Measurement of pulse width and amplitude jitter noises of gigahertz optical pulse trains by time-domain demodulation

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We propose a technique for measuring both pulse width and amplitude jitter noises of high-repetition-rate optical pulse trains and the cross correlation between these noises as well. The technique is based on time-domain amplitude demodulation of three harmonic components of the detected pulse train. We applied this technique to characterize noises of a gigahertz optical pulse train generated by an actively mode-locked Er-doped fiber laser. Correlation between pulse width jitter and pulse amplitude jitter was observed at low frequencies in this laser. Unlike relaxation oscillation noise, low-frequency noise is free from pulse energy jitter. Owing to its ability to measure pulse width jitter in addition to amplitude and phase jitters, this technique is of great interest for characterizing noises of a wide variety of optical pulse train sources. © 2001 Optical Society of America

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Characterization of the noises of gigahertz-repetition-rate optical pulse trains is an important issue for applications such as ultrafast optical telecommunications and all-optical analog-to-digital conversion. Phase noise (i.e., pulse timing jitter) measurement techniques were developed that rely on optical cross correlation,[1] time-interval analysis,[2] or phase-encoded optical sampling.[3] For measurement of both amplitude and phase noises, a well-known technique, originally proposed by von der Linde,[4] permits retrieval of the power spectral densities (PSDs) of amplitude and phase noises by measuring the rf spectrum of the detected pulse train about two harmonics of the repetition rate. Unfortunately, extraction of amplitude and phase noises is subject to assumptions, i.e., smallness of amplitude and phase noises, absence of cross correlation between them, and absence of pulse width jitter, whose validity can be seriously questioned. To get rid of the two last-named assumptions, Chen *et al.*[5] proposed an extension of the original technique in which pulse width jitter, in addition to amplitude and phase jitters, is determined. Unfortunately, a complicated measurement setup including optical second-harmonic generation is required. Recently, a time-domain technique based on demodulation, in amplitude and in phase, of the first harmonic of the detected pulse train was proposed by Tsuchida.[6,7] This technique extracts the instantaneous amplitude and phase of the pulses, which are not accessible with the techniques mentioned above. From amplitude and phase time series, it is then straightforward to calculate PSDs of amplitude and phase noises by Fourier transform. Moreover, the cross-spectral density between amplitude and phase noises can be calculated. Unfortunately, this technique gives no information about pulse width jitter.

In this Letter we first derive a mathematical expression of the harmonics of the detected pulse train in the presence of pulse width jitter. Based on this result, we propose an extension of the time-domain demodulation technique for measuring pulse width jitter in addition

to pulse amplitude jitter. Then we use this technique to characterize the noise of an optical pulse train generated by an actively mode-locked fiber laser.

In the presence of amplitude and phase noises, the nth harmonic of the detected pulse train, $V_n(t)$, is expressed by

$$V_n(t) = V_{n0}[1 + \epsilon(t)]\sin[n\omega_r t + n\phi(t)], \quad (1)$$

where V_{n0} is the average amplitude, $\epsilon(t)$ and $\phi(t)$ are the amplitude and phase jitters of the pulses, respectively, $\omega_r = 2\pi f_r$, and f_r is the pulse repetition rate. From Eq. (1) we see that one can retrieve $\epsilon(t)$ and $\phi(t)$ by amplitude and phase demodulation, respectively,[7] of the nth harmonic. To minimize bandwidth requirements, the first harmonic ($n = 1$) is usually chosen. We now derive the expression of $V_n(t)$ in the presence of pulse width jitter. Let us first define $H(\Omega)$ as the Fourier transform of the optical intensity pulse shape function $h(\zeta)$, where Ω and ζ are normalized frequency and time, respectively. If pulse width T undergoes random fluctuations ΔT, then the Fourier transform of $h[t/(T + \Delta T)]$ is $(T + \Delta T) \times H[(T + \Delta T)\omega]$. Expanding $(T + \Delta T) \times H[(T + \Delta T)\omega]$ in Taylor series about $\omega = 0$ up to the quadratic term, and keeping only terms of first order in ΔT, we find [remembering that $H(\Omega)$ is real and even] that

$$(T + \Delta T) \times H[(T + \Delta T)\omega]$$
$$= T \times H(T\omega)[1 + (1 - \beta T^2 \omega^2)\tau], \quad (2)$$

where $\tau = \Delta T/T$ is the relative pulse width jitter and $\beta = \int_{-\infty}^{+\infty} \zeta^2 h(\zeta)\mathrm{d}\zeta / \int_{-\infty}^{+\infty} h(\zeta)\mathrm{d}\zeta$ is a pulse-shape-dependent constant.[8] Equation (2) is the expression of the spectrum of one pulse and also represents the envelope of the harmonics of the optical pulse train and thus of the detected pulse train (or photocurrent), i.e., $V_{n0} = T \times H(Tn\omega_r)$, for frequencies $n\omega_r$ lower than the upper frequency limit of the detection system.[5] Therefore Eq. (2) shows that,

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in the presence of pulse width jitter, the amplitude $V_{n0} = T \times H(Tn\omega_r)$ of the nth-harmonic component is multiplied by $1 + (1 - \beta T^2 n^2 \omega_r{}^2)\tau$. Hence, if we consider only first-order perturbation terms, Eq. (1) can be rewritten as

$$V_n(t) = V_{n0}[1 + \epsilon(t) + (1 - \beta T^2 n^2 \omega_r{}^2)\tau(t)]$$
$$\times \sin[n\omega_r t + n\phi(t)]. \qquad (3)$$

Equation (3) shows that the instantaneous amplitude of $V_n(t)$ is no longer equal to the amplitude jitter of the pulse train itself, $\epsilon(t)$, as was the case in the absence of pulse width jitter [Eq. (1)]. Hence, whereas phase demodulation of $V_n(t)$ still yields $\phi(t)$ (actually $n \times \phi$), amplitude demodulation now yields not $\epsilon(t)$ but instead $\epsilon_n(t)$, which is defined as

$$\epsilon_n(t) = \epsilon(t) + (1 - \beta T^2 n^2 \omega_r{}^2)\tau(t). \qquad (4)$$

In Eq. (4), $\epsilon_n(t)$, $\epsilon(t)$, and $\tau(t)$ are random variables. In experiments, however, one has access only to their realizations in the form of time series. If one could acquire simultaneously time series $\epsilon_n(t)$ that correspond to two different values of n, then time series $\epsilon(t)$ and $\tau(t)$ could be determined by solution of the set of two equations such as Eq. (4). In practice, simultaneous acquisition of the time series $\epsilon_n(t)$ implies performing amplitude demodulation simultaneously on two harmonics of the pulse train, which significantly complicates the experimental setup. The approach that we propose hereafter is less demanding, as we assume that time series are acquired successively on the different harmonics. Assuming that noise processes are cyclostationary, we calculate the PSD of the time series $\epsilon_n(t)$:

$$S_{\epsilon_n}(f) = S_\epsilon(f) + (1 - Kn^2)^2 S_\tau(f)$$
$$+ 2(1 - Kn^2)\mathrm{Re}[S_{\epsilon\tau}(f)], \qquad (5)$$

where $K = \beta T^2 \omega_r{}^2$; $S_\epsilon(f)$ and $S_\tau(f)$ are the PSDs of $\epsilon(t)$ and $\tau(t)$, respectively; and $S_{\epsilon\tau}(f)$ is the cross-spectral density of $\epsilon(t)$ and $\tau(t)$. If we perform amplitude demodulation on three harmonics successively and calculate PSDs $S_{\epsilon_n}(f)$ from measured time series $\epsilon_n(t)$, we obtain a set of three equations such as Eq. (5) from which we can retrieve $S_\epsilon(f)$, $S_\tau(f)$ and $\mathrm{Re}[S_{\epsilon\tau}(f)]$, i.e., the useful information.

We apply this technique to the case of a gigahertz optical pulse train generated by an actively mode-locked Er-doped fiber laser.[9] The optical pulse train is detected by a 20-GHz photodiode and is fed into a vector signal analyzer (Agilent Technologies, 89441A). Successively, time-domain amplitude demodulation of first, second, and third harmonics of the detected pulse train is performed by the vector signal analyzer (details of its operation were described previously[7]). The time series $\epsilon_1(t)$, $\epsilon_2(t)$, and $\epsilon_3(t)$ are then retrieved from the vector signal analyzer's memory, and we use them to calculate the PSDs $S_{\epsilon_1}(t)$, $S_{\epsilon_2}(t)$ and $S_{\epsilon_3}(t)$, respectively, using a fast-Fourier-transform-based algorithm. A Hanning window was chosen [average over 50 similar traces of

$S_{\epsilon_1}(t)$, $S_{\epsilon_2}(t)$, and $S_{\epsilon_3}(t)$]. The pulse train repetition rate was set to 0.866 GHz to permit measurements up to the third harmonic (2.6 GHz) with a 2.65-GHz downconverter unit (Agilent 89431A). This limitation is in no way intrinsic to the measurement technique but is due only to the available equipment used. To determine K in Eq. (5), we measured the autocorrelation of the pulse and found that pulses were Gaussian: $h(t/T_{\mathrm{FWHM}}) = \exp\{-[t/(1.2 \times T_{\mathrm{FWHM}})]^2\}$, with $T_{\mathrm{FWHM}} = 14.5$ ps. As $\beta = 0.72$ for a Gaussian pulse and $f_r = 0.86$ GHz in the present case, we found that $K = 0.0045 \ll 1$.

Figure 1 shows the PSDs calculated from measured time series $\epsilon_1(t)$, $\epsilon_2(t)$, and $\epsilon_3(t)$. The total frequency span ranges from 10 Hz to 500 kHz. Figure 1 results from a concatenation of four measurements with spans (inverse of sampling periods) of 500 Hz, 5 kHz, 30 kHz, and 500 kHz. The resolution (the inverse of measurement duration) was equal to one thousandth of the span in all cases. Data below 10 Hz were dropped to avoid the influence of the phase-locked loop of the analyzer on the measurement.[7] From Fig. 1 we observe that, at low frequencies (below 3 kHz), all PSDs have almost the same shape, whereas their level increases with the harmonic n (note the peaks at 10 Hz and its harmonics, originating from the length-stabilizing feedback loop of the laser[9]). At frequencies higher than 3 kHz, all PSDs decrease to measurement noise floors, except the presence of a peak at $\approx$23 kHz, which is due to relaxation oscillation (RO) noise. The intensity of the RO peak, however, is identical in the three cases, i.e., does not depend on n. Note that the measurement noise floor increases with the harmonic n because the amplitude V_{n0} of the successive harmonics decreases. In what follows, we consider low-frequency noise (10 Hz–3 kHz) and RO noise ($\approx$23 kHz). In the case of low-frequency noise, Fig. 2 shows the PSDs $S_\epsilon(f)$, $S_\tau(f)$, and $-\mathrm{Re}[S_{\epsilon\tau}(t)]$ that were determined by solution of the set of Eqs. (5) from the data of Fig. 1. We can observe that the three curves are perfectly superimposed: $S_\epsilon(f) = S_\tau(f) = -\mathrm{Re}[S_{\epsilon\tau}(f)]$. This
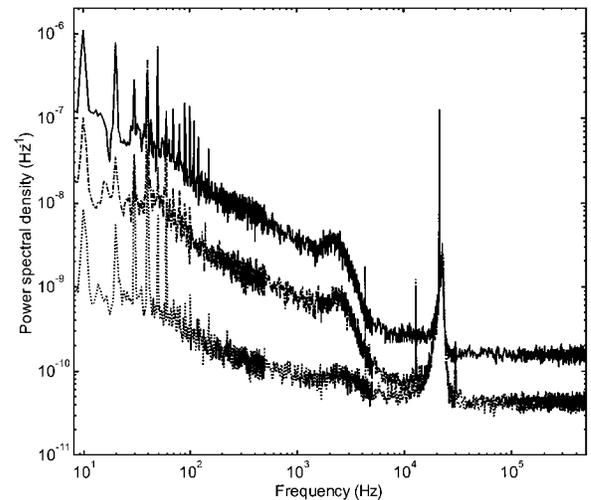
Fig. 1.   Bottom to top, PSDs $S_{\epsilon_1}(f)$, $S_{\epsilon_2}(f)$, and $S_{\epsilon_3}(f)$ obtained from time series $\epsilon_1(t)$, $\epsilon_2(t)$, and $\epsilon_3(t)$, respectively.
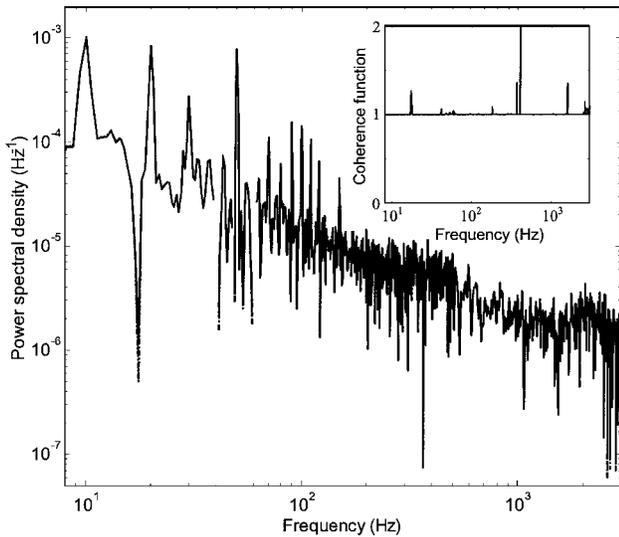
Fig. 2.   PSDs $S_\epsilon(f)$, $S_\tau(f)$, and $-\mathrm{Re}[S_{\epsilon\tau}(f)]$ in the frequency range 10 Hz–3 kHz. Inset, coherence function $\gamma_{\epsilon\tau}{}^2(f)$ in the same frequency range.

implies that pulse amplitude jitter and pulse width jitter are equal in magnitude and of opposite sign, i.e., $\epsilon(t) = -\tau(t)$ [$\epsilon(t)$ and $\tau(t)$ are totally correlated]. Moreover, remembering that $\mathrm{Re}[S_{\epsilon\tau}]^2 \le |S_{\epsilon\tau}|^2 \le S_\epsilon S_\tau$, we have $|S_{\epsilon\tau}|^2 = S_\epsilon S_\tau$, and the coherence function of ϵ and τ, defined as $\gamma_{\epsilon\tau}{}^2 = |S_{\epsilon\tau}|^2/(S_\epsilon \times S_\tau)$, equal to 1 (Fig. 2, inset). Unphysical values of $\gamma_{\epsilon\tau}{}^2$ greater than 1 are assumed to be due to slight perturbations between consecutive measurements of $\epsilon_n(t)$. Using $S_\epsilon = S_\tau = -\mathrm{Re}[S_{\epsilon\tau}]$ in Eq. (5), we find that $S_{\epsilon_n}(f) = K^2 n^4 S_\epsilon(f)$, yielding $S_{\epsilon_n}(f) = n^4 S_{\epsilon_1}(f)$, a result that can be verified from Fig. 1 for $n = 2, 3$. The large correlation between low-frequency pulse width and amplitude jitters is not an astonishing result: Indeed, it was already observed by Chen et al.[5] in the case of a passively mode-locked Nd:YLF laser. This is nevertheless what we believe is the first measurement of pulse width jitter and pulse amplitude width cross correlation in an actively mode-locked fiber laser. Moreover, the technique that we used, based on time-domain amplitude demodulation of the detected pulse train, is more versatile than the technique described in Ref. 5, which requires detection of the optically frequency-doubled pulse train by use of a nonlinear crystal, along with direct detection of the pulse train. $\epsilon(t) = -\tau(t)$ means that low-frequency pulse amplitude fluctuations are exactly compensated for by low-frequency pulse width fluctuations of identical relative magnitude and opposite sign. In other words, the relative pulse energy jitter $\Delta E/E = \epsilon + \tau$ is equal to zero at low frequencies. This result also indicates that the generated pulses are not solitons, because in that case the well-known relation between soliton energy and duration ($E \propto 1/T$)

would lead to $\Delta E/E = -\Delta T/T = -\tau$, which was not observed in our experiment. In the case of RO noise, $S_{\epsilon_1}(f) \cong S_{\epsilon_2}(f) \cong S_{\epsilon_3}(f)$ (Fig. 1); i.e., the left-hand sides of Eqs. (5) are almost identical. Moreover, given the smallness of the parameter K, coefficients on the right-hand sides of Eqs. (5) are almost identical. Therefore, solving the set of equations leads to large errors in S_ϵ, S_τ, and $-\mathrm{Re}[S_{\epsilon\tau}]$. The relation $S_{\epsilon_n}(f) = n^4 S_{\epsilon_1}(f)$, which stands if $\epsilon(t) = -\tau(t)$, is clearly not satisfied in the frequency range of RO noise (Fig. 1). As a consequence, pulse energy jitter exists in this frequency range. This result is of course expected, as RO noise consists in an energy transfer from intracavity optical power to population inversion of Er ions and vice versa.

In conclusion, we have demonstrated that, in the presence of pulse width jitter, the instantaneous amplitude of any harmonic component of the detected pulse train is a combination of both pulse amplitude jitter and pulse width jitter. Separation between the two jitter contributions can be achieved by time-domain amplitude demodulation of three harmonics of the detected pulse train. Using this technique, we were able to measure pulse width and amplitude jitter noises of a gigahertz pulse train generated by an actively mode-locked fiber laser, as well as the cross correlation between these noises. At low frequencies, cross correlation is maximal; pulse width fluctuations exactly compensate for pulse amplitude fluctuations. We believe that the absence of pulse energy fluctuations is due to the stability of the pump laser diode and to the laser operation in the Gaussian-pulse (linear) mode-locking regime. Owing to its ability to measure pulse width jitter in addition to amplitude and phase jitters, we believe that the proposed technique will be of great interest for characterizing noises of a wide variety of optical pulse train sources.

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