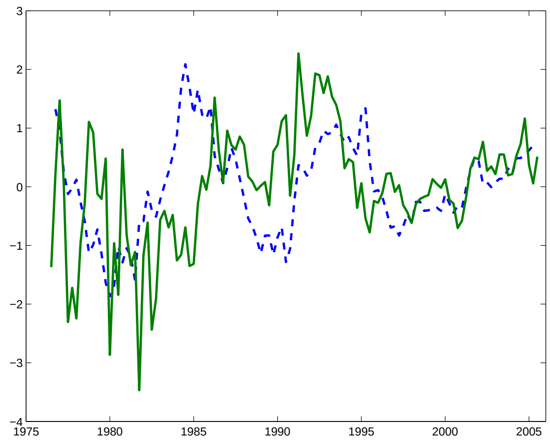


(b) Germany

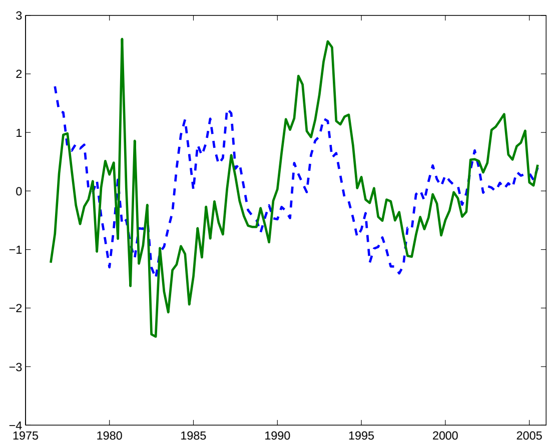


(c) Japan

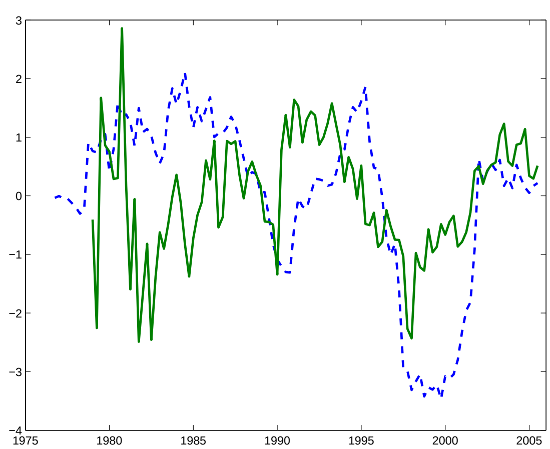
Figure 3: Forward premium (solid line) and domestic minus foreign consumption volatility (dashed line).



(a) UK



(b) Germany



(c) Japan

Figure 4: Ex-ante excess return in foreign bonds (solid line) and domestic minus foreign consumption volatility (dashed line).

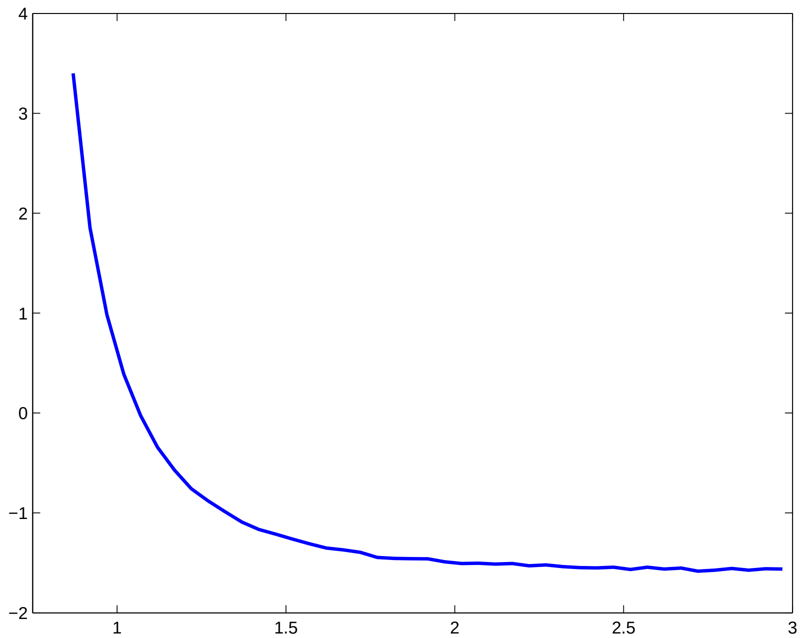


Figure 5: Model-implied slope coefficient in nominal foreign exchange regression for different values of the IES.