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A Formal Theory of Feature Binding in Object Perception

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Visual objects are perceived correctly only if their features are identified and then bound together. Illusory conjunctions result when feature identification is correct but an error occurs during feature binding. A new model is proposed that assumes feature binding errors occur because of uncertainty about the location of visual features. This model accounted for data from 2 new experiments better than a model derived from A. M. Treisman and H. Schmidt's (1982) feature integration theory. The traditional method for detecting the occurrence of true illusory conjunctions is shown to be fundamentally flawed. A reexamination of 2 previous studies provided new insights into the role of attention and location information in object perception and a reinterpretation of the deficits in patients who exhibit attentional disorders.

A description of visual object identification in terms of registering visual stimulus features has a long history (see Boring, 1950). This description of identification is implicit in many popular models, including the pandemonium model of Selfridge (1959), the recognition-by-components model of Biederman (1987), and models based on spatial frequency analysis (e.g., DeValois & DeValois, 1988; see also Prinzmetal & Keysar, 1989; Wolford & Shum, 1980). Formal models of visual feature analysis have been developed by many investigators (e.g., Ashby & Perrin, 1988; Ashby & Townsend, 1986; Graham, 1989; Thomas & Olzak, 1992; Wickens & Olzak, 1992; Townsend & Ashby, 1982).

In the normal environment, however, observers are not confronted with single objects. Hence, the registration of features may not be sufficient for veridical object recognition. When several different objects are presented, not only must features be correctly registered, they also must be combined correctly (Treisman & Gelade, 1980). Treisman and her colleagues have shown that under limited exposure conditions, observers will report *illusory conjunctions*—percepts in which visual features are identified correctly but combined incorrectly (Treisman & Schmidt, 1982). The problem of correctly combining or integrating features has been called the *binding problem* (e.g., Crick, 1984).

Treisman and Schmidt (1982) found that the illusory conjunction of features occurred in whole report, partial report, detection, and same–different tasks. In these experiments, participants were briefly presented multielement displays of colored letters. Participants sometimes perceived the colors and letters correctly but in the wrong combination. For example, a participant might briefly be presented a display consisting of a blue *N* and a green *T* but occasionally perceive the *N* as being green. Illusory conjunctions between the dimensions of color and shape have been found in many studies (e.g., Cohen & Ivry, 1989; Ivry & Prinzmetal, 1991; Keele, Cohen, Ivry, Liotti, & Yee, 1988; Prinzmetal, 1992; Prinzmetal, Hoffman, & Vest, 1991; Prinzmetal & Keysar, 1989; Prinzmetal & Mills-Wright, 1984; Rapp, 1992; Seidenberg, 1987). In addition, illusory conjunctions have been obtained with other stimulus dimensions (e.g., Butler, Mewhort, & Browse, 1991; Prinzmetal, 1981; Gallant & Garner, 1988; Lasaga & Hecht, 1991; Treisman & Paterson, 1984; Treisman & Schmidt, 1982). For example, Prinzmetal (1981) found that participants sometimes perceived illusory plus signs when presented with nonoverlapping vertical and horizontal line segments (see also Maddox, Prinzmetal, Ivry, & Ashby, 1994).

At present, there are no formal theories of the feature binding process.[1] This study develops and tests such a theory. In addition to the benefits provided by any new theory, a formal approach has

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[1] There have been two previous formal accounts of illusory conjunctionlike phenomena. Wolford (1975) and Maddox et al. (1994) developed models of the incorrect combination of shape features, such as vertical and horizontal lines. Wolford (1975) modeled whole-report letter identification experiments by assuming that features of letters were extracted with some probability. Feature location "perturbated" with time until a central processor could read out display features. Thus, errors resulted both from failures in feature extraction and feature perturbation. Maddox et al. proposed a model for detection tasks in which feature location may be incorrectly registered. The Maddox et al. and Wolford models share the assumptions that performance is related to both feature registration and location. They differ in that Wolford's model assumes location information decays with time, whereas Maddox et al. assumed feature locations may be incorrectly registered.

at least three extra advantages. First, a formal theory forces one to make assumptions explicit. For example, in previous studies, errors of feature binding were detected by operational methods based on feature error rates for color, shape, or both (e.g., Cohen & Ivry, 1989; Prinzmetal & Mills-Wright, 1984; Treisman & Schmidt, 1982). As we will see, a formal analysis indicates that the logic behind such methods is often murky or incorrect.

Second, formal models allow us to compare various theoretical approaches to feature binding. For example, we will propose that one cause of illusory conjunctions is the perceptual mislocation of visual features (see also Maddox et al., 1994). On the other hand, Treisman and Schmidt's (1982) theory does not embody this assumption. If a formal theory of feature binding that attributes illusory conjunctions to poor location information provides a better account of data than one that does not, it would strengthen our belief in the psychological validity of the location uncertainty theory.

Third, a formal account of feature binding permits the estimation of underlying psychological parameters from overt behavior. For example, in signal detection theory the psychological parameters of sensitivity and bias are estimated from hit and false-alarm rates (Green & Swets, 1966). In experiments on feature binding, we would like to estimate the probability of correctly perceiving the various features and the probability of correctly binding the features.

This article develops a formal theory of feature binding for conventional illusory conjunction tasks involving the dimensions of color and shape. This new theory, which we call *location uncertainty theory,* is developed in the next two sections. As noted earlier, location uncertainty theory assumes that illusory conjunctions occur because of uncertainty about the location of visual features. In the fourth section, a set of multinomial models are developed to account for the effects of guessing in illusory conjunction experiments. The fifth section develops a formal model, called the *random binding model,* that is consistent with Treisman and Schmidt's (1982) feature integration theory. The sixth and seventh sections describe the results of two new experiments. A model based on location uncertainty theory is shown to account for the data from both experiments significantly better than the random binding model and two models that assume illusory conjunction reports are always due to guessing. Also, it is shown that the traditional method for detecting the occurrence of true illusory conjunctions is fundamentally flawed. The eighth section reexamines two previous studies of illusory conjunctions and demonstrates how our modeling approach can provide new insights into the role of attention and location information in object perception, as well as a reinterpretation of the deficits in patients who exhibit attentional disorders. Finally, we discuss limitations of our approach, and we compare it with another popular method for studying attention that uses a visual search task.

Distance and Similarity Effects on Illusory Conjunctions

Treisman and Schmidt (1982) proposed that the different features of an object are identified automatically (i.e., without attention) and in parallel. Object recognition, therefore, is a process of correctly binding or conjoining the various features that have been identified. This feature binding stage is assumed to require focused attention and to be performed in a serial fashion. If attention is not focused, features that were identified correctly may be combined incorrectly to form illusory conjunctions.

Treisman and Schmidt (1982) found that the distance or similarity between items had little effect on the likelihood of an illusory conjunction. As a result, they postulated that without focused attention, features are completely free-floating and may combine with any feature that has been identified. Further, they argued that "the internal representation on which conscious experience depends contains discrete labels of values on each dimension separately. The whole object must be resynthesized from a set of these feature labels" (p. 139). According to this interpretation, the continuous nature of the information available in the physical stimulus is lost very early in perceptual processing. As a result, decision processes in perception are assumed to operate on a set of discrete feature values.

At the time Treisman and Schmidt (1982) developed their feature integration theory, the available data indicated that neither interitem distance nor similarity affects the probability of an illusory conjunction. Subsequently, a number of studies have challenged this position. First, a number of investigators found that illusory conjunctions are more likely between items that are close together than between items that are far apart (e.g., Chastain, 1982; Cohen & Ivry, 1989; Ivry & Prinzmetal, 1991; Prinzmetal & Keysar, 1989; Prinzmetal & Mills-Wright, 1984; Prinzmetal, Treiman, & Rho, 1986; Wolford & Shum, 1980). Second, Ivry and Prinzmetal (1991) found that illusory conjunctions occurred more often between letters that were similar in color than between letters that were dissimilar in color. Thus, the internal representation does not seem to depend on discrete labels of values. Rather, there is abundant evidence that distance is an important variable in the formation of illusory conjunctions and that similarity also may be important.

Location Uncertainty Theory

One well-documented phenomenon that might account for the effects of distance on feature binding errors is uncertainty about the true location of a perceived stimulus feature. There is good evidence for trial-by-trial variability in the perceived location of visual features (e.g., Chastain, 1982; Estes, 1975; Klein & Levi, 1987; Levi & Klein, 1989; Wolford & Shum, 1980). For example, Wolford and Shum (1980) briefly presented squares with a tick mark in the middle of one side. When asked to report the location of the tick mark, participants frequently reported that the tick marks had migrated to adjacent squares.

To see how location uncertainty could lead to an illusory conjunction, consider a task in which two colored letters are presented on each trial, one of which is designated as the target and the other as the nontarget. If there is independent location uncertainty about form and color, then an ambiguous percept might be formed, even on trials when the target and nontarget identities are correctly identified. In this case, the percept would be of two letters and two colors, but the colors and letters would not necessarily be perceived in their veridical locations. The perceptual system must now decide which color belongs with the target letter. An obvious decision rule is to choose the color that appears closest to the target letter location. An illusory conjunction occurs when location uncertainty makes the target letter appear closer to the nontarget color than to the target color.

Location uncertainty theory depends critically on the assumption that uncertainty about form location is largely independent of uncertainty about color location. There is a large physiological literature relevant to this issue. First, there is abundant evidence that the tasks of identifying the shape and color of a target letter are largely performed by separate visual systems (Lennie, 1984; Livingstone & Hubel, 1984; Zrenner et al., 1990). The most important pathway in color perception appears to pass through the interlaminar layers of lateral geniculate nucleus (LGN), then through the "blobs" of area V1, the thin stripes in area V2, and into area V4. In contrast, the most important pathway in form perception apparently passes through the parvocellular layers of LGN, then through the interblobs of area V1 and the interstripes of area V2.

Although there is good evidence for separate form and color systems, they clearly are not independent. For example, the color system clearly passes information about form (e.g., DeValois & DeValois, 1988). Even so, the best evidence indicates that in traditional illusory conjunction experiments, where the target and nontarget letters are small and displayed peripherally, the color system would be of little help in identifying the target letter. With foveal presentation, the form (i.e., luminance sensitive) system responds to spatial frequencies about three times higher than the color (i.e., chromatic sensitive) system (Mullen, 1985; Sekiguchi, Williams, & Brainard, 1993a, 1993b). With peripheral displays, the upper limits for both systems are lowered considerably. Thus, with small letters and peripheral displays, the color system is likely to be blind to target letter identity. In this case, therefore, location uncertainty in the two systems should be approximately independent.

One advantage of this location uncertainty theory of feature binding is that much is known about the neural mechanisms responsible for location uncertainty and about experimental variables that might increase or decrease such uncertainty. A cell, say in the LGN, will fire if any rod or cone in its receptive field is stimulated. Thus, with respect to that cell, location uncertainty is equal to the size of its receptive field. It is well known that receptive field size increases with eccentricity (e.g., Fiorentini, Baumgartner, Magnussen, Schiller, & Thomas, 1990), so there should be a corresponding increase in location uncertainty with eccentricity. As a result, illusory conjunctions should be more likely with peripheral presentation than with foveal presentation.

Of course, the visual system uses more than one cell to determine object location. Location uncertainty can be reduced significantly if many cells with overlapping but nonidentical receptive fields fire in the presence of a feature. If every cell containing the stimulus in its receptive field fires, then the stimulus must necessarily have fallen on the retinal location corresponding to the intersection of the receptive fields of all active cells. With brief exposure, however, only some of the cortical cells will fire, and there is no guarantee that the intersection of their receptive fields will mark the true retinal location of the stimulus. Thus, another prediction of location uncertainty theory is that as exposure duration decreases, location uncertainty will increase, and hence illusory conjunctions will become more likely. We know of no direct tests of this hypothesis, but with unlimited exposure duration and foveal viewing, illusory conjunctions do not seem to occur.

A third factor that can affect the size of receptive fields is attention (Colby, 1991). In particular, there is evidence that receptive fields may shrink around an attended stimulus (Moran & Desimone, 1985). If so, then location uncertainty theory predicts that illusory conjunctions are more likely if attention is focused elsewhere. In addition to affecting location uncertainty, however, attention may influence feature binding by affecting perceptual organization (e.g., Gogel & Sharkey, 1989; Hochberg & Peterson, 1987; Tsal & Kolbet, 1985; Wong & Weisstein, 1982). For example, Prinzmetal and Keysar (1989) presented participants with an evenly spaced matrix of items. The perceptual organization of the matrix into rows or columns was determined by whether participants were attending to digits that were horizontally or vertically aligned with respect to the matrix. The resulting perceptual organization affected the pattern of illusory conjunctions obtained. Hence, attention may operate in several ways to affect feature binding and help prevent the occurrence of illusory conjunctions.

The effects of location uncertainty on feature binding can be modeled in a straightforward manner. Consider the task described earlier in which the relevant display consists of a target and a nontarget letter (see Figure 5 for some examples). Suppose the subject's task is to report the identity and color of the target letter. Because the stimuli are displayed on a two-dimensional surface (e.g., a computer monitor), we assume the only relevant location uncertainty is in the two-dimensional frontal plane. Let the vector $X_{\mathrm{TL}} = [X_{\mathrm{TL}}, Y_{\mathrm{TL}}]'$ denote the coordinates in this plane of the perceived location of the target letter (where the prime denotes matrix transpose). Similarly, let X_{TC} and X_{NC} denote the coordinates of the perceived locations of the target and nontarget colors, respectively. Location uncertainty theory assumes that illusory conjunctions occur because the perceived locations of the target and nontarget features vary from trial to trial. We can model this trial-by-trial variability by assuming that X_{TL}, X_{TC}, and X_{NC} vary probabilistically across trials. On each trial in which all features in the display are correctly identified, the participant is assumed to bind the target letter to the nearer of the two colors. Let $D_{\mathrm{TL,TC}}$ be the distance between the perceived locations of the target letter and target color and $D_{\mathrm{TL,NC}}$ be the distance between the perceived locations of the target letter and the nontarget color. Then the participant will solve the binding problem correctly with probability,

$$P(\text{correct binding}) = P(D_{\mathrm{TL,TC}} < D_{\mathrm{TL,NC}}). \quad (1)$$

A few more assumptions are needed before we can actually compute the Equation 1 probability. First, we assume that the perceived location of a feature is normally distributed across trials in both the vertical and horizontal dimensions. As a result, X_{TL}, X_{TC}, and X_{NC} each have a bivariate normal distribution.[2]

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[2] Although location uncertainty theory does not stand on the assumption of a bivariate normal distribution, we conducted a pilot study that makes us feel this is a reasonable assumption. In the pilot study, a small target was presented briefly in the periphery, and the participant's task was to indicate where he or she thought the stimulus appeared by mov-

Second, the mean perceived locations of the target letter and target color are assumed to be equal. Let d denote the perceived distance between the mean perceived target location and the mean perceived location of the nontarget color. Third, the variability in perceived location is assumed to be the same for all features and directions. Call this common variance σ^2. The Appendix establishes an efficient method for computing the Equation 1 probability, given these three assumptions. It also shows that the probability of correct binding depends only on the ratio d/σ, or in other words, only on the standardized distance between the mean perceived locations of the target and nontarget letters.

A conceptual illustration of this model is given in Figure 1. The plane illustrated in Figure 1 is the frontal plane in which the target and nontarget features are perceived (when the stimuli are presented on a display such as a computer screen). The three circles represent the distributions of perceived locations of the target letter, the target color, and the nontarget color. The center of each circle is the mean perceived location of that feature. Thus, the two superimposed circles that are centered at (0, 0) represent the distributions of perceived locations of the target letter and target color. The circle centered at (d, 0) represents the distribution of perceived locations of the nontarget color. Each point on a circle is an equal number of standard deviation units away from the mean. As a consequence, the resulting circles are contours of equal variability. They are circles because of the assumption that variability is equal in all directions.

According to location uncertainty theory, the processes the participant uses to solve the binding problem can be mimicked in the following way. First, a random sample is drawn from each of the three distributions illustrated in Figure 1. This process provides a perceived location for each of the three relevant features. In Figure 1, these three samples are indicated by the dots labeled *TC*, *TL*, and *NC*, for the target color, target letter, and nontarget color, respectively. Thus, in the Figure 1 example, the target letter is registered between the two colors. Next, two distances are computed. The first is between the sample from the target letter distribution and the sample from the target color distribution (i.e., between the TL and TC points), and the second is between the samples from the target letter distribution and the nontarget color distribution (i.e., between the TL and NC points). The participant correctly binds the target letter to the target color if the first distance is less than the second.

Two parameters of the Figure 1 model will have a large effect on whether the participant correctly solves the binding problem. First, illusory conjunctions will decrease as the distance, d, between the target and nontarget letters increases, because as d increases, it becomes less likely that a sample from the nontarget color distribution will be closer than a sample from the target color distribution to a sample from the target letter distribution. Thus, the model naturally predicts distance effects in illusory conjunction experiments. Second, illusory conjunctions increase with σ^2, the variance or uncertainty in location information. As the variances increase in the Figure 1 distributions, the probability that the participant will correctly solve the binding problem decreases. As discussed earlier, the variance, σ^2, should increase with eccentricity and decrease with exposure duration and attention.

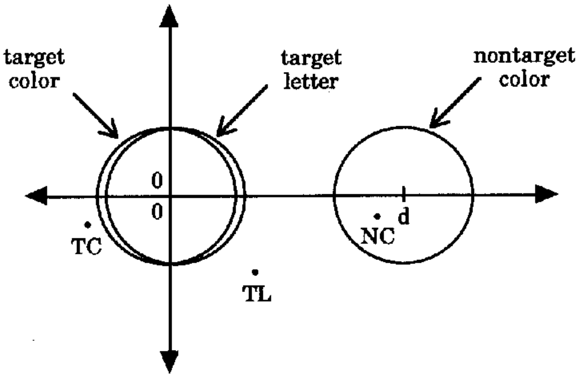

Figure 1. Contours of equal variability for the location uncertainty distributions of the target letter, target color, and nontarget color as predicted by the location uncertainty model. The points labeled *TC*, *TL*, and *NC* represent hypothetical random samples from the target color, target letter, and nontarget color distributions, respectively.

The model illustrated in Figure 1 assumes equal location uncertainty for color and shape and in both the horizontal and vertical directions (the only uncertainty parameter is σ^2). Alternative versions of the model could be constructed that relax this assumption. For example, in principle, the model could be used to test whether there is more location uncertainty for color than for shape. For the applications described in this article, however, the assumption of equal location uncertainty works well. This is because, in the experiments we consider, moderate differences in location uncertainty, across features or directions, would cause only small changes in the observed response proportions. Thus, although the model allows for tests of whether there are location uncertainty differences across features or directions, the experimental designs focused on in this article do not address this issue.

Our choice of a (bivariate) normal distribution with a mean equal to the true feature position to model the participant's perception of feature location assumes no systematic bias in feature localization. A number of studies suggest this assumption may not be true in general. In particular, memory for target location is systematically affected by boundaries or reference points near the target (Holyoak & Mah, 1982; Sadalla, Burroughs, & Staplin, 1980; Thorndyke, 1981). For example, systematic biases occur when participants are asked to report the location of a single dot, when it is always presented within a circle (Huttenlocher, Hedges, & Duncan, 1991; Nelson & Chaiklin, 1980). In traditional illusory conjunction experiments, the only features that might play the role of a reference point or boundary are the fixation point and the edge of the screen on which the stimuli are displayed. In most such experiments, including those described in this article, the target location is several degrees of visual angle from either the fixation point or the edge of the screen. In such cases, the effect of the reference point or boundary is negligible (Huttenlocher et al., 1991).

If one attempts to test the Equation 1 prediction of location uncertainty theory, a problem is immediately encountered. An

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ing a cursor to the stimulus location. Over trials, the distribution of perceived locations was well fit by a bivariate normal distribution.

error in feature binding leads to an illusory conjunction and an observable response that we call a *conjunction response.* Unfortunately, however, the observed proportion of conjunction responses is not a good estimate of the probability of a feature binding error. This is because conjunction responses can occur frequently by guessing. The next section considers this problem in more detail and suggests a method that corrects for guessing in standard illusory conjunction experiments.

The Role of Guessing in Illusory Conjunction Experiments

In a typical illusory conjunction experiment, the participant must report the identity and color of a target letter that is presented simultaneously with distractors that are also colored letters. In such a task, the participant can make a number of possible responses. These are described in Table 1. A true illusory conjunction, that is, a trial on which the participant perceives an identity and a color but incorrectly combines these two features, will always lead to a conjunction response. Unfortunately, however, conjunction responses can also occur by guessing, so the proportion of conjunction responses is usually not a good estimate of the probability of an illusory conjunction.

The significant contribution of guessing to conjunction responses can be seen by considering an experimental design typical of those used to investigate feature binding. For example, in our Experiment 1 (reported later), two colored letters were presented on every trial. The target letter was always a *T* or an *X*, and the nontarget letter was always a *C* or an *S*. The two colors displayed on each trial were sampled randomly and without replacement from a set of three colors (i.e., red, yellow, and blue). Participants were asked to report the identity and color of the target letter. (See Figure 5 for examples of the stimulus display.) In this design, a conjunction response could occur for any of the following reasons: (a) because of a true illusory conjunction; (b) because the participant correctly identified the target letter, failed to identify the target color, and happened to guess the color of the nontarget letter; and (c) because the participant was unable to identify any colors or letters in the display and happened to guess the correct identity of the target and the color of the nontarget. Because of the small number of possible target identities (i.e., 2) and colors (i.e., 3), these guessing probabilities are significant. With other designs (e.g., whole report), there are even more possibilities that guessing will cause a conjunction response.

Table 1
Some Response Categories in Illusory Conjunction Experiments

Response category	Participant's response on target identity	Participant's response on target color
C	Correct	Correct
CR	Correct	Names a color appearing elsewhere in the display
CR	Names a letter appearing elsewhere in the display	Correct
CF	Correct	Names a color not appearing in the display
LF	Names a letter not appearing in the display	Correct
LFCR	Names a letter not appearing in the display	Names a color appearing elsewhere in the display
CLF	Names a letter not appearing in the display	Names a color not appearing in the display

Note. C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; LFCR = letter feature error, conjunction response; CLF = color–letter feature error.

Before one can formulate and test theories of the feature binding process, it is necessary to account for the significant effect of guessing in illusory conjunction experiments. The most widely known methods of correcting for guessing are based on the assumption that information gain is all-or-none (e.g., Link, 1982; Lord & Novick, 1968). For example, consider a multiple-choice test in which the student must choose among four alternative answers to every question. The classical method used to correct for guessing assumes the student either knows the correct answer or guesses randomly among the four alternatives. If the student knows the correct answer with probability *p*, then the probability of a correct response equals

$$P(\text{correct}) = p + (1 - p)^{1/4}. \qquad (2)$$

An estimate of the probability that the student knows the correct answer is readily obtained by solving Equation 2 for *p*. This model assumes no partial information, so it is almost surely an oversimplification. Even so, it provides a useful estimate of the student's true state of knowledge.

A generalization of this all-or-none guessing model can be developed for the illusory conjunction experiment described earlier. An especially simple guessing model assumes the participant gains all-or-none information about the identity of the target and about the target color and that feature binding is always perfect. The model also assumes that the participant treats the identity and color information independently. Let T_L and T_C denote the probabilities that the participant perfectly perceives[3] the target identity and target color, respectively. The model is most easily described by the tree diagram illustrated in Figure 2. The outcome of every trial is represented by a path from the node at the top to a response category listed at the bottom. The probability that the participant will follow a particular path equals the product of all probabilities found on that path. The first bifurcation specifies whether the participant perceives the identity of the target letter. With probability T_L, the participant perfectly perceives the target identity and the left branch is taken; with probability $1 - T_L$, no information about target identity is obtained and the right branch is taken. The second bifurcation specifies whether the color of the target letter is perceived perfectly (left branch) or whether no color information is obtained (right branch). The first two bifurcations lead to four possible paths. The leftmost of these corresponds to

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[3] The word *perceives* is used somewhat loosely in this article. Because we expect decision processes also to be involved, it would be more accurate to say "the participant perfectly identifies the target letter and target color, respectively."

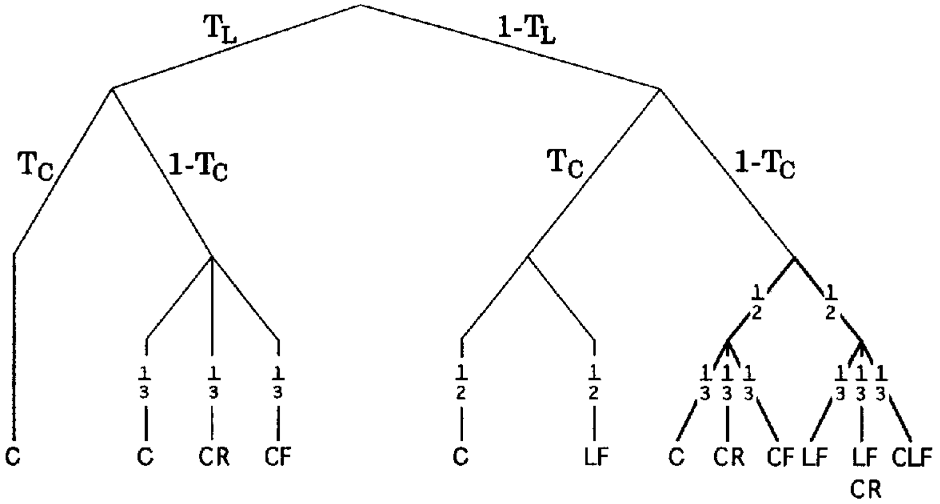

Figure 2. Tree diagram of the simple null model. (T_C = probability of perceiving the target color; T_L = probability of perceiving the target letter; C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; CLF = color–letter feature error; LFCR = letter feature error, conjunction response.)

trials on which the participant perceives the target identity and color perfectly. On these trials a correct response is inevitable because the model assumes that feature binding is always perfect. The next path corresponds to trials on which the target identity is perceived perfectly but no color information is obtained. As a consequence, the participant must guess a color response. The three possible guesses (i.e., red, yellow, or blue) lead to three different response categories: correct (C), conjunction response (CR), or color feature error (CF). The third path corresponds to trials on which the color is perceived perfectly but the participant has no letter identity information and so must guess the target identity (i.e., either a *T* or an *X*). This path does not necessarily assume that the participant perceives an amorphous color. Rather, a more reasonable assumption might be that the participant perceives a smudge of color but is unable to identify the shape of the smudge. Finally, the rightmost path corresponds to trials on which the participant has no information about target identity or color, so both must be guessed.

The overall probability of a particular response category is just the sum of the probabilities associated with each branch leading to that response. For example, in the Figure 2 model the probability of a CR equals the following:

$$P(\mathrm{CR}) = {}^{1}\!/_{3}T_L(1 - T_C) + ({}^{1}\!/_{2})({}^{1}\!/_{3})(1 - T_L)(1 - T_C). \quad (3)$$

Tree models like those in Figure 2 are called *multinomial models* (e.g., Riefer & Batchelder, 1988; Batchelder & Riefer, 1990). Their statistical properties are well understood (e.g., Bishop, Fienberg, & Holland, 1975; Riefer & Batchelder, 1988). For example, a straightforward algorithm, called the *EM* (expectation maximization) *algorithm* (e.g., Bishop et al., 1975), can be used to find maximum likelihood estimates of the unknown parameters (in the Figure 2 model, the unknown parameters are T_L and T_C).

We call the model described in Figure 2 the *simple null model* because it represents a null hypothesis that feature binding errors do not occur. Nonetheless, it contains two paths that terminate in a CR. For both paths, the CR is assumed to be the result of guessing that follows the failure to perceive one or more target features.

Figure 2 shows the decision about the identity of the target letter preceding the decision about the target's color. A model with these two stages in reverse order makes exactly the same predictions, so multinomial models cannot be used to order processing stages. In fact, it probably makes the most sense to think that the features are identified in parallel.

The simple null model illustrated in Figure 2 assumes that the participant guesses in a naive fashion. Alternative versions of the model can be constructed that assume sophisticated guessing. For example, suppose the participant sees the target letter but not the target color. According to the simple null model, the participant guesses among the three alternative colors in this situation. However, suppose the participant had also seen the nontarget letter and color. The simple null model assumes no location uncertainty, so the participant would know that the nontarget color could be ruled out as an alternative for the target color (because the target and nontarget colors are always different). As a consequence, rather than guess the target color among all three color alternatives, a sophisticated subject would guess only between the two remaining colors. This is an "exclusionary" guessing strategy because the nontarget color was excluded from the guessing set. Exclusionary guessing was optimal because the target and nontarget were always different colors in our experiments.

The tree diagram for the null model with sophisticated guessing is illustrated in Figure 3. It assumes that the participant makes use of both the nontarget letter and the nontarget color, so it has two more free parameters than the simple null model: (a) the probability that the nontarget letter will be identified correctly, denoted by N_L; and (b) the probability that the non-

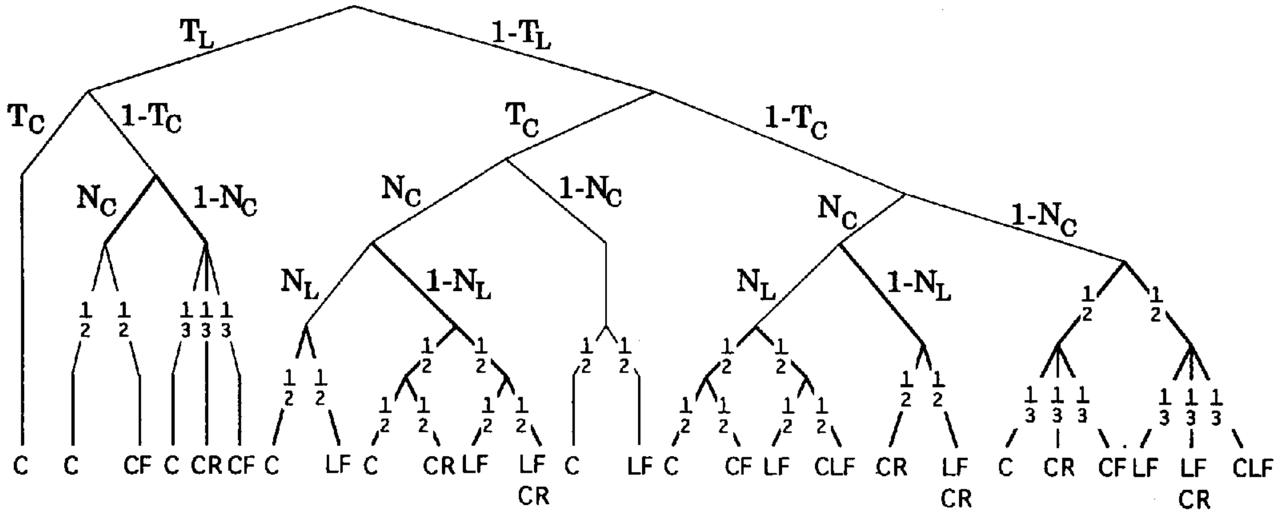

Figure 3. Tree diagram of the sophisticated null model. (T_C = probability of perceiving the target color; T_L = probability of perceiving the target letter; N_C = probability of perceiving the nontarget color; C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; CLF = color–letter feature error; LFCR = letter feature error, conjunction response.)

target color will be identified correctly, denoted by N_C. It also assumes that if the nontarget color is the only feature perceived by the participant, then he or she will respond with that color. This state of affairs occurs in the sophisticated null model on the branch $1 - T_L$, $1 - T_C$, N_C, $1 - N_L$. As Figure 3 indicates, on such trials the participant responds with a CR if the target letter is guessed correctly or with a simultaneous letter feature error and conjunction response (LFCR) if the target letter is guessed incorrectly. The exclusionary guessing assumption of the sophisticated null model is tested in Experiment 2.

If the target letter and target color are both perceived (the T_L, T_C branch), the sophisticated and simple null models both predict that a correct response will be made. If the target letter is perceived but not the target color, then according to the sophisticated null model the guessing strategy depends on whether the nontarget color is perceived. If it was (the T_L, $1 - T_C$, N_C branch), then the nontarget color is excluded as a possibility and the participant guesses between the two remaining colors. If the nontarget color is not perceived (the T_L, $1 - T_C$, $1 - N_C$ branch), the participant guesses among all three colors (as in the simple null model).

If the target color is perceived but not the target letter, then the sophisticated guessing strategy depends on what nontarget information was perceived. If the nontarget letter and color are both perceived (the $1 - T_L$, T_C, N_C, N_L branch), then the color corresponding to the target will appear in a different location from the nontarget letter. Thus, the participant will know that this color could not be from the nontarget. As a result, the participant need only guess the identity of the target letter. However, if the nontarget color is perceived but not the nontarget letter (the $1 - T_L$, T_C, N_C, $1 - N_L$ branch), the participant has no way of knowing which of the two colors was associated with the target. As a result, the participant is forced to guess between the two display colors and between the two possible target letters (resulting in four possible outcomes). Finally, suppose the nontarget color is not perceived (the $1 - T_L$, T_C, $1 - N_C$ branch). There are two possibilities, and both lead to the same two outcomes. If the nontarget letter is perceived, then the target color will appear in a different location from the nontarget letter, so the participant will know that the single perceived color is from the target. As a consequence, only the target letter must be guessed. If the nontarget letter is not perceived, then the participant will not know whether the single color perceived is from the target or nontarget. However, because there is no other information available, the optimal strategy is to guess the color that was seen (which, in this case, is the correct choice).

Illusory Conjunction Models

Both the simple and sophisticated null models assume participants incorrectly combine features only because of guessing. True illusory conjunctions never occur. It is straightforward, however, to augment the models to allow illusory conjunctions. This augmentation can be done in many different ways. This article generalizes the sophisticated null model in two ways. First, we develop a model that is consistent with the feature integration theory proposed by Treisman and Schmidt (1982). Second, we develop a model consistent with location uncertainty theory.

According to Treisman and Schmidt (1982), attention is required to solve the feature binding problem. If attention is unavailable, expectations may be used to bind features together. For example, we expect clouds to be white and trees to be green, so even when our attention is directed elsewhere, we rarely see green clouds and white trees. In the experiments reported later, however, the stimulus features (i.e., color and shape) co-occur independently, so the participant has no idea of what stimulus to expect. In this case, Treisman and Schmidt say that if attention is unavailable "some of the features detected . . . may be randomly conjoined" (p. 111) and that "illusory conjunctions will be generated . . . by random couplings" (p. 111).

A strict interpretation of the statement that features are randomly conjoined is that, on trials when all features are perceived, any letter identity is equally likely to bind to any color, and thus illusory conjunctions should be just as likely as correct perceptions. This prediction is not supported by the data, so an alternative interpretation of "random feature conjoining" is required. Another possibility, which seems consistent with Treisman and Schmidt's ideas, is that on trials when binding does occur, features are bound correctly with some probability α and are bound incorrectly with probability $1 - \alpha$. Feature binding of this type is random in the sense that it does not depend on any physical or perceptual characteristics of the stimulus display.

Figure 4 shows a simple generalization of the sophisticated null model that incorporates this notion of random feature binding. The branches that are different from the sophisticated null model are highlighted. This new model, which we call the *random binding model,* assumes that a true illusory conjunction occurs on trials when the target letter and the nontarget color are perceived and incorrect binding occurs (i.e., as on the T_L, T_C, N_C, $1 - \alpha$ branch or the T_L, $1 - T_C$, N_C, $1 - \alpha$ branch). The random binding model has five free parameters (T_L, T_C, N_C, N_L, and α). The sophisticated null model occurs as a special case in which $\alpha = 1$.

The location uncertainty model is identical to the random binding model except that correct binding is assumed to occur with the probability specified in Equation 1 rather than according to some fixed probability α. The location uncertainty model also has five free parameters (T_L, T_C, N_C, N_L, and σ, or alternatively, d/σ), and the sophisticated null model occurs as a special case in which $\sigma = 0$.

For any fixed distance between the target and nontarget letters, the random binding and location uncertainty models make identical predictions. Exact equivalence occurs if $\alpha = P(D_{\text{TL,TC}} < D_{\text{TL,NC}})$. However, suppose the random binding model is fit separately to data collected from a number of experimental conditions in which the distance between the target and nontarget letters is varied. If location uncertainty theory is correct, the resulting alpha estimates (computed separately for each distance) should increase with distance. In contrast, if Treisman and Schmidt's (1982) feature integration theory is correct, the alpha estimates should be invariant with distance.

Thus, when fit to data from a single experimental condition, the random binding model is fairly atheoretical. For example, it no longer allows a test of the assumption of random feature binding. Consequently, when the random binding model is fit to data from a single experimental condition, we will refer to it simply as the *alpha model* (after the name of its binding parameter). The alpha model, or something like it, could be used to estimate the probability of correct feature binding, regardless of whether one considers feature binding errors a failure of attention (i.e., Treisman, 1988), poor temporal linking (Crick, 1984; Strong & Whitehead, 1989), or inaccurate location information (Cohen & Ivry, 1989; Maddox et al., 1994; Prinzmetal & Keysar, 1989). Recently, Prinzmetal, Henderson, and Ivry (1995) used this model to compare the probability of correctly binding features in a variety of circumstances, such as with and without diverting attention.

The modeling approach proposed here has a number of advantages over the methods traditionally used to analyze data from illusory conjunction experiments. First, unlike previous methods, the probability of correctly binding features (i.e., α) can be estimated independently of the probability of correctly perceiving the features (i.e., T_L, T_C, N_L, and N_C). Second, the models provide a rigorous method to assess whether illusory conjunctions actually occur. For the partial report design used in our Experiment 1 (i.e., 3 possible colors and a 2-object display), the accepted criterion for the occurrence of illusory conjunctions is that CRs are more frequent than color feature errors (CF; e.g., Cohen & Ivry, 1989). If there are no true illusory conjunctions, then on trials when the participant fails to perceive the target color, any of the possible colors should be reported with equal probability. In the present design, there are three colors, two of which are presented on a trial. Suppose the participant identifies the target letter but not the target color. One of the colors must be selected as the report of the target color. If the subject guesses randomly, one third of the trials will result in a correct response. On the remaining two thirds of the trials, the participant will make an incorrect color response. If the nontarget color is reported, the response will be recorded as a conjunction response. If the color that is not included in the display is reported, the response will be recorded as a color feature error. If the data indicate significantly more conjunction responses than color feature errors, then it was thought that some mechanism other than chance must be at work. Feature binding errors seemed the most likely candidate.

This intuition is justified theoretically by the simple null model. Figure 2 indicates that in the simple null model CR and CF terminal nodes are always paired, so the simple null model always predicts $P(\text{CR}) = P(\text{CF})$. Thus, if the simple null model accurately describes participants' guessing strategies, then data with significantly more conjunction responses than color feature errors can safely be interpreted as indicative of true illusory conjunctions. In other words, the accepted criterion for the occurrence of illusory conjunctions would be valid. The simple null model of Figure 2, then, can be viewed as a formalization of the guessing strategy implicitly assumed in the literature. On the other hand, an examination of Figure 3 indicates that in the sophisticated null model, the pairing of conjunction responses and color feature errors is broken, and as a result it is unclear whether the current test is valid if the participant adopts a sophisticated guessing strategy.

The alpha model can be used to answer this question. When binding is perfect (i.e., $\alpha = 1$), the alpha model reduces to the sophisticated null model, and conjunction responses occur only by guessing. If the prevailing method of testing for illusory conjunctions is valid, then regardless of the values of T_L, T_C, N_C, and N_L, it should be true that $P(\text{CR}) \leq P(\text{CF})$ whenever $\alpha = 1$ and $P(\text{CR}) > P(\text{CF})$ whenever $\alpha < 1$. We tested this hypothesis by computing predicted $P(\text{CR})$ and $P(\text{CF})$ values from the alpha model for a number of different parameter settings. (We did not systematically search the parameter space.) A few of these results are shown in Table 2. First, consider the results when $\alpha = 1$. In this case, traditional logic predicts that $P(\text{CR})$

≤ $P(\mathrm{CF})$. In the Table 2 examples, this prediction is supported only when letter perception is better than color perception (i.e., when T_L and N_L are greater than T_C and N_C). When color perception is better, conjunction responses are more common than color feature errors, even though all conjunction responses are by guessing (because $\alpha = 1$). When true illusory conjunctions are occurring (i.e., when $\alpha = 0.9$), traditional logic predicts $P(\mathrm{CR}) > P(\mathrm{CF})$. In Table 2, this prediction is true when color perception is better than letter perception and false when letter perception is better than color perception.

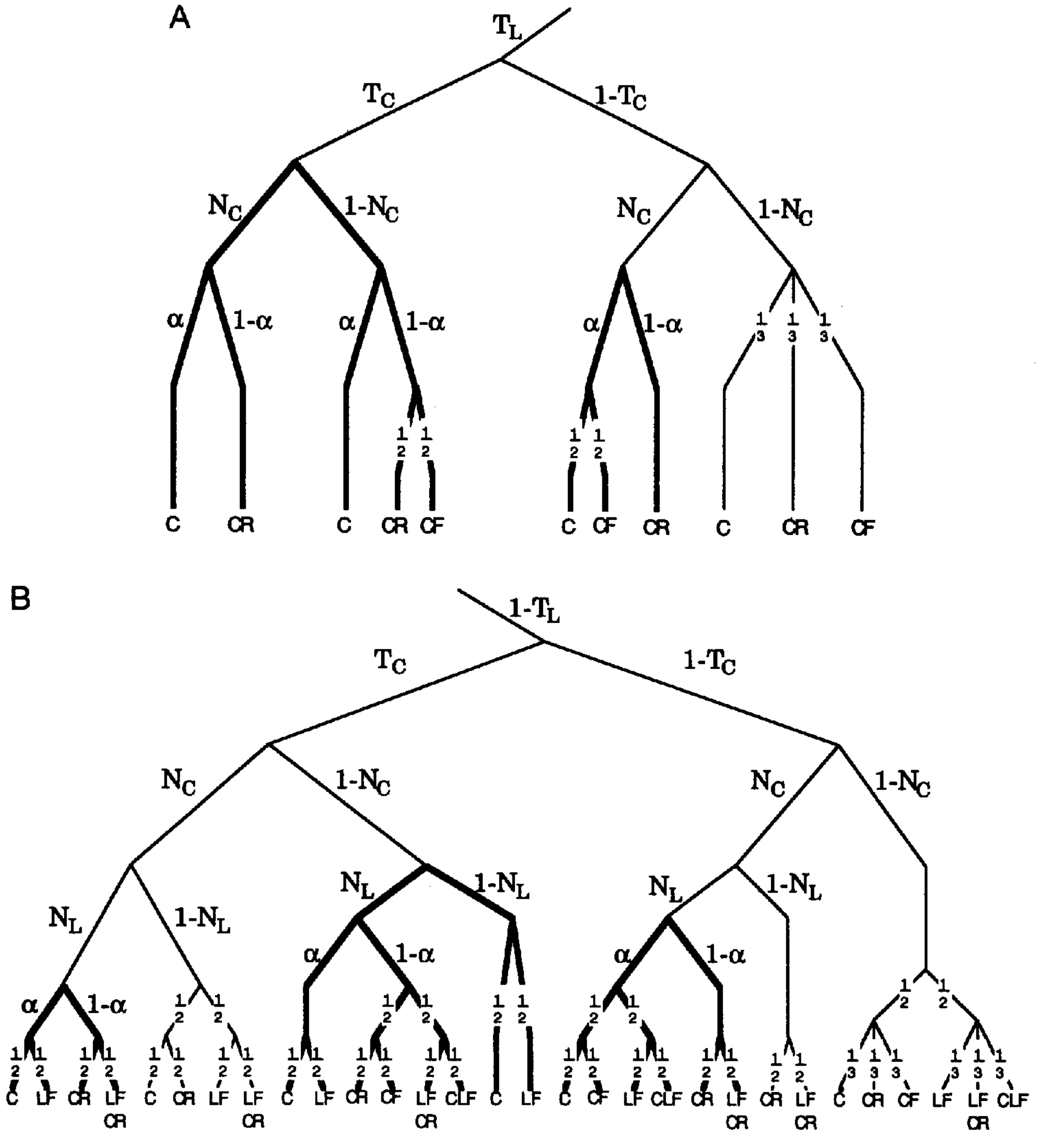

Figure 4. Tree diagram of the random binding model (called the *alpha model* when fit to data from a single experimental condition). The T_L half of the tree is given in Figure 4a and the $1 - T_L$ half is given in 4b. The branches not in bold are identical to those appearing in the sophisticated null model. (T_C = probability of perceiving the target color; T_L = probability of perceiving the target letter; N_C = probability of perceiving the nontarget color; N_L = probability of perceiving the nontarget letter; C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; LFCR = letter feature error, conjunction response; CLF = color–letter feature error.)

The results in Table 2 emphasize the importance of the assumptions about guessing strategy that underlie the currently popular test for assessing whether illusory conjunctions occur. Table 2 indicates that the test is valid only if participants adopt the naive guessing strategies embodied in the simple null model. If participants use more sophisticated strategies whereby they make use of information from the nontarget object, then the standard test is inadequate. In particular, with sophisticated guessing the test is susceptible to two kinds of errors: (a) to a conclusion of no illusory conjunctions when illusory conjunctions did occur and (b) to a conclusion that illusory conjunctions occurred when all conjunction responses were by guessing. Because of this problem, it is important to determine what type of guessing strategies participants use in typical illusory conjunction experiments. If they are found to use sophisticated strategies, the currently popular test for the occurrence of illusory conjunctions should not be used. In any case, we recommend replacing the current illusory conjunctions test with a model-based analysis of the sort described here.

With other experimental designs, different tests for the occurrence of true illusory conjunctions have been proposed. For example, in the case of a full report design, Treisman and Schmidt (1982, Experiment 1) argued that true illusory conjunctions occur if the proportion of conjunction responses exceeds the proportion of color feature errors plus letter feature errors. It is straightforward to show that this test implicitly assumes the naive guessing strategy of the simple null model.[4] As such, its validity must also be questioned if participants adopt a sophisticated guessing strategy.

Experiment 1

In Experiment 1, participants were briefly presented with a display that contained two colored letters: a target and a nontarget. The distance between the target and nontarget letters varied across trials. The task was to indicate which target was present and to indicate its color (e.g., red *X*). The participant did not report either the identity or the color of the nontarget letter. Unlike a full report task, memory requirements are minimized in this experiment (see Estes & Taylor, 1964). In contrast to Wolford's (1975) study, Experiment 1 was not concerned with the loss of position information in memory but instead with inaccurate feature identification and combination.

Table 2
Probabilities of Conjunction Responses (CRs) and Color Feature Errors (CFs) as Predicted by the Alpha Model for Various Parameter Values

α	$T_L = N_L$	$T_C = N_C$	*P*(CR)	*P*(CF)
1.0	.9	.6	.053	.164
1.0	.6	.9	.042	.035
0.9	.9	.6	.121	.164
0.9	.6	.9	.110	.035

Note. T_L = target letter; N_L = nontarget letter; T_C = target color; N_C = nontarget color.

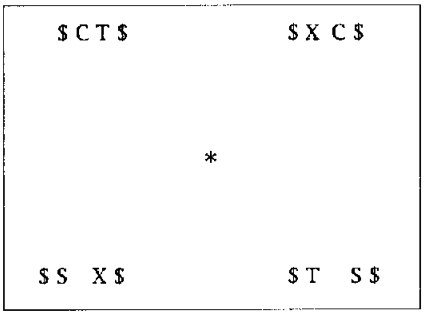

Figure 5. Four possible stimulus displays used in Experiment 1 (without the colors). *T* or *X* = target letter set; *C* or *S* = nontarget letter set.

Method

Procedure. Each trial consisted of the following sequence of events. First, an asterisk, serving as both a fixation point and warning signal, appeared at the center of a large white rectangle. The white rectangle covered most of the black background of the monitor. After 1 s, the asterisk and white rectangle were replaced by the stimulus display. This display consisted of four characters: two colored letters surrounded by two achromatic dollar signs ($). An example of four possible displays is illustrated in Figure 5 (without the colors). The colored letters were flanked with dollar signs because Treisman (1982, Experiment 4) found that such an arrangement increased the number of illusory conjunctions. The four characters were located either above or below the fixation point (see Figure 5). There were four possible interletter distances. The two colors were selected from a set of three colors: red, yellow, or blue. One of the letters was selected from a target letter set (*T* or *X*), and the other letter was selected from a nontarget letter set (*C* or *S*). The stimulus was briefly presented and was followed by a white masking rectangle.

The participant's task was to report the color and identity of the target letter (e.g., red *X*). Responses were made on the computer keyboard. The participant typed in the target color using the *1*, *2*, and *3* keys to indicate red, yellow, and blue, respectively. The participant then indicated the target identity by typing a *1* or a *2* for *T* or *X*, respectively. The following feedback was provided. A high-pitch tone (1,000 Hz) was played for 150 ms if the color response was incorrect. A low-pitch tone (200 Hz) was played for 150 ms if the letter response was incorrect. If both responses were incorrect, both tones were played with a 100-ms intertone interval. No feedback was given following correct responses. The fixation point, indicating the onset of the next trial, was displayed immediately after correct responses and 500 ms after the last tone on incorrect trials.

Seven participants were tested. For two of the participants (Participants 1 and 2), the viewing distance was approximately 90 cm. For the other participants, a chin rest was used to fix the viewing distance at 70 cm. Each participant completed three sessions. At the beginning of the first session, a series of short practice blocks were run to establish the exposure duration for the stimulus displays. Each practice

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[4] The proof that the Treisman and Schmidt (1982) whole-report test for illusory conjunctions is based on the same assumptions as the simple null model involves the following steps. First, the assumptions embodied in the simple null model (shown in Figure 2) are used to construct a model of the Treisman and Schmidt whole-report task. Second, this model is used to derive equations for the predicted probabilities of CR, CF, and LF responses. Finally, it is shown that *P*(CR) = *P*(CF) + *P*(LF).

block consisted of 16 trials, with 2 trials for each of the eight conditions (top or bottom × 4 interletter distances).

The exposure duration was set to 500 ms for the first practice block to familiarize the participants with the task and displays. The exposure durations for the next five practice blocks were fixed at 300 ms, 200 ms, 143 ms, 86 ms, and 57 ms. Following the sixth practice block, the exposure duration was adjusted according to the performance of each participant. If the participant was correct on 13 or more of the 16 trials, the exposure duration was reduced by 1 tick of the vertical raster (approximately 14.3 ms). If the participant was correct on 11 or fewer of the trials, the exposure duration was increased by 1 tick. No change in exposure duration was made if the participant was correct on 12 of the 16 trials. The minimum exposure duration in both practice and test blocks was 2 ticks (approximately 29 ms). The session began after one practice block at the minimum speed or after the participant had had two practice blocks in which the speed required no further adjustment.

Each test block consisted of 80 trials (10 trials for each of 8 conditions). The exposure duration for the first test block was equal to that of the last practice block. Following each test block, an adjustment was made on the basis of the following criteria: If the participant was correct on 80% or more of the trials, the exposure duration was lowered (if not at the minimum duration); if the participant was correct on 65% or less of the trials, the exposure duration was increased.

Each test session consisted of 10 blocks. Three sessions were completed within a 1-week period, yielding a total of 2,400 trials per participant for the data analyses. Participants completed two practice blocks to begin Sessions 2 and 3 beginning with the exposure duration from the last test block on the preceding session and using the same adjustment criteria. The exposure durations ranged from a high of 4 ticks (57 ms) to a low of 2 ticks (29 ms). The median duration was 2 ticks for 5 of the participants and 3 ticks for the other 2 participants.

Participants were paid a base rate of $5 per session plus a monetary bonus based on their performance. The bonus was calculated by adding $.01 for each correct response and subtracting $.02 for each incorrect response. On average, participants earned an additional $2.50 in bonus payment for each session.

Apparatus and stimuli. The stimuli were presented on a Sony Multi-Scan HG monitor (ZCM-1490) controlled by a C^2 386 PC equipped with VGA graphics. The width and height of the letters, dollar signs, and fixation asterisk spanned 0.8 and 1.0 cm, respectively. The stimulus string (2 colored letters flanked on each side by the dollar signs) was located on either the upper or lower side of a 9-cm square centered around the fixation point. The white masking rectangle extended beyond this region, measuring 20 cm × 13.5 cm. The edge-to-edge distance between the dollar sign and its adjacent colored letter was fixed at 0.4 cm. This spacing was used to keep any lateral masking effect from the dollar signs fairly constant. Because the spacing between the two colored letters was varied, the distance between the two dollar signs varied in a correlated manner.

As noted earlier, 2 of the participants were situated 90 cm from the monitor and 5 were situated 70 cm from the monitor. For the former group, the four interletter distances were 1.6 mm, 7.6 mm, 13.7 mm, and 19.8 mm. These corresponded to visual angles of approximately 6′, 29′, 52′, and 76′ (1.3°) of arc, respectively. At 90 cm, the colored letters subtended visual angles of approximately 30′ × 38′ in the horizontal and vertical directions, and the distance from the center of the fixation point to the center of the colored letters ranged from 2.9° to 5.3°.

The viewing distance and interletter distances were adjusted for the second group of 5 participants to yield a larger range of performance. The four distances were 3.8 mm, 7.6 mm, 15.2 mm, and 22.8 mm. At a viewing distance of 70 cm, these correspond to visual angles of 19′, 44′, 75′ (1.25°), and 112′ (1.87°) of arc, respectively. The visual angles subtended by the colored letters was 39′ × 49′, and the distance from the fixation marker to the colored letters ranged from 3.7° to 7.1°.

The stimuli were viewed under normal lighting. The CIE coordinates (measured with a Minolta Chroma meter, Model CS100) of the red, yellow, and blue stimuli were $x = .499$, $y = .340$; $x = .437$, $y = .472$; and $x = .196$, $y = .165$, respectively. The luminance values were 27.5 cd/m^2 (red), 67.5 cd/m^2 (yellow), and 29.2 cd/m^2 (blue). The luminance of the white mask and dollar signs was 127.0 cd/m^2, and the luminance of the black fixation marker and background was 6.0 cd/m^2.

The stimulus display was set according to the following constraints. On half of the trials the stimulus string was located above the fixation mark; on the other half, the string was below the fixation mark. The letters could appear at any position along the selected side as long as those positions fell within the boundaries just described. For each side, the four interletter distances were used on an equal number of trials. The left–right order of the target and distractor letters was randomly determined. The colors for each letter were randomly selected with the constraint that different colors were assigned to the target and distractor on each trial.

Participants. Four female and 3 male students selected from the undergraduate and graduate population at University of California, Berkeley were tested. All had normal or corrected-to-normal vision. On the basis of self-report, all had normal color vision.

Results

Response categories. The mean response proportions (across participants) are shown in Table 3 for each distance and response category. The proportion of all types of errors decreased as the distance increased. This effect was especially clear for conjunction responses (CRs), which reliably decreased as the distance between the target and nontarget increased, $F(3, 18) = 66.08$, $p < .01$. The effect of distance on color feature errors (CFs), letter feature errors (LFs), and simultaneous letter feature errors and conjunction responses (LFCRs), though smaller, was also reliable, $F(3, 18) =$ 6.29, 3.43, and 4.95, respectively; all ps $< .05$. The position of the letter string (above or below the fixation point) did not significantly affect any of the response measures, and there was no significant interaction between distance and position.

When collapsed over distance, the responses of each participant showed a higher proportion of CRs than CFs (0.113 vs. 0.049, $t(6) = 9.269$, $p < .001$, two-tailed). This finding is interesting because the simple null model (Figure 2) predicts $P(\mathrm{CR}) = P(\mathrm{CF})$. As noted earlier, several investigators have used the violation of this prediction as an indication of feature binding errors (e.g., Cohen & Ivry, 1989; Cohen & Rafal, 1991).

Table 3
Mean Response Proportions From Experiment 1

Response category	Distance				
	1	2	3	4	*M*
C	.67	.75	.81	.83	.77
CR	.17	.13	.08	.07	.11
CF	.06	.05	.04	.04	.05
LF	.04	.03	.03	.03	.03
LFCR	.04	.03	.03	.02	.03
CLF	.01	.01	.01	.01	.01

Note. C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; LFCR = letter feature error, conjunction response; CLF = color–letter feature error.

Table 4
Goodness-of-Fit Values ("A" Information Criterion) for Experiment 1

	Model				
Participant	Simple null	Sophisticated null	Random binding	Location uncertainty	Location uncertainty with spreading attention
1	4,665	4,559	4,498	4,422	4,424
2	4,236	4,167	4,113	4,079	4,074
3	4,583	4,511	4,421	4,393	4,374
4	3,824	3,791	3,679	3,700	3,659
5	4,835	4,811	4,697	4,727	4,672
6	3,126	3,121	3,061	2,972	2,970
7	4,717	4,679	4,553	4,533	4,512
M	4,284	4,234	4,146	4,118	4,098

Theoretical analysis. Each of the four models described earlier were fit separately to the data collected from each of the 7 participants. For each model, maximum likelihood estimates of the free parameters were obtained using an iterative search procedure. If f_i is the observed response frequency in cell i of the data matrix, and P_i is the probability of this type of response as predicted by the model in question, then the maximum likelihood estimates (e.g., Ashby, 1992; Wickens, 1982) are those values of the unknown parameters that minimize

$$L = -\sum_i f_i \log P_i. \qquad (4)$$

In Experiment 1, there were four target-to-nontarget distances, two target positions (above or below fixation), and six response categories, so there were 48 cells in the data matrix for each participant (4 × 2 × 6) and 40 degrees of freedom. None of the models had more than six free parameters, so after parameter estimation, there were still at least 34 degrees of freedom to test the validity of the models. To compare models with different numbers of free parameters, we used the A Information Criterion (AIC) statistic (Akaike, 1974; Takane & Shibayama, 1992):

$$AIC = L + 2v, \qquad (5)$$

where v is the number of free parameters. The AIC statistic penalizes a model for extra free parameters in such a way that the smaller the AIC, the closer a model is to the "true model," regardless of the number of free parameters. Thus, to find the best model among a given set of competitors, one simply computes an AIC value for each model and chooses the model associated with the smallest AIC value.

The AIC values for each participant under the four models are shown in Table 4. (Ignore the last column for now.) Three conclusions stand out. First, for every participant, the worst fit is provided by the simple null model. Apparently, participants do use information from the nontarget letter to aid their identification of the target. As discussed in the introduction, this finding raises serious questions about the validity of the currently popular test for illusory conjunctions. Second, the sophisticated null model always fits worse than either the random binding or location uncertainty models. The sophisticated null model assumes feature binding is perfect on every trial, whereas both the random binding and location uncertainty models assume at least some errors in the feature binding process. Thus, the poor performance of the sophisticated null model supports the hypothesis that at least some errors in feature binding did occur. Third, the location uncertainty model fits better than the random binding model for 5 of the 7 participants. The superior performance of the location uncertainty model is not surprising given the strong effect of distance on the frequency of conjunction responses. Finally, although it is not apparent from Table 4, the absolute fits of the location uncertainty model are excellent.[5] For each participant, the location uncertainty model accounts for about 99% of the variance in the data.

Table 5 shows the parameter estimates from the best-fitting versions of each model. There is strong agreement among all models on the value of T_L. Although agreement is also good on the value of T_C, some model-specific differences are apparent. First, the simple null model predicts the lowest value of T_C for every participant. Presumably, these estimates are artificially low because this is the only way the simple null model can account for the high proportion of conjunction responses. To see this, note from Figure 2 that only two branches lead to conjunction responses (CRs) in the simple null model. Both of these require a misperception of the target color. The sophisticated null model produces larger estimates of T_C, but in every case, these estimates are smaller than predicted by either the random binding or location uncertainty models. Figure 3 indicates that conjunction responses can occur in the sophisticated null model on trials when the target color is correctly perceived (through the $1 - T_L$, T_C, $1 - N_L$, N_C branch), but the majority of paths that terminate in a CR require misperception of the target color. The random binding and location uncertainty models agree almost perfectly on the value of T_C.

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[5] It is important to note that a perfect fit does not result in an AIC value of 0. For example, for the random binding and location uncertainty models, a perfect fit to the data of Participant 7 would yield an AIC of 4,409. The fit for the location uncertainty model for this participant is 4,533, an increase of only 124, or 2.8%.

Table 5
Parameter Estimates From Best Fitting Models for Experiment 1

	Participant						
Parameter/model	1	2	3	4	5	6	7
T_L							
Simple null	0.82	0.84	0.81	0.86	0.84	0.94	0.85
Sophisticated null	0.80	0.83	0.78	0.84	0.83	0.94	0.83
Random binding	0.82	0.84	0.81	0.86	0.84	0.94	0.85
Location uncertainty	0.82	0.84	0.80	0.84	0.83	0.94	0.84
T_C							
Simple null	0.67	0.71	0.69	0.77	0.63	0.78	0.64
Sophisticated null	0.73	0.74	0.74	0.79	0.64	0.78	0.66
Random binding	0.83	0.84	0.86	0.92	0.82	0.89	0.83
Location uncertainty	0.83	0.84	0.86	0.92	0.82	0.91	0.84
N_C							
Sophisticated null	0.44	0.35	0.36	0.23	0.17	0.11	0.24
Random binding	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Location uncertainty	0.77	0.65	0.79	0.63	0.72	0.63	0.69
N_L							
Sophisticated null	0	0	0	0	0	0	0
Random binding	0.45	0.49	0.55	0.61	0.62	0.53	0.46
Location uncertainty	0.29	0.17	0.37	0.20	0.39	0.29	0.22
α							
Random binding	0.91	0.91	0.89	0.91	0.86	0.92	0.87
σ							
Location uncertainty	12.4	11.5	13.2	12.1	16.0	12.3	15.5

Note. T_L = probability of perceiving the target letter; T_C = probability of perceiving the target color; N_C = probability of perceiving the nontarget color; N_L = probability of perceiving the nontarget letter.

In contrast, there is strong disagreement among all the models on the values of N_C and N_L. In fact, only the location uncertainty model produces plausible estimates of these parameters. According to the random binding model, N_C is always 1. This seems extremely unlikely, given that the largest T_C estimate is .92 and we expect T_C to be larger than N_C. On the other hand, the sophisticated null model predicts that N_L is always zero. Intuition predicts that N_L should be smaller than T_L, but it seems unlikely that the participant would never correctly perceive the identity of the nontarget letter.

According to the location uncertainty model, N_C and especially N_L are significantly smaller than T_C and T_L. At first this might seem implausible, but the low N_C and N_L estimates are consistent with the hypothesis that participants dynamically allocated their attention to the target location (e.g., Eriksen & Hoffman, 1972; Eriksen & Rohrbaugh, 1970; Wolfe, Cave, & Franzel, 1989). In other words, as the processing of shape information makes clear which object is the target, the participant may narrow attention to this object, facilitating the identification of both target features at the expense of accuracy on the nontarget features. The fact that the location uncertainty model estimates of N_L are much lower than N_C also makes sense. The nontarget letter provides no information about either target feature (i.e., letter or color), but the nontarget color eliminates one of the three target color alternatives. Thus, an optimal strategy might be to allocate some attention to the nontarget color but no attention to the nontarget letter.

Physiological evidence of such narrowing of attention has been found. Chelazzi, Miller, Duncan, and Desimone (1993) recorded from cells in the inferotemporal cortex of two monkeys while the animals performed a simplified version of our Experiment 1. The monkeys were first shown a target cue (e.g., a square or triangle) and then, after a delay of 1.5 to 3 s, a peripheral display containing the target and a distractor in an unknown location. The animals then made an eye movement to the target location to obtain a reward. Chelazzi et al. found that when the target and distractor were first displayed, cells tuned to the target and cells tuned to the distractor both responded strongly. About 200 ms after display onset, however, and still 100 ms before eye movement, the activity of cells tuned to the distractor was suppressed, whereas activity of cells tuned to the target was unaffected. Thus, 100 ms before a response was initiated, attention was narrowed from the full display to the target location only. Activity in the cells tuned to the target was maintained during the delay between presentation of the cue and target-distractor display, so it is plausible that the short display durations used in our Experiment 1 would not prevent such narrowing of attention. On the other hand, it is unknown whether this sort of focusing of attention can occur before identification of the target features has been completed.

To get an even clearer understanding of these data, we also fit the simple null, sophisticated null, and alpha models separately to the data collected from each target-to-nontarget distance. A number of interesting findings emerged. First, for every participant, the sophisticated null model fit better than the simple null model at the farthest distance. In other words, even when participants are not making illusory conjunctions, the simple null model does not provide a very good account of the data. This finding turns out not to depend critically on specific details of the sophisticated null model. We tried several versions of the sophisticated null model, including versions with and without the exclusionary guessing assumption and with and without the parameter N_L. The results were always the same. At the farthest distance, any sophisticated null model with a parameter representing nontarget color information always fit better than the simple null model. We saw earlier that the logic of the currently popular test of whether illusory conjunctions occur, that is, comparing $P(\mathrm{CR})$ with $P(\mathrm{CF})$, depends critically on the validity of the simple null model. Therefore, our results indicate that the traditional practice of focusing exclusively on conjunction responses and color feature errors is fundamentally flawed.

When the alpha model is fit separately to the data from each distance, the resulting α estimates provide an atheoretical estimate of the probability of correct binding at each distance. Treisman and Schmidt's (1982) feature integration theory (and the random binding model) predicts these probabilities will be invariant with distance, whereas location uncertainty theory predicts they will increase according to Equation 1. Figure 6 shows these estimated probabilities of correct binding as a function of distance for 4 of the 7 participants. Also shown are the predicted probabilities according to the location uncertainty model for two different values of σ. The plots for the other 3 participants followed a similar pattern. Three conclusions stand out. First, as expected, the probability of correct binding increases with distance for all participants. Second, the primary effect of increasing σ is to decrease the predicted probability of correct binding by a uniform amount at all but the largest distances. Third, the observed probability of correct binding versus distance functions were shallower than predicted by the location uncertainty model. The flattest functions are from Participants 4 and 5. For both of these participants, a better fit is provided by a flat function (as predicted by the random binding model) than by any function predicted by the location uncertainty model.

According to the location uncertainty model, the probability of correct binding is completely determined by d/σ, where d is the perceived distance between the target and nontarget and σ is a measure of location uncertainty. In the present application, we assumed d equals physical distance and σ is invariant with distance. The model could be made to predict shallower probability of correct binding versus distance functions if either d increases more slowly than physical distance or σ increases with physical distance. Under certain conditions, both of these possibilities seem plausible. For example, it makes sense that σ increases with physical distance because attention must be spread over a larger and larger region of visual space as the distance between target and nontarget increases. We argued earlier that any factor that spreads attention should increase location uncertainty (and hence σ). Let σ_d be the location uncertainty when the target-to-nontarget distance is d. There are many possible models relating σ_d to d. One plausible alternative is that

$$\sigma_d = \sigma d^{\gamma}, \qquad (6)$$

where σ is the location uncertainty when the distance equals one unit and γ is some arbitrary constant. This spreading attention location uncertainty model has one more parameter than the simple location uncertainty model (i.e., γ).

When fit to the data of Experiment 1, the location uncertainty model with spreading attention produces the AIC values shown in the last column of Table 4. In six of seven cases, adding the assumption of spreading attention significantly improves

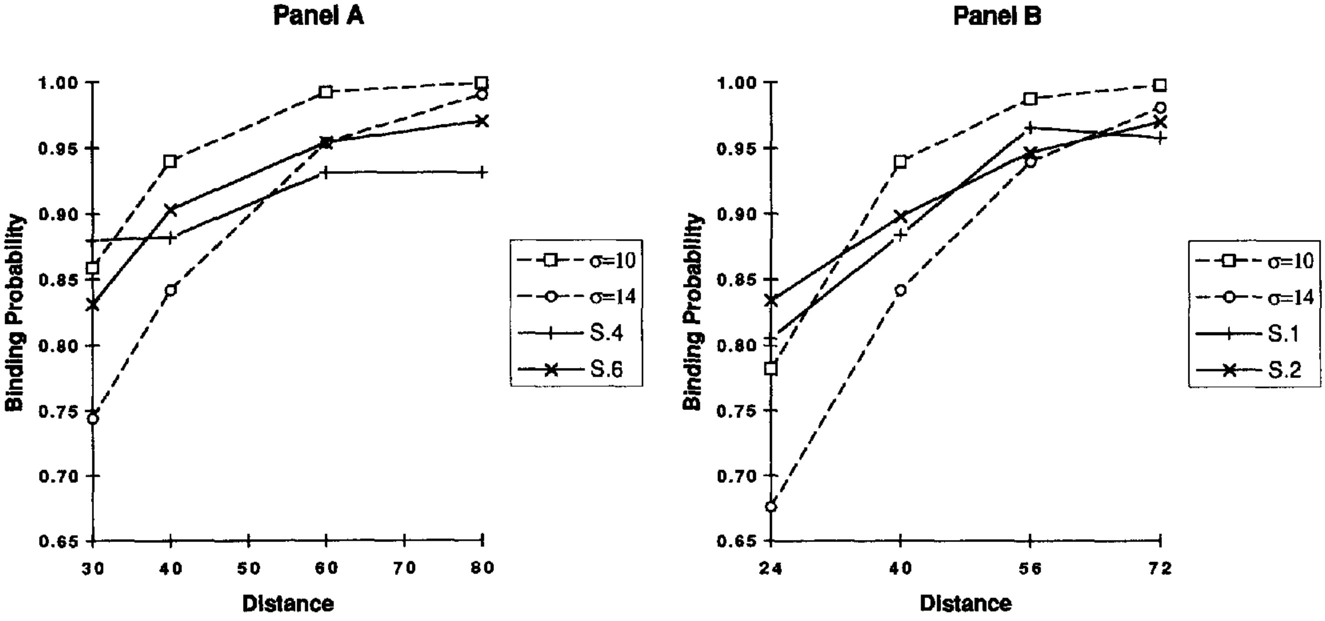

Figure 6. Predicted probability of feature binding versus distance for 4 participants in Experiment 1. Panel A shows predictions for Participants (S.) 6 and 4, and Panel B shows predictions for Participants 1 and 2. The predicted probabilities are from the alpha model and the location uncertainty model (for $\sigma = 10$ and $\sigma = 14$).

the fit of the location uncertainty model. The only exception is Participant 1. Figure 6 indicates that for this participant, the probability of correct binding increases with distance at about the same rate as predicted by the original location uncertainty model (i.e., when $\sigma = 12.4$). With spreading attention, the location uncertainty model always fits better than the random binding model.

Discussion

The theoretical analysis compared models that assumed all of the observed conjunction responses in the experiment were due to guessing (the simple and sophisticated null models) with models that assumed participants sometimes incorrectly combine features (the random binding and location uncertainty models). The latter class significantly outperformed the former. Second, we compared a model that assumed feature binding errors were due to uncertainty in the perceptual location of features (the location uncertainty model) with a model that did not make this assumption (the random binding model). The success of the location uncertainty model supports the hypothesis that feature-binding errors are, in part, due to errors in the perceptual location of features.

Although the location uncertainty model fit the data well, especially when it incorporated the notion of spreading attention, it systematically mispredicted some aspects of the data. For example, the location uncertainty model does not predict a distance effect in the response categories LF (letter feature error), LFCR (simultaneous letter feature error and conjunction response), and CF (color feature error) because it assumes these categories do not involve feature binding. Nevertheless, performance improved with distance for all response categories (see Table 3). There is good reason to believe that this improvement was due to a decrease in lateral masking at the letter level as the distance between target and nontarget increased (Banks, Larson, & Prinzmetal, 1979; Bouma, 1973; Woodworth, 1938). In the models considered earlier, this could be handled by assuming a different value for T_L for each distance, although this solution would not be parsimonious. A more attractive solution is suggested by Wolford's (1975) hypothesis that lateral masking is the result of feature perturbations from adjacent letters. If Wolford's explanation is correct, then perhaps the lateral masking at the letter level could be modeled by a mechanism similar to one used by the location uncertainty model to account for feature-binding errors.

Despite the success of the location uncertainty model, the design of Experiment 1 has fundamental limitations. In particular, the design is not suited to studying the participants' perception of the nontarget letter. The tree diagram in Figure 3 indicates that in the sophisticated null model, the branches involving N_C and N_L tend to make a relatively small contribution to the predicted response probabilities. To explore this issue further, we examined the stability of the model fits by computing changes in goodness of fit (i.e., in AIC) for the location uncertainty model with spreading attention when the parameter estimates were increased or decreased by 0.15 (0.50 for the σ parameter). As an example, for Participant 5, changes in the T_L, T_C, σ, and γ parameter estimates caused the AIC statistic to increase by an average of 418, 183, 284, and 436, respectively, whereas changes in the N_C and N_L estimates caused average increases in AIC of only 5 and 4, respectively. This pattern was found in all cases tested. Thus, the partial report design of Experiment 1 places tight constraints on the parameter estimates for T_L, T_C, σ, and γ, and only weak constraints on the estimates of N_C and N_L.

Experiment 2

Experiment 2 was identical to Experiment 1 in every respect except that in addition to identifying the target letter and color, participants also identified the nontarget color. As in Experiment 1, they were not required to identify the nontarget letter. Participants 1, 2, 6, and 7 from Experiment 1 participated in Experiment 2.

Experiment 2 had two goals. First, having participants report both the target and nontarget colors allows a direct test of the exclusionary guessing strategy assumed by the sophisticated null, random binding, and location uncertainty models. In particular, we can observe how frequently participants give the same response for both the target and nontarget colors. If the exclusionary assumption is correct, color repetition responses should be rare. Second, Experiment 2 tests the generality of our theoretical approach to modeling illusory conjunctions. Because there are more response categories in Experiment 2 than Experiment 1, the models developed for Experiment 1 must be modified. Even so, the same strategy used to develop the Experiment 1 models can be applied in a straightforward fashion to develop models for the Experiment 2 task. If our approach is valid, the results of fitting the revised models to the Experiment 2 data should be similar to the results for Experiment 1.

Method

The displays in Experiment 2 were the same as in Experiment 1. A pair of colored letters framed within two achromatic dollar signs was presented on each trial. The stimuli were located either above or below a fixation point, and the distance between the two letters was varied. The only change in Experiment 2 was in the participants' responses. After reporting the target color and target letter, the participants were required to make a third response indicating the nontarget color. The same keys (*1*, *2*, or *3*) used to report the target color were used for the nontarget color. No feedback was provided regarding accuracy on the nontarget color responses. Changes in the exposure duration were based on the accuracy in reporting the color and letter of the target object, and bonus payments were calculated solely from performance on these features. In this manner, we sought to ensure that the main requirements of the task were identical to those of Experiment 1.

Experiment 2 consisted of two sessions of 800 trials each. Participants 6 and 7 from Experiment 1 participated in these sessions within a couple of days of completing Experiment 1. The other two participants had also been in Experiment 1 (Participants 1 and 2) but had completed that experiment a few months earlier. These participants were tested on one session exactly as in Experiment 1 before completing the two sessions for Experiment 2. The distances between the colored letters were identical to those used in Experiment 1. For Participants 6 and 7, this range was 6′ to 76′ of arc; for Participants 1 and 2, this range was 19′ to 112′ of arc.

Results

Response categories. There were two response alternatives for the identity of the target letter (*T* or *X*), three for the identity of the target color (red, yellow, or blue), and three for the identity of the nontarget color (red, yellow, or blue). Thus, there were a total of 18 different response categories (2 × 3 × 3). The data consisted of these 18 response categories at four different distances and two display conditions (top vs. bottom). The total number of trials (i.e., the total response frequency) was therefore distributed across 144 different cells (18 × 4 × 2). To increase the sample size in each cell, the data were averaged across display position. Thus, the basic data consisted of an 18 × 4 matrix of response proportions for each of the four participants (with 68 degrees of freedom). The mean of these four matrices is shown in Table 6. Each response category is identified by a three-letter code. The first letter describes the participant's response on the identity of the target letter (*C* for correct and *E* for error). The second letter describes the target color response, and the third letter describes the nontarget color response. In both cases, a *T* indicates the participant responded with the target color, an *N* indicates the response was of the nontarget color, and an *O* indicates the participant responded with the color not present in the display. Thus, *CTN* indicates that all three responses were correct, and *CNT* is a classic conjunction response.

As in Experiment 1, there were strong effects of distance on the proportion of correct responses (CTN) and on the proportion of conjunction responses (CNT). In fact, the distance effect on conjunction responses appears to be even greater in Experiment 2 than it was in Experiment 1. However, unlike Experiment 1, the proportion of letter feature errors (ETN, ETT, and ETO) did not decrease with distance, nor did color feature errors on the nontarget letter (CTO). Table 6 also provides preliminary support for the assumption of exclusionary guessing made by the Experiment 1 models. Six of the 18 response categories violate exclusionary guessing (CTT, CNN, COO, ETT, ENN, and EOO). The mean proportions of each of these response types was less than .004, indicating that participants had a strong bias against color-repetition responses.

Table 6
Mean Response Proportions for Experiment 2

Response category	Distance 1	2	3	4	*M*
CTN	.14	.16	.17	.18	.17
CTT	.00	.00	.00	.00	.00
CTO	.03	.03	.03	.03	.03
CNN	.00	.00	.00	.00	.00
CNT	.03	.02	.01	.01	.02
CNO	.01	.01	.01	.01	.01
CON	.01	.01	.01	.01	.01
COT	.00	.00	.00	.00	.00
COO	.00	.00	.00	.00	.00
ETN	.01	.00	.01	.01	.01
ETT	.00	.00	.00	.00	.00
ETO	.00	.00	.00	.00	.00
ENN	.00	.00	.00	.00	.00
ENT	.01	.00	.01	.00	.01
ENO	.00	.00	.00	.00	.00
EON	.00	.00	.00	.00	.00
EOT	.00	.00	.00	.00	.00
EOO	.00	.00	.00	.00	.00

Note. Response category code is as follows. First position in the acronym describes response on target identity (*C* = correct, *E* = error). Second position describes response on target color (*T* = target color, *N* = nontarget color, *O* = other color). Third position describes response on nontarget color (*N* = nontarget color, *T* = target color, *O* = other color).

Theoretical analysis. Our first step in the theoretical analysis was to check the reliability of the modeling that was done on the Experiment 1 data. We did this by ignoring the nontarget color reports in Experiment 2. In this way the 18 × 4 matrix of response frequencies collapses into a 6 × 4 matrix of exactly the same type[6] as in Table 3. The sophisticated null model, the random binding model, and the location uncertainty model (with and without spreading attention) were fit to the collapsed 6 × 4 matrices separately for each of the 4 participants. The simple null model was not included because the Experiment 1 results proved it to be psychologically implausible. The AIC values for the best-fitting versions of each model are shown in Table 7.

Table 7 has an especially simple form because the four models are ordered in the same way for every participant. The location uncertainty model with spreading attention always fits better than the simple location uncertainty model, which always fits better than the random binding model, which always fits better than the sophisticated null model. This ordering is consistent with the Experiment 1 results. In absolute terms, the fits were again impressive. The location uncertainty model with spreading attention accounted for 98.5%, 96.8%, 99.8%, and 98.3% of the variance in the data for Participants 1, 2, 6, and 7, respectively.

The estimated parameter values for all four models also were similar to those obtained in Experiment 1. For the location uncertainty model, the estimates of T_{L} were high, ranging from .77 to .98. The estimated values for T_{C} were also high (ranging from .80 to .94), indicating that participants were generally accurate in perceiving the target features. The N_{C} estimates were lower (ranging from .38 to .81), so as in Experiment 1, the best fits were obtained by assuming that the participants were poorer in perceiving the nontarget color than the target color.

In the second analysis, we generalized the Experiment 1 models to make predictions on the full set of 18 response categories that result when participants simultaneously report the target letter, target color, and nontarget color. For the most part, this was a straightforward process of translating the response categories from Experiment 1 into the three letter codes used to describe the 18 response categories of Experiment 2. For example, Figure 7 shows the T_{L} half of the tree diagram that characterizes the two-color report version of the random binding model. A comparison with Figure 4 (the Experiment 1 version of the random binding model) indicates that the first two terminal nodes of the two trees are identical, except that the C and CR response category labels are replaced in Figure 7 with the labels CTN and CNT, respectively.

The most important case in which simple translation of the response codes fails is with response categories that involve

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[6] Collapsing the data across nontarget color reports increases the sample size in each cell. As a result, for these analyses we did not average over display position (i.e., top vs. bottom). Thus, as in Experiment 1, the models were fit to separate 6 × 4 matrices for the top and bottom display positions.

Table 7

Goodness-of-Fit Values ("A" Information Criterion) for Experiment 2 When the Models Were Fit to Target Letter and Target Color Data Only

	Model			
Participant	Sophisticated null	Random binding	Location uncertainty	Location uncertainty with spreading attention
1	3,349	3,315	3,279	3,273
2	2,901	2,858	2,854	2,838
6	1,257	1,243	1,221	1,212
7	3,278	3,235	3,220	3,206
M	2,696	2,663	2,644	2,632

color repetitions. Because of its exclusionary guessing assumption, the one-color report version of the random binding model (Figure 4) assumed that color repetitions never occur. The two-color report version of the random binding model shown in Figure 7 assumes that the participant guesses that the second color is identical to the first with some probability equal to g. The exclusionary guessing assumption is that $g = 0$. The two-color report models also assume that the participant guesses each of the colors that are different from the first color reported with probability $b = (1 - g)/2$. The two-color report version of the sophisticated null model was generated by setting $\alpha = 1$, and the location uncertainty models were generated by replacing α with the probability specified by Equation 1.

Two-color report versions of the sophisticated null model, the random binding model, and the location uncertainty models were fit separately to the 18 × 4 response matrix collected from each participant in Experiment 2. The results are described in Table 8. As in Experiment 1, the random binding model always outperforms the sophisticated null model, thus suggesting that feature binding was not perfect. The strong distance effect observed in Table 6 for CNT responses suggests that feature binding may have improved with distance. This hypothesis is sup-

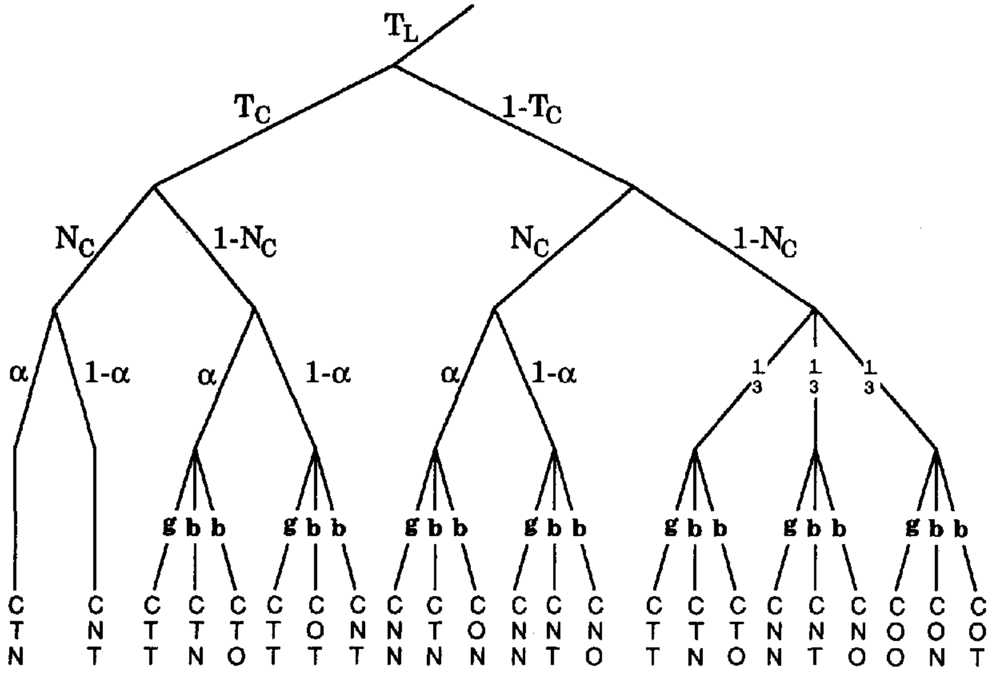

Figure 7. Tree diagram of the T_L half of the two-color report version of the alpha model. (N_L = probability of perceiving the nontarget letter; T_C = probability of perceiving the target color; T_L = probability of perceiving the target letter; N_C = probability of perceiving the nontarget color; C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; CLF = color–letter feature error; g = probability of perceiving the second color identical to the first; b = probability of perceiving colors different than the first; C = correct; T = target color; N = nontarget color; O = other color.)

Table 8

Goodness-of-Fit Values ("A" Information Criterion) When the Two-Color Report Models Were Fit to the Complete Experiment 2 Data Set

	Model			
Participant	Sophisticated null	Random binding	Location uncertainty	Location uncertainty with spreading attention
1	5,531	5,519	5,484	5,486
2	4,182	4,104	4,103	4,081
6	2,325	2,218	2,182	2,174
7	5,318	5,311	5,289	5,291
M	4,339	4,288	4,265	4,258

ported by the fact that the location uncertainty model provided better fits than the random binding model for all participants. Finally, when the notion of spreading attention is incorporated into the location uncertainty model, the fit is significantly improved for 2 of the 4 participants. This model accounts for 98.4%, 99.4%, 99.9%, and 98.6% of the variance in the data of Participants 1, 2, 6, and 7, respectively.

Table 9 shows parameter estimates of the best-fitting versions of the various models. First, note the uniformly small estimates of g. The largest estimate is only .04. To appreciate this value,

Table 9

Parameter Estimates From Best-Fitting Two-Color Report Models for Experiment 2

	Participant			
Parameter/Model	1	2	6	7
T_{L}				
Sophisticated null	0.74	0.79	0.93	0.77
Random binding	0.76	0.86	0.98	0.79
Location uncertainty	0.77	0.85	0.98	0.79
Location uncertainty with SA	0.77	0.86	0.98	0.79
T_{C}				
Sophisticated null	0.71	0.76	0.92	0.69
Random binding	0.73	0.79	0.94	0.71
Location uncertainty	0.73	0.78	0.94	0.71
Location uncertainty with SA	0.73	0.79	0.94	0.71
N_{C}				
Sophisticated null	0.49	0.79	0.85	0.51
Random binding	0.51	0.80	0.85	0.52
Location uncertainty	0.51	0.80	0.85	0.53
Location uncertainty with SA	0.52	0.80	0.85	0.53
N_{L}				
Sophisticated null	0.02	0	0	0
Random binding	0.17	0.25	0.96	0.02
Location uncertainty	0.22	0.15	0.52	0.05
Location uncertainty with SA	0.23	0.27	0.97	0.05
g				
Sophisticated null	0.04	0.01	0.04	0.02
Random binding	0.04	0.01	0.04	0.03
Location uncertainty	0.04	0.01	0.04	0.03
Location uncertainty with SA	0.04	0.01	0.04	0.03
α				
Random binding	0.95	0.91	0.95	0.96
σ				
Location uncertainty	9.4	10.7	9.6	9.5
Location uncertainty with SA	5.1	1.4	2.1	9.5
γ				
Location uncertainty with SA	0.96	0.99	0.43	0

Note. SA = Spreading attention; T_{L} = probability of perceiving the target letter; T_{C} = probability of perceiving the target color; N_{L} = probability of perceiving the nontarget letter; N_{C} = probability of perceiving the nontarget color.

consider a trial on which the participant saw nothing. Suppose he or she guessed the target color to be red. Then if g is .04, the participant will guess the nontarget color to be red with probability .04, to be yellow with probability .48, and to be blue with probability 0.48. This is very close to an exclusionary guessing strategy.

All four models strongly agree as to their estimates of T_L, T_C, N_C, and g. In particular, all models agree there were individual differences in the perception of the nontarget color. For Participants 1, 6, and 7, the probability of perceiving the nontarget color is considerably lower than the probability of perceiving the target color. Participant 2 appears to be equally proficient in perceiving the two colors, perhaps because of a strategy difference. Participants 1, 6, and 7 may have narrowed attention rapidly to the target location, thereby reducing the likelihood of correctly perceiving the nontarget color. In contrast, Participant 2 may have adopted a strategy in which attention was distributed more evenly over the display. It should be possible in future experiments to explore these hypotheses by varying the importance of perceiving the nontarget color.

There is major disagreement among the models only about the value of N_L. The disagreement could result from the failed attempt of invalid models to fit the data. However, even for the best model—the location uncertainty model with spreading attention—the estimates of N_L are highly variable. In fact, they seem more variable than one would expect on the basis of individual difference. A second possibility is that the data just do not constrain the N_L parameter enough. This was the problem in Experiment 1 for both the N_L and N_C parameters. To test this hypothesis, we again examined the effects on AIC of separately incrementing and decrementing each parameter in the location uncertainty model with spreading attention by .15 (.50 for σ). These modest changes in the N_L parameter estimates had almost no effect on AIC (AIC increased an average of 2 or 3 points), which confirms the hypothesis that the data do not sufficiently constrain this parameter. Changes in all other parameters caused large increases in AIC. For example, for Participant 2, changing N_C by .15 caused an average increase in AIC of 130. For Participant 7, the average increase was 51.

The results of Experiments 1 and 2 indicate that data from partial report designs are sufficiently rich to allow precise estimation of all parameters describing the participant's perceptual experience with stimulus features that must be reported (assuming, of course, that one uses a valid model). In Experiment 1 this included only the identity of the target letter and its color. In Experiment 2 this included the identity of the target letter and the target and nontarget colors. Partial report data do not appear rich enough to allow accurate estimation of parameters describing the participant's experience with features that are not reported (e.g., the identity of the nontarget letter). This suggests that when modeling partial report data, simplifying assumptions can be made about these latter parameters (e.g., $N_L = 0$), with only a minor penalty in goodness of fit. An alternative might be to constrain the values of parameters corresponding to nonreported features to equal that of corresponding reported features (e.g., $N_L = T_L$).

Previous Research

Experiments 1 and 2 demonstrate the efficacy of multinomial models at accounting for the complex guessing strategies participants adopt in the partial report designs used to study feature binding. In addition, the experiments support the basic assumptions of location uncertainty theory over the notion of random binding. This section demonstrates the potency of our approach by reexamining published data from two studies. In both examples, formal modeling provides insights that go beyond those available from traditional methods of analysis.

Cohen and Ivry (1989)

Cohen and Ivry (1989) were interested in the effects of interitem distance and attention on feature binding. In their Experiments 3 and 4, which we focus on here, they hypothesized that the effect of distance would be modulated by attention, so that (a) within the span of spatial attention there would be no effect of interitem distance and (b) feature binding errors would occur only between items that were both within the span of attention.

The basic paradigm used by Cohen and Ivry (1989) was the same as in our Experiment 1: Two colored letters were presented on each trial, and the participants had to report the color and identity of a target letter with no report required of the nontarget features. The locations of the colored letters were varied across trials, although one letter was always presented to the left of fixation, and one letter was always presented to the right. The possible letter locations are shown by the boxes in Figure 8. One letter was always presented in position P_1, P_2, or P_3, and the second letter appeared in position P_4, P_5, or P_6. In addition to the colored letters, two digits were presented on each trial, and the participants had to report the identity of the two digits before describing the target letter. In an effort to manipulate the span of attention, two conditions were run. In the "small spotlight" condition, the digits were separated by 3.3° and flanked the two center locations (the location of the *SS*s in Figure 8). In the "large spotlight" condition, the digits were separated by 6.6° (the *LS*s in Figure 8). Cohen and Ivry only collected about 10 trials per condition per participant, so the data they presented were averaged across participants and left-right symmetrical pairs (e.g., data from pairs P_2–P_4 and P_3–P_5 were averaged).

We fit a number of models to these data (i.e., to the data reported in their Tables 3 and 4), including the random binding and location uncertainty models. The data from the small spotlight and large spotlight conditions were fit separately, and all data were modeled, including conditions that were within and outside the spotlight. In both experiments, the location uncertainty model (without spreading attention) provided much better fits than the random binding model for both the small and large spotlight conditions (the smallest AIC difference was 211). Thus, these data provide another example of distance effects on illusory conjunctions.

The parameter estimates from the location uncertainty model showed several interesting attentional effects. First, σ was much greater in the large than in the small spotlight condition. The average σ estimate across experiments was 0.53° (of visual angle) in the small spotlight condition and 1.01° in the large spotlight condition. Thus, as expected, there is more location uncertainty when attention is distributed over a larger area, and consequently, there are more feature binding errors. This result supports the notion of spreading attention that we incorporated

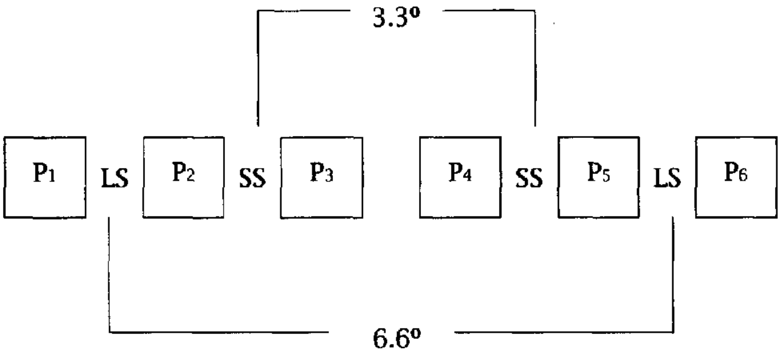

Figure 8. Display conditions of Experiments 3 and 4 of Cohen and Ivry (1989). P_i = position i; LS = location of digits in large spotlight condition; SS = location of digits in small spotlight condition.

into the location uncertainty model through Equation 6. In addition, attention not only affected the probability of binding but also the probability of feature perception. The average estimates of T_L were .65 and .53 for the small and large spotlight conditions, respectively. For T_C, the values were .51 and .45. These differences suggest there is a cost to spreading attention over a large area. In addition to increased location uncertainty, feature perception also suffers. One of the advantages of a formal modeling approach is that the simultaneous effects of an independent variable, such as attention, can be examined on both feature perception and feature binding. In fact, our approach makes it impossible to analyze feature binding without taking into account feature perception.

Cohen and Ivry (1989) might question our model-fitting procedure because we simultaneously fit data from letter pairs that were both within and outside the spotlight of attention. They might argue that the real issue is whether there is a distance effect within the spotlight. Specifically, they argued that there is not. This claim was also made by Treisman and Schmidt (1982) using a paradigm with similar requirements for a secondary digit-report task. To test this hypothesis, we fit the random binding and location uncertainty models to the data from the letter pairs that fell within the large spotlight (i.e., Pairs 3–4, 2–5, 2–4, and 3–5). The location uncertainty model provided much better fits in both experiments (AIC values were 3,836 and 3,689 in Experiment 3 and 3,719 and 3,574 in Experiment 4 for the random binding and location uncertainty models, respectively). Thus, there does seem to be a distance effect, even within the spread of attention.

Cohen and Rafal (1991)

Cohen and Rafal (1991) tested a woman with a mild attention disorder on an illusory conjunction task. This patient had suffered a stroke 12 years before testing, and the resulting lesion encompassed a large portion of the left parietal lobe. The patient still showed mild signs of extinction under laboratory testing conditions; in particular, she was slow to detect a stimulus in the contralesional hemifield when a competing stimulus was presented simultaneously in the ipsilesional hemifield. The parietal lobe has been implicated in visual attention (e.g., Posner, Walker, Friedrich, & Rafal, 1984), and Treisman has claimed that attention is necessary for feature binding (Treisman & Gelade, 1980; Treisman & Schmidt, 1982). Hence, Cohen and Rafal predicted that the patient would make more illusory conjunctions for stimuli presented in the contralesional hemifield relative to stimuli presented in the ipsilesional hemifield. Furthermore, because Treisman claimed that feature perception is a preattentive process, Cohen and Rafal expected that the patient would be as adept in perceiving features presented to either hemifield.

The method used by Cohen and Rafal (1991) was similar to that used in our Experiment 1. Two colored letters were presented on each trial, and the participant had to report the color and identity of the target letter. There were two possible target letters and four possible colors. The colored letters were always presented to a single hemifield. The letters were vertically aligned and were separated by visual angles of approximately 0.5° (near condition) or 4.0° (far condition). Two digits of different sizes were also presented at the center of the display. The participant was required to report the identity of the larger of the two digits (in size). This secondary task was included to increase the processing demands and to ensure the patient maintained fixation at the center of the display.

Cohen and Rafal's (1991) data are reproduced in Table 10. We fit the alpha model separately to the data from each experimental condition. To reduce the number of free parameters,[7] we set $N_L = T_L$. For the far conditions, the best fit was with α = .95 on the contralesional side and α = .92 on the ipsilesional side. Thus, feature binding was accurate in both hemifields.

The most interesting data came from the near conditions. Cohen and Rafal (1991) made two claims. First, they argued that feature integration errors were more numerous on the contralesional side than the ipsilesional side. Our α estimates supported this conclusion. For the contralesional hemifield, the estimated value of α was only .55, indicating the patient was only slightly better than chance at binding the target letter to the target color. In contrast, on the ipsilesional side, the α estimate was .90, indicating accurate binding. These results appear to

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[7] The $N_L = T_L$ version of the alpha model had a lower AIC value than the version with N_L free, for all four conditions.

Table 10
Response Proportions From Cohen and Rafal (1991)

Hemifield	Condition	Response category C	CR	LF	CF	CLF	LFCR
Ipsilesional	Near	.35	.34	.12	.03	.01	.13
Contralesional	Near	.70	.10	.06	.10	.03	.03
Ipsilesional	Far	.39	.20	.09	.08	.07	.07
Contralesional	Far	.68	.09	.09	.11	.02	.02

Note. C = correct response; CR = conjunction response; CF = color feature error; LF = letter feature error; CLF = color–letter feature error; LFCR = letter feature error, conjunction response.

support the claim that the patient has a contralesional binding deficit, but we will present an alternative explanation shortly.

The second claim, motivated by Treisman's theory, was that the patient should be equally adept at identifying features in the ipsilesional and contralesional hemifields. Cohen and Rafal (1991) claimed that, whereas the patient was impaired in identifying the letter when the stimuli were presented in the contralesional hemifield, there was no hemispheric difference in color perception. To the contrary, our analysis indicated that both color and letter feature perception differed in the two hemifields but in a rather surprising way. In support of the Cohen and Rafal claim, the model fits indicated that the probability of detecting the target letter (T_L) in the ipsilesional hemifield was higher than in the contralesional field—.78 versus .47. However, the estimates of T_C were actually higher in the contralesional than in the ipsilesional field—.86 versus .71. Thus, in the near condition, the alpha model indicates that the patient's color perception was actually better in the contralesional hemifield!

A direct examination of the Cohen and Rafal (1991) data (i.e., Table 10) supports these conclusions about the accuracy of the participant's feature reports. If the participant really was poorer at identifying contralesional than ipsilesional targets, then the total number of trials on which an incorrect letter response was made should be higher on the contralesional side. The total number of letter errors can be computed by adding the frequencies of LF (letter feature error), CLF (color-letter feature error), and LFCR (simultaneous letter feature error and conjunction response) responses. These sums indicate there were significantly more errors in the contralesional field than in the ipsilesional field, $\chi^2(1, N = 306) = 25.24, p < .01$. We also computed the total errors on which participants responded with a color that was not part of the display (the sum of CF and CLF response frequencies). In agreement with our T_C estimates, the participant was significantly more accurate in the contralesional field, $\chi^2(1, N = 325) = 19.76, p < .01$.

The fits of the alpha model provide converging evidence that the patient's deficit is most pronounced on tasks requiring feature binding rather than feature perception. However, we were quite surprised by the finding in both the raw data and in the parameter estimates that suggested the patient may have better color perception in the lesioned hemifield. It is easy to understand why letter perception is worse in the contralesional field, but it is curious why color perception should be better in the contralesional field. This counterintuitive result led us to consider an alternative model to account for the patient's performance.

Our alternative model was inspired by the phenomenon of simultaneous extinction—perhaps the most characteristic symptom of this type of lesion. When presented with more than one object simultaneously, extinction occurs if the patient is unable to perceive one of the objects (Bender, 1945). A simple model that assumes extinction is illustrated in Figure 9. The model assumes that the patient never perceives the same features from both stimulus letters. For example, the patient may either perceive a single color (from either the target or nontarget letter) or no colors. Similarly, the patient will either perceive a single letter or no letters. The extinction model has only three free parameters. The parameter T_L represents the probability that the patient perceives a single letter *and* that it is the target letter. Thus, $1 - T_L$ is the probability that the patient perceives only the nontarget letter *or* that no letter is perceived. The parameter P_C represents the probability the patient perceives one color, and $1 - P_C$ is the probability that no colors are perceived. Finally, T_C is the probability that the one color perceived is from the target letter, and $1 - T_C$ is the probability that it is from the nontarget letter. The model assumes that whenever a letter and a color are perceived, they will bind together, regardless of whether the two features are from the same object.

For the data from the contralesional hemifield (near condition), the extinction model provided a slightly better fit than did the alpha model (AIC = 918 vs. 920). Thus, here are two strikingly different interpretations of the Cohen and Rafal (1991) data. According to the alpha model, there were many failures of feature binding in the contralesional hemifield ($\alpha = .55$), whereas according to the extinction model, the many conjunction responses in this condition are really a result of the extinction phenomenon. Because of extinction, there will be many trials in which the nontarget color is the only color seen by the patient. Consequently, the nontarget color will be reported more frequently than colors not present in the display, and as a result, more conjunction responses will occur. Moreover, because the perceived color is reported, be it from the target or nontarget letter, the participant will make few feature errors on the contralesional side in comparison with the ipsilesional side. The modeling results provide at least as much support for the extinction hypothesis as for the failure of feature binding hypothesis.

The parameter estimates from the extinction model were $T_L = .47$, $P_C = .93$, and $T_C = .51$. The color parameters suggest that the patient was very good at perceiving at least one color on each trial ($P_C = .93$), and the color she perceived was equally likely from the target or nontarget ($T_C = .51$). T_L is slightly smaller than T_C, but this may be because T_L is the probability that the patient perceives a single letter and that it is the target letter. The analogous probability for color—that is, the probability that she perceives a single color and that it is the target color—is equal to the product of P_C and T_C. It is interesting that $P_C \times T_C = .47$, exactly the T_L estimate. The data, therefore, do not contradict the hypothesis that the patient was as equally likely to see one letter as to see one color and that, on trials when she did perceive a single letter, it was equally likely to be the target or nontarget. Finally, it should be noted that the extinction model provides a poor fit of the data from our Experiment 1. Thus, it is not

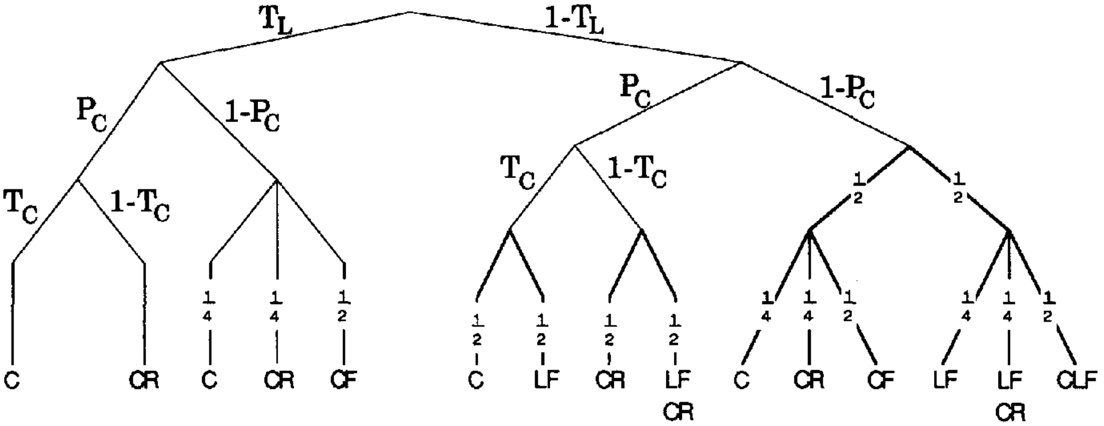

Figure 9. Tree diagram of an alternative interpretation of the Cohen and Rafal (1991) data that assumes conjunction responses are due to simultaneous extinction (i.e., the extinction model). (T_L = probability of perceiving one letter and it is the target letter; P_C = probability of perceiving one color; T_C = probability that the one color perceived is from the target letter; C = correct responses; CR = conjunction response; CF = color feature error; LF = letter feature error; LFCR = letter feature error, conjunction response; CLF = color–letter feature error.)

appropriate for modeling data from participants with normal feature perception.

The extinction model and the resulting parameter estimates seem to match nicely the clinical description of simultaneous extinction. When simultaneously presented with more than one object, the patient is unable to perceive all of the information. Although the typical extinction paradigm has involved the simultaneous presentation of ipsi- and contralesional stimuli, the current results suggest that extinction may also occur when more than one stimulus is presented contralesionally. Moreover, the results imply that extinction may occur within a dimension (e.g., color) rather than for whole objects. Baylis, Driver, and Rafal (1993) have also recently found evidence that extinction may operate more strongly within a dimension (e.g., as when a color extinguishes a similar color) than between dimensions (e.g., as when a color extinguishes a letter).

Cohen and Rafal (1991) claimed that the patient had intact feature perception in the contralesional hemifield and a deficit in feature binding. The modeling work leads to new insights into what is meant by a deficit in feature binding. By traditional interpretations, one would infer that the patient has perceived the feature information in a normal manner, but that she is unable to bind together those features that emanate from the same object. However, we propose an alternative interpretation in which the emphasis returns to a problem in feature perception. By our account, the patient's primary problem is that she is unable to perceive more than one feature within each dimension on a given trial. Illusory conjunctions for this patient are not the result of jumbling up a set of colors and letters but rather result from the fact that the patient appears to respond with the limited information available to her.[8]

General Discussion

Assumptions of the Multinomial Modeling Approach

All models described in this article rest on two key assumptions that cannot be expected to hold in general. The first is that feature perception is all or none; the second is an assumption of feature-sampling independence—namely, that the simultaneous perception of different features occurs in a statistically independent fashion. These two assumptions have a long history in the pattern perception literature, and a good deal is known about the conditions under which they might be valid.

The models assume feature perception is all or none because each major fork in the trees that describe the models has only two branches. For example, consider the T_L versus $1 - T_L$ forks. The T_L branch assumes the target letter is perceived perfectly, whereas the $1 - T_L$ branch assumes there is no target letter information at all. No branches in any models allow partial feature information.

All-or-none models of pattern perception were first formally tested about 25 years ago (Townsend, 1971). A decade later they were rejected as a complete description of human pattern perception (Smith, 1980; Townsend & Ashby, 1982). There is abundant evidence that partial information is frequently available, even in the most impoverished viewing conditions (e.g., Townsend & Ashby, 1982; see also Kinchla, 1994). In fact, the best current models of pattern identification assume some information is obtained on every trial (at least with foveal display; e.g., Ashby & Lee, 1991). Nevertheless, under certain conditions, all-or-none models can provide reasonably good fits to data. Specifically, all-or-none models can fit well if the competing stimulus features are highly dissimilar, because in this case

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[8] Arguin, Cavanagh, and Joanette (1994) recently tested several patients with clinical neglect in a presence–absence task, in which participants were to respond *present* only when the display contained a target defined by a color and a form (e.g., red circle). They found a large number of false positives when displays presented in the contralesional hemifield contained the target color in one element and the target shape in another (e.g., red square and green circle). We are now developing models of the presence–absence task, which should make it possible to compare random binding and extinction interpretations of this phenomenon.

partial information will likely lead to the correct response (Townsend, 1971; Townsend & Ashby, 1982). For example, in Experiment 1, the target letter was always a *T* or an *X*. If the participant perceives any evidence of horizontalness or verticalness, the target was almost certainly a *T*, whereas any evidence of diagonality strongly implicates an *X*. Thus, any partial information leads to a correct response, so it seems reasonable to assume that letter feature errors in Experiment 1 occur only on trials when the participant has no information available about the identity of the target letter. As a consequence, the T_L parameter might more appropriately be interpreted as the probability that at least some target letter information is perceived.

In experiments where the similarity among the alternative feature values is high, the all-or-none assumption will not be valid. In this case, the treelike structure of the location uncertainty model might be retained, but the model will require considerable generalization. For example, suppose we replicate Experiment 1, but instead of using *T* and *X* for the target letters we use *B* and *D*. In this case, $1 - T_L$ and its associated branches could have exactly the same meaning as before—namely, the probability of no target letter information of any kind. However, T_L could now equal the probability that at least some information about the target letter was obtained. Two new branches would sprout from the T_L branch. The first would correspond to the probability that the target is identified as a *B*, and the second would correspond to the probability the target is identified as a *D* (where the two probabilities add to 1). The computation of these two probabilities would differ depending on whether the display contained a *B* or a *D*. Ideally they would be derived from some underlying similarity-based identification model (e.g., Ashby & Perrin, 1988).

One consequence of substituting the letters *B* and *D* for the letters *T* and *X* is that many more letter feature errors would be expected. If we had also replaced the colors red, yellow, and blue with red, yellow, and orange, then we would also expect more conjunction responses. For example, on a trial when the target was an orange *T* and the nontarget was yellow, the participant might misidentify the color of the target as yellow. This would lead to a CR, but the CR would not be the result of incorrectly combining features. Rather, the CR would be the result of misperceiving the features. A formal modeling approach has the potential to separate the effects of similarity on feature perception from the effects of similarity on feature binding (see Ivry & Prinzmetal, 1991, for a different solution to this problem).

The models developed in this article all assume feature-sampling independence because the probability of following two successive branches is assumed to be the product of the probabilities associated with each single branch. In general, the perception of multiple stimulus features does not satisfy feature-sampling independence (Ashby & Townsend, 1986; Townsend & Ashby, 1982; Townsend, Hu, & Ashby, 1981). Nevertheless, with some features the assumption may be valid. Ashby and Townsend (1986) showed that feature-sampling independence will hold if the separate features are perceived independently and if the participant makes separate decisions about the level of the various features. Because there is growing evidence that shape and color perception occur in different visual systems, at least with small targets and peripheral displays (e.g., Lennie, 1984; Livingstone & Hubel, 1984; Zrenner et al., 1990), it seems reasonable to assume that in our experiments, shape and color were perceived independently.

The assumption that participants make separate decisions about the level of each stimulus feature seems reasonable, but it is not guaranteed to be optimal (i.e., maximize overall response accuracy), even if perceptual independence holds (Ashby & Gott, 1988). Ashby and Maddox (1994) showed that two other properties are critically important. First, the stimulus features must also be perceptually separable—that is, the perception of one must be unaffected by the level of the other (Ashby & Townsend, 1986). Second, the stimulus ensemble must contain all possible combinations of the levels of the various features (i.e., a factorial design). Under these conditions, the optimal response strategy is to make separate decisions about the level of each feature.[9] Both of these conditions were satisfied in the present experiments. First, hue and shape are known to be perceptually separable (e.g., Garner, 1974; Handel & Imai, 1972; Imai & Garner, 1965); second, six possible targets were created by factorially crossing two levels of shape (*T* or *X*) with three levels of hue (red, yellow, or blue). Thus, the multinomial models presented here might reasonably be interpreted as maximum likelihood models: Given the processed information available from a display, the participant computes the likelihood of each possible stimulus display and chooses the alternative associated with the largest likelihood.

It is not difficult to imagine experiments, however, where the feature-sampling independence assumption would almost surely be invalid. For example, consider a version of Experiment 1 in which the target and nontarget were both colored squares that varied across trials in hue and brightness. Suppose the target was red or green and the nontarget was always blue and, furthermore, the target and nontarget brightnesses were sampled (without replacement) from three different brightness levels (e.g., dim, moderate, bright). The participant's task is to name the target color and identify the level of the target brightness. The location uncertainty model could be fit, without alteration, to the results of such an experiment, but it might not fit well. Perceptual interactions between hue and brightness are well known (e.g., Garner & Felfoldy, 1970; Hyman & Well, 1968), so it is likely that feature-sampling independence would fail. If so, then the probability of perceiving a particular hue and brightness would not equal the probability of perceiving that hue times the probability of perceiving that brightness.

In the absence of feature-sampling independence, the simultaneous perception of different features must be modeled as a single event, not as a sequence of independent events, as is possible if feature-sampling independence holds. One possibility is to assume that the participant perceives the target as a gestalt, with values on different perceptual dimensions that may covary over trials and that the decision about the level of a particular feature may depend on the values along several perceptual dimensions. Exactly such a model is currently popular in the pattern identification literature (Ashby & Lee, 1991; Lee & Ashby, 1995).

If feature-sampling independence holds for form and color,

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[9] The Ashby and Maddox (1994) proof also assumed that the percepts have a trial-by-trial multivariate normal distribution, but this condition is much less important than perceptual independence, perceptual separability, and a factorial stimulus design.

then perception of the target letter will be independent of perception of the target color. Even in this case, however, it might be unreasonable to expect the simultaneous perception of the target and nontarget colors to be independent. How would the possible failure of feature-sampling independence for the target and nontarget colors affect the analyses reported in this article? To answer this question, we simulated a number of data sets from a generalization of the alpha model in which a dependence was introduced between the perception of the target and nontarget colors (using the Ashby & Townsend, 1986, model of perceptual dependence). We then fit each data set using the standard alpha model (i.e., the version assuming independence). When the perceptual dependence was moderately strong, the standard alpha model was unable to estimate T_C and N_C accurately. However, the standard model was able to estimate α accurately. Thus, our conclusions about feature binding are relatively unaffected by a possible perceptual dependence between the target and nontarget colors.

Relations to the Visual Search Literature

Another popular method for studying attention uses a visual search task, in which participants search for either a target defined by the conjunction of features (e.g., red X) or defined by a single unique feature (e.g., any red letter). These two tasks are called *conjunction* and *feature search*, respectively. The original finding was that response time (RT) for feature search is almost unaffected by display size (i.e., by the number of distractors), a result that is highly suggestive of massive parallel processing (Treisman & Gelade, 1980). In contrast, it was found that RT for conjunction targets increases sharply with display size. Treisman has argued that feature search requires only feature perception, whereas conjunction search also requires feature binding (Treisman, 1982, 1991; Treisman & Gelade, 1980; Treisman & Gormican, 1988; Treisman & Sato, 1990). Location uncertainty theory assumes that feature binding is required for the correct perception of multielement displays, so in this respect, we agree with Treisman.

A number of recent empirical findings have seriously complicated this story. First, for feature search, mean RT almost always increases with display size to some extent. For example, Cavanagh, Arguin, and Treisman (1990) examined feature search for a number of stimulus attributes. In 9 of 10 cases, mean RT increased with display size (i.e., Tables 1, 2, and 3, positive trials). Second, it has been known for a long time that visual search RT increases with target-distractor similarity (e.g., Estes, 1972). In fact, if the stimuli are confusable, the display-size effect can be quite large, even for feature search (e.g., Duncan & Humphreys, 1989; Nagy & Sanchez, 1990). Third, under certain conditions, search time for conjunctions is minimally affected by display size (Cohen & Ivry, 1991; McLeod, Driver, & Crisp, 1988; Nakayama & Silverman, 1986; Steinman, 1987; Treisman & Sato, 1990; Wolfe et al., 1989).

Although the study of visual search behavior is important in its own right, as a paradigm for the study of feature binding, it is problematic. Because both feature search and conjunction search can yield either large or small display-size effects, a particular data set is not diagnostic as to whether the limits in performance are due to feature perception, feature binding, or both. For example, feature binding is not logically necessary for feature search, but this does not imply that feature binding does not occur. To complicate matters more, there is convincing evidence that other processes are also involved. For example, decision processes become more complex as display size increases (e.g., Palmer, Ames, & Lindsey, 1993). Unfortunately, no method has been proposed to separate the contributions of these various components. In contrast, within the context of the partial report paradigm, the models developed in this article allow us to estimate separately the effects of feature perception and feature binding on human performance. Thus, we believe that partial report is a better paradigm than visual search if one is interested in studying the feature binding process.

Conclusions

The formal modeling approach described in this article has a number of important advantages over past methods of data analysis in feature binding experiments. The problem with traditional methods of data analysis has always been that many of the responses labeled CRs are errors due to guessing rather than to incorrect feature binding. This has made it difficult to test competing theories of feature binding and even to decide whether feature binding errors have occurred. The multinomial modeling approach described in this article solves both of these problems. In particular, it allowed us to formalize the assumptions made by previous investigators. The resulting simple null model provided a poor account of our data, even in those conditions where participants did not make illusory conjunctions. Our analyses indicated that, in contrast to traditional assumptions, participants make use of information from the nontarget letter, even when they are asked only to identify features of the target letter.

Another advantage of the formal modeling approach is that it provides estimates of a number of psychologically meaningful quantities. For example, the alpha model provides simultaneous estimates that feature binding is successful (i.e., α) and that each stimulus feature in the display is identified correctly (i.e., T_L, T_C, N_C, and N_L). This allows us to study feature perception and feature binding within the same model. An examination of these parameter estimates from two separate experiments indicated that (a) when the target and nontarget letters were sufficiently close together, true illusory conjunctions did occur; and (b) contrary to the feature integration theory of Treisman and Schmidt (1982), the probability of correct binding increased with interitem distance. We proposed a location uncertainty model that accounted for both of these phenomena and that fit the data from two experiments better than a model consistent with Treisman and Schmidt's (1982) feature integration theory (i.e., the random binding model).

A formal modeling approach also can provide new insights into old data. For example, our reanalysis of the Cohen and Ivry (1989) data confirmed some of their conjectures but disconfirmed others. Perhaps most important, our reanalysis indicated that there was a distance effect on illusory conjunctions within the spotlight of attention, in contradiction to the claims of both Cohen and Ivry (1989) and Treisman and Schmidt (1982).

Finally, the models developed in this article are easy to modify. As such, the same basic approach can be used to formalize other ideas about illusory conjunction experiments. For example, we were able to formulate and test an alternative account of the performance of a patient with parietal lobe damage that assumed the large number of conjunction responses she produced were the result of simultaneous extinction rather than a failure of feature binding. It is the very general and flexible nature of the multinomial modeling approach that is perhaps its greatest asset. We believe that the formulation and testing of models of the type introduced here will lead to a rapid increase in our understanding of the psychological phenomena underlying object perception.

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Appendix

Computing the Probability of Correct Binding in the Location Uncertainty Model

This appendix derives an efficient method for computing the probability of correct binding according to location uncertainty theory. From Equation 1,

$$\begin{aligned} P(\text{correct binding}) &= P(D_{\mathrm{TL,TC}} < D_{\mathrm{TL,NC}}) \\ &= P(D^2_{\mathrm{TL,TC}} < D^2_{\mathrm{TL,NC}}) \\ &= P[(X_{\mathrm{TL}} - X_{\mathrm{TC}})'(X_{\mathrm{TL}} - X_{\mathrm{TC}}) \\ &\quad < (X_{\mathrm{TL}} - X_{\mathrm{NC}})'(X_{\mathrm{TL}} - X_{\mathrm{NC}})]. \qquad \text{(A1)} \end{aligned}$$

By assumption, X_{TL}, X_{TC}, and X_{NC} are all mutually independent, and each has a bivariate normal distribution with variance–covariance matrix equal to $\sigma^2 I$, where I is the identity matrix. The coordinate system in the perceived plane can be set arbitrarily, so without loss of generality we assume the point (0, 0) corresponds to the coordinates of the mean perceived location of the target letter. We can also arbitrarily set the coordinates of the mean perceived location of the nontarget letter to $(d, 0)$, where d is the perceived distance between target and nontarget. As a consequence, the means of the three random vectors are

$$E(X_{\mathrm{TL}}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, E(X_{\mathrm{TC}}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } E(X_{\mathrm{NC}}) = \begin{bmatrix} d \\ 0 \end{bmatrix}. \qquad \text{(A2)}$$

Now let $V_{\mathrm{T}} = X_{\mathrm{TL}} - X_{\mathrm{TC}}$ and let $V_{\mathrm{N}} = X_{\mathrm{TL}} - X_{\mathrm{NC}}$. Next define the 4 × 1 dimensional random vector,

$$Y = \begin{bmatrix} V_{\mathrm{T}} \\ V_N \end{bmatrix}. \qquad \text{(A3)}$$

Note that Y has a multivariate normal distribution with mean vector and variance–covariance matrix equal to

$$\mu_{\mathrm{Y}} = \begin{bmatrix} 0 \\ 0 \\ -d \\ 0 \end{bmatrix} \quad \text{and} \quad \Sigma_{\mathrm{Y}} = \begin{bmatrix} 2\sigma^2 I & \sigma^2 I \\ \sigma^2 I & 2\sigma^2 I \end{bmatrix} = \sigma^2 \begin{bmatrix} 2I & I \\ I & 2I \end{bmatrix}, \qquad \text{(A4)}$$

where I is a 2 × 2 identity matrix. Therefore,

$$\begin{aligned} P(\text{correct binding}) &= P(V_{\mathrm{T}}{}'V_{\mathrm{T}} < V_{\mathrm{N}}{}'V_{\mathrm{N}}) \\ &= P(V_{\mathrm{T}}{}'V_{\mathrm{T}} - V_{\mathrm{N}}{}'V_{\mathrm{N}} < 0) \\ &= P(Y'JY < 0), \end{aligned}$$

where

$$J = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{bmatrix}. \qquad \text{(A5)}$$

(*Appendix continues on next page*)

Because $\boldsymbol{Y}$ has a multivariate normal distribution, $\boldsymbol{Y}'J\boldsymbol{Y}$ is a quadratic function of normal variables. Unfortunately, however, the entries in $\boldsymbol{Y}$ are not mutually independent. To proceed, we must rotate the variables to achieve independence.

To begin, let A be the unique lower triangular matrix with the property that $AA' = \Sigma_Y$ (e.g., Ashby, 1992; Graybill, 1976). Now define $\boldsymbol{W} = A^{-1}(\boldsymbol{Y} - \boldsymbol{\mu}_Y)$. Note that $\boldsymbol{W}$ has a multivariate z distribution (i.e., multivariate normal with mean $\boldsymbol{0}$ and variance–covariance matrix equal to the identity) and that $\boldsymbol{Y} = A\boldsymbol{W} + \boldsymbol{\mu}_Y$. Therefore,

$$\begin{aligned} P(\text{correct binding}) &= P(\boldsymbol{Y}'J\boldsymbol{Y} < 0) \\ &= P[(A\boldsymbol{W} + \boldsymbol{\mu}_Y)'J(A\boldsymbol{W} + \boldsymbol{\mu}_Y) < 0] \\ &= P[(A\boldsymbol{W} + AA^{-1}\boldsymbol{\mu}_Y)'\,J(A\boldsymbol{W} + AA^{-1}\boldsymbol{\mu}_Y) < 0] \\ &= P[(\boldsymbol{W} + A^{-1}\boldsymbol{\mu}_Y)'A'JA(\boldsymbol{W} + A^{-1}\boldsymbol{\mu}_Y) < 0]. \end{aligned} \tag{A6}$$

Next, let P denote the matrix whose columns are the eigenvectors of $A'JA$, and let D denote the diagonal matrix whose entries on the main diagonal are the corresponding eigenvalues of $A'JA$. Therefore, $A'JA = PDP'$. Finally, define $\boldsymbol{Z} = P'\boldsymbol{W}$. Note that $\boldsymbol{Z}$ also has a multivariate z distribution and that $\boldsymbol{W} = P\boldsymbol{Z}$. The probability of correct binding can now be written as

$$\begin{aligned} P(\text{correct binding}) &= P[(P\boldsymbol{Z} + A^{-1}\boldsymbol{\mu}_Y)'A'JA(P\boldsymbol{Z} + A^{-1}\boldsymbol{\mu}_Y) < 0] \\ &= P[(P\boldsymbol{Z} + PP'A^{-1}\boldsymbol{\mu}_Y)'A'JA(P\boldsymbol{Z} + PP'A^{-1}\boldsymbol{\mu}_Y) < 0] \\ &= P[(\boldsymbol{Z} + P'A^{-1}\boldsymbol{\mu}_Y)'P'A'JAP(\boldsymbol{Z} + P'A^{-1}\boldsymbol{\mu}_Y) < 0] \\ &= P[(\boldsymbol{Z} + P'A^{-1}\boldsymbol{\mu}_Y)'D(\boldsymbol{Z} + P'A^{-1}\boldsymbol{\mu}_Y) < 0]. \end{aligned} \tag{A7}$$

Denote the ith entry of the $\boldsymbol{Z}$ vector by z_i. Now if we perform all the matrix operations specified in Equation A7, the probability of correct binding reduces to

$$\begin{aligned} &P(\text{correct binding}) \\ &= P\left[\left(z_1 - \frac{.2113d}{\sigma}\right)^2 - \left(z_2 - \frac{.7887d}{\sigma}\right)^2 + z_3^2 - z_4^2 < 0\right] \\ &= P\left[\left(z_1 - \frac{.2113d}{\sigma}\right)^2 + z_3^2 < \left(z_2 - \frac{.7887d}{\sigma}\right)^2 + z_4^2\right] \\ &= P[\chi^{2\prime}(2,\delta_1) < \chi^{2\prime}(2,\delta_2)] \\ &\qquad = P\left[\frac{\chi^{2\prime}(2,\delta_1)/2}{\chi^{2\prime}(2,\delta_2)/2} < 1\right] = P[F''(2,2,\delta_1,\delta_2) < 1], \end{aligned} \tag{A8}$$

where $\chi^{2\prime}(2,\delta_1)$ and $\chi^{2\prime}(2,\delta_2)$ are independent random variables with noncentral chi-squared distributions with two degrees of freedom each and noncentrality parameters $\delta_1 = (.2113d/\sigma)^2$ and $\delta_2 = (.7887d/\sigma)^2$, respectively. Their ratio defines a random variable $F''(2,2,\delta_1,\delta_2)$ that has a doubly noncentral F distribution (e.g., Johnson & Kotz, 1970). Thus, the probability of correct binding is equal to the probability that a random variable with a doubly noncentral F distribution is less than 1.

Unfortunately, there is no closed form expression for the cumulative doubly noncentral F distribution function. Using a result of Imhof's (1961), Ennis and Johnson (1993) showed that

$$P[F''(2,2,\delta_1,\delta_2) < 1] = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\sin\left[\frac{(\delta_1 - \delta_2)x}{1 + 4x^2}\right]\exp\left[-\frac{2(\delta_1 + \delta_2)x^2}{1 + 4x^2}\right]}{x + 4x^3}\,dx. \tag{A9}$$

Thus, the probability of correct binding can be computed exactly as a single integral. The integral can be evaluated numerically using a method such as Simpson's rule (e.g., Press, Flannery, Teukolsky, & Vetterling, 1988).

Several approximations to the doubly noncentral F distribution function are available. The simplest of these is a normal approximation due to Patnaik (1949; see also Zinnes & Griggs, 1974). Let $r_i = 2 + \delta_i$ and let $b_i = \delta_i/(2 + \delta_i)$, for $i = 1$ and 2. Then Patnaik showed that

$$U_i = [\chi^{2\prime}(2,\delta_i)/r_i]^{1/3} \tag{A10}$$

is approximately normally distributed with mean

$$\mu_i = 1 - \frac{2}{9}\left(\frac{1 + b_i}{r_i}\right) \tag{A11}$$

and variance

$$\sigma_i^2 = \frac{2}{9}\left(\frac{1 + b_i}{r_i}\right). \tag{A12}$$

Now

$$\begin{aligned} P[F''(2,2,\delta_1,\delta_2) < 1] &= P\left[\frac{\chi^{2\prime}(2,\delta_1)}{\chi^{2\prime}(2,\delta_2)} < 1\right] \\ &= P\left\{\left[\frac{\chi^{2\prime}(2,\delta_1)/r_1}{\chi^{2\prime}(2,\delta_2)/r_2}\right]^{1/3} < \left(\frac{r_2}{r_1}\right)^{1/3}\right\} \\ &= P\left[\frac{U_1}{U_2} < \left(\frac{2 + \delta_2}{2 + \delta_1}\right)^{1/3}\right] \\ &= P(U_1 < CU_2) \\ &= P(U_1 - CU_2 < 0), \end{aligned} \tag{A13}$$

where $C = [(2 + \delta_2)/(2 + \delta_1)]^{1/3}$. From Patnaik's result, $U_1 - CU_2$ is approximately normally distributed with mean $\mu_1 - C\mu_2$ and variance $\sigma_1^2 + C^2\sigma_2^2$. Therefore,

$$P[F''(2,2,\delta_1,\delta_2) < 1] = P\left[Z < \frac{C\mu_2 - \mu_1}{\sqrt{\sigma_1^2 + C^2\sigma_2^2}}\right], \tag{A14}$$

where Z has a z distribution (i.e., normal with mean 0 and variance 1). Several excellent approximations to the cumulative z distribution are available (e.g., see Ashby, 1992).

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