Compressed Wideband Sensing in Cooperative Cognitive Radio Networks

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***Abstract*— In emerging cognitive radio (CR) networks with spectrum sharing, the first cognitive task preceding any dynamic spectrum access is the sensing and identification of spectral holes in wireless environments. This paper develops a distributed compressed spectrum sensing approach for (ultra-)wideband CR networks. Compressed sensing is performed at local CRs to scan the very wide spectrum at practical signal-acquisition complexity. Meanwhile, spectral estimates from multiple local CR detectors are fused to collect spatial diversity gain, which improves the sensing quality especially under fading channels. New distributed consensus algorithms are developed for collaborative sensing and fusion. Using only one-hop local communications, these distributed algorithms converge fast to the globally optimal solutions even for multi-hop CR networks, at low communication and computation load scalable to the network size.**

Keywords: ***Cognitive Radio, Compressed Spectrum Sensing, Distributed Fusion, Collaborative Sensing, Consensus***

I. INTRODUCTION

In cognitive radio (CR) networks adopting hierarchical spectrum access, unlicensed secondary CR users dynamically seek transmission opportunities over spectral bands that are temporarily unoccupied by primary communication systems holding the license of the spectrum [1]. The first cognitive task is hence to sense and identify those unused spectral holes. The increasingly popular (ultra-)wideband wireless networks not only offer high throughput and user capacity for primary communication systems, but also purport pronounced dynamic spectrum access opportunities for secondary CR users. On the other hand, spectrum sensing in the wideband regime also faces considerable technical challenges.

A primary challenge in wideband sensing stems from the high RF signal acquisition costs of current analog-to-digital hardware technology. Very high sampling rates are required by conventional spectral estimation methods which have to operate at or above the Nyquist rate. Meanwhile, the stringent timing requirements for monitoring the dynamically changing spectrum only allow for a limited number of measurements to be collected for sensing, which makes it challenging to reliably perform high-resolution signal reconstruction.

Furthermore, wireless fading constitutes a major factor of performance degradation to traditional spectrum detection techniques [2]. A CR user may not be able to accurately sense and detect the transmission of a primary system due to several channel fading effects, including both large-scale path loss and small-scale deep fades that are random and unpredictable. When a missed detection arises, the CR may unwittingly transmit over the same channels used by active primary users, causing detrimental interference to legacy services.

To provide reliable spectrum sensing at affordable complexity, this paper presents a distributed compressed sensing framework for wideband communication networks. First, we recognize that the wireless signals in open-spectrum networks are typically *sparse* in the frequency domain. This is due to the low percentage of spectrum occupancy by active radios – a fact motivating dynamic spectrum access. For sparse signals, recent advances in compressive sampling have demonstrated the principle of sub-Nyquist-rate sampling and reliable signal recovery via computationally feasible algorithms [4]–[6]. We develop a compressed sensing technique for detecting wideband signals at reduced signal sampling and acquisition costs.

Next, we deal with the wireless channel fading effects by performing collaborative fusion among multiple spatially distributed CRs in the network. Because channels fade independently among CRs, collaborative fusion enables spatial diversity gain and improves the sensing accuracy. Current research on collaborative sensing focuses on small-size one-hop networks operating in flat fading channels; e.g., [3]. In contrast, this paper contributes to develop distributed fusion techniques for multi-hop large networks operating in frequency selective fading channels. Consensus-based techniques [10], [11] are utilized to derive new distributed fusion solutions that converge to the globally optimal solutions, using only one-hop local communications among CR neighbors. The computational complexity of these distributed fusion techniques is scalable to the network size. Corroborating simulation results are presented to testify the effectiveness of the proposed compressed sensing and distributed fusion techniques in identifying and locating the spectral hole opportunities in wireless fading environments.

II. SIGNAL MODELING AND PROBLEM STATEMENT

Consider a (ultra-)wide frequency band that hosts both primary communication systems and secondary CR users. Different from the compressed sensing scheme in [9], we consider a slotted frequency segmentation structure in which the entire wideband channel is divided into M non-overlapping narrowband subchannels (a.k.a. slots) centered at $\{f_m\}_{m=0}^{M-1}$.

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The locations of these slots are pre-defined and known, as in multi-band radios and OFDM systems, but their power spectral levels are unknown and dynamically varying, depending on whether they are occupied by primary users or not in a particular geographical region and time. Those temporarily idle subchannels are termed spectral holes and are available for opportunistic spectrum access by secondary users. There are J spatially distributed CRs that collaboratively sense the wide band in order to identify the spectral holes, giving rise to the spectrum sensing and detection problem.

During each detection period, we assume for simplicity that higher layer protocols (e.g., the medium access control layer) can guarantee that all CRs stay silent such that only the primary users are emitting spectral power. Suppose that there are I active primary users during the detection interval, whose transmitted signals are denoted by $s_i(t)$, $i = 1, \ldots, I$. After propagating through a wireless fading channel, the signal $s_i(t)$ reaches the j-th CR receiver in the form $h_{ij}(t) \star s_i(t)$, where $\star$ denotes convolution and $h_{ij}(t)$ is the channel impulse response that is typically frequency selective over the wide band. We assume that the channels are slowly varying and can be treated as time-invariant during the detection interval. The received signal at CR j is thus given by

$$r_j(t) = \sum_{i=1}^{I} h_{ij}(t) \star s_i(t) + w_j(t)$$

where the ambient noise $w_j(t)$ is white Gaussian with zero mean and power spectral density (PSD) σ_w^2.

To reflect the discretized signal response on the M subchannels, we take an M-point discrete Fourier transform (DFT) on $r_j(t)$, where M is larger than the channel memory length. Collecting the frequency-domain samples into an $M \times 1$ vector $\mathbf{r}_f^{(j)}$, we have

$$\mathbf{r}_f^{(j)} = \sum_{i=1}^{I} \mathbf{D}_h^{(ij)} \mathbf{s}_f^{(i)} + \mathbf{w}^{(j)} \tag{1}$$

where $\mathbf{D}_h^{(ij)} = \text{diag}(\mathbf{h}_f^{(ij)})$ is an $M \times M$ diagonal channel matrix, and $\mathbf{h}_f^{(ij)}$, $\mathbf{s}_f^{(i)}$ and $\mathbf{w}_f^{(j)}$ are the frequency-domain discrete versions of $h_{ij}(t)$, $s_i(t)$ and $w_j(t)$ respectively. This signal model can be written in a general form as

$$\mathbf{r}_f^{(j)} = \mathbf{H}_f^{(j)} \bar{\mathbf{s}}_f^{(j)} + \mathbf{w}^{(j)}. \tag{2}$$

In the absence of the channel state information (CSI) $\{\mathbf{h}_f^{(ij)}\}_i$ at each CR receiver j, it is useful to lump all the transmitted signals as follows ($\mathbf{I}_M$ is an identity matrix):

$$\mathbf{H}_f^{(j)} = \mathbf{H}_f := \mathbf{I}_M; \quad \bar{\mathbf{s}}_f^{(j)} := \sum_{i=1}^{I} \mathbf{D}_h^{(ij)} \mathbf{s}_f^{(i)}. \tag{3}$$

When each CR knows the channels $\mathbf{h}_f^{(ij)}$, one can adopt

$$\begin{aligned} \mathbf{H}_f^{(j)} &:= \left[\mathbf{D}_h^{(1j)}, \cdots, \mathbf{D}_h^{(Ij)}\right]; \\ \bar{\mathbf{s}}_f^{(j)} &= \bar{\mathbf{s}}_f := \left[(\mathbf{s}_f^{(1)})^T, \cdots, (\mathbf{s}_f^{(I)})^T\right]^T, \quad \forall j. \end{aligned} \tag{4}$$

At each CR, *spectrum sensing* boils down to estimating $\bar{\mathbf{s}}_f^{(j)}$ in (2) from $r_j(t)$. In the absence of the CSI, the CR cannot decouple each product $\mathbf{D}_h^{(ij)} \mathbf{s}_f^{(i)} = \mathbf{h}_f^{(ij)} \odot \mathbf{s}_f^{(i)}$ ($\odot$ denotes element-by-element multiplication); hence, the estimated $\bar{\mathbf{s}}_f^{(j)}$ in (3) entails the unknown channel gain. With CSI, it is possible to estimate individual sources $\{\mathbf{s}_f^{(i)}\}_{i=1}^{I}$ from $\bar{\mathbf{s}}_f^{(j)}$ in (4). It is of interest to investigate the impact of the channel knowledge on spectrum sensing quality, yet the sensing techniques we will develop are applicable for both cases, based on the general expression in (2).

Depending on the spectrum sharing protocol adopted, the CRs might not be interested in the signal strength $\bar{\mathbf{s}}_f^{(j)}$, but simply want to know which of the M subchannels are unoccupied spectral holes. All CRs avoid transmitting at any occupied subchannels, but dynamically share the spectral holes among themselves. In this case, the spectrum sensing task is reduced to *spectrum detection*. The goal is to determine the frequency occupancy of primary users by detecting a binary state vector $\mathbf{d} \in \{0, 1\}^{M \times 1}$, whose m-th element is defined by

$$d[m] = \begin{cases} 1, & \text{if } f_m \text{ is occupied, i.e., } \exists i : \mathbf{s}_f^{(i)}[m] \neq 0 \\ 0, & \text{if all primary users are silent on } f_m \end{cases} \tag{5}$$

Spectrum detection is feasible even in the absence of the channel knowledge. In (3), when $\bar{\mathbf{s}}_f^{(j)}[m] \neq 0$, at least one of the sources $s_i(f)$ is emitting on f_m, and the channel fading $h_{ij}(f_m)$ on this frequency is non-zero. In another words, occupied subchannels would correspond to non-zero elements in $\bar{\mathbf{s}}_f^{(j)}$, whereas the rest zero elements in $\bar{\mathbf{s}}_f^{(j)}$ reflect idle frequency opportunities for CRs. Exceptions arise when a deep channel fade $h_{ij}(f_m)$ makes it inaccurate to detect a non-zero $\mathbf{s}_f^{(i)}[m]$ from $\bar{\mathbf{s}}_f^{(j)}[m]$, resulting in missed detection.

Hence, the spectral detection problem can be tackled by finding the (non-)zero elements in the noise-free version $\bar{\mathbf{s}}_f^{(j)}$ of the received signal spectrum $\mathbf{r}_f^{(j)}$. Because detection by one CR receiver is subject to missed detection due to channel fading, it is necessary for a network of spatially diverse CRs to collaborate during the spectral detection phase.

III. Collaborative Compressed Spectrum Sensing

To achieve high-performance spectrum sensing at practical complexity, this section contributes to develop collaborative sensing schemes that utilize compressive sampling in the temporal-frequency domain and fused detection in the spatial domain. Our schemes consist of two steps: i) compressed spectrum sensing at individual CRs to estimate $\bar{\mathbf{s}}_f^{(j)}$, and make local decisions $\{\hat{\mathbf{d}}^{(j)}\}_j$ if needed, at low sampling complexity; and, ii) collaborative, distributed spectral detection/estimation across the network to collect spatial diversity gain.

A. Compressed Spectrum Sensing at Individual CRs

Let us now turn to each of the CR receivers j, $j = 1, \ldots, J$. In this subsection we drop the index j for notational simplicity. Locally, the goal is to estimate $\bar{\mathbf{s}}_f$ in (2) given $\mathbf{H}_f$ and $r(t)$. To this end, we develop a compressive sampling technique that reduces the sampling costs in the wideband regime. Our development bears resemblance to the compressed sensing approach in [9], but with different goals: this work seeks to estimate the spectral shape $\bar{\mathbf{s}}_f$ given the slotted subchannel structure, whereas [8], [9] aims to find the unknown frequency locations of occupied spectrum segments via edge detection.

The first step of compressive sampling is to collect time-domain samples. Motivated by the need to reduce the sampling burden in the wideband regime, we adopt a linear random sampler at each CR to collect a $K \times 1$ time-domain sample vector $\mathbf{x}_t$ from $r(t)$, $K \leq M$, as follows:

$$\mathbf{x}_t = \mathbf{S}^T \mathbf{r}_t \tag{6}$$

where the $M \times 1$ vector $\mathbf{r}_t$ is the discrete-time representation of $r(t)$ and $\mathbf{S}$ is an $M \times K$ projection matrix. Columns $\{\mathbf{s}_k\}_{k=1}^K$ of $\mathbf{S}$ can be viewed as a set of basis functions or matched filters used to collect the time-domain samples, while the measurements $\{x_t[k]\}_{k=1}^K$ are in essence the projection of $r(t)$ onto the bases. The model in (6) subsumes all sampling schemes yielding linear measurements. For example, $\mathbf{S} = \mathbf{I}_M$ represents Nyquist-rate uniform sampling, while reduced-rate linear sampling arises when $K < M$. We adopt a simple selection matrix $\mathbf{S} = \mathbf{S}_c$ that randomly retains $K(< M)$ columns of the size-M identity matrix $\mathbf{I}_M$. It amounts to collecting samples on the Nyquist sampling grid but skipping randomly $(K - M)$ time instants to reduce the average sampling rate.

With the K measurements $\mathbf{x}_t = \mathbf{S}_c^T \mathbf{r}_t$, we now estimate the frequency response $\bar{\mathbf{s}}_f$ in (2). Noting $\mathbf{r}_t = \mathbf{F}_M^{-1} \mathbf{r}_f$, we have

$$\mathbf{x}_t = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{r}_f = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{H}_f \bar{\mathbf{s}}_f + \tilde{\mathbf{w}}_f \tag{7}$$

where $\tilde{\mathbf{w}}_f = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{w}_f$ is the noise sample vector that remains to be white Gaussian. Because the spectrum utilization by the primary network is low – a fact motivating dynamic spectrum access at the outset, the unknown vector $\bar{\mathbf{s}}_f$ is sparse with only a small number of non-zero elements. The sparsity measure is given by the l-norm $||\bar{\mathbf{s}}_f||_l$, $l \in [0, 2)$, where the l_0-norm ($l = 0$) indicates exact sparsity [5]. Recent literature has seen the emergence of signal reconstruction techniques developed under the compressive sampling framework [6]. For example, the Basis Pursuit (BP) technique [7] solves the following linear convex optimization problem:

$$\min_{\bar{\mathbf{s}}_f} ||\bar{\mathbf{s}}_f||_1, \quad s.t. \;\; \mathbf{x}_t = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{H}_f \bar{\mathbf{s}}_f. \tag{8}$$

In general, we use $\mathbf{s} = \mathrm{CS}(\mathbf{x}; \mathbf{A})$ to represent a signal recovery algorithm (e.g., BP, OMP, LASSO etc. [6]) for solving the sparse vector $\mathbf{s}$ in a linear regression model $\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{w}$, where $\mathbf{w}$ is Gaussian noise. In this notation, the local spectral estimation solution to (7) can be expressed as

$$\hat{\bar{\mathbf{s}}}_f = \mathrm{CS}(\mathbf{x}_t; \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{H}_f). \tag{9}$$

B. Distributed Collaborative Spectrum Detection

To collect spatial diversity gain, we first consider CR collaboration for *spectrum detection* in the absence of channel knowledge. The signal model in (3) becomes relevant to reach the detection decision in (5).

Locally at each CR receiver, a decision on the spectrum state vector $\mathbf{d}$ can be made by comparing the local spectral estimate $\hat{\bar{\mathbf{s}}}^{(j)}$ obtained in (9) with a decision thresold η_j:

$$\hat{\mathbf{d}}^{(j)} = \left(|\hat{\bar{\mathbf{s}}}_f^{(j)}| \geq \eta_j \right), \quad j = 1, \ldots, J. \tag{10}$$

The threshold η_j can be chosen based on a desired level of probability of false alarms P_{fa}, using the well-known Neyman-Pearson binary hypothesis test rule.

Globally at the network level, the sufficient statistic for optimal decision fusion is the average value $\bar{\mathbf{c}} = \frac{1}{J} \sum_{j=1}^J \hat{\mathbf{d}}^{(j)}$. This can be done by a spectrum controller that collects all local detection outputs and compute $\bar{\mathbf{c}}$ in a centralized manner. However, centralized sensing is sensitive to node failure and incurs heavy communication overhead. Especially for a multi-hop CR network, extra routing is needed to convey local CR decisions to the spectrum controller.

Our goal here is to design a distributed and decentralized fusion rule that is scalable to the network size J. Local one-hop broadcasting is allowed among neighboring CRs, but multi-hop routing is to be avoided. To compute $\bar{\mathbf{c}}$ in a distributed manner, we propose to adopt the average-consensus technique [10], which is an algorithm design technique for distributed computing over a large network.

To represent an average consensus problem, Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be an undirected connected graph with node set $\mathcal{N} = \{1, \ldots, J\}$ and edge set $\mathcal{E}$, where each edge $(j, k) \in \mathcal{E}$ is an unordered pair of distinct nodes within the one-hop communication range. Let $\mathbf{c}_j(0) := \hat{\mathbf{d}}^{(j)}$ be a real vector associated with CR node j at time $t = 0$. The (distributed) average consensus problem is to compute the average $(1/J) \sum_{j=1}^J \mathbf{c}_j(0)$ at every node, via local communication and computation on the graph. Specifically, each node j broadcasts $\mathbf{c}_j(t)$ to its neighbors $\mathcal{N}_j = \{k | (j, k) \in \mathcal{E}\}$ and updates itself by adding a weighted sum of the local discrepancies, i.e., the differences between neighboring node values and its own [10]:

$$\mathbf{c}_j(t+1) = \mathbf{c}_j(t) + \sum_{k \in \mathcal{N}_j} w_{jk} \left(\mathbf{c}_k(t) - \mathbf{c}_j(t) \right), \quad \forall j, t \tag{11}$$

where w_{jk} is a weight associated with the edge (j, k). These weights are algorithm parameters, and some design rules for $\{w_{jk}\}$ are delineated in [10]. With properly designed weights, it can be guaranteed that

$$\lim_{t \to \infty} \mathbf{c}_j(t) = \frac{1}{J} \sum_{k=1}^J \mathbf{c}_k(0) = \frac{1}{J} \sum_{k=1}^J \hat{\mathbf{d}}^{(k)} = \bar{\mathbf{c}}, \quad \forall j = 1, \ldots, J. \tag{12}$$

Thus, through local one-hop communications, each CR obtains the averaged statistic $\bar{\mathbf{c}}$ of the entire multi-hop network.

Subsequently, each CR can make the fusion decision on $\mathbf{d}$ straightforwardly by comparing $\mathbf{c}_j(t)$ with a threshold c_{th} at a sufficiently large t. The choice of c_{th} reflects how conservative the network is in taking spectrum opportunities. A conservative CR network decides the presence of a primary user as long as one of the J CRs claims detection, which corresponds to $c_{th} = 1/J$. A more aggressive network may take a majority vote by setting $c_{th} = 1/2$.

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Putting together, the distributed spectrum detection algorithm can be summarized below:

- s1) each CR makes its local decision $\hat{\mathbf{d}}^{(j)}$ via compressed sensing in (9) and thresholding in (10), $j = 1, \ldots, J$;
- s2) each CR sets $\mathbf{c}_j(0) = \hat{\mathbf{d}}^{(j)}$, $j = 1, \ldots, J$, and collaboratively iterates through (11) until convergence $\mathbf{c}_j(t) = \bar{\mathbf{c}}$;
- s3) each CR makes the fusion decision via thresholding

$$\hat{\mathbf{d}} = (\mathbf{c}_j(t) \geq c_{th}).$$

C. Distributed Collaborative Spectral Estimation

When each CR acquires its own CSI $\mathbf{H}_f^{(j)}$ in (4), it is sensible for the network to fuse all the local measurements $\{\mathbf{x}_t^{(j)}\}_{j=1}^J$ in (7) to estimate the common transmit spectrum vector $\bar{\mathbf{s}}_f$ in (4), giving rise to the collaborative *spectral estimation* problem. Channel occupancy decisions made from $\bar{\mathbf{s}}_f$ is immune to channel fading effects, at the expense of CSI estimation efforts at local CRs.

When a fusion center is present, a globally optimal fusion estimate of $\bar{\mathbf{s}}_f$ can be obtained by stacking all measurements into a $KJ \times 1$ vector and solving the corresponding linear regression problem as follows:

$$\hat{\bar{\mathbf{s}}}_f = \mathrm{CS}\left(\begin{bmatrix} \mathbf{x}_t^{(1)} \\ \vdots \\ \mathbf{x}_t^{(J)} \end{bmatrix}; \begin{bmatrix} (\mathbf{S}_c^{(1)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(1)} \\ \vdots \\ (\mathbf{S}_c^{(J)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(J)} \end{bmatrix} \right) \quad (13)$$

Apparently, this fusion formula is costly to implement, because the fusion center needs to know, in addition to the measurements, the sampling matrices $\{\mathbf{S}_c^{(j)}\}_j$ and channel matrices $\{\mathbf{H}_f^{(j)}\}_j$ from all CRs, not mention the computational load.

To overcome the implementation challenges in centralized fusion, we develop a distributed sensing algorithm using consensus techniques in conjunction with the alternating direction method of multipliers. We define local copies $\bar{\mathbf{s}}_f^{(j)}$ of $\bar{\mathbf{s}}_f$ and constrain them to consent with one-hop neighbors, as in [11]. Using the BP algorithm for illustration, each CR locally carries out the following optimization:

$$\begin{aligned} \min_{\bar{\mathbf{s}}_f^{(j)}} \quad & ||\bar{\mathbf{s}}_f^{(j)}||_1, \\ s.t. \quad & \mathbf{x}_t^{(j)} = (\mathbf{S}_c^{(j)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(j)} \bar{\mathbf{s}}_f^{(j)} \\ & \bar{\mathbf{s}}_f^{(j)} = \bar{\mathbf{s}}_f^{(k)}, \quad \forall k \in \mathcal{N}_j \end{aligned} \quad (14)$$

Here CR j hears the broadcasts of $\bar{\mathbf{s}}_f^{(k)}$ from all one-hop neighbors $k \in \mathcal{N}_j$ and uses the constraints (14) to enforce consensus. There is no need to exchange CSI or sampling matrices among CRs.

While (14) is conceptually illuminating, we still need to design a distributed algorithm that is amenable to implementation. The distributed consensus algorithm in (11) is not directly applicable: it simply performs data averaging, whereas our spectral estimation problem in (14) requires linear regression on a sparse vector. We combine consensus averaging with sparsity-constrained linear regression to develop a new iterative procedure for distributed collaborative sensing, as follows:

- s1) at $t = 0$, each CR initializes its local spectral estimate as $\mathbf{c}_j(0) = \mathrm{SC}(\mathbf{x}_t^{(j)}; (\mathbf{S}_c^{(j)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(j)})$, $j = 1, \ldots, J$;
- s2) each CR locally iterates through following steps over t:
  - a) compressing sensing on enhanced measurements:

$$\hat{\bar{\mathbf{s}}}_f^{(j)}(t) = \mathrm{CS}\left(\begin{bmatrix} \mathbf{x}_t^{(j)} \\ \mathbf{c}_j(t) \end{bmatrix}; \begin{bmatrix} (\mathbf{S}_c^{(1)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(1)} \\ \mathbf{I}_M \end{bmatrix} \right) \quad (15)$$

  - b) consensus averaging: set $\mathbf{c}_j(t) = \hat{\bar{\mathbf{s}}}_f^{(j)}(t)$ and update $\mathbf{c}_j(t+1) = \mathbf{c}_j(t) + \sum_{k \in \mathcal{N}_j} w_{jk}(\mathbf{c}_j(t) - \mathbf{c}_k(t))$;
  - c) broadcasting of $\mathbf{c}_j(t+1)$ to one-hop neighbors;
- s3) at $t \to \infty$, each $\mathbf{c}_j(t)$ converges to the globally optimal $\hat{\bar{\mathbf{s}}}_f$ in (13), $\forall j$; then, each CR detects the frequency occupancy state via ($\oplus$ denotes logical OR operation and η_{th} is the detection threshold):

$$\hat{\mathbf{d}}[m] = \oplus_i \left(|\hat{\bar{\mathbf{s}}}_f[iM + m]| \geq \eta_{th} \right), \quad \forall m. \quad (16)$$

In (15), an update on $\mathbf{s}_f^{(j)}(t)$ is sought not only to satisfy the measurement equation in (7), but also to enforce consensus with the most recent average value through the constraint $\mathbf{c}_j(t) = \mathbf{I}_M \mathbf{s}_f^{(j)}(t)$. In (16), each CR separates the I transmitted sources $\{\mathbf{s}_f^{(i)}\}_{i=1}^I$ and claims an occupied subchannel as long as there exists one source i with a non-zero element $\mathbf{s}_f^{(i)}[m] \neq 0$.

IV. Performance simulations

We consider a wide band of interest that is partitioned into $M = 32$ equal-bandwidth subchannels. Primary users randomly occupy some of the subchannels, with an average spectrum occupancy ratio of 20%. The wideband channel experiences frequency-selective fading, which is modeled as a multipath channel with N_p time-delayed taps and independent Rayleigh fading gains on these taps.

The signal to noise ratio (SNR) is defined to be the signal energy of the wideband signal over the entire spectrum, scaled by the power of the white noise. The compression ratio K/M reflects the reduced number of samples used with reference to the number M needed in full-rate Nyquist sampling. For the spectral hole detection problem, performance metrics of interest include the probability of detection P_d and the probability of false alarms P_{fa}, which we average over all subchannels as follows:

$$P_d = \mathrm{E}\left\{ \frac{\mathbf{d}^T(\mathbf{d} = \hat{\mathbf{d}})}{\mathbf{1}^T \mathbf{d}} \right\}, \qquad P_{fa} = \mathrm{E}\left\{ \frac{(\mathbf{1} - \mathbf{d})^T(\mathbf{d} \neq \hat{\mathbf{d}})}{M - \mathbf{1}^T \mathbf{d}} \right\}$$

where $\mathbf{d} \in \{0, 1\}^{M \times 1}$ is the true frequency occupancy state and $\mathbf{1}$ denotes the all-one vector.

CR collaboration enables diversity gain, which we quantify via simulations in Fig. 1. The detection fusion scheme is used for illustration, in the absence of any channel knowledge. It

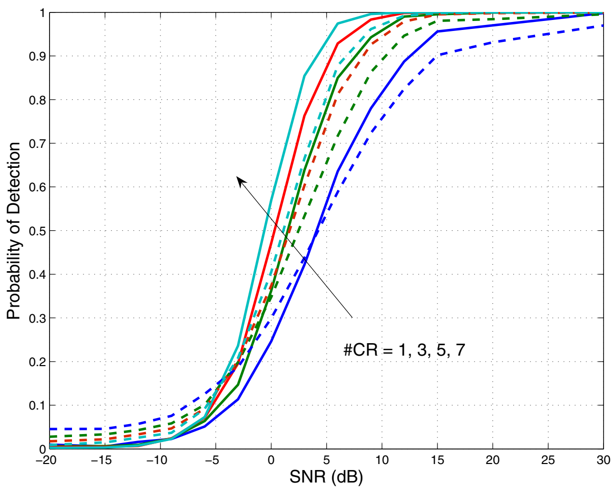

Fig. 1. Probability of detection for various number of collaborating CRs, for $P_{fa} = 0.01$. *Solid lines*: no compression; *dash lines*: compression at $K/M = 50\%$.

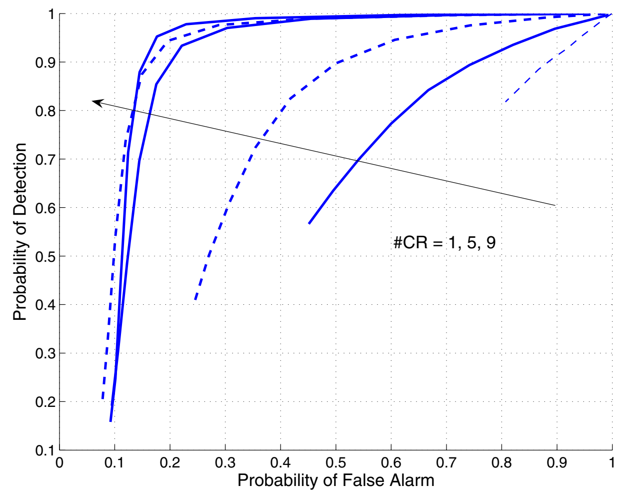

Fig. 2. Receiver Operating Characteristic (ROC), for SNR = 0 dB. *Solid lines*: no compression; *dash lines*: compression at $K/M = 60\%$.

can be shown that the detection performance improves as the number of collaborating CRs J increases. Fig. 1 also illustrates the effect of compressive sampling. Encouragingly, the compressed sensing scheme is able to detect the primary users at strong compression ($K/M = 0.5$), when the number of samples used (K) is much less than that required by Nyquist rate sampling (M). When K/M increases, the robustness to noise improves, so does the detection performance. It is the inherent spectral occupancy sparsity in the transmitted signals that enables reduced-rate sampling, which in turn alleviate the sampling burden and energy consumption of CRs in the wideband regime. On the other hand, Compression incurs performance degradation, especially in the presence of ambient noise and channel fading. It is observed that compression can offset some of the spatial diversity gain enabled by collaboration, but the degradation is relatively small.

These observations are corroborated by the receiver operating characteristics (ROC) depicted in Fig. 2. Interestingly, when the number of collaborating CRs is large, the degradation caused by compression becomes trivial.

The developed distributed algorithms in Section III converge quite fast. The distributed sensing algorithm in Section III-C converges within 10 iterations in our simulations. The distributed detection algorithm in Section III-B converges even faster, typically in 2-3 iterations.

V. Summary and Future Work

This paper presents distributed collaborative sensing techniques in combination with local compressed sensing to save the overall sensing costs for cognitive radios. Consensus techniques for both spectral detection fusion and spectral estimation fusion are developed, which converge to their respective globally optimally solutions using only distributed computation and local communications among one-hop neighbors.

In general, the collected diversity gains vary, depending on the amount of compression and whether the channel knowledge is available. While the CSI knowledge enhances sensing performance, it also requires local channel estimation which may be costly or even difficult in multiuser wireless systems. Hence, the cost of channel estimation needs to be justified by the sensing performance gain it offers in collaborative fusion. Furthermore, the sensitivity of the fusion results to channel estimation errors need to quantified in order to properly allocate systems resources to the channel estimation task. Analysis on the theoretical spatial diversity gain and the value of channel knowledge will be investigated in future work.

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