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Numerical and arithmetical deficits in learning-disabled children: Relation to dyscalculia and dyslexia

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Cognitive research on the number, counting, and arithmetic competencies of children with a learning disability in arithmetic (AD) is reviewed, and similarities between the associated deficits of AD children and the deficits of individuals afflicted with dyscalculia are highlighted. It is concluded that the defining features of AD and most dyscalculias are difficulties with the procedural features associated with the solving of complex arithmetic problems and difficulties in remembering basic arithmetic facts. The procedural deficits and one form of retrieval deficit appear to be associated with functioning of the prefrontal cortex, while a second form of retrieval deficit appears to be associated with the functioning of the left parieto-occipito-temporal areas and several subcortical structures. The review ends with a discussion of the potential relation between this second form of retrieval deficit and dyslexia.

Acquired and developmental dyscalculia refer to deficits in the processing of numerical and arithmetical information that are associated with overt brain injury or presumed neurodevelopmental abnormalities, respectively. Research in this area has yielded a wealth of insights into the architecture of the cognitive and associated neural systems that support basic quantitative abilities and has provided insights into the core deficits of dyscalculia (Dehaene & Cohen, 1995, 1997; Girelli, Delazer, Semenza, & Denes, 1996; Hittmair-Delazer, Sailer, & Benke, 1995; Levin et al., 1996; McCloskey, Caramazza, & Basili, 1985; McCloskey & Macaruso, 1995; Pesenti, Seron, & Van der Linden, 1994; Semenza, Miceli, & Girelli, 1997; Temple, 1989, 1991). Although the two domains of study are not often linked, the cognitive deficits of children with a learning disability in arithmetic (AD) are very similar to those associated with acquired and developmental dyscalculia (Geary, 1993). The goals here are to provide a review of the numerical and arithmetical deficits of AD children, to illustrate empirical and conceptual links between these deficits and those associated with dyscalculia, and, finally, to explore the relation between AD and dyslexia.

Large-scale studies suggest that AD children constitute between 6% and 7% of the school-age population (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974), and indicate that the number of children affected by AD is comparable to the number of children affected by reading disabilities (RD), or dyslexia. In fact, AD and dyslexia are comorbid in many children (Ackerman & Dykman, 1995). Cognitive studies of AD

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children have, to a large degree, been based on the conceptual models and experimental measures used to study the development of number, counting, and arithmetic competencies in normal children (Geary, 1990, 1994; Jordan, Levine, & Huttenlocher, 1995; Siegler, 1996; Siegler & Shrager, 1984). The use of this approach has provided theoretical coherence to the study of AD children and has enabled their deficits to be understood within the broader context of normal numerical and arithmetical development. The sections that follow provide a brief overview of normal development in the areas of number, counting, and arithmetic, along with discussion of any deficits found with AD children and related deficits in dyscalculia. The final section presents discussion of the potential relation between AD and dyslexia.

NUMERICAL AND ARITHMETICAL COGNITION

Number

Models of children's ability to understand and produce numbers have been informed by research on normally developing children and by research on disruptions in the ability to process and understand the meaning of numbers following brain injury (Fuson, 1988; McCloskey et al., 1985; McCloskey & Macaruso, 1995; Seron & Fayol, 1994). Across these areas, number production and comprehension are understood to require the ability to process verbal (e.g., ''three hundred forty two'') and Arabic representations of numbers (e.g., ''342''), and to transcode, or translate, numbers from one representation to another (e.g., ''three hundred forty two'' to ''342''; Dehaene, 1992; McCloskey, 1992). Number comprehension also requires that the individual understand the meaning of the processed numbers. Early in development, number comprehension is evidenced when, among other things, children associate number names and Arabic representations with the associated quantities and understand ordinal relations among these quantities (e.g., that $3 > 2$; Fuson, 1988; Geary, 1994). Later, children are expected to understand more complex relations among numbers; for instance, that the 3 in 342 represents 3 sets of 100s.

Although not typical (e.g., Semenza et al., 1997; Temple, 1991), disruptions in the ability to understand, produce, or transcode numbers are sometimes evident with acquired (McCloskey, Sokol, & Goodman, 1986) and developmental (Temple, 1989) dyscalculia. The associated deficits can involve difficulties in lexical access—stating a number word that is numerically close to the correct number word (e.g., stating ''nine'' when presented with ''7'', or twenty-two when presented with ''28'')—or in number syntax. Syntax refers to the base-10 structure of the Arabic number system; that is, that the values of the integers in different columns of complex numbers, such as 342, differ by a factor of ten. An understanding of the meaning of the structure of the associated sequence of number words is equally important, such as ''three hundred fifty six'' (e.g., ''six'' refers to 6 units, not 6 tens). Disruptions in the syntactic structure of number processing might be reflected, for instance, in errors in transcoding verbal to Arabic representations, such as transcoding ''eighty four'' as ''804'' (McCloskey et al., 1986; Seron & Fayol, 1994). Difficulties in lexical access and number syntax are often associated with damage to the left hemisphere, while deficits in number comprehension are sometimes associated with damage to the inferior parietal cortex of either hemisphere (Dehaene & Cohen, 1995, 1997; see also Temple & Posner, 1998).

Number production and comprehension skills have not been as systematically studied in AD children as they have in individuals afflicted with dyscalculia. The few studies that have been conducted suggest that the cognitive and neural systems that support number production and comprehension are generally intact in AD children, at least for the processing of simple numbers (e.g., 6 9; Badian, 1983; Geary, 1993; Gross-Tsur et al., 1996; Russell & Ginsburg, 1984). Nonetheless, some first-grade AD children do have difficulties in identifying and producing numbers greater than 10 and in determining which of two consecutive Arabic numbers, such as 8 9, represents the larger quantity (Geary, Hoard, & Hamson, 1999). These difficulties appear to be restricted to first grade (Geary, Hamson, & Hoard, 2000). Nonetheless, it is not currently known whether AD children have deficits in the ability to comprehend and produce more complex numbers, such as 342, above and beyond the difficulties found in normal children (Seron & Fayol, 1994).

Counting

The study of children's counting competencies has largely focused on their understanding of the underlying concepts rather than the ability to count in a rote fashion. Children's counting knowledge appears to emerge from a combination of inherent and experiential factors (Briars & Siegler, 1984; Geary, 1995; Gelman & Gallistel, 1978). Early inherent constraints can be represented by Gelman and Gallistel's five implicit principles; one–one correspondence (one and only one word tag, e.g., ''one'', ''two'', is assigned to each counted object); the stable order principle (the order of the word tags must be invariant across counted sets); the cardinality principle (the value of the final word tag represents the quantity of items in the counted set); the abstraction principle (objects of any kind can be collected together and counted); and, the order-irrelevance principle (items within a given set can be tagged in any sequence). The principles of one–one correspondence, stable order, and cardinality define the ''how to count'' rules, which, in turn, provide constraints on the nature of preschool children's counting behaviour and provide the skeletal structure for children's emerging knowledge of counting.

Children also appear to make inductions about the basic characteristics of counting, by observing standard counting behaviour (Briars & Siegler, 1984; Fuson, 1988). This induced knowledge reflects both essential features of counting, such as those identified by Gelman and Gallistel (1978), and unessential features of counting (Briars & Siegler, 1984). These unessential features include standard direction (counting starts at one of the end points of an array of objects); adjacency (a consecutive count of contiguous objects); pointing (counted objects are typically pointed at but only once); and, start at an end (counting proceeds from left to right). By 5 years of age, many children know the essential features of counting but also believe that adjacency and start at an end are essential features of counting. The latter beliefs indicate that young children's counting knowledge is immature and influenced by the observation of counting procedures.

Individuals with acquired or developmental dyscalculia are generally able to count arrays of objects and recite the correct sequence of number words during the act of counting (e.g., counting from 1 to 20; Hittmair-Delazer et al., 1995; Pesenti et al., 1994; Temple, 1989). However, individuals with damage to the right hemisphere sometimes show difficulties with the procedural component of counting; specifically, difficulties in systematically pointing to successive objects as they are enumerated (Seron et al., 1991). Individuals with damage to the left hemisphere sometimes have difficulties in producing number names. Even with such difficulties, most of these individuals appear to understand many of the basic principles, such as cardinality, identified by Gelman and Gallistel (1978; Seron et al., 1991).

Similarly, Geary and his colleagues found that first-grade children with comorbid AD and dyslexia understand most of the essential features of counting, such as stable order and cardinality (Geary, Bow-Thomas, & Yao, 1992). However, these children consistently made errors on tasks that assess adjacency and order-irrelevance. A more recent study confirmed this finding (Geary et al., 1999; Geary et al., 2000). Using IQ as a covariate, Geary et al. showed that children with comorbid AD and dyslexia and AD-only children made similar errors on order-irrelevance or adjacency tasks in both first and second grade. In contrast, dyslexic children with average or better mathematics achievement scores did not differ from normal children on the associated tasks. The results suggests that many young AD children, regardless of their reading or IQ status, do not understand the order-irrelevance principle, or, from Briars and Siegler's (1984) perspective, they believe that adjacency is an essential feature of counting. The overall pattern suggests that young AD children, as a group, understand counting as a rote, mechanical activity, although it is not currently known whether this immature counting knowledge extends beyond second grade.

In these same studies, it was found that some AD children have difficulties on counting tasks that involve detecting double-counting errors, although the magnitude of this effect is not as large as that found for the order-irrelevance task (Geary et al., 1992; Geary et al., 1999). Specifically, many of these children fail to detect errors that involve double counting the first, but not the last, item (e.g., pointing at the first item twice in succession and stating ''one, two''). Detection of an error when the item is double counted suggests that these AD children understand the one–one correspondence principle. At same time, the failure to note that the double counting of the first item is an error suggests that many of these children cannot retain an ''error notation'' in working memory while monitoring the counting process (see also Hitch & McAuley, 1991; Hoard, Geary, & Hamson, 1999).

Arithmetic

The cognitive competencies associated with the solving of simple arithmetic problems and the development of these competencies have been extensively studied during the past 25 years (e.g., Ashcraft, 1982, 1995; Ashcraft & Fierman, 1982; Carpenter & Moser, 1984; Geary, 1994; Groen & Parkman, 1972; Seigler, 1996; Siegler & Shrager, 1984). Developmental and schooling-based improvements in basic arithmetical competencies are reflected in changes in the distribution of procedures, or strategies, used in problem solving and in advances in children's conceptual understanding of arithmetic and related domains, such as counting (Geary, 1994).

When first learning to solve simple arithmetic problems, such as $5 + 3$, children rely on their knowledge of counting and counting procedures; that is, children typically count the addends to solve such problems. These counting procedures are sometimes executed with the aid of fingers—the finger counting strategy—and sometimes without them—the verbal counting strategy (Siegler & Shrager, 1984). The two most commonly used counting procedures, whether children use their fingers or not, are termed min (or counting-on) and sum (or counting-all; Fuson, 1982; Groen & Parkman, 1972). The min procedure involves stating the larger-valued addend and then counting a number of times equal to the value of the smaller addend, such as counting 5, 6, 7, 8 to solve $5 + 3$. The sum procedure involves counting both addends starting from 1. Occasionally, children will state the value of the smaller addend and then count the larger addend, which is termed the max procedure. The development of procedural competencies is reflected in a gradual shift from heavy reliance on the sum and max procedures to frequent use of min counting (Siegler, 1987). This shift is related, in part, to improvements in the children's conceptual understanding of counting (Geary et al., 1992).

Simple arithmetic problems are also solved by means of memory-based processes, specifically direct retrieval of arithmetic facts, decomposition, and fingers. With direct retrieval, children state an answer that is associated in long-term memory with the presented problem, such as stating ''/eyt/'' (i.e., eight) when asked to solve $5+3$. Decomposition involves reconstructing the answer based on the retrieval of a partial sum. For instance, the problem $6+7$ might be solved by retrieving the answer to $6+6$ (i.e., 12) and then adding 1 to this partial sum. With the fingers strategy, children uplift a number of fingers corresponding to the addends and then state an answer without counting their fingers. The uplifted fingers appear to prompt retrieval of the answer.

The use of memory-based processes appears to follow from the use of counting procedures; that is, the frequent use of counting procedures eventually leads to the formation of associations between problems and the answers generated by means of counting. The problem/answer associations, in turn, provide the basis for direct retrieval, decomposition, and fingers (see Siegler, 1996, for a more thorough discussion). However, the use of retrieval-based processes is moderated by a confidence criterion. The confidence criterion represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer. Children with a rigorous confidence criterion only state answers that they are certain are correct, whereas children with a lenient criterion state any retrieved answer, correct or not (Siegler, 1988).

These developmental findings and the accompanying conceptual models have provided a useful framework for the study of AD children's difficulties in learning basic arithmetic. Studies conducted in the United States, Europe, and Israel have revealed consistent differences in the procedural and memory-based processes used by normal and AD children to solve simple arithmetic problems (e.g., Barrouillet, Fayol, & Lathulière, 1997; Geary, 1990; Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991; Gross-Tsur et al., 1996; Jordan & Montani, 1997; Ostad, 1997, 1998; Räsänen & Ahonen, 1995; Svenson & Broquist, 1975). In fact, the procedural and memory-based deficits that are common in AD children are also common in acquired and developmental dyscalculia (e.g., Geary, 1993; Pesenti et al., 1994; Temple, 1991). Double dissociations between procedural and memory-based processes are often found with dyscalculia (Dehaene & Cohen, 1997; Semenza et al., 1997; Temple, 1991) and are sometimes found in AD children (Geary et al., 1991; Jordan & Montani, 1997). For these reasons, procedural and memory-based deficits are presented in separate sections here.

Procedural deficits. Much of the research on AD children has focused on their use of counting procedures to solve arithmetic problems. When using these procedures, young AD children commit more errors than do their normal peers (Geary, 1990; Jordan et al., 1995; Jordan & Montani, 1997). The errors result as these children miscount or lose track of the counting process. As a group, AD children also rely on finger counting and use the sum procedure more frequently than do normal children. Although it is not yet certain, their use of finger counting appears to be a working memory aid, in that it helps these children to keep track of the counting process. Their delayed use of min counting appears to be related, in part, to their immature counting knowledge (Geary, 1990; Geary et al., 1992). However, many, but not all, of these children show more normal procedural skills by the middle of the elementary school years (Geary et al., 1991; Geary & Brown, 1991; Geary et al., 1999; Jordan & Montani, 1997). For these children, their error-prone use of immature procedures represents a developmental delay rather than a long-term cognitive deficit (Geary & Brown, 1991; Russell & Ginsburg, 1984).

In an assessment of skill at solving more complex arithmetic problems, such as 45×12 or $126 + 537$, Russell and Ginsburg (1984) found that fourth-grade AD children committed more errors than did their IQ-matched normal peers. These errors involved the misalignment of numbers while writing down partial answers or errors while carrying or borrowing from one column to the next. At the same time, these AD children appeared to understand the base-10 system as well as did the normal children, and thus the errors could not be attributed to a poor conceptual understanding of the structure of the problems. Rather, many of the errors appeared to result from difficulties in monitoring the sequence of problem-solving steps and from poor skills in detecting and then self-correcting errors. Similarly, Geary et al. (1992) found that many first-grade children with comorbid AD and dyslexia did not detect and self-correct counting errors as readily as did their normal peers. Although it is not certain, it appears that for many AD children these deficits persist at least through the elementary school years and probably longer (Geary, 1994).

Difficulties in the solving of complex arithmetic problems are also common with acquired and developmental dyscalculia (Semenza et al., 1997; Temple, 1991). As an example, in an extensive assessment of the counting, number, and arithmetic competencies of a 17-year-old (MM) with severe congenital damage to the right frontal and parietal cortices, Semenza and his colleagues reported deficits very similar to those found by Russell and Ginsburg (1984) with AD children. Basic number and counting skills were intact, as was the ability to retrieval basic facts (such as 8 for $5 + 3$) from memory. However, MM had difficulty solving complex division and multiplication problems, such as 32×67. Of particular difficulty was tracking the sequence of partial products. Once the first step was completed (2×7), difficulties in placing the partial product (4) in the correct position and carrying to the next column were evident. Thus, the primary deficit of MM appeared to involve difficulties in sequencing the order of operations and in monitoring the problem-solving process, as is often found with damage to the frontal cortex (Luria, 1980). Temple (1991) reported a similar pattern of procedural difficulties for an individual with neurodevelopmental abnormalities in the right-frontal cortex.

Memory retrieval deficits. Many AD children do not show the shift from procedural-based problem solving to memory-based problem solving that is commonly found in normal children (Geary, Widaman, Little, & Cormier, 1987; Ostad, 1997), suggesting difficulties in storing or accessing arithmetic facts in or from long-term memory. Indeed, disrupted memory-based processes are consistently found with comparisons of AD and normal children (Barrouillet et al., 1997; Bull & Johnston, 1997; Garnett & Fleischner, 1983; Geary, 1993; Geary & Brown, 1991; Geary et al., 1987; Jordan & Montani, 1997; Ostad, 1997, 1998) and are very frequently a feature of dyscalculia (Dehaene & Cohen, 1991, 1997; Hittmair-Delazer et al., 1995; Levin et al., 1996; Pesenti et al., 1994). Disruptions in the ability to retrieve basic facts from long-term memory might, in fact, be considered the defining feature of AD and most, but not all, cases of dyscalculia (Geary, 1993). However, most of these individuals can retrieve some facts, and disruptions in the ability to retrieve facts associated with one operation (e.g., multiplication) are sometimes found with intact retrieval of facts associated with another operation, at least with dyscalculia (e.g., subtraction; Pesenti et al., 1994).

When they retrieve arithmetic facts from long-term memory, AD children commit many more errors than do their normal peers, and show error and reaction time (RT) patterns that often differ from the patterns found with younger, normal children (Geary, 1993; Geary et al., 2000). The RT patterns are similar to the patterns found with children who have suffered from an early (before age 8 years) lesion to the left-hemisphere or associated subcortical regions (Ashcraft, Yamashita, & Aram, 1992). Although this pattern does not necessarily indicate that AD children have suffered from some form of overt brain injury, it does suggest that the memory-based deficits of many AD children may reflect the same mechanisms underlying the retrieval deficits associated with dyscalculia (Geary, 1993; Rourke, 1993).

However, the cognitive and neural mechanisms underlying these deficits are not completely understood. At this point, it appears that there might be two different forms of retrieval deficit, each reflecting a disruption to different cognitive and neural systems (Barrouillet et al., 1997; Dehaene & Cohen, 1997; Geary et al., 2000). The work of Dehaene and his colleagues suggests that the retrieval of arithmetic facts is supported by a system of neural structures, including the left basal ganglia, thalamus, and the left parieto-occipito-temporal areas (Dehaene & Cohen, 1995, 1997). Damage to either the subcortical or cortical structures in this network is associated with difficulties in accessing previously known arithmetic facts (Dehaene & Cohen, 1991, 1997). Cognitive studies of AD children also support the view that their retrieval deficits are due, in part, to difficulties in accessing facts from long-term memory (Geary, 1993). However, it is not currently known if these deficits are associated with damage to or neurodevelopmental abnormalities in the regions identified by Dehaene and Cohen (1995, 1997).

More recent studies of AD children suggest a second form of retrieval deficit; specifically, disruptions in the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations. This form of retrieval deficit was first discovered by Barrouillet et al. (1997), based on the memory model of Conway and Engle (1994), and was recently confirmed by Geary and his colleagues (Geary et al., 2000; see also Koontz & Berch, 1996). In the latter study, children with comorbid AD and dyslexia, AD only, and dyslexia only were compared to their normal peers on an array of number, counting, arithmetic, and memory tasks in first and second grade. One of the arithmetic tasks administered in second grade required the children to only use retrieval—the children were instructed not to use counting strategies—to solve simple addition problems. Children in all of the learning-disability groups committed more retrieval errors than did their normal peers, even after controlling for IQ. The most common of these errors was a counting string associate of one of the addends. For instance, common retrieval errors for the problem $6 + 2$ were 7 and 3, the numbers following 6 and 2, respectively, in the counting sequence.

The pattern in this study and that of Barrouillet et al. (1997) is in keeping with Conway and Engle's (1994) position that individual differences in working memory and retrieval efficiency are related, in part, to inefficient inhibition of irrelevant associations. In this model, the presentation of a to-be-solved problem results in the activation of relevant information in working memory, including problem features—such as the addends in a simple addition problem—and information associated with these features. Problem solving is efficient when irrelevant associations are inhibited and prevented from entering working memory. Insufficient inhibition results in activation of irrelevant information, which functionally lowers working memory capacity. In this view, AD children make retrieval errors, in part, because they cannot inhibit irrelevant associations from entering working memory. Once in working memory these associations either suppress or compete with the correct association for expression. Whatever the cognitive mechanism, these results suggest that the retrieval deficits of some AD children result either from delayed development of the prefrontal cortex or from neurodevelopmental abnormalities in these regions (Bull, Johnston, & Roy, 1999; Luria, 1980; Welsh & Pennington, 1988).

Summary

Most AD children and most individuals afflicted with dyscalculia perform as well as their normal peers on tasks of number production and comprehension. When deficits are found, they often involve difficulties in accessing the appropriate number name, difficulties with the syntax of complex numbers, or in accessing the quantities associated with verbal or Arabic number representations. The former deficits are often associated with lesions to the left hemisphere, while the latter is often associated with damage to the inferior parietal cortices of either hemisphere (Dehaene & Cohen, 1997; Geary, 1993; Seron et al., 1991). At this point, it is not known if these same neural systems are involved in the number comprehension and production difficulties of some young AD children.

For the most part, AD children and individuals afflicted with dyscalculia understand the basic counting principles identified by Gelman and Gallistel (1978; Geary et al., 1992; Seron et al., 1991). Difficulties in the procedural components of counting—difficulties in systematically pointing to objects as they are counted—are sometimes found with lesions to the right hemisphere. Difficulties in retrieving number words during the act of counting are associated with lesions to the left hemisphere (Seron et al., 1991). Although AD children understand the basic principles of counting, they also believe that counting *must* involve counting and tagging with a number word each object in succession; that is, skipping around during the count is not acceptable (Geary et al., 1992; Geary et al., 2000). This finding suggests that many of these children view counting as a rote, mechanical activity, which, in turn, appears to contribute to their delayed use of min counting; because the task that is sensitive to this knowledge has not been administered in the dyscalculia studies, it is not currently known if the same pattern will emerge with these individuals.

All in all, difficulties in solving simple and complex arithmetic problems are the defining feature of dyscalculia and AD (Geary, 1993; Temple, 1991). These difficulties are subsumed under procedural and retrieval deficits. For both AD children and individuals afflicted with dyscalculia, procedural deficits are typically evident during the solving of complex arithmetic problems, such as $362 + 973$, but do not appear to be due to a poor understanding of the associated concepts (e.g., the base-10 system; Semenza et al., 1997). Rather, errors during the solving of these problems appear to reflect difficulties in sequencing the associated component processes (e.g., keeping track of partial sums) and in detecting and self-correcting errors (Russell & Ginsburg, 1984; Semenza et al., 1997; Temple, 1991). With dyscalculia, these difficulties are typically associated with damage to the prefrontal cortex (Luria, 1980; Semenza et al., 1997; Temple, 1991). However, it is not currently known if AD children have neurodevelopmental abnormalities or delayed maturation of the prefrontal cortex, although either of these is a distinct possibility (Bull et al., 1999).

In young AD children, procedural errors are often found during the solving of simple arithmetic problems. When using counting procedures to solve these problems, AD children often miscount, lose track of the counting process, and use developmentally immature procedures (e.g., sum counting on their fingers; Geary, 1990). The latter appears to be related to their rather rigid understanding of counting (Geary et al., 1992). For most of these children, these procedural difficulties appear to reflect a developmental delay rather than a cognitive deficit (Geary et al., 1991) and, given this, it is not likely that the same mechanisms are involved in these procedural difficulties and those associated with the solving of more complex problems.

Difficulties in remembering arithmetic facts and a high frequency of errors and unusual RT patterns for those facts that are retrieved are common in AD and dyscalculia (Dehaene & Cohen, 1991; Garnett & Fleischner, 1983; Geary, 1993; Temple, 1991). These memory deficits appear to take two forms, one involving difficulties in the accessing of facts from long-term memory and the other involving irrelevant associations interfering with the retrieval process (Barrouillet et al., 1997; Geary, 1993). The first of these deficits appears to be associated with damage to or neurodevelopmental abnormalities in the left parieto-occipito-temporal areas or associated subcortical structures, specifically the basal ganglia and thalamus (Dehaene & Cohen, 1995, 1997). The second form of retrieval deficit is most likely related to the functioning of the prefrontal cortex. However, a definitive association between the integrity of these neural areas and AD has yet to be established.

ARITHMETIC DISABILITIES AND DYSLEXIA

For most dyslexic readers, the core deficit involves the neural and cognitive systems that support the processing of language sounds or phonemes (e.g., Morris et al., 1998). This basic deficit is manifested as a lack of phonetic awareness, as well as difficulties in segmenting language sounds and in retrieving words from long-term memory (Denckla & Rudel, 1976; Shankweiler et al., 1995). These difficulties are moderately heritable and are often associated with neurodevelopmental abnormalities in the cortical and subcortical systems that support language processing (Hynd & Semrud-Clikeman, 1989; Olson et al., 1989). The issue here is whether there is a relation between difficulties in processing language sounds and the comorbidity of AD and dyslexia (Rourke & Finlayson, 1978).

Theoretically, difficulties in the processing of language sounds could also result in AD, specifically difficulties in accessing arithmetic facts from long-term memory. As described earlier, the long-term memory representations between arithmetic problems and the associated answers appear to form as children use counting procedures to solve the problems (Siegler & Shrager, 1984). In other words, the repeated use of counting to solve problems, such as $5 + 3$, eventually leads to the formation of an association between the problem and the answers generated by means of counting. After many counts, the presentation of the problem leads to the automatic retrieval of the associated answer. Because counting involves the articulation of number words—that is, the use of the basic phonetic and language systems—the associations in long-term memory between problems and answers should be represented, at least in part, in the same phonetic and semantic memory systems that support word processing and word retrieval. Any neurodevelopmental disruptions in the functioning of these systems might then place the individual at risk for dyslexia and for difficulties in the arithmetical processes that are supported by the same systems, such as fact retrieval.

On the basis of these theoretical considerations and on the comorbidity of AD and dyslexia, Geary (1993, p. 356) argued that these disorders co-occur

> because of a common underlying neuropsychological deficit, perhaps involving the posterior regions of the left hemisphere. At the cognitive level, this deficit manifests itself as difficulties in the representation and retrieval of semantic information from long-term memory. This would include fact retrieval problems in simple arithmetic and, for instance, word-recognition and phonological-awareness difficulties in reading.

Although it now appears that fact-retrieval deficits are more varied than originally believed, recent results confirm that children with comorbid AD and dyslexia are slower at accessing number and word names from long-term memory than are their normal peers and show arithmetic fact-retrieval deficits (Geary et al., 2000). Based on the work of Dehaene and Cohen (1995, 1997) and others (Hynd & Semrud-Clikeman, 1989), it appears that difficulties in accessing arithmetic facts and words from long-term memory might also result from damage to or neurodevelopmental abnormalities in subcortical regions, particularly the basal ganglia and perhaps the thalamus. It still remains to be demonstrated, however, that the neural systems underlying the word retrieval difficulties associated with dyslexia and the fact-retrieval problems associated with AD are one and the same.

CONCLUSION

Acquired and developmental dyscalculia share many features with one another and with AD. The most common of these features are difficulties with the procedural components of solving complex, and sometimes simple, arithmetic problems and with remembering basic arithmetic facts. The procedural deficits as well as one form of retrieval deficit appear to result from damage to or neurodevelopmental abnormalities in the prefrontal cortex, resulting in difficulties in sequencing problem-solving steps, detecting and self-correcting errors, and inhibiting irrelevant associations from entering working memory. The other form of retrieval deficit—difficulties in fact access—appears to result from damage to or neurodevelopmental abnormalities in the posterior regions of the left hemisphere and some subcortical structures, such as the basal ganglia. However, with AD children and learning-disabled children in general, neurological deficits are typically presumed and often not concretely demonstrated. Neuroimaging studies with older children or adults with AD would add greatly to the understanding of the brain systems that support basic quantitative abilities in general and AD in particular.

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