

# Capacity of Opportunistic Routing in Multi-Rate and Multi-Hop Wireless Networks

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**Abstract**—Opportunistic routing (OR) copes with the unreliable transmissions by exploiting the broadcast nature of the wireless medium and spatial diversity of the multi-hop wireless networks. In this paper, we carry out a comprehensive study on the impacts of multiple rates, interference, candidate selection and prioritization on the maximum end-to-end throughput or capacity of OR. Taking into account the wireless interference and unique properties of OR, we introduce the concept of concurrent transmitter sets to represent the constraints imposed by the transmission conflicts of OR, and formulate the maximum end-to-end throughput problem as a maximum-flow linear programming subject to the transmission conflict constraints. We also propose two multi-rate OR metrics: *expected medium time* (EMT) and *expected advancement rate* (EAR), and the corresponding distributed and local rate and candidate set selection schemes, one of which is Least Medium Time OR (LMTOR) and the other is Multi-rate Geographic OR (MGOR). We compare the capacity of multi-rate OR with single-rate ones under different settings. We show that our proposed multi-rate OR schemes achieve higher throughput bound than any single-rate GOR. We observe some insights of OR: 1) although involving more forwarding candidates increases the end-to-end capacity, the capacity gained from involving more forwarding candidates decreases; 2) there exists a node density threshold, higher than which 24Mbps GOR performs better than 12Mbps GOR, and vice versa.

**Index Terms**—Multi-hop wireless networks, opportunistic routing, multi-rate, capacity.

## I. INTRODUCTION

MULTI-HOP wireless networks, such as mobile ad hoc networks (MANETs), wireless sensor networks (WSNs), and wireless mesh networks (WMNs), have received increasing attention in the past decade due to their easy deployment at low cost and broad applications, ranging from tactical communication in a battlefield, disaster rescue after an earth quake, to wildlife monitoring and tracking, last-mile network access, etc.

Routing in multi-hop wireless networks presents a great challenge mainly due to the following facts. First, wireless links are unreliable because of channel fading [1]. Second, achievable channel rates may be different at different links since link quality depends on distance and path loss between two neighbors. Third, as the wireless medium is broadcast in

nature, the transmission on one link may interfere with the transmissions on the neighboring links.

A new routing paradigm, known as opportunistic routing (OR) [2]–[5], has recently been proposed to mitigate the impact of unreliable wireless links by exploiting the broadcast nature of the wireless medium and spatial diversity of the multi-hop wireless networks. OR basically runs in such a way that for each local packet forwarding, a set of next-hop forwarding candidates are selected according to some criteria, e.g. the neighbors which are geographically closer to the destination than the transmitter are selected as the candidates; then the transmitter broadcasts the packet to the forwarding candidates; based on which forwarding candidate(s) receiving the packet correctly, some MAC coordination mechanism [2]–[4] selects one candidate to actually forward this packet. As multiple forwarding candidates are involved to help relay the packet, the probability of at least one forwarding candidate correctly receiving the packet increases compared to the traditional routing that only involves one neighbor. The increase of forwarding reliability in one transmission reduces the retransmission overhead, which in turn improves the throughput [3], [4], [6], [7] and energy efficiency [2], [8].

The existing works on OR mainly focused on a single-rate system. Researchers have proposed several candidate selection and prioritization schemes to improve throughput or energy efficiency. However, there is a lack of theoretical analysis on the performance limit or the throughput bounds achievable by OR. In addition, one of the current trends in wireless communication is to enable devices to operate using multiple transmission rates. For example, many existing wireless networking standards such as IEEE 802.11a/b/g include this multi-rate capability. The inherent rate-distance trade-off of multi-rate transmissions has shown its impact on the throughput performance of traditional routing [9]–[11]. Generally, low-rate communication covers a long transmission range, while high-rate communication must occur at short range. Different transmission ranges also imply different neighboring node sets, which results in different spatial diversity opportunities. These rate-distance-diversity trade-offs will no doubt affect the throughput of OR, which deserves a careful study.

In this paper, we bridge these two gaps by studying the throughput bound of OR and the performance of OR in a multi-rate scenario. First, for OR, we propose the concept of concurrent transmitter sets which captures the unique transmission conflict constraints of OR. Then, for a given network with a given opportunistic routing strategy (i.e., forwarder selection and prioritization), we formulate the maximum end-to-end throughput problem as a maximum-flow linear pro-

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gramming subject to the constraints of transmitter conflict. The solution of the optimization problem provides the performance bound of OR. The proposed method establishes a theoretical foundation for the evaluation of the performance of different variants of OR with various forwarding candidate selection, prioritization policies, and transmission rates. We also propose two OR metrics: *expected medium time* (EMT) and *expected advancement rate* (EAR), and the corresponding distributed and local rate and candidate set selection schemes, one of which is Least Medium Time OR (LMTOR) and the other is Multi-rate Geographic OR (MGOR). Simulation results show that for OR, by incorporating our proposed multi-rate OR schemes, system operating at multiple rates achieves higher throughput than that operating at any single rate. Several insights of OR are observed: 1) although involving more forwarding candidates increases the end-to-end capacity, the capacity gained from involving more forwarding candidates decreases; 2) there exists a node density threshold, higher than which 24Mbps GOR performs better than 12Mbps GOR, and vice versa.

The rest of this paper is organized as follows. Section II introduces the system model. We propose the framework of computing the throughput bounds of OR in Section III. Section IV studies the impact of multi-rate capability and forwarding strategy on the throughput of OR. We then propose the OR metrics, and rate and candidate selection schemes for multi-rate systems in Section V. Simulation results are presented and analyzed in Section VI. Section VII discusses the related work, and conclusions are drawn in Section VIII.

## II. SYSTEM MODEL

We consider a multi-hop wireless network with  $N$  nodes arbitrarily located on a plane. Each node  $n_i$  ( $1 \leq i \leq N$ ) can transmit a packet at  $J$  different rates  $R^1, R^2, \dots, R^J$ . We say there is a **usable** directed link  $l_{ij}$  from node  $n_i$  to  $n_j$ , when the **packet reception ratio** (PRR), denoted as  $p_{ij}$ , from  $n_i$  to  $n_j$  is larger than a non-negligible positive threshold  $p_{td}$ . The PRR we consider is an average value of the link quality in a long-time scale (e.g. in tens of seconds). There exist several link quality measurement mechanisms [1], [12] to obtain the PRR on each link. In this paper, we assume that there is no power control scheme and the PRR on each link for each rate is given. We define the **effective transmission range**  $L_m$  at rate  $R^m$  ( $1 \leq m \leq J$ ) as the sender-receiver distance at which the PRR equals  $p_{td}$ .

The basic module of opportunistic routing is illustrated in Fig. 1. Assume node  $n_i$  is forwarding a packet to a remote sink/destination  $n_d$ . We denote the set of nodes within the effective transmission range of node  $n_i$  as the **neighboring node set**  $\mathcal{C}_i$  (e.g., all the five nodes around  $n_i$  in Fig. 1). Note that, for different transmission rates, the corresponding effective transmission ranges are different, then we have different neighboring node sets of node  $n_i$ , and the PRR on the same link  $l_{ij}$  may be different at different rates. We define the set  $\mathcal{F}_i := \langle n_{i_1}, \dots, n_{i_r} \rangle$  (e.g.,  $\langle n_{i_1}, n_{i_2}, n_{i_3} \rangle$  in Fig. 1) as **forwarding candidate set**, which is a subset of  $\mathcal{C}_i$  and includes  $r$  nodes selected to be involved in the local opportunistic forwarding based on a particular selection

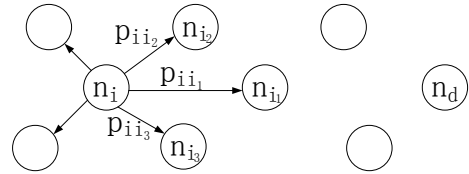


Fig. 1. Node  $n_i$  is forwarding a packet to a remote destination  $n_d$  with a forwarding candidate set  $\mathcal{F}_i = \langle n_{i_1}, n_{i_2}, n_{i_3} \rangle$  at some transmission rate.

strategy.  $\mathcal{F}_i$  is an ordered set, where the order of the elements corresponds to their priority in relaying a received packet.

The opportunistic routing works by the sender node  $n_s$  forwarding the packet to the nodes in its forwarding candidate set  $\mathcal{F}_s$ . One of the candidate nodes continues the forwarding based on their relay priorities – If the first node in the set has received the packet successfully, it forwards the packet towards the destination while all other nodes suppress duplicate forwarding. Otherwise, the second node in the set is arranged to forward the packet if it has received the packet correctly. Otherwise the third node, the fourth node, etc. A forwarding candidate will forward the message only when all the nodes with higher priorities fail to do so. When no forwarding candidate has successfully received the packet, the sender will retransmit the packet if retransmission is enabled. The sender will drop the packet when the number of retransmissions exceeds the limit. The forwarding reiterates until the packet is delivered to the destination. Several MAC protocols have been proposed in [2]–[4], [13] to coordinate the forwarding candidates and ensure the relay priority among them. In this paper, since our objective is to study the performance bound and capacity limit, we assume that packet transmissions at the individual nodes can be finely controlled and carefully scheduled by an omniscient and omnipotent central entity. So here we do not concern ourselves with issues such as MAC contention or coordination overhead that may be unavoidable in a distributed network. This is a very commonly used assumption for such theoretical study [11], [14].

## III. COMPUTING THROUGHPUT BOUND OF OR

The first fundamental issue we want to address is the maximum end-to-end throughput when OR is used. Any traffic load higher than the throughput capacity is not supported and even deteriorates the performance as a result of excessive medium contention. The knowledge of throughput capacity can be used to reject any excessive traffic in the admission control for real-time services. It can also be used to evaluate the performance of different OR variants. Furthermore, the derivation of the capacity of OR may suggest novel and efficient candidate selection and prioritization schemes.

In this section we present our methodology to compute the throughput bound between two end nodes in a given network with a given OR strategy (i.e., given each node's forwarding candidate set, node relay priority, and transmission/broadcast rate at each node). We first introduce two concepts, transmitter based conflict graph and concurrent transmitter set, which are used to represent the constraints imposed by the interference among wireless transmissions in a multi-hop wireless network.

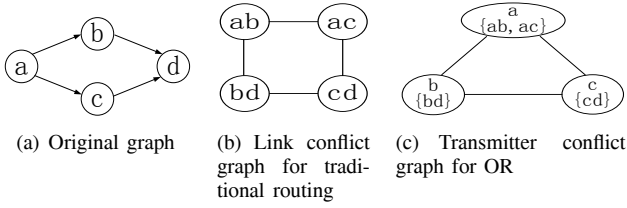


Fig. 2. Conflict graph.

We then present methods for computing bounds on the optimal throughput that a network can support when OR is used.

### A. Transmission Interference and Conflict

Wireless interference is a key issue affecting throughput. Existing wireless interference models generally fall into two categories: *protocol model* and *physical model* [15]. Under the protocol model, a transmission is considered successful when both of the following conditions hold: 1) The receiver is in the effective transmission range of the transmitter; and 2) No other node that is in the carrier sensing range of the receiver is transmitting. Under the physical model, for a successful transmission, the aggregate power at the receiver from all other ongoing transmissions plus the noise power must be less than a certain threshold so that the SNR requirement at the ongoing receiver is satisfied. In this paper, we use the term “usable” to describe a link when it is able to make a successful transmission based on either the protocol model or the physical model. When two (or more) links are not able to be usable at the same time, they are having a “conflict”.

Link conflict graphs have been used to model such interference [11], [14]. As shown in Fig. 2(b), in a link conflict graph, each vertex corresponds to a link in the original connectivity graph. There is an edge between two vertices if the corresponding two links may not be active simultaneously due to interference (e.g., having a “conflict”). However, this link-based conflict graph cannot be directly applied to study capacity problem of OR networks because by the nature of opportunistic routing, for one transmission, throughput may take place on any one of the links from the transmitter to its forwarding candidates. The throughput dependency among multiple outgoing links from the same transmitter makes the subsequent maximum-flow optimization problem very difficult (if it is still possible). Therefore, in this paper, we propose a new construction of conflict graph to facilitate the computation of throughput bounds of OR. Instead of creating link conflict graph, we study the conflict relationship by transmitters (or nodes) associated with their forwarding candidates. As shown in Fig. 2(c), in the node conflict graph, each vertex corresponds to a transmitter in the original connectivity graph. Each vertex is associated with a set of links, e.g., the links to its selected forwarding candidates. There is an edge (conflict) between two vertices if the two nodes cannot be transmitting simultaneously due to a conflict caused by one or more unusable links as we will define in section III-B.

### B. Concurrent Transmitter Sets

We define the concepts of **concurrent transmitter sets (CTS’s)** for OR as follows. These concepts capture the impact

of interference of wireless transmissions and OR’s opportunistic nature. They are the foundation of our method of computing the end-to-end throughput bound.

1) **Conservative CTS:** According to a specific OR policy, when one node is transmitting, the packet is broadcast to all the nodes in its forwarding candidate set. The links from a transmitter to all its forwarding candidates are defined as links associated with the transmitter. We define a conservative CTS (CCTS) as a set of transmitters, when all of them are transmitting simultaneously, all links associated with them are still usable.

The conservative CTS actually requires all the opportunistic receivers to be interference-free for one transmission. This is probably true for certain protocols [4] where Ready To Send/Clear To Send-like mechanism is used to clear certain range within transmitter/receiver or confirm a successful reception. But this is a stricter requirement than necessary and will only give us a lower bound of end-to-end capacity. We define the following greedy CTS to compute the maximum end-to-end throughput.

2) **Greedy CTS:** In order to maximize the throughput, we permit two or more transmitters to transmit at the same time even when some links associated with them become unusable. The idea is to allow a transmitter to transmit as long as it can deliver some throughput to one of the next-hop forwarder(s). Therefore, we define a greedy CTS as a set of transmitters, when all of them are transmitting simultaneously, at least one link associated with each transmitter is usable.

### C. Effective Forwarding Rate

After we find a CTS, we need to identify the capacity on every link associated with a node in the CTS. We introduce the concept of **effective forwarding rate** on each link associated with a transmitter according to a specified OR strategy. Assume node  $n_i$ ’s forwarding candidate set  $\mathcal{F}_i = \langle n_{i_1}, \dots, n_{i_r} \rangle$ , with relay priorities  $n_{i_1} > \dots > n_{i_r}$ . Let  $\psi_q$  denote the indicator function on link  $l_{ii_q}$  when it is in a particular CTS:  $\psi_q = 1$  indicating link  $l_{ii_q}$  is usable, and  $\psi_q = 0$  indicating that link  $l_{ii_q}$  is not usable. Then the effective forwarding rate of link  $l_{ii_q}$  in that particular CTS is defined in Eq. (1):

$$\tilde{R}_{ii_q} = R_i \cdot \psi_q \cdot p_{ii_q} \prod_{k=0}^{q-1} (1 - \psi_k \cdot p_{ii_k}) \quad (1)$$

where  $R_i$  is the broadcast rate of transmitter  $n_i$ , and  $p_{ii_0} := 0$ .  $p_{ii_q} \prod_{k=0}^{q-1} (1 - \psi_k \cdot p_{ii_k})$  is the probability of candidate  $n_{i_q}$  receiving the packet correctly but all the higher-priority candidates not. Note that the candidate (with  $\psi_k = 0$ ), which is interfered by other transmissions, is not involved in the opportunistic forwarding, and has no effect on the effective forwarding rate from the transmitter to lower-priority candidates, as  $(1 - \psi_k \cdot p_{ii_k}) = 1$ .

In a conservative CTS, all the receptions are interference-free. Therefore, in each CCTS, every link associated with a transmitter is usable, i.e.  $\psi = 1$ , and the effective forwarding rate on each link is non-zero. And the effective forwarding rate for a particular link remains same when the link is in a different CCTS. The effective forwarding rate indicates that according to the relay priority, only when a usable higher

$$\begin{aligned}
& \text{Max} \sum_{l_{si} \in \mathbf{E}} f_{si} \\
& \quad \text{s.t.} \\
& \sum_{l_{ij} \in \mathbf{E}} f_{ij} = \sum_{l_{ji} \in \mathbf{E}} f_{ji} \quad \forall n_i \in \mathbf{V} - \{n_s, n_d\} \quad (2) \\
& \sum_{l_{is} \in \mathbf{E}} f_{is} = 0 \quad (3) \\
& \sum_{l_{di} \in \mathbf{E}} f_{di} = 0 \quad (4) \\
& f_{ij} \geq 0 \quad \forall l_{ij} \in \mathbf{E} \quad (5) \\
& f_{ij} = 0 \quad \forall l_{ij} \in \mathbf{E}, n_j \notin \mathcal{F}_i \quad (6) \\
& \sum_{\alpha=1}^M \lambda_\alpha \leq 1 \quad (7) \\
& \lambda_\alpha \geq 0, 1 \leq \alpha \leq M \quad (8) \\
& f_{ij} \leq \sum_{n_i \in T_\alpha, n_j \in \mathcal{F}_i, 1 \leq \alpha \leq M} \lambda_\alpha \tilde{R}_{ij}^\alpha \quad \forall l_{ij} \in \mathbf{E} \quad (9)
\end{aligned}$$

Fig. 3. LP formulations to optimize the end-to-end throughput of OR.

forwarding candidate does not receive the packet correctly, a usable lower priority candidate may have a chance to relay the packet if it receives the packet correctly. Note that this definition generalizes the effective rate for unicast in traditional routing, that is, when there is only one forwarding candidate, the effective forwarding rate reduces to the unicast effective data rate.

While for the greedy mode, some link(s) associated with one transmitter may become unusable, thus having zero effective forwarding rate. Furthermore, the effective forwarding rate on the links may be different when they are in different GCTS's. To indicate this possible difference, we use  $\tilde{R}_{ij}^\alpha$  to denote the effective forwarding rate of link  $l_{ij}$  when it is in the  $\alpha^{\text{th}}$  GCTS.

#### D. Lower Bound of End-to-End Throughput of OR

Assume we have found all the CCTS's  $\{T_1, T_2, \dots, T_M\}$  in the network. At any time, at most one CTS can be scheduled to transmit. When one CTS is scheduled to transmit, all the transmitters in that set can transmit simultaneously. Let  $\lambda_\alpha$  denote the time fraction scheduled to CCTS  $T_\alpha$  ( $1 \leq \alpha \leq M$ ). Then the maximum throughput problem can be converted to an optimal scheduling problem that schedules the transmission of the maximum CTS's to maximize the end-to-end throughput. Therefore, considering communication between a single source,  $n_s$ , and a single destination,  $n_d$ , with opportunistic routing, we formulate the maximum achievable throughput problem between the source and the destination as a linear programming corresponding to a maximum-flow problem under additional constraints in Fig. 3.

In Fig. 3,  $f_{ij}$  denotes the amount of flow on link  $l_{ij}$ ,  $\mathbf{E}$  is a set of all links in the connected graph  $G$ , and  $\mathbf{V}$  is the set of all nodes. The maximization states that we wish to maximize the sum of flow out of the source. The constraint (2) represents flow-conservation, i.e., at each node, except the source and the destination, the amount of incoming flow is equal to the

amount of outgoing flow. The constraint (3) states that the incoming flow to the source node is 0. The constraint (4) indicates that the outgoing flow from the destination node is 0. The constraint (5) restricts the amount of flow on each link to be non-negative. The constraint (6) says there is no flow from the node to the neighboring nodes that are not selected as the forwarding candidates of it. The constraint (7) represents at any time, at most one CTS will be scheduled to transmit. The constraint (8) indicates the scheduled time fraction should be non-negative. The constraint (9) states the actual flow delivered on each link is constrained by the total amount of flow that can be delivered in all activity periods of the OR modules which contain this link.

The key difference of our maximum flow formulations from the formulations for traditional routing in [11], [14] lies in the methodology we use to schedule concurrent transmissions. With the construction of concurrent transmitter sets, we are able to schedule the transmissions based on node set (with each node associated with a set of forwarding candidates) rather than link set in traditional routing. When we schedule a transmitter, we effectively schedule the links from the transmitter to its forwarding candidates at the same time according to OR strategy. While for traditional routing, any two links share the same sender can not be scheduled simultaneously. When a packet is not correctly received by the intended receiver but opportunistically received by some neighboring nodes of the transmitter, traditional routing will retransmit that packet instead of making use of the correct receptions on the other links. OR takes advantage of the correct receptions. That's why OR achieves higher throughput than traditional routing. Our proposed model accurately captures OR's capability of delivering throughput opportunistically.

#### E. Maximum End-to-end Throughput of OR

The throughput bound we find based on the conservative CTS's in section III-D is a lower bound of maximum end-to-end throughput. The CCTS's can be constructed based on either the protocol model or the physical model. However, the interference freedom at every intended receiver is a stricter requirement than necessary. It may be applicable under some protocol scenario but it fails to take full advantage of opportunistic nature of OR, because it excludes the situations where concurrent transmission is able to deliver throughput on some of the links even though some other links are suffering conflicts. In order to compute the exact capacity, we apply the same optimization technique to the greedy CTS's. Since greedy CTS's include all the possible concurrent transmission scenarios that generate non-zero throughput, the bound found by the optimization technique based on all greedy CTS's will be the maximum end-to-end throughput of OR.

Similar to the construction of CCTS's, GCTS's can be constructed based on either the protocol model or the physical model. Under the protocol model, the conflict between two links is binary, either conflict or no conflict. It is not difficult to construct the GCTS's under the protocol model with the proposed transmitter conflict graph. On the other hand, it is well known that the physical model captures the interference property more accurately. However, it is more complicated to

represent the interference when multiple transmitters are active at the same time. In this section, we discuss the construction of GCTS's based on the physical interference model.

Under the physical interference model, a link  $l_{ij}$  is usable if and only if the signal to noise ratio at receiver  $n_j$  is no less than a certain threshold, e.g.,  $\frac{Pr_{ij}}{P_N} \geq SNR_{th}$ , where  $Pr_{ij}$  is the average signal power received at  $n_j$  from  $n_i$ 's transmission,  $P_N$  is the interference+noise power, and  $SNR_{th}$  is the SNR threshold, under which the packet can not be correctly received and above which the packet can be received at least with probability  $p_{td}$ . Note that,  $SNR_{th}$  is different for different data rates.

Under the physical model, the interference gradually increases as the number of concurrent transmitters increases, and becomes intolerable when the interference+noise level reaches a threshold. We define a weight function  $w_{ijq}$ , to capture the impact of a transmitter  $n_i$ 's transmission on a link  $l_{jjq}$ 's reception. Link  $l_{jjq}$  represents the data forwarding from node  $n_j$  to one of its forwarding candidate  $n_{jq}$ .

$$w_{ijq} = \frac{Pr_{ijq}}{\frac{Pr_{jjq}}{SNR_{th}} - P_{noise}} \quad (10)$$

where  $Pr_{ijq}$  and  $Pr_{jjq}$  are the received power at node  $n_{jq}$  from the transmissions of nodes  $n_i$  and  $n_j$ , respectively,  $P_{noise}$  is the ambient noise power, and  $\frac{Pr_{jjq}}{SNR_{th}} - P_{noise}$  is the maximum allowable interference at node  $n_{jq}$  for keeping link  $l_{jjq}$  usable.

Then given a transmitter set  $T$  and  $n_j \in T$ , a link  $l_{jjq}$  is usable if and only if  $\sum_{n_i \in T, i \neq j} w_{ijq} < 1$ . It means that link  $l_{jjq}$  is usable even when all the transmitters in set  $T$  are simultaneously transmitting. For conservative mode, if this condition is true for every link associated with each transmitter in  $T$ , this set  $T$  is a CCTS. For greedy mode, if this condition is true for at least one link associated with each transmitter in  $T$ , the set  $T$  is a GCTS.

After finding all the GCTS's, we can apply the same optimization technique to the maximum flow problem based on all the GCTS's. The result is the exact bound of maximum end-to-end throughput.

When each node has only one forwarding candidate, OR degenerates to the traditional routing. Therefore, finding all the concurrent transmitter sets is at least as hard as the NP-hard problem of finding the independent sets in [11], [14] for traditional routing. However, it may not be necessary to find all of them to maximize an end-to-end throughput. Some heuristic algorithm similar to that in [16], or column generation technique [17] can be applied to find a good subset of all the CTS's. In addition, complexity can be further reduced by taking into consideration that interferences/conflicts always happen for nodes within certain range. How to efficiently find all the CTS's is out of the scope of this paper. We simply apply a greedy algorithm to find all the CTS's, say each time we add new transmitters into the existing CTS's to create new CTS's, until no any additional transmitter can be added into any of the existing CTS's.

## F. Multi-flow Generalization

Our formulations in Fig. 3 can be extended from a single source-destination pair to multiple source-destination pairs using a multi-commodity flow formulation augmented with OR transmission constraints. By assigning a unique connection identifier to each source-destination pair, we introduce the variable  $f_{ij}^k$  to denote the amount of flow for connection  $k$  on link  $l_{ij}$ . For each flow  $k$ , according to some OR routing strategy, the corresponding transmitters and their forwarding candidates can be decided. Then the CCTS or GCTS can be constructed over the union of all the OR modules. Referring to Fig. 3, the objective is now to maximize the summation of all the flows out of all the sources; the flow conservation constraints at each node apply on a per-connection basis (constraint (2)); the total incoming flow into a source node is zero only for the connection(s) originating at that node (constraint (3)); similarly, the total outgoing flow from a destination node is zero only for the connection(s) terminating at that node (constraint (4));  $f_{ij}^k$  is non-negative (constraint (5));  $f_{ij}^k$  is equal to zero if the flow  $k$  is not routed by link  $l_{ij}$  (constraint (6)); and the sum of all the flows traversing on a link is constrained by the total amount of flow that can be delivered in all activity periods of the OR modules which contain this link (constraint (9)).

## IV. IMPACT OF TRANSMISSION RATE AND FORWARDING STRATEGY ON THROUGHPUT

The impact of the transmission rate on the throughput of OR is twofold. On the one hand, different rates have different transmission ranges, which lead to different neighborhood diversity. High rate usually has short transmission range. In one hop, there are few neighbors around the transmitter, which presents low neighborhood diversity. Low-rate is likely to have long transmission range, therefore achieves high neighborhood diversity. From the diversity point of view, low rate may be better. On the other hand, although low rate brings the benefit of larger one-hop distance which results in higher neighborhood diversity and fewer hop counts to reach the destination, it may still end up with a low effective end-to-end throughput because the low rate disadvantage may overwhelm all other benefits. It is nontrivial to decide which rate is indeed better.

We now use a simple example in Fig. 4 to illustrate transmitting at lower rate may achieve higher throughput than transmitting at higher rate for OR. In this example, we assume all the nodes operate on a common channel, but each node can transmit at two different rates  $R$  and  $R/2$ . We compare the throughput from source  $a$  to destination  $d$  when the source transmits the packets at the two different rates. Fig. 4(a) shows the case when all the nodes transmit at rate  $R$ , and the packet delivery ratio on each link is 0.5. So the effective data rate on each link is  $0.5R$ . There is no link from  $a$  to  $d$  because  $d$  is out of  $a$ 's effective transmission range when  $a$  operates on rate  $R$ . Assume the four nodes are in the carrier sensing range of each other, so they can not transmit at the same time. Assuming  $b$  and  $c$  are the forwarding candidates of  $a$ , and  $b$  has higher relay priority than  $c$ . Then link  $l_{ac}$  has effective forwarding rate of  $0.25R$ . By using the formulations in Fig. 3, we obtain an

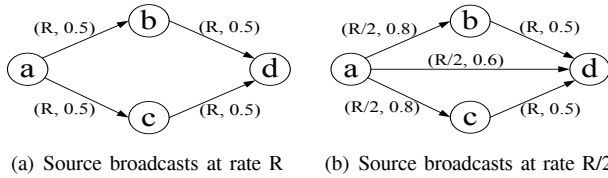


Fig. 4. End-to-end throughput comparison at different transmission rates.

optimal transmitter schedule such that  $a$ ,  $b$  and  $c$  are scheduled to transmit for a fraction of time 0.4, 0.4 and 0.2, respectively. So the maximum end-to-end throughput from  $a$  to  $d$  is  $0.3R$ . While in Fig. 4(b), when  $a$  is transmitting at a lower rate  $R/2$ , we assume it can reach  $d$  directly with packet delivery ratio of 0.6, also we get higher packet deliver ratio from  $a$  to  $b$  and  $c$  as 0.8. Then in this case, lower rate achieves longer effective transmission range and brings more spacial diversity chances. Assume  $d$ ,  $b$ , and  $c$  are forwarding candidates of  $a$ , and with priority  $d > b > c$ . Similarly, we calculate the maximum throughput from  $a$  to  $d$  as  $0.36R$ , which is 20% higher than the scenario in Fig. 4(a) where system operates on a single rate.

Besides the inherent rate-distance, rate-diversity and rate-hop tradeoffs which affect the throughput of OR, the forwarding strategy will also have an impact on the throughput. For example, different forwarding candidates may achieve different throughput, and even for the same forwarding candidate set, different forwarding priority will also result in different throughput, etc.. We refer readers to [6] for detail analysis on the impact of forwarding strategy on the OR throughput.

## V. RATE AND CANDIDATE SELECTION SCHEMES

How to efficiently select the transmission rates and forwarding strategy for each node such that the network capacity can be globally optimized is still an open research issue. We have shown the example in Fig. 4 that nodes transmitting at a lower rate may lead to a higher end-to-end throughput than that when nodes are transmitting at a higher rate. Then, what criteria should node  $a$  follow to select transmission rate, forwarding candidates and candidate priority to approach the capacity? It is non-trivial to answer this question. Towards the development of distributed and localized OR protocol that maximize the capacity, in this section, we propose two rate and candidate selection schemes, one is enlightened by least-cost opportunistic routing (LCOR) proposed in [5], and the other is inspired by geographic opportunistic routing (GOR) [2], [4], [6]–[8].

### A. Least Medium Time Opportunistic Routing

In traditional routing, the medium time metric (MTM) [9] and expected transmission time (ETT) [18] have shown to be good metrics to achieve high throughput. For OR, we define the opportunistic ETT (OETT) as the expected transmission time to send a packet from  $n_i$  to any node in its forwarding candidate set  $\mathcal{F}_i$ .

$$OETT_{n_i}^{\mathcal{F}_i} = \frac{L_{pkt}}{R_i P_{\mathcal{F}_i}} \quad (11)$$

where  $L_{pkt}$  is the packet length,  $P_{\mathcal{F}_i}$  is the probability of at least one candidate in  $\mathcal{F}_i$  receiving the packet sent by  $n_i$  correctly:

$$P_{\mathcal{F}_i} = 1 - \prod_{q=1}^r (1 - p_{ii_q}) \quad (12)$$

Note that this metric actually generalizes the unicast ETT, that is, for  $|\mathcal{F}_i| = 1$ , the OETT reduces to the unicast ETT.

Denote by  $D_i$  the expected medium time (EMT) to reach the destination  $n_d$  from a node  $n_i$ . Assume that  $n_i$ 's forwarding candidates are prioritized according to their expected medium time  $D_{i_q}$ , such that  $D_{i_1} < D_{i_2} \dots < D_{i_r}$ . Then we define the remaining EMT to the destination  $n_d$  when node  $n_i$  chooses forwarding candidate set  $\mathcal{F}_i$  as following:

$$EMT_{\mathcal{F}_i}^{n_d} = \frac{1}{P_{\mathcal{F}_i}} \sum_{q=1}^r D_{i_q} p_{ii_q} \prod_{k=0}^{q-1} (1 - p_{ii_k}) \quad (13)$$

where  $p_{ii_0} := 0$ .

$p_{ii_q} \prod_{k=0}^{q-1} (1 - p_{ii_k})$  is the probability of candidate  $n_{i_q}$  receiving the packet correctly but all the higher-priority candidates do not. That is, it is the probability of  $n_{i_q}$  becoming the actual forwarder. So the summation is the expected remaining medium time needed for a packet to travel to the destination from the set  $\mathcal{F}_i$ .

Note that like the OETT, the EMT generalizes the single-path case: when  $|\mathcal{F}_i| = 1$ , it simply becomes the delay from the next-hop to the destination. We should also notice that for any two different transmitters,  $n_i$  and  $n_j$ , even if  $\mathcal{F}_i = \mathcal{F}_j$ , they may have different EMT, since this EMT is affected by the delivery probabilities from the transmitter to its each forwarding candidate. In other words, the remaining EMT from a forwarding candidate set to the destination depends not only on the candidate set itself, but also on the predecessor node of this set.

We now define the least EMT of node  $n_i$  to the destination  $n_d$  in a multi-rate scenario:

$$D_i = \min_{\mathcal{F}_i^m \in \mathcal{C}_i^m, 1 \leq m \leq J} (OETT_{n_i}^{\mathcal{F}_i^m} + EMT_{\mathcal{F}_i^m}^{n_d}) \quad (14)$$

where  $\mathcal{C}_i^m$  is the neighboring node set of node  $n_i$  when  $n_i$  transmits at rate  $R^m$ ,  $\mathcal{F}_i^m$  is the corresponding forwarding candidate set.

We enumerate all the possible  $\mathcal{F}_i^m$  to get the optimal one. This equation represents the steady-state of the least medium time OR (LMTOR), that selects the forwarding candidates and transmission rate for each node to achieve the minimum end-to-end EMT. A distributed algorithm running like Bellman-Ford can solve the LMTOR problem. That is, in one iteration, each node  $n_i$  updates its value  $D_i^k$ , where  $k$  is the iteration index. This  $D_i^k$  is the estimated EMT from  $n_i$  to the destination at the  $k^{th}$  iteration; it converges toward  $D_i$ .  $D_d^k = 0, \forall k$ . One iteration step consists of updating the estimated EMT to the destination from each node:

$$D_i^{k+1} = \min_{\mathcal{F}_i^m \in \mathcal{C}_i^m, 1 \leq m \leq J} (OETT_{n_i}^{\mathcal{F}_i^m} + EMT_{\mathcal{F}_i^m}^{n_d}(k)) \forall n_i \neq n_d \quad (15)$$

where  $EMT_{\mathcal{F}_i^m}^{n_d}(k)$  is the remaining EMT computed using the costs  $D_{i_q}^k(n_{i_q} \in \mathcal{F}_i^m)$  from the previous iteration.

The rate and candidates selected by  $n_i$  are determined as a byproduct of minimizing the Eq. (15). The algorithm terminates when:  $D_i^{k+1} = D_i^k \forall n_i \neq n_d$ . Similar to the proof in [5], this algorithm converges after at most  $N$  iterations, where  $N = |V|$  is the number of nodes in the network. Although this algorithm needs to enumerate all the combinations of neighboring nodes of each node, which is in exponential complexity, it is feasible when the number of neighbors per node is not large. In a denser network, we propose another local rate and candidate selection scheme by using node's location information as in GOR.

### B. Per-hop greedy: Most Advancement per Medium Time

**A local metric: Expected Advancement Rate** The location information is available to the nodes in many applications of multi-hop wireless networks, such as sensor networks for monitoring and tracking purposes [2] and vehicular networks [4]. GOR has been proposed as an efficient routing scheme in such networks. In GOR, nodes are aware of the location of itself, its one-hop neighbors, and the destination. A packet is forwarded to neighbor nodes that are geographically closer to the destination. In [8], we have proposed a local metric, *expected packet advancement (EPA)* for GOR to achieve efficient packet forwarding. It represents the expected packet advancement achieved by opportunistic routing in one transmission without considering the transmission rate. In this paper, we extend it into a bandwidth adjusted metric, *expected advancement rate (EAR)*, by taking into account various transmission rates.

Given a transmitter  $n_i$ , one of its forwarding candidates  $n_{i_q}$ , and the destination  $n_d$ , we define the **packet advancement**  $a_{ii_q}$  in Eq. (16), which is the Euclidean distance between the transmitter and destination subtracting the Euclidean distance between the candidate  $n_{i_q}$  and the destination.

$$a_{ii_q} = \text{dist}(n_i, n_d) - \text{dist}(n_{i_q}, n_d) \quad (16)$$

This definition represents the advancement in distance made toward the destination when  $n_{i_q}$  forwards the packet sent by  $n_i$ . Then we define the EAR as follows.

$$EAR_{n_i}^{\mathcal{F}_i} = R_i \sum_{q=1}^r a_{ii_q} p_{ii_q} \prod_{k=0}^{q-1} (1 - p_{ii_k}) \quad (17)$$

The physical meaning of EAR is the *expected bit advancement per second* towards the destination when the packet is forwarded according to the opportunistic routing procedure introduced in section II.

The definition of EAR is the rate  $R_i$  multiplying the EPA proposed in [8]. According to the proved relay priority rule for EPA [8], we have the following theorem for EAR:

**Theorem 5.1: (Relay priority rule)** For a given transmission rate at  $n_i$  and  $\mathcal{F}_i$ , the maximum EAR can only be achieved by giving the candidates closer to the destination higher relay priorities.

This Theorem indicates how to prioritize the forwarding candidates when a transmission rate and the forwarding candidate set are given. From the definition of EAR, it is also not difficult to find that adding more neighboring nodes with

positive advancement into the existing forwarding candidate set will lead to a larger EAR. Therefore, we conclude that *an OR strategy that includes all the neighboring nodes with positive advancement into the forwarding candidate set and gives candidates with larger advancement higher relay priorities will lead to the maximum EAR* for a given rate.

Then a straightforward way to find the best rate is: for node  $n_i$ , at each transmission rate  $R^m$  ( $1 \leq m \leq J$ ), we calculate the largest EAR according to the above conclusion, then we pick the rate that yields the maximum EAR. This would be the local optimal transmission rate and the corresponding forwarding candidate set. Note that for a node  $n_i$ , it is possible that no neighboring nodes are closer to the destination than itself. In this case we need some mechanism like face routing [19] to contour the packet around the void. However, solving the communication voids problem is out of the scope of this paper.

Note that the above discussion does not take into consideration of protocol overhead. As we have shown in [6]–[8], including as many as possible nodes might not be the optimal strategy when overheads, such as the time used to coordinate the relay contention at MAC layer, are taken into consideration. To consider the protocol overhead, the EAR can be extended to the metric EOT (expected one-hop throughput) proposed in [7]. However, in this paper, since our goal is on studying the end-to-end throughput bound of OR, we apply EAR as the local metric, which is the upper bound of the packet advancement rate that can be made by any GOR.

## VI. PERFORMANCE EVALUATION

In this section, we use Matlab to investigate the impact of different factors on the end-to-end throughput bound of opportunistic routing, such as source-destination distances, node densities, and number of forwarding candidates. Both line and square topologies are studied for each factor. We also compare the performance of single rate opportunistic routing and multi-rate ones, and the performance of OR with traditional routing (TR). We call a routing scheme “traditional” when there is only one forwarding candidate selected for each packet relay at each hop.

The OR schemes we investigate include single-rate ExOR [3], single/multi-rate GOR and single/multi-rate LMTOR introduced in Section V. For ExOR [3], each transmitter selects the neighbors with lower ETX (Estimated Transmission count) to the destination than itself as the forwarding candidates, and neighbors with lower ETX have higher relay priorities. For GOR, the forwarding candidates of a transmitter are those neighbors that are closer to the destination, and candidates with larger advancement to the destination have higher relay priorities. The EAR metric proposed in Section V-B is used to select the transmission rate for each node in the multi-rate scenario. For multi-rate LMTOR, the algorithm and metric proposed in Section V-A is used to choose transmission rate and forwarding candidates at each node. All the evaluations are under the protocol model [15].

### A. Simulation Setup

The simulated network has 20 stationary nodes randomly uniformly distributed on a line with length  $L$  or in a  $W \times Wm^2$

square region. The data rates 24, 12, and 6 Mbps (chosen from 802.11a) are studied. We use one of the most common models - log-normal shadowing fading model to characterize the signal propagation. The received signal power is:

$$P_r(d)_{dB} = P_r(d_0)_{dB} - 10\beta \log\left(\frac{d}{d_0}\right) + X_{dB} \quad (18)$$

where  $P_r(d)_{dB}$  is the received signal power at distance  $d$  from the transmitter,  $\beta$  is the path loss exponent, and  $X_{dB}$  is a Gaussian random variable with zero mean and standard deviation  $\sigma_{dB}$ .  $P_r(d_0)_{dB}$  is the receiving signal power at the reference distance  $d_0$ , which is calculated by Eq. (19):

$$P_r(d_0)_{dB} = 10 \log\left(\frac{P_t G_t G_r c^2}{(4\pi)^2 d_0^2 f^2 l}\right) \quad (19)$$

where  $P_t$  is the transmitted signal power,  $G_t$  and  $G_r$  are the antenna gains of the transmitter and the receiver respectively,  $c$  is velocity of light,  $f$  is the carrier frequency, and  $l$  is the system loss.

In our simulation,  $d_0 = 1m$ ,  $\beta = 3$ ,  $\sigma_{dB} = 6$ ,  $G_t$ ,  $G_r$  and  $l$  are all set to 1,  $P_t = 15dbm$ ,  $c = 3 \times 10^8 m/s$ , and  $f = 5GHz$ .

We assume a packet is received successfully if the received signal power is greater than the receiving power threshold ( $P_{Th}$ ). For 802.11a, the  $P_{Th}$  for 24, 12, and 6Mbps is -74, -79, and -82dbm, respectively. Then according to Eq. (18) and (19), the packet reception ratio for each rate at a certain distance  $d$  can be derived. We set the PRR threshold  $p_{td}$  as 0.1, so the effective transmission radius for each rate (24, 12, and 6Mbps) is 47, 70 and 88m, respectively. As discussed in [10], 802.11 systems have very close interference ranges for different channel rates, so we use a single interference range 120m for all channel rates for simplicity.

### B. Impact of Source-Destination Distances

In this subsection, we evaluate the impact of the source-destination distance on the end-to-end throughput bound of OR and TR in line and square topologies. For line topology, the length  $L$  is set as 400m. We fix the left-end node as the destination, and calculate the throughput bounds from all other nodes to it under different OR and TR variants. For square topology, the side length is set as 150m. We fix the node nearest to the lower left corner as the destination, and calculate the throughput bounds from all the other nodes to it. We evaluate the performance under both single-rate and multi-rate scenarios. The average numbers of neighbors per node under line topology at rate 24, 12 and 6 Mbps is 3.5, 5.5 and 6.8, respectively, and under square topology, it is 3.5, 7.0 and 10.0, respectively.

In the single-rate scenario, for TR, we compute the exact end-to-end throughput bounds between the source-destination pairs according to the LP formulations in [14], which normally result in multiple paths from the source to the destination. So we call it ‘‘Multipath TR’’. We also compute the end-to-end throughput bound of a single path that is found by minimizing the medium time (delay), and we call it ‘‘Single-path TR’’. The bound of single-path TR is calculated according to the formulations in [11]. For the three OR variants, we compute

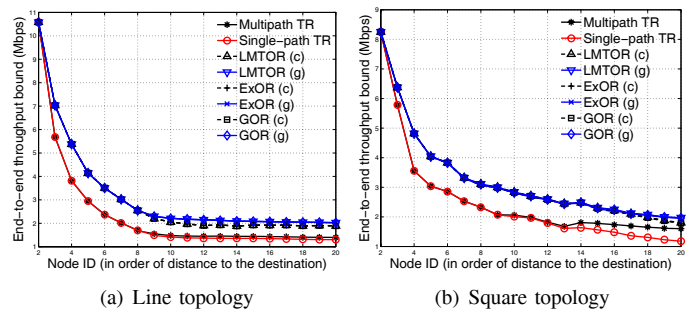


Fig. 5. End-to-end throughput bound of OR and TR in a single rate (12Mbps) network under different topologies.

the throughput bounds under both conservative (indicated as ‘c’) and greedy (indicated as ‘g’) modes as we discussed in Section III-B.

Fig. 5(a) shows the simulation results of LMTOR, ExOR, GOR and TR in a single rate (12Mbps) system under line topology. We have the following observations: 1) when the distance between the source and destination increases, the end-to-end throughput bound of each routing scheme decreases. 2) the OR achieves higher throughput bound than TR under different source-destination distances. 3) all the OR variants achieve the same performance under the same mode. 4) when source-destination distance is larger than 2 hops, OR in greedy mode results in higher end-to-end throughput than that in conservative mode, while when the source-destination distance is smaller than 2 hops, they represent the same performance. 5) the multipath TR achieves almost the same throughput bound as single-path TR.

In the line topology, the throughput gain of OR over TR mainly comes from the opportunistic property. That is, OR increases the reliability of a successful transmission by involving multiple forwarding candidates. The increased reliability reduces the retransmission overhead, and saves the medium time for each packet forwarding, thus improves the throughput.

By tracing into the simulation, we find that the three OR variants result in the same forwarding candidate selection and prioritization at each forwarding node, although they follow different criteria to select the candidates and prioritize them. That’s why we have the observation 3), which indicates that in the line topology the per-hop greedy behavior in GOR can approach the same end-to-end performance as that obtained by a distributed scheme like LMTOR.

For observation 4), when the source is near to the destination, all the nodes along the paths are in the interference range of each other, thus there is no concurrent transmission allowed in either greedy or conservative mode. Therefore, OR in both modes achieves the same performance when the source-destination distance is smaller than 2 hops. When the source-destination distance is larger than 2 hops, concurrent transmission becomes possible. Since conservative mode requires interference free at all the forwarding candidates, for each transmission, it consumes more space than greedy mode. That is, greedy mode achieves higher spatial reuse ratio than conservative mode and allows more concurrent transmissions, thus results in higher throughput.

The observation 5) indicates that multipath TR does not re-



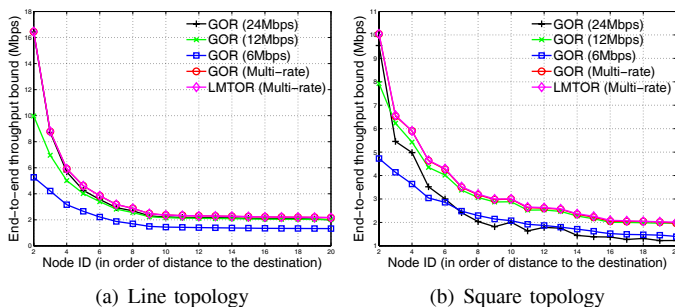


Fig. 6. End-to-end throughput bound of OR in single-rate and multi-rate networks under different topologies.

ally improve the wireless network throughput over the single-path TR in the line topology. The reason is that even when there are multiple paths between the source and destination, the links on different paths can not be scheduled at the same time due to interference. OR does make real use of multiple paths, in the sense that throughput can take place on any one of the outgoing links from the transmitter to its forwarding candidates for each transmission.

Fig. 5(b) shows the simulation results of LMTOR, ExOR, GOR and TR in a single rate (12Mbps) system under square topology. One interesting observation is that the multipath TR achieves (up to 60%) higher throughput bound than single-path TR, and it can achieve comparable or even higher throughput than OR in conservative mode when the source-destination distance is larger than 2 hops. In the square topology, when the source and destination is far apart, real multipath routing becomes feasible. That is, different links on different paths can be activated at the same time, thus improve the throughput. This observation also indicates that it is not a good idea to include as many as possible forwarding candidates into opportunistic routing when some protocol requires interference free at all the forwarding candidates. As we can see in Fig. 5(b) that OR in greedy mode still achieves higher throughput than OR in conservative mode and multipath TR. So the advantage of OR over TR is still validated.

Since OR in greedy mode always achieves higher throughput bound than that in conservative mode, in the following evaluation, the throughput bound of OR is only calculated under greedy mode. As the performance of ExOR is nearly the same as that of GOR, we will not show the simulation result of ExOR in the following figures. Now, we compare the throughput bounds of OR in multi-rate and single-rate systems.

Fig. 6(a) shows the simulation results of multi-rate LMTOR, multi-rate GOR, and single-rate GOR under line topology. We can see that generally multi-rate OR achieves better performance than any single-rate OR. When the distance between the source and destination is shorter than the interference range (corresponding to node ID 7), the system operating on 24Mbps achieves better performance than that on 12Mbps. However, the difference becomes smaller and smaller when the source-destination distance becomes larger, since more forwarding candidates are involved for 12Mbps and the spatial diversity is increased. When the source-destination distance is larger than the interference range, the performance of 24Mbps is as the same as that of 12Mbps. Fig. 6(b) shows the simulation results

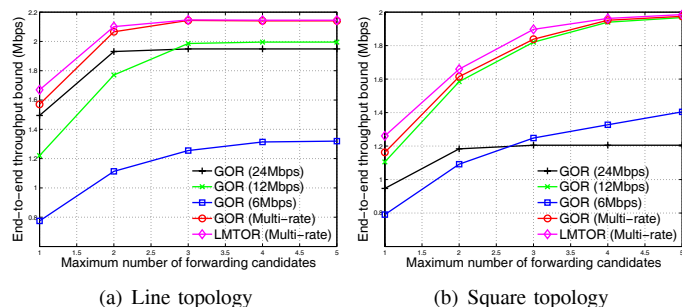


Fig. 7. End-to-end throughput bound of OR with different number of forwarding candidates under different topologies.

under square topology. An interesting difference from line topology is that the system operating at 24Mbps shows lower throughput bound than those operating at 12Mbps and 6Mbps for most of the source-destination pairs. The disadvantage of short transmission range and lower spacial diversity of 24Mbps overwhelms its higher data rate advantage in the square topology.

### C. Impact of Forwarding Candidate Number

In this subsection, we study the impact of the number of forwarding candidates on the performance of OR. For line topology, we examine the throughput bound between the two end nodes on the line. For square topology, we examine the throughput bound between the two end nodes on the diagonal. The topology sizes are set as the same as those in the previous simulation.

For a transmitter, given a maximum number of forwarding candidates, the single-rate GOR selects the forwarding candidates by applying the algorithm proposed in [8] to maximize the EPA. For multi-rate GOR, we select the optimal forwarding candidates (no larger than the maximum number) for each single-rate GOR, then select the data rate which yields the highest EAR. For LMTOR, we apply the distributed algorithm proposed in Section V-A. For the local search in Eq. (14) and (15), we test all the subsets of one-hop neighbors with cardinality no larger than the maximum number of forwarding candidates.

Fig. 7(a) and 7(b) show the simulation results under line and square topologies, respectively. Generally, multi-rate OR achieves better performance than any single-rate OR, and multi-rate LMTOR achieves better performance than multi-rate GOR. In the square topology (Fig. 7(b)), GOR on 12Mbps is always the best among all the single-rate GOR for all the different candidate sizes. The 24Mbps GOR performs even worse than 6Mbps GOR in square topology when the maximum forwarding candidate number is larger than 3. Since 24Mbps has the shortest transmission range, which results in the lowest node density, GOR on 24Mbps actually does not have 3 or more forwarding candidates to choose. Note that, the maximum number of forwarding candidates being equal to 1 corresponds to the TR. Although 6Mbps geographic TR (GTR) achieves lower throughput bound than 24Mbps GTR, it is not necessarily the truth for GOR. Since lower data rates have longer transmission ranges, thus result in higher neighborhood

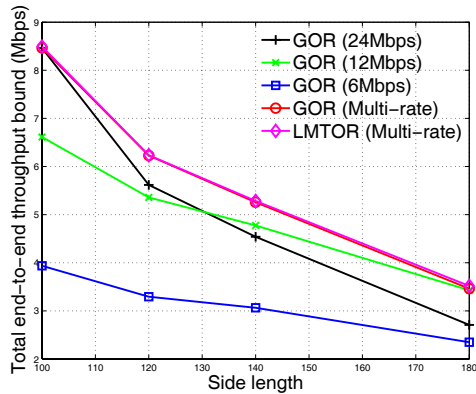


Fig. 8. Total end-to-end throughput bound of OR under square topology with different side lengths in multi-flow case.

diversities, which can help to increase the effective forwarding rate for each transmission when OR is used. In the line topology (Fig. 7(a)), when the forwarding candidate number is greater than 3, GOR on 12Mbps achieves better performance than that on 24 Mbps, which can be explained by the same reason. However, in the line topology, the disadvantage of low data rate of 6Mbps overwhelms its advantage on higher spatial diversity. Therefore, GOR on 6Mbps shows the worst performance.

An interesting observation in both Fig. 7(a) and 7(b) is the concavity of each curve, which indicates that although involving more forwarding candidates improves the end-to-end throughput bound of OR, the capacity increase gained from involving more candidates decreases when we keep doing so. This end-to-end throughput observation is consistent with the local behavior found in [6], [8]. For a realistic MAC for OR, the coordination overhead is likely to increase when more forwarding candidates are involved. Since the throughput gain decreases when the number of forwarding candidates is increased, considering the MAC overhead, it may not be optimal or necessary to involve as many as forwarding candidates in OR.

#### D. Impact of Node Density

The impact of the node density on the performance of OR is investigated in this subsection. Instead of single flow, we investigate multi-flow case by randomly selecting four source-destination pairs in the network. The settings of the network terrain size and the corresponding number of neighbors per node at different data rates under square topology are summarized in Table I.

Fig. 8 shows the simulation results under square topology. There exists a threshold on the node density, higher than which, the GOR on 24Mbps performs better than that on 12Mbps, and vice versa. The threshold is about 10.9 neighbors per node on 12Mbps. Our proposed multi-rate GOR and LMTOR can adapt to the different node densities, and choose the proper transmission rate and forwarding candidate set to achieve the best performance than any single-rate GOR. Since we obtain the same performance trend under line topology, we do not show the corresponding result.

TABLE I  
AVERAGE NUMBER OF NEIGHBORS PER NODE AT EACH RATE UNDER SQUARE TOPOLOGY WITH DIFFERENT SIDE LENGTHS

Data rate (Mbps)	Square side length (m)			
	100	120	140	180
24	7.7	5.5	4.1	2.8
12	13.8	10.9	8.7	5.8
6	17	14.5	11.9	8.6

## VII. RELATED WORK

### A. Capacity of Multi-hop Wireless Networks

Jain *et al.* proposed a framework to calculate the throughput bounds of traditional routing between a pair of nodes by adding wireless interference constraints into the maximum flow formulations [14]. Zhai and Fang studied the path capacity of traditional routing in a multi-rate scenario [11]. Distinguished from the previous works, we propose a method to compute the end-to-end throughput bounds of OR. Our framework can be used as a tool to calculate the end-to-end throughput bound of different OR variants, and is an important theoretical foundation for the performance study of OR.

### B. Opportunistic Routing

Existing study on OR mainly focuses on protocol design and candidate selection schemes. One variant of OR relies on path cost to select and prioritize forwarding candidates, such as ExOR [3], opportunistic any-path forwarding [20], and least-cost opportunistic routing (LCOR) [5]. The other variant of OR uses the location information of nodes to define the candidate set and relay priority, such as GeRaF [2] and [4]. However, there is no theoretical work on determining the end-to-end throughput bounds of OR.

### C. Multi-rate Routing

Multi-rate wireless network has started attracting research attention recently. Several metrics are proposed for multi-rate traditional routing, such as weighted cumulative expected transmission time (WCETT) [18], medium time metric (MTM) [9], bandwidth distance product [10], and interference clique transmission time [11].

However, these metrics are proposed for routing on a fixed path following the concept of the traditional routing. Our previous work [7] on multi-rate OR discusses the impact of the protocol overhead and multi-rate capability on the performance of GOR. While in this paper, by extending our study in [21], we focus on identifying the end-to-end throughput bound of OR for different OR schemes under both single and multi-rate scenarios.

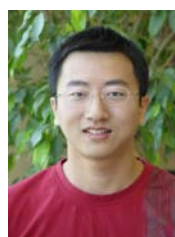
## VIII. CONCLUSION

In this paper, we studied the impact of multiple rates, interference, candidate selection and prioritization on the maximum end-to-end throughput of OR. Taking into account the wireless interference and unique property of OR, we proposed a new method of constructing transmission conflict graphs, and presented a methodology for computing the end-to-end throughput bounds (capacity) of OR. We formulate

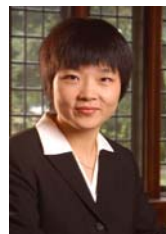
the maximum end-to-end throughput problem of OR as a maximum-flow linear programming subject to the transmission conflict constraints. We also proposed two metrics for OR under multi-rate scenario, one is *expected medium time* (EMT), and the other is *expected advancement rate* (EAR). Based on these metrics, we proposed the distributed and local rate and candidate selection schemes: LMTOR and MGOR, respectively. We compared the throughput capacity of multi-rate OR with single-rate ones under different settings, such as different topologies, source-destination distances, number of forwarding candidates, and node densities. We show that OR has great potential to improve the end-to-end throughput under different settings, and our proposed multi-rate OR schemes achieve higher throughput bound than any single-rate GOR. We observe some insights of OR: 1) although involving more forwarding candidates increases the end-to-end capacity, the capacity gained from involving more forwarding candidates decreases; 2) there exists a node density threshold, higher than which 24Mbps GOR performs better than 12Mbps GOR, and vice versa.

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