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Particle swarm approach based on quantum mechanics and harmonic oscillator potential well for economic load dispatch with valve-point effects

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ABSTRACT

Particle swarm optimization (PSO) algorithm is population-based heuristic global search algorithm inspired by social behavior patterns of organisms that live and interact within large groups. The PSO is based on researches on swarms such as fish schooling and bird flocking. Inspired by the classical PSO method and quantum mechanics theories, this work presents a quantum-inspired version of the PSO (QPSO) using the harmonic oscillator potential well (HQPSO) to solve economic dispatch problems. A 13-units test system with incremental fuel cost function that takes into account the valve-point loading effects is used to illustrate the effectiveness of the proposed HQPSO method compared with the simulation results based on the classical PSO, the QPSO, and other optimization algorithms reported in the literature.

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ENERGY Conversion and Management

1. Introduction

Electric power utilities are expected to generate their power at minimum cost. The economic dispatch problem (EDP) of electric power generation pertains to the optimum generation scheduling of available generators in the power system to minimize the cost of generation subject to systems constraints [1].

Previous efforts at economic dispatch by engineers and researchers have applied various mathematical programming methods based on several optimization techniques, such as linear programming, dynamic programming, homogenous linear programming, and nonlinear programming techniques [2–8]. In recent years, as an alternative to the conventional mathematical approaches, modern heuristic optimization techniques such as simulated annealing [9], taboo search [10], evolutionary algorithms [11], artificial neural networks [12], fuzzy systems [13], and ant colony [14] have been given much attention by many researchers due to their ability to find an almost global optimal solution in EDPs. One of these heuristic optimization paradigms is particle swarm optimization (PSO) [15–17].

Inspired from nature, PSO is a stochastic search algorithm based on population cooperation and competition of individuals and is motivated by the simulation of social behavior instead of the survival of the fittest individual. It is based on the simulation of simplified social models, such as bird flocking, fish schooling, and the swarming theory [17]. PSO is generally considered to be an evolutionary computation paradigm or swarm intelligence algorithm. In a classical PSO system, a swarm of individuals (called *particles*) fly through the search space. Each particle represents a candidate solution to the optimization problem. The position of a particle is influenced by the best position visited by itself (i.e. its own experience) and the position of the best particle in its neighborhood (i.e. the experience of neighboring particles).

Recently, the concepts of quantum mechanics and computation have motivated the generation of optimization methods, see [18– 20]. Quantum-behaved particle swarm optimization (QPSO), which was proposed in [21,22], is a novel optimization algorithm inspired by the fundamental theory of particle swarm and features of quantum mechanics such as the use of Schrödinger equation and potential field distribution. In the QPSO proposed in [21,22], the deltapotential-well is chosen to be a suitable attractive potential distribution.

This work presents a QPSO approach using another potential distribution, the harmonic oscillator potential well (HQPSO), to solve optimization problems. The harmonic oscillator is a very common potential distribution, and also it is one of the most important model systems in quantum mechanics.

An EDP is employed to demonstrate the performance of the HQPSO. In this context, a 13-units test system [23] with incremental fuel cost function that takes into account the valve-point loading effects is used to illustrate the effectiveness of the proposed HQPSO method. The results obtained with the proposed HQPSO approach were analyzed and compared with the classical PSO, the QPSO [21,22] and other optimization results reported in the literature.

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The remainder of this paper is organized as follows: Section 2 describes the formulation of an EDP, while Section 3 explains the classical PSO and the proposed QPSO, respectively. Section 4 presents the optimization results for the 13-units test system. Lastly, Section 5 outlines our conclusions and future research.

2. Formulation of economic dispatch problem

The primary concern of an economic dispatch problem is to minimize the total fuel cost at thermal power plants subjected to the operating constraints of a power system. Therefore, it can be formulated mathematically with an objective function and two constraints. The equality and inequality constraints are represented by Eqs. (1) and (2) given by

$$\sum_{i=1}^{n} P_i - P_{\rm L} - P_{\rm D} = \mathbf{0},\tag{1}$$

$$P_i^{\min} \leqslant P_i \leqslant P_i^{\max}.$$
 (2)

In the power balance criterion, an equality constraint must be satisfied, as shown in Eq. (1). The generated power should be the same as the total load demand plus total line losses. The generating power of each generator should lie between maximum and minimum limits represented by Eq. (2), where P_i is the power of generator *i* (in MW); *n* is the number of generators in the system; P_D is the system's total demand (in MW); P_L represents the total line losses (in MW) and P_i^{min} and P_i^{max} are, respectively, the output of the minimum and maximum operation of the generating unit *i* (in MW). The total fuel cost function is formulated as follows:

$$\min f = \sum_{i=1}^{n} F_i(P_i), \tag{3}$$

where F_i is the total fuel cost for the generator unity *i* (in \$/h), which is defined by

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i, \tag{4}$$

where a_i , b_i and c_i are cost coefficients of generator *i*.

A cost function is obtained based on the ripple curve for more accurate modeling. This curve contains higher order nonlinearity and discontinuity due to the valve-point effect, and should be refined by a sine function. Therefore, Eq. (4) can be modified [24], as

$$\widetilde{F}_{i}(P_{i}) = F(P_{i}) + \left| e_{i} \sin \left(f_{i} \left(P_{i}^{\min} - P_{i} \right) \right) \right| \text{ or }$$
(5)

$$\widetilde{F}_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \sin\left(f_i \left(P_i^{\min} - P_i\right)\right) \right|,\tag{6}$$

where e_i and f_i are constants of the valve-point effect of generators. Hence, the total fuel cost that must be minimized, according to Eq. (3), is modified to

$$\min f = \sum_{i=1}^{n} \widetilde{F}_i(P_i),\tag{7}$$

where \tilde{F}_i is the cost function of generator *i* (in \$/h) defined by Eq. (6). In the case study presented here, we disregarded the transmission losses, P_L (mentioned in Eq. (1)), i.e., in this work $P_L = 0$.

3. Particle swarm optimization approaches to solve the economic dispatch problem

3.1. Classical particle swarm optimization

The PSO was first introduced by Kennedy and Eberhart in the middle of 90s [15,16]. It is an evolutionary population-based algorithm, where each member is seen as a particle, and each particle is a potential solution to the problem under analysis. Each particle in

PSO has a randomized velocity associated to it, which moves through the space of the problem.

Each particle in PSO keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) it has achieved so far. This value is called pbest (personal best). Another "best" value that is tracked by the global version of the particle swarm optimizer is the overall best value and its location obtained so far by any particle in the population. This location is called gbest (global best).

The PSO concept consists of, in each time step, changing (accelerating) the velocity of each particle flying toward its pbest and gbest locations (global version of PSO). Acceleration is weighted by random terms, with separate random numbers being generated for acceleration toward pbest and gbest locations, respectively. In this work, the gbest version of PSO is adopted. The gbest (star structure) version is a fully connected neighborhood relation. Each particle has all the other particles as neighbors; this implies that the global best particle-position for all particles is identical [25].

The procedure for implementing the global version of PSO is given by the following steps [26–28]:

- Step 1. *Initialization of positions and velocities*: Initialize a population (array) of particles with random positions and velocities in the *n*-dimensional problem space using a uniform probability distribution function.
- Step 2. *Evaluation of particle's fitness*: Evaluate each particle's fitness value.
- Step 3. *Comparison to pbest*: Compare each particle's fitness with the particle's pbest. If the current value is better than pbest, then set the pbest value equal to the current value and the pbest location equal to the current location in *n*-dimensional space.
- Step 4. *Comparison to gbest*: Compare the fitness with the population's overall previous best. If the current value is better than gbest, then reset gbest to the current particle's array index and value.
- Step 5. Updating of each particle's velocity and position: Change the velocity, v_i, and position of the particle, x_i, according to Eqs. (8) and (9):

$$v_{ij}(t+1) = w \cdot v_{ij}(t) + c_1 \cdot r_1 \cdot [p_{ij}(t) - x_i(t)] + c_2 \cdot r_2 \cdot [p_{gj}(t) - x_{ij}(t)],$$

$$(8)$$

$$v_i(t+1) = v_i(t) + At_i \cdot v_i(t+1)$$

 $x_{ij}(t+1) = x_{ij}(t) + \Delta t \cdot v_{ij}(t+1),$ (9) where i = 1, 2, ..., N indicates the particles of population

(swarm); j = 1, 2, ..., n indicates the dimension; $t = 1, 2, ..., t_{max}$ indicates the iterations, w is defined as inertia weight factor; $v_{i,j}(t+1)$ stands for the velocity of the *i*th particle with respect to the *j*th dimension in iteration *t*; and $p_{i,j}(t+1)$ represents the best previous position of the *i*th particle to the *j*th dimension. The variable $p_{g,j}(t)$ is the best previous position among all the particles along the *j*th dimension in iteration t. The first part in Eq. (8) is the momentum part of the particle. The inertia weight w represents the degree of the momentum of the particles. The second part is the 'cognition' part, which represents the independent thinking of the particle itself. Positive constants c_1 and c_2 are the cognitive and social components, respectively, which are the positive acceleration constants responsible for varying the particle speed towards pbest and gbest, respectively. Index g represents the index of the best particle among all the particles in the swarm. Variables r_1 and r_2 are vectors with components uniformly distributed in the range [0,1]. Eq. (9) represents the position update, according to its previous position and its velocity, considering $\Delta t = 1$.

Step 6. *Repeating the evolutionary cycle*: Return to Step 2 until a stop criterion is met, usually a sufficiently good fitness or a maximum number of iterations (generations), t_{max} . Maximum number of iterations criterion is adopted in this work.

Particle velocities in each dimension are clamped to a maximum velocity V_{max} . If the sum of accelerations causes the velocity in that dimension to exceed V_{max} , which is a parameter specified by the user, then the velocity in that dimension is limited to V_{max} . Previous experience with PSO (trial and error, mostly) led us to set the V_{max} to 20% of the dynamic range of the variable in each dimension.

3.2. HQPSO

2

Quantum mechanics is a fundamental approach in the description and understanding of physical and chemical phenomena. In terms of classical mechanics, a particle is depicted by its position vector x and velocity vector v, which determine the trajectory of the particle. The particle moves along a determined trajectory in Newtonian mechanics, but this is not the case in quantum mechanics. In quantum world, the term *trajectory* is meaningless, because x and v of a particle can not be determined simultaneously according to uncertainty principle [22,29]. The quantum particle swarm optimization algorithm allows all particles to move under quantum-mechanical rules rather than the classical Newtonian random motion [30].

On the basis of the fundamental hypotheses proposed by Heisenberg, Schrödinger, Bohr, Dirac, and others established the theory of linear quantum mechanics which describes the properties and motions of microscopic particle systems. This theory states that once the externally applied potential fields and initial states of the particles are given, the states of the particles at any time later and any position can be determined by the Schrödinger equation [31].

In the proposed version of the QPSO algorithm, we apply an attractive potential field that will eventually pull all particles to the location defined by local attractors. In quantum mechanics, this implies that the potential field will generate *bound states*. Details of the bound states and its formulations are presented in [30,32].

In the quantum model of a PSO called here QPSO, the state of a particle is associated with an appropriate time-dependent Schrödinger equation [32,33], instead of position and velocity, given by

$$jh\frac{\partial}{\partial t}\psi(x,t) = \hat{H}(x)\psi(x,t), \qquad (10)$$

where $\hat{H}(x)$ is a time-independent Hamiltonian operator of the system given by

$$\hat{H}(\mathbf{x}) = -\frac{\hbar^2}{2m} \nabla^2 + \mathbf{v}(\mathbf{x}),\tag{11}$$

where h is the Planck's constant, *m* is the mass of the particle, and v(x) is the potential energy distribution. The solutions of the Schrödinger satisfy the linear superposition principle. Solutions $\psi(x,t)$ of the Eq. (10) are called wavefunctions or state functions and are used to describe the behavior of the system. In particular, the product of the wavefunction and its complex conjugate is considered to be a probability density for position.

The dynamic behavior of the particle is widely divergent form that of that the particle in classical PSO systems in that the exact values of *x* and *v* cannot be determined simultaneously. In this context, the probability of the particle's appearing in position *x* from probability density function $|\psi(x,t)|^2$, the form of which depends on the potential field the particle lies in [34].

The design step in deriving the HQPSO algorithm of QPSO proposed in [21,22] is the choice of a suitable attractive potential field that can guarantee bound states for the particles moving in the quantum environment. A potential distribution, which is very common in quantum mechanics, is the harmonic oscillator potential well given by

$$\nu(\mathbf{x}) = \frac{k \cdot \mathbf{x}^2}{2},\tag{12}$$

where k is a parameter defining the well "depth" or "strength." This problem has the following well-known analytical solution

$$\psi_m(\mathbf{x}) = \left(\frac{\alpha}{2^m m! \pi^{1/2}}\right)^{1/2} H_m(\alpha \cdot \mathbf{x}) \cdot e^{-0.5\alpha^2 \mathbf{x}^2},$$
(13)

where $\alpha = (mk/h)^{1/4}$ and H_m is the Hermite polynomial. Eq. (13) shows that multiple possible eigen-states exits in this system, each with integer index *m*. However, we may simplify the problem considerably by assuming that only the lowest possible mode (the ground state m = 0) is available. In this case, the Gaussian probability distribution can be obtained. In this case, the Gaussian probability distribution is given by

$$Q(x) = \frac{\alpha}{\sqrt{\pi}} e^{-x^2 x^2}.$$
 (14)

Again, the characteristic length of this well can be seen to be $\alpha/\sqrt{\pi}$, a quantity that is directly controlled by the strength of the well *k*.

The derivation of the corresponding iterative equation for the harmonic oscillator is presented in [30]. In this context, employing the Monte Carlo method, the particles move according to the following iterative equation:

$$\begin{cases} x_{ij}(t+1) = P_j + \beta \cdot \frac{1}{0.47694 \cdot g} \cdot |\text{Mbest}_j - x_{ij}(t)| \cdot \sqrt{\ln(1/u)}, \\ \text{if } z \ge 0.5 \\ x_{ij}(t+1) = P_j - \beta \cdot \frac{1}{0.47694 \cdot g} \cdot |\text{Mbest}_j - x_{ij}(t)| \cdot \sqrt{\ln(1/u)}, \\ \text{if } z < 0.5 \end{cases}$$
(15)

where i = 1, 2, ..., N indicates the particles of population, β is a design parameter called contraction–expansion coefficient [29]; g is a constant value (g > 1); u and z are values generated according to a uniform probability distribution in the range [0, 1]. In this work, g = 2 is adopted.

The global point called Mainstream Thought or Mean Best (Mbest) of the population is defined as the mean of the pbest positions of all particles and it is given by

$$Mbest = \frac{1}{N} \sum_{j=1}^{N} p_{i,j}(t)$$

= $\left(\frac{1}{N} \sum_{j=1}^{N} p_{i,1}(t), \frac{1}{N} \sum_{j=1}^{N} p_{i,2}(t), \dots, \frac{1}{N} \sum_{j=1}^{N} p_{i,n}(t)\right)$
= $(Mbest_{1}, Mbest_{2}, \dots, Mbest_{n}),$ (16)

where $p_{i,j}$ is the pbest position of *i*th particle to the *j*th dimension. In this case, the local attractor [17,35] to guarantee convergence of the optimization method presents the following coordinates:

$$P_j = \frac{c_1 \cdot p_{ij} + c_2 \cdot p_{gj}}{c_1 + c_2}.$$
(17)

In this case, the potential well will be centered around the vector *P* given by Eq. (17). The procedure for implementing the HQPSO is given by the following steps:

Step 1. *Initialization of swarm positions*: Initialize a population (array) of particles with random positions in the *n*-dimensional problem space using a uniform probability distribution function.

- Step 2. *Evaluation of particle's fitness*: Evaluate the fitness value of each particle.
- Step 3. *Comparison to pbest (personal best)*: Compare each particle's fitness with the particle's pbest. If the current value is better than pbest, then set the pbest value equal to the current value and the pbest location equal to the current location in *n*-dimensional space.
- Step 4. *Comparison to gbest (global best)*: Compare the fitness with the population's overall previous best. If the current value is better than gbest, then reset gbest to the current particle's array index and value.
- Step 5. Updating of global point: Calculate the Mbest using Eq. (16).
- Step 6. Updating of particles' position: Change the position of the particles where c_1 and c_2 are two random numbers generated using a uniform probability distribution in the range [0,1].
- Step 7. *Repeating the evolutionary cycle*: Loop to Step 2 until a stop criterion is met, usually a sufficiently good fitness or a maximum number of iterations (generations), *t*_{max}.

4. Case study of 13 thermal units and analysis of optimization results

This case study consisted of 13 thermal units of generation with the valve-point effects, as given in Table 1. The system data shown in Table 1 is also available in [23,36]. In this case, the load demand expected to be determined was $P_D = 1800$ MW.

Each optimization method was implemented in Matlab (Math-Works). All the programs were run on a 3.2 GHz Pentium IV processor with 2 GB of random access memory. In each case study, 50 independent runs were made for each of the optimization methods involving 50 different initial trial solutions for each optimization method.

In this case study, the population size *N* was 20 particles and the stopping criterion t_{max} was 800 generations for the classical PSO, QPSO and HQPSO approaches. The parameters used for verifying the performance of the classical PSO in searching the decision variables of EDP are:

- acceleration constant c_1 = 2.05 and c_2 = 2.05;
- *w* decreases linearly from about 0.9–0.4 during a run.The setup of QPSO and HQPSO approaches used was the following:
- QPSO: QPSO proposed in [21,22] using a constant contraction– expansion coefficient given by *β* = 0.6;
- HQPSO(1): HQPSO using a constant contraction–expansion coefficient given by β = 0.6;
- HQPSO(2): HQPSO using $\beta = 0.7$;
- HQPSO(3): QPSO using $\beta = 0.8$;

Table 1				
Data for	the	13	thermal	units

Thermal unit	P_i^{\min}	P_i^{\max}	а	b	С	е	f
1	0	680	0.00028	8.10	550	300	0.035
2	0	360	0.00056	8.10	309	200	0.042
3	0	360	0.00056	8.10	307	150	0.042
4	60	180	0.00324	7.74	240	150	0.063
5	60	180	0.00324	7.74	240	150	0.063
6	60	180	0.00324	7.74	240	150	0.063
7	60	180	0.00324	7.74	240	150	0.063
8	60	180	0.00324	7.74	240	150	0.063
9	60	180	0.00324	7.74	240	150	0.063
10	40	120	0.00284	8.60	126	100	0.084
11	40	120	0.00284	8.60	126	100	0.084
12	55	120	0.00284	8.60	126	100	0.084
13	55	120	0.00284	8.60	126	100	0.084

- HQPSO(4): QPSO with adaptive control parameter using a linear reduction of β with initial and final values of 0.8 and 0.6, respectively:
- HQPSO(5): QPSO using a sinusoidal function for the contractionexpansion coefficient given by $\beta = \alpha + |A \cdot \sin(\omega \cdot t)|$, where *A* is the amplitude of signal and ω is the angular frequency of signal. The choice of values was $\alpha = 0.6$, A = 0.2 and $\omega = 0.1 \cdot t$.

A key factor in the application of optimization methods is how the algorithm handles the constraints relating to the problem. In this work, the penalty-based method proposed in [11] was used.

The results in Table 2 illustrated that HQPSO(4) and HQPSO(5) has the highest probability of achieving better solutions among these tested PSO algorithms. The results obtained for this case study are given in Table 2, which shows that the QPSO(5) has both a better economic cost and lower mean cost than the other tested PSO approaches. However, the QPSO(4) outperformed the other tested methods in terms of maximum cost.

The best results obtained for solution vector P_i , i = 1, ..., 13 with HQPSO(5) with minimum cost of 17963.9571 \$/h is given in Table 3. Furthermore, considering the accurate cost model, the proposed HQPSO(5) algorithm not only has the best economic dispatch of tested methods, but also completely satisfies the system constraints.

Table 4 compares the results obtained in this paper with those of other studies reported in the literature. Note that in studied case,

Table 2

Convergence results (50 runs) of a case study of 13 thermal units with valve-point and $P_{\rm D}$ = 1800 MW

Optimization	Maximum	Minimum	Mean cost	Standard
method	cost (\$/h)	cost (\$/h)	(\$/h)	deviation (\$/h)
PSO	18878.8271	18239.7537	18589.1527	128.3804
QPSO	18760.2152	18321.4745	18291.4029	126.7042
HQPSO(1)	18693.5371	18146.7234	18391.3499	118.7776
HQPSO(2)	18542.4122	18083.6341	18278.8418	111.2173
HQPSO(3)	18795.8387	18134.1893	18280.3750	116.9973
HQPSO(4)	18524.2866	18092.7130	18274.9907	112.9353
HQPSO(5)	18633.0435	17963.9571	18273.8610	123.2242

Table 3

Best result (50 runs) obtained for the case study using HQPSO(5)

Power	Generation (MW)	Power	Generation (MW)
P_1	628.3180	P_8	60.0000
P_2	149.1094	P_9	109.8664
P ₃	223.3226	P ₁₀	40.0000
P_4	109.8650	P ₁₁	40.0000
P_5	109.8618	P ₁₂	55.0000
P_6	109.8656	P ₁₃	55.0000
P ₇	109.7912	$\sum_{i=1}^{13} P_i$	1800.0000

Table 4

Comparison of best results for fuel costs presented in the literature

Optimization method	Total cost (\$/h) for the case study with 13 thermal units
Evolutionary programming [23]	17994.07
Particle swarm optimization [37]	18030.72
Hybrid evolutionary programming with SQP ^a [37]	17991.03
Hybrid particle swarm with SQP [37]	17969.93
Improved genetic algorithm [38]	17963.9848
Modified particle swarm optimization [39]	17973.34
Best result of this paper using HQPSO(5)	17963.9571

^a SQP: sequential quadratic programming.

the best result reported here using HQPSO(5) is comparatively lower than recent studies presented in the literature.

5. Conclusions and future research

Most power system optimization problems, including EDP, have complex and nonlinear characteristics with heavy equality and inequality constraints. Traditionally, optimization methods involved derivative-based techniques. Such techniques can encounter difficulties in EDPs such as getting trapped in local minima, increasing computational complexity, and not being applicable to certain classes of objective functions. This led to the need of developing a new class of solution methods that can overcome these shortcomings. Optimization techniques, such as PSO, are fast growing tools that can overcome most of the limitations found in derivative-based techniques [40]. Recently, many researchers attempted to apply the PSO algorithm in solving optimization problems in electric power systems [37,39–52].

This paper discusses the use of a HQPSO to solve economic dispatch problems. The classical PSO, QPSO and HQPSO methodologies were validated for a test system consisting of 13 thermal units whose incremental fuel cost function takes into account the valve-point loading effects.

The proposed HQPSO(5) has the lowest cost of all methods tested, demonstrating that the proposed algorithm is more effective than other methods for the case study consisted of 13 thermal units of generation. The tested HQPSO techniques provide new approaches with promising new features. In future work, we plan to study the HQPSO techniques applied to multi-objective optimization in EDPs.

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