Broadcasting and topology control in wireless ad hoc networks

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1 Introduction

Network wide broadcasting in Mobile Ad Hoc Networks (MANET) provides important control and route establishment functionality for a number of unicast and multicast protocols. In this chapter, we present an overview of the recent progress of broadcast and multicast in wireless ad hoc networks. We discuss two energy models that could be used for broadcast: one is non-adjustable power and one is adjustable power. If the power consumed at each node is not adjustable, minimizing the total power used by a reliable broadcast tree is equivalent to the minimum connected dominating set problem (MCDS), i.e., minimize the number of nodes that relay the message, since all relaying nodes of a reliable broadcast form a connected dominating set (CDS). If the power consumed at each node is adjustable, we assume that the power consumed by a relay node u is $\|uv\|^\beta$, where real number $\beta \in [2,5]$ depends on transmission environment and v is the farthest neighbor of u in the broadcast tree. For both models, we review several centralized methods that compute broadcast trees such that the broadcast based on them consumes the energy within a constant factor of the optimum if the original communication graph is unit disk graph. Since centralized methods are expensive to implement, we further review several localized methods that can approximate the minimum energy broadcast tree for non-adjustable power case. For adjustable power case, no localized methods can approximate the minimum energy broadcast tree within a constant factor, thus we review several currently best possible heuristics. Several local improvement methods and activity scheduling of nodes (active, idle, sleep) are also discussed in this chapter.

Wireless Ad Hoc Networks: Due to its potential applications in various situations such as battlefield, emergency relief, environment monitoring, and so on, wireless ad hoc networks [1, 2, 3, 4] have recently emerged as a premier research topic. Wireless networks consist of a set of wireless nodes which are spread over a geographical area. These nodes are able to perform processing as well as capable of communicating with each other by means of a wireless ad hoc network. With coordination among these wireless nodes, the network together will achieve a larger task both in urban environments and in inhospitable terrain. For example, the sheer numbers of wireless sensors and the expected dynamics in these environments present unique challenges in the design of wireless sensor networks. Many excellent researches have been conducted to study problems in this new field [1, 2, 5, 3, 6, 4].

In this chapter, we consider a wireless ad hoc network consisting of a set V of n wireless nodes distributed in a two-dimensional plane. Each wireless node has an omni-directional antenna. This is attractive because a single transmission of a node can be received by many nodes within its vicinity which, we assume, is a disk centered at the node. We call the radius of this disk the *transmission range* of this wireless node. In other words, node v can receive the signal from node u if node v is within the transmission range of the sender u. Otherwise, two nodes communicate through multi-hop wireless links by using intermediate nodes to relay the message. Consequently, each node in the wireless network also acts as a router, forwarding data packets for other nodes. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes

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define a *unit disk graph* UDG(V) in which there is an edge between two nodes if and only if their Euclidean distance is at most one.

In addition, we assume that each node has a low-power Global Position System (GPS) receiver, which provides the position information of the node itself. If GPS is not available, the distance between neighboring nodes can be estimated on the basis of incoming signal strengths. Relative co-ordinates of neighboring nodes can be obtained by exchanging such information between neighbors [7]. With the position information, we can apply computational geometry techniques to solve some challenging questions in wireless networks.

Power-Attenuation Model: Energy conservation is a critical issue in wireless network for the node and network life, as the nodes are powered by batteries only. Each wireless node typically has a portable set with transmission and reception processing capabilities. To transmit a signal from a node to the other node, the power consumed by these two nodes consists of the following three parts. First, the source node needs to consume some power to prepare the signal. Second, in the most common power-attenuation model, the power needed to support a link uv is $\|uv\|^{\beta}$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 dependent on the transmission environment. This power consumption is typically called *path loss*. Finally, when a node receives the signal, it needs consume some power to receive, store and then process that signal. For simplicity, this overhead cost can be integrated into one cost, which is almost the same for all nodes. Thus, we will use c to denote such constant overhead. In most results surveyed here, it is assumed that $c = 0$, i.e., the path loss is the major part of power consumption to transmit signals. The power cost $p(e)$ of a link $e = uv$ is then defined as the power consumed for transmitting signal from u to node v, i.e., $p(uv) = \|uv\|^{\beta}$.

Broadcasting and Multicasting: Broadcasting is a communication paradigm that allows to send data packets from a source to multiple receivers. In one-to-all model, transmission by each node can reach *all* nodes that are within radius distance from it, while in the one-to-one model, each transmission is directed toward only one neighbor (using, for instance, directional antennas or separate frequencies for each node). The broadcasting in literature has been studied mainly for one-to-all model and we will use that model in this chapter. Broadcasting is also frequently referred to as *flooding*.

Broadcasting and multicasting in wireless ad hoc networks are critical mechanisms in various applications such as information diffusion, wireless networks, and also for maintaining consistent global network information. Broadcasting is often necessary in MANET routing protocols. For example, many unicast routing protocols such as Dynamic Source Routing (DSR), Ad Hoc On Demand Distance Vector (AODV), Zone Routing Protocol (ZRP), and Location Aided Routing (LAR) use broadcasting or a derivation of it to establish routes. Currently, these protocols all rely on a simplistic form of broadcasting called *flooding*, in which each node (or all nodes in a localized area) retransmits each received unique packet exactly one time. The main problems with Flooding are that it typically causes unproductive and often harmful bandwidth congestion, as well as inefficient use of node resources. Broadcasting is also more efficient than sending multiple copies the same packet through unicast. It is highly important to use power-efficient broadcast algorithms for such networks since wireless devises are often powered by batteries only.

Recently, a number of research groups have proposed more efficient broadcasting techniques [8, 9, 10, 11, 12, 13, 14] with various goals such as minimizing the number of retransmissions, minimizing the total power used by all transmitting nodes, minimizing the overall delay of the broadcasting, and so on. Williams and Camp [13] classified the broadcast protocols into four categories: simple (blind) flooding, probability based, area based, and neighbor knowledge methods. Wu and Lou [15] classified broadcasting protocols based on neighbor knowledge information: global, quasi-global, quasi-local, and local. The global broadcast protocol, centralized or distributed, is based on global state information. In quasi-global broadcasting, a broadcast protocol is based on partial global state information. For example, the approximation algorithm in [16] is based on building a global spanning tree (a form of partial global state information) that is constructed in a sequence of sequential propagations. In quasi-local broadcasting, a distributed broadcast protocol is based on mainly local state information and occasionally partial global state information. Cluster networks are such examples: while clusters can be constructed locally for most of the time, the chain reaction does occur occasionally. In local broadcasting, a distributed broadcast protocol is based on solely local state information. All protocols that select forward nodes locally (based on 1-hop or 2-hop neighbor set) belong to this category. It has been recognized that scalability in wireless networks cannot be achieved by relying on solutions where each node requires global knowledge about the network. To achieve scalability, the concept of localized algorithms was proposed, as distributed algorithms where simple local node behavior, based on local knowledge, achieves a desired global objective.

In this chapter, we categorize previously proposed broadcasting protocols into several families: centralized methods, distributed methods, localized methods. Centralized methods calculate a tree used for broadcasting with various optimization objectives of the tree. In localized methods, each node has to maintain the the state of its local neighbors (within some constant hops). After receiving a packets that needed to be relayed, the node decides whether to relay the packet only based on its local neighborhood information. Majority of the protocols are in this family. In distributed methods, a node may need some information more than a constant hop away to decide whether to relay the message. For example, broadcasting based on MST constructed in a distributed manner is a distributed method, but not localized method since we cannot construct MST in a localized manner.

Distributed or Localized Algorithms? Distributed algorithms and architectures have been commonly used terms for a long time in computer science. Unfortunately none of the already proposed approaches are applicable to wireless ad-hoc networks. In order to address the needs of distributed computing in wireless ad hoc networks, one has to address how key goals, such as power minimization, low latency, security and privacy, are affected by the algorithms used. Some common denominators are almost always present, such as high relative cost of communication to computation in wireless networks.

Due to the limited capability of processing power, storage, and energy supply, many conventional algorithms are too complicated to be implemented in wireless ad hoc networks. Thus, the wireless ad hoc networks require efficient distributed algorithms with low computation complexity and low communication complexity. More importantly, we expect the distributed algorithms for wireless ad hoc networks to be localized: each node running the algorithm only uses the information of nodes within a constant number of hops. However, localized algorithms are difficult to design or impossible sometimes. For example, we cannot construct the minimum spanning tree (MST) locally. Recently, several challenging questions [17, 18, 19, 20, 21, 28, 29, 25] have been solved by using efficient localized algorithms for energy efficient topology control and broadcast and multicast in wireless ad hoc networks. In all these algorithms, they proved that the total communication cost of all nodes constructing the structure together is $O(n \log n)$ bits.

MAC Specification: Collision avoidance is inherently difficult in MANETs; one often cited difficulty is overcoming the hidden node problem, where a node cannot decide whether some of its neighbors are busy receiving transmissions from an uncommon neighbor. The 802.11 MAC follows a Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) scheme. For unicast, it utilizes a Request To Send (RTS) / Clear To Send (CTS) / Data / Acknowledgment (ACK) procedure to account for the hidden node problem. However, the RTS/CTS/Data/ACK procedure is too cumbersome to implement for broadcast packets as it would be difficult to coordinate and bandwidth expensive: a relay node has to perform RTS/CTS individually with all its neighbors that should receive the packets. Thus, the only requirement made for broadcasting nodes is that they assess a clear channel before broadcasting. Unfortunately, clear channel assessment does not prevent collisions from hidden nodes. Additionally, no resource is provided for collision when two neighbors assess a clear channel and transmit simultaneously. Ramifications of this environment are subtle but significant. Unless specific means are implemented at the network layer, a node has no way of knowing whether a packet was successfully reached by its neighbors. In congested networks, a significant amount of collisions occur leading to many dropped packets. The most effective broadcasting protocols try to limit the probability of collisions by limiting the number of rebroadcasts in the network. Thus, it is often imperative the underlying structure for broadcasting is degree bounded and the links are at similar lengths. By using a power adjustment at each node, the collision of packets and contention for channel will be alleviated. Notice that, if the underlying structure for broadcasting is degree bounded, we can either use RTS/CTS scheme to avoid hidden node problem, or we can rebroadcast the dropped packets (such rebroadcast will be less since the number of intended receiving neighbors is bounded by a small constant).

Reliability: Reliability is the ability of a broadcast protocol to reach all the nodes in the network. It can be considered at the network or at the medium access layer. We will classify protocols according to their network layer performance. That is, assuming that MAC layer is ideal (every message sent by a node reaches all its neighbors), location update protocol provides accurate desired information to all nodes about their neighborhood, and the network is connected. Broadcast protocols can be *reliable* or *unreliable*. In a reliable protocol, every node in the network is reached, while in unreliable broadcast protocols, some nodes may not receive the message at all.

Message Contents: The broadcast schemes may require different neighborhood information, which is reflected in the contents of messages sent by nodes when they move, react to topological changes, change activity status, or simply send periodically update messages. For example, commonly seen *hello* message may contain (all or a subset of) the following information: its own ID, its position, one bit for dominating set status (informing neighbors whether or not the node itself is in dominating set), list of 1-hop neighbors, its degree. Other content is also possible, such as list of 1-hop neighbors with their positions, or list of 2-hop neighbors, or even global network information.

The broadcast message sent by the source, or retransmitted, may contain broadcast message only. In addition, it may contain a various of information needed for proper functioning of broadcast protocol, such as the same type of information already listed for *hello* messages, some constant bits of the system requirements (such as the maximum broadcast delay), or list of forwarding neighbors of current relaying node, informing them whether or not to retransmit the message.

Jitter and RAD: Suppose a source node originates a broadcast packet. Given that radio waves propagate at the speed of light, all neighbors will receive the transmission almost simultaneously. Assuming similar hardware and system loads, the neighbors will process the packet and rebroadcast at the same time. To overcome this problem, broadcast protocols jitter the scheduling of broadcast packets from the network layer to the MAC layer by some uniform random amount of time. This (small) offset allows one neighbor to obtain the channel first, while other neighbors detect that the channel is busy (clear channel assessment fails) and thus delay their transmissions to avoid collision. Since the node has to backup all received broadcast packets within RAD, the RAD cannot be too large also. On the other hand, if RAD is small, this node may repeatedly broadcast a same packet, and thus causing the infinity loop of rebroadcast.

Many of the broadcasting protocols require a node to keep track of redundant packets received over a short time interval in order to determine whether to rebroadcast. That time interval, which were termed *Random Assessment Delay* (RAD) [13], is randomly chosen from a uniform distribution between 0 and *Tmax* seconds, where Tmax is the highest possible delay interval. This delay in transmission accomplishes two things. First it allows nodes sufficient time to receive redundant packets and assess whether to rebroadcast. Second, the randomized scheduling prevents the collisions of transmission.

Performance Measurement: The performance of broadcast protocols can be measured by variety of metrics. A commonly used metric is the number of message retransmissions with respect to the number of nodes. In case of broadcasting with adjusted transmission power (thus adjusted disk that the message can reach), the total power can be used as performance metrics. The next important metric is reachability, or the ratio of nodes connected to the source that received the broadcast message. Time delay or latency is sometimes used, which is the time needed for the last node to receive broadcast message initiated at the source. Note that retransmissions at MAC layer are normally deferred, to avoid message collisions. Some authors consider alternative more restricted indicator, whether or not the path from source to any node is always following a shortest path. This measure may be important if used as part of routing scheme, since route paths are created during the broadcast process.

Brief Literature Review: In the minimum energy broadcasting problem, each node can adjust its transmission power in order to minimize total energy consumption but still enable a message originated from a source node to reach all the other nodes in an ad-hoc wireless network. The problem is known to be NP-complete [93]. There exists a number of approximate solutions in literature where each node requires global network information (including distances between any two neighboring nodes in the network) in order to decide its own transmission radius. Three greedy heuristics were proposed in [33] for the minimum-energy broadcast routing problem: MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). It was shown that the total energy consumed by MST or BIP methods are no more than 12 times larger than the optimum

[39]. Cartigny, Simplot and Stojmenovic [77] described a localized protocol where each node requires only the knowledge of its distance to all neighboring nodes and distances between its neighboring nodes (or, alternatively, geographic position of itself and its neighboring nodes). In addition to using only local information, the protocol is shown experimentally to be even competitive with the best-known globalized BIP solution [33], which is a variation of Dijkstra's shortest path algorithm. The solution [77] is based on the use of RNG that preserves the network connectivity and is defined in a localized manner. The transmission range for each node is equal to the distance to its furthest RNG neighbor, excluding the neighbor from which the message came from. Localized energy efficient broadcast for wireless networks with directional antennas are described in [95], and are also based on RNG. Messages are sent only along RNG edges, requiring about 50% more energy than BIP based [33] globalized solution. However, when the communication overhead for maintenance is added, localized solution becomes superior. Localized minimum spanning tree can replace RNG to improve energy efficiency, as proposed in [76, 96]. Their simulations show the performance is comparable to that of BIP. Li *et al.* [21, 22, 26, 27] recently proposed several methods with further improvements. They described several low weight planar structures (IMRG and LMST_k) that can be constructed by localized methods with total communication costs $O(n)$ and the simulations showed a significant improvement of energy consumption compared with [95, 76]. Although the structures IMRG and LMST_k ($k \geq 2$) have total edge length within a constant factor of the MST, the broadcasting based on these locally constructed structures could still consume energy arbitrarily large than the optimum, when we assume that the power needed to support a link uv is $\|uv\|^\beta$. It has been proved that the broadcasting based on MST consumes energy within a constant factor of the optimum, but MST cannot be constructed locally. They also showed that there is no structure that can be constructed locally and the broadcasting based on it consumes energy within a constant factor of the optimum.

Organization: The rest of the chapter is organized as follows. In Section 2, we discuss in detail several centralized broadcasting methods. Note that both minimizing the number of retransmissions and minimizing the total power used by transmitting nodes are NP-complete problems even for unit disk graphs. We thus discuss in detail several methods that can achieve constant approximation ratio in polynomial time. Since centralized methods are expensive to implement for wireless ad hoc networks, in Section 3, we then review several protocols that use only localized information. In Section 4, we discuss how to judiciously assign operation model to each node thus saving overall energy consumptions in the network. In Section 5, we conclude this chapter with discussion of some possible future works.

2 Centralized Methods

We assume that two energy models could be used for broadcast: one is non-adjustable power and one is adjustable power. If the power consumed at each node is not adjustable, minimizing the total power used by a reliable broadcast tree is equivalent to the minimum connected dominating set problem (MCDS), i.e., minimize the number of nodes that relay the message, since all relaying nodes of a reliable broadcast form a connected dominating set (CDS). If the power consumed at each node is adjustable, we assume that the power consumed by a relay node u is $\|uv\|^\beta$, where real number $\beta \in [2, 5]$ depends on transmission environment and v is the farthest neighbor of u in the broadcast tree. In the rest of the section, for these two energy models respectively, we reviewed several centralized methods that can build some broadcast tree whose energy consumption is within a constant factor of the optimum if the original communication graph is modelled by unit disk graph.

2.1 Assumptions

We first study the adjustable power model. Minimum-energy broadcast/multicast routing in a simple ad hoc networking environment has been addressed by the pioneering work in [30, 31, 32, 33]. To assess the complexities *one at a time*, the nodes in the network are assumed to be randomly distributed in a two-dimensional plane and there is no mobility. Nevertheless, as argued in [33], the impact of mobility can be incorporated into this static model because the transmitting power can be adjusted to accommodate the new locations of the nodes as necessary. In other words, the capability to adjust the transmission power provides considerable "elasticity" to the topological connectivity, and hence may reduce the need for hand-offs and tracking. In addition, as assumed in [33], there are sufficient bandwidth and transceiver resources. Under these assumptions, centralized (as opposed to distributed) algorithms were presented by [33, 34, 35, 36] for minimum-energy broadcast/multicast routing. These centralized algorithms, in this simple networking environment, are expected to serve as the basis for further studies on distributed algorithms in a more practical network environment, with limited bandwidth and transceiver resources, as well as the node mobility.

2.2 Based on MST and Variations

Some centralized methods are based on optimization. The scheme proposed in [37] is built upon an alternate search based paradigm in which the minimum-cost broadcast/multicast tree is constructed by a search process. Two procedures are devised to check the viability of a solution in the search space. Preliminary experimental results show that this method renders better solutions than BIP, though at a higher computational cost. Liang [34] showed that the minimum-energy broadcast tree problem is NP-complete, and proposed an approximate algorithm to provide a bounded performance guarantee for the problem in the general setting. Essentially they reduce the minimum-energy broadcast tree problem to an optimization problem on an auxiliary weighted graph and solve the optimization problem so as to give an approximate solution for the original problem. They also proposed another algorithm that yields better performance under a special case. Das *et al.* [35] proposed an evolutionary approach using genetic algorithms. The same authors also presented in [38] three different integer programming models which can be used to find the solutions to the minimum-energy broadcast/ multicast problem. The major drawback of optimization based schemes are, however, that they are centralized and require the availability of global topological information.

Some centralized methods are based on greedy heuristics. Three greedy heuristics were proposed in [33] for the minimum-energy broadcast routing problem: MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). The MST heuristic first applies the Prim's algorithm to obtain a MST, and then orient it as an arborescence rooted at the source node. The SPT heuristic applies the Dijkstra's algorithm to obtain a SPT rooted at the source node. The BIP heuristic is the node version of Dijkstra's algorithm for SPT. It maintains, throughout its execution, a single arborescence rooted at the source node. The arborescence starts from the source node, and new nodes are added to the arborescence one at a time on the minimum incremental cost basis until all nodes are included in the arborescence. The incremental cost of adding a new node to the arborescence is the minimum additional power increased by some node in the current arborescence to reach this new node. The implementation of BIP is based on the standard Dijkstra's algorithm, with one fundamental difference on the operation whenever a new node q is added. Whereas the Dijkstra's algorithm updates the node weights (representing the current knowing distances to the source node), BIP updates the cost of each link (representing the incremental power to reach the head node of the directed link). This update is performed by subtracting the cost of the added link pq from the cost of every link qr that starts from q to a node r not in the new arborescence. They have been evaluated through simulations in [33], but little is known about their analytical performances in terms of the approximation ratio. Here, the approximation ratio of a heuristic is the maximum ratio of the energy needed to broadcast a message based on the arborescence generated by this heuristic to the least necessary energy by any arborescence for any set of points.

For a pure illustration purpose, another slight variation of BIP was discussed in detail in [39]. This greedy heuristic is similar to the Chvatal's algorithm [40] for the set cover problem and is a variation of BIP. Like BIP, an arborescence, which starts with the source node, is maintained throughout the execution of the algorithm. However, unlike BIP, many new nodes can be added one at a time. Similar to the Chvatal's algorithm [40], the new nodes added are chosen to have the minimal *average* incremental cost, which is defined as the ratio of the minimum additional power increased by some node in the current arborescence to reach these new nodes to the number of these new nodes. They called this heuristic as the Broadcast Average Incremental Power (BAIP). In contrast to the $1 + \log m$ approximation ratio of the Chvatal's algorithm [40], where m is the largest set size in the Set Cover Problem, they showed that the approximation ratio of BAIP is at least $\frac{4n}{\ln n} - o(1)$, where n is the number of receiving nodes.

Wan *et al.* [39] showed that the approximation ratios of MST and BIP are between 6 and 12 and between $\frac{13}{3}$ and 12 respectively; on the other hand, the approximation ratios of SPT and BAIP are at least $\frac{n}{2}$ and $\frac{4n}{\ln n} - o(1)$ respectively, where n is the number of nodes. We then discuss in detail of their proof techniques in next subsection.

The Iterative Maximum-Branch Minimization (IMBM) algorithm was another effort [36] to construct power-efficient broadcast trees. It begins with a basic broadcast tree in which the source directly transmits to all other nodes. Then it attempts to approximate the minimum-energy broadcast tree by iteratively replacing the maximum branch with less-power, more-hop alternatives.

Both BIP and IMBM operate under the assumption that the transmission power of each node is unconstrained, i.e., every node can reach every other node. Both algorithms are centralized in the sense that they require: (a) the source node needs to know the position/distance of every other node; and (b) each node needs to know its downstream, on-tree neighbors so as to propagate broadcast messages. As a result, it may be difficult to extend both algorithms into distributed versions, as a significant amount of information is required to be exchanged among nodes.

2.3 Theoretical Analysis

Any broadcast routing is viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, that spans all nodes. Let $f_T(\mathbf{p})$ denote the transmission power of the node $\mathbf{p}$ required by T. For any leaf node $\mathbf{p}$ of T, $f_T(\mathbf{p}) = 0$. For any internal node $\mathbf{p}$ of T,

$$f_T(\mathbf{p}) = \max_{\mathbf{pq} \in T} \|\mathbf{pq}\|^{\beta},$$

in other words, the β-th power of the longest distance between $\mathbf{p}$ and its children in T. The total energy required by T is $\sum_{\mathbf{p} \in P} f_T(\mathbf{p})$. Thus the minimum-energy broadcast routing problem is different from the conventional link-based minimum spanning tree (MST) problem. Indeed, while the MST can be solved in polynomial time by algorithms such as Prim's algorithm and Kruskal's algorithm [41], the minimum-energy broadcast routing problem cannot be solved in polynomial time unless P=NP [30]. In its general graph version, the minimum-energy broadcast routing can be shown to be NP-hard [42], and even worse, it can not be approximated within a factor of $(1-\epsilon)\log\Delta$, unless $NP \subseteq DTIME\left[n^{O(\log\log n)}\right]$, where Δ is the maximal degree and ϵ is any arbitrary small positive constant. However, this hardness of its general graph version does not necessarily imply the same hardness of its geometric version. In fact, as shown later in the chapter, its geometric version can be approximated within a constant factor. Nevertheless, this suggests that the minimum-energy broadcast routing problem is considerably harder than the MST problem. Recently, Clementi *et al.* [30] proved that the minimum-energy broadcast routing problem is a NP-hard problem and obtained a parallel but weaker result to those of [39].

Wan *et al.* [39] gave some lower bounds on the approximation ratios of MST and BIP by studying some special instances in [39]. Their deriving of the upper bounds relies extensively on the geometric structures of Euclidean MSTs. A key result in [39] is an upper bound on the parameter $\sum_{e \in mst(P)} \|e\|^2$ for any finite point set P of radius one. Note that the supreme of the total edge lengths of $mst(P)$, $\sum_{e \in mst(P)} \|e\|$, over all point sets P of radius one is infinity. However, the parameter $\sum_{e \in mst(P)} \|e\|^2$ is bounded from above by a constant for any point set P of radius one. They use c to denote the supreme of $\sum_{e \in mst(P)} \|e\|^2$ over all point sets P of radius one. The constant c is at most 12; see [39]. The proof of this theorem involves complicated geometric arguments; see [39] for more detail. Note that for any point set P of radius one, the length of each edge in $mst(P)$ is at most one. Therefore, for any point set P of radius one and any real number $\beta \geq 2$,

$$\sum_{e \in mst(P)} \|e\|^{\beta} \leq \sum_{e \in mst(P)} \|e\|^2 \leq c \leq 12.$$

The next theorem proved in [39] explores a relation between the minimum energy required by a broadcasting and the energy required by the Euclidean MST of the corresponding point set.

Lemma 1 *[39] For any point set P in the plane, the total energy required by any broadcasting among P is at least $\frac{1}{c}\sum_{e \in mst(P)} \|e\|^{\beta}$.*

Proof. Let T be an arborescence for a broadcasting among P with the minimum energy consumption. For any none-leaf node $\mathbf{p}$ in T, let $T_{\mathbf{p}}$ be an Euclidean MST of the point set consisting $\mathbf{p}$ and all children of $\mathbf{p}$ in T.

Suppose that the longest Euclidean distance between $\mathbf{p}$ and its children is r. Then the transmission power of node $\mathbf{p}$ is r^β, and all children of $\mathbf{p}$ lie in the disk centered at $\mathbf{p}$ with radius r. From the definition of c, we have

$$\sum_{e \in T_\mathbf{p}} \left(\frac{\|e\|}{r} \right)^\beta \leq c,$$

which implies that

$$r^\beta \geq \frac{1}{c} \sum_{e \in T_\mathbf{p}} \|e\|^\beta .$$

Let T^* denote the spanning tree obtained by superposing of all $T_\mathbf{p}$'s for non-leaf nodes of T. Then the total energy required by T is at least $\frac{1}{c} \sum_{e \in T^*} \|e\|^\beta$, which is further no less than $\frac{1}{c} \sum_{e \in mst(P)} \|e\|^\beta$. This completes the proof. □

Consider any point set P in a two-dimensional plane. Let T be an arborescence oriented from some $mst\,(P)$. Then the total energy required by T is at most $\sum_{e \in T_\mathbf{p}} \|e\|^\beta$. From Lemma 1, this total energy is at most c times the optimum cost. Thus the approximation ratio of the link-based MST heuristic is at most c. Together with $c \leq 12$, this observation leads to the following theorem.

Theorem 2 *[39] The approximation ratio of the link-based MST heuristic is at most c, and therefore is at most 12.*

In addition, they derived an upper bound on the approximation ratio of the BIP heuristic. Once again, the Euclidean MST plays an important role.

Lemma 3 *[39] For any broadcasting among a point set P in a two-dimensional plane, the total energy required by the arborescence generated by the BIP algorithm is at most $\sum_{e \in mst(P)} \|e\|^\beta$.*

2.4 Centralized Clustering

We then study the non-adjustable power model case. The set of nodes that rebroadcast message in a reliable broadcasting scheme define a connected dominating set. A subset S of V is a *dominating set* if each node u in V is either in S or is adjacent to some node v in S. Nodes from S are called dominators, while nodes not is S are called dominatees. A subset C of V is a *connected dominating set* (CDS) if C is a dominating set and C induces a connected subgraph. Consequently, the nodes in C can communicate with each other without using nodes in $V - C$. A dominating set with minimum cardinality is called *minimum dominating set*, denoted by MDS. A connected dominating set with minimum cardinality is denoted by *minimum connected dominating set* (MCDS). A broadcasting based on connected dominating set only uses the nodes in CDS to relay the message. We first review several methods in the literature to build a connected dominating set.

If every nodes cannot adjust its transmission power accordingly, then we need find the minimum connected dominating set to save the total power consumption of the broadcasting protocol. Unfortunately, the problem of finding connected dominating set of minimal size is NP-complete even for unit disk graphs. Guha and Khuller [43] studied the approximation of the connected dominating set problem for general graphs. They gave two different approaches, both of them guarantee approximation ratio of $\Theta(H(\Delta))$. As their approaches are for general graphs and thus do not utilize the geometry structure if applied to the wireless ad hoc networks. One approach is to grow a spanning tree that includes all nodes. The internal nodes of the spanning tree is selected as the final connected dominating set. This approach has approximation ratio $2(H(\Delta)+1)$. The other approach is first approximating the dominating set and then connecting the dominating set to a connected dominating set. They [43] proved that this approach has approximation ratio $\ln \Delta + 3$.

One can also use the Steiner tree algorithm to connect the dominators. This straightforward method gives approximation ratio $c(H(\Delta)+1)$, where c is the approximation ratio for the unweighted Steiner tree problem. Currently, the best ratio is $1 + \frac{\ln 3}{2} \simeq 1.55$, due to Robins and Zelikovsky [44].

By definition, any algorithm generating a maximal independent set is a clustering method. We first review the methods that approximates the maximum independent set, the minimum dominating set, and the minimum connected dominating set. Hunt *et al.* [45] and Marathe *et al.* [46] studied the approximation of the maximum independent set and the minimum dominating set for unit disk graphs. They gave the first PTASs for MDS in UDG. The method is based on the following observations: a maximal independent set is always a dominating set; given a square Ω with a fixed area, the size of any maximal independent set is bounded by a constant C. Assume that there are n nodes in Ω. Then, we can enumerate all sets with size at most C in time $\Theta(n^C)$. Among these enumerated sets, the smallest dominating set is the minimum dominating set. Then, using the shifting strategy proposed by Hochbaum [47], they derived a PTAS for the minimum dominating set problem.

Since we have PTAS for minimum dominating set and the graph $VirtG$ connecting every pair of dominators within at most 3 hops is connected [48], we have an approximation algorithm (constructing a minimum spanning tree $VirtG$) for MCDS with approximation ratio $3+\epsilon$. Notice that, Berman *et al.* [49] gave an $\frac{4}{3}$ approximation method to connect a dominating set and Robins *et al.* [44] gave an $\frac{4}{3}$ approximation method to connect an independent set. Thus, we can easily have an $\frac{8}{3}$ approximation algorithm for MCDS, which was reported in [50]. Recently, Cheng *et al.* [51] designed a PTAS for MCDS in UDG. However, it is difficult to distributize their method efficiently.

3 Localized Methods

3.1 Based on Distributed CDS

A natural structure for broadcasting is *connected dominating set.* Many distributed clustering (or dominating set) algorithms have been proposed in the literature [52, 53, 54, 55, 56, 57]. All algorithms assume that the nodes have distinctive identities (denoted by ID hereafter).

In the rest of section, we will interchange the terms cluster-head and dominator. The node that is not a cluster-head is also called *dominatee.* A node is called *white* node if its status is yet to be decided by the clustering algorithm. Initially, all nodes are white. The status of a node, after the clustering method finishes, could be *dominator* with color *black* or *dominatee* with color *gray.* The rest of this section is devoted for the distributed methods that approximates the minimum dominating set and the minimum connected dominating set for unit disk graph.

3.1.1 Clustering without Geometry Property

For general graphs, Jia *et al.* [58] described and analyzed some randomized distributed algorithms for the minimum dominating set problem that run in polylogarithmic time, independent of the diameter of the network, and that return a dominating set of size within a logarithmic factor from the optimum with high probability. Their best algorithm runs in $O(\log n \log \Delta)$ rounds with high probability, and every pair of neighbors exchange a constant number of messages in each round. The computed dominating set is within $O(\log \Delta)$ in expectation and within $O(\log n)$ with high probability. Their algorithm works for weighted dominating set also.

The method proposed by Das *et al.* [59, 60] contains three stages: approximating the minimum dominating set, constructing a spanning forest of stars, expanding the spanning forest to a spanning tree. Here the *stars* are formed by connecting each dominatee node to one of its dominators. The approximation method of MDS is essentially a distributed variation of the the centralized Chvatal's greedy algorithm [40] for set cover. Notice that the dominating set problem is essentially the set cover problem which is well-studied. It is then not surprise that the method by Das *et al.* [59, 60] guarantees a $H(\Delta)$ for the MDS problem, where H is the harmonic function and Δ is the maximum node degree.

While the algorithm proposed by Das *et al.* [59, 60] finds a dominating set and then grows it to a connecting dominating set, the algorithm proposed by Wu and Li [61, 62] takes an opposite approach. They first find a connecting dominating set and then prune out certain redundant nodes from the CDS. The initial CDS $\mathbb{C}$ contains all nodes that have at least two non-adjacent neighbors. A node u is said to be *locally redundant* if it has either a neighbor in $\mathbb{C}$ with larger ID which dominates all other neighbors of u, or two adjacent neighbors with larger ID which together dominates all other neighbors of u. Their algorithm then keeps removing all locally redundant nodes from $\mathbb{C}$. They showed that this algorithm works well in practice when the nodes are distributed uniformly and randomly, although no any theoretical analysis is given by them both for the worst case and for the average approximation ratio. However, it was shown by Alzoubi *et al.* [52] that the approximation ratio of this algorithm could be as large as $\frac{n}{2}$.

Recently, Dai and Wu have proposed a distributed dominant pruning algorithm [63]. Each node has a priority which can be simply its unique identifier or a combination of remaining battery, degree or identifier. A node u is "fully covered" by a subset S of its neighboring nodes if and only if the following three conditions hold:

- the subset S is connected,
- any neighbor of u is neighbor of at least one node from S,
- all nodes in S have higher priority than u.

A node belongs to the dominating set if and only if there is no subset that fully covers it. The advantage of using CDS as defined in [61, 62, 63] is that each node can decide whether or not it is in dominating set without any additional communication steps involved, other than those needed to maintain neighborhood information. The neighborhood information needed is either 2-hop neighbors knowledge, or 1-hop neighbor knowledge with their position.

Stojmenovic *et al.* [64] observed that distributed constructions of connecting dominating set can be obtained following the clustering scheme of Lin and Gerla [53]. Connecting dominating set consists of two types of nodes: clusterhead and border-nodes (also called gateway or connectors elsewhere). The clusterhead nodes are decided as follows. At each step, all white nodes which have the lowest *rank* among all white neighbors are colored black, and the white neighbors are colored gray. The ranks of the white nodes are updated if necessary. The clustering method uses two messages which can be called IamDominator and IamDominatee. A white node claims itself to be a dominator if it has the smallest ID among all of its white neighbors, if there is any, and broadcasts IamDominator to its 1-hop neighbors. A white node receiving IamDominator message marks itself as dominatee and broadcasts IamDominatee to its 1-hop neighbors. The set of dominators generated by the above method is actually a maximal independent set. Here, we assume that each node knows the IDs of all its 1-hop neighbors, which can be achieved if each node broadcasts its ID to its neighbors initially. This approach of constructing MIS is well-known. The following rankings of a node are used in various methods: the ID only [54, 53], the ordered pair of degree and ID [65], and an ordered pair of degree and location [64]. In [66, 67], Basagni *et al.*, used a general *weight* as a ranking criterion for selecting the node as the clusterhead, where the weight is a combination of mentioned criteria and some new ones, such as mobility or remaining energy. After the clusterhead nodes are selected, border-nodes are selected to connect them. A node is a border-node if it is not a clusterhead and there are at least two clusterheads within its 2-hop neighborhood. It was shown by [52] that the worst case approximation ratio of this method is also $\frac{n}{2}$, although it works well in practice.

3.1.2 Clustering with Geometry Property

Notice that none of the above algorithm utilizes the geometry property of the underlying unit disk graph. Recently, several algorithms were proposed with a constant worst case approximation ratio by taking advantage of the geometry properties of the underlying graph. It is used to connect the clusterheads constructed as described above into a CDS with fewer additional nodes. During this second step of backbone formation, some *connectors* (also called *gateways*) are found among all the dominatees to connect the dominators. Then the connectors and the dominators form a *connected dominating set*. Recently, Wan, *et al.* [68] and Wu and Lou [15] proposed a communication efficient algorithm to find connectors based on the fact that there are only a constant number of dominators within k-hops of any node. The following observation is a basis of several algorithms for CDS. After clustering, one dominator node can be connected to many dominatees. However, it is well-known that a dominatee node can only be connected to at most *five* dominators in the unit disk graph model. Generally, it was shown in [68, 48, 15] that for each node v (dominator or dominatee), the number of dominators inside the disk centered at v with radius k-units is bounded by a constant $\ell_k < (2k+1)^2$.

Given a dominating set S, let $VirtG$ be the graph connecting all pairs of dominators u and v if there is a path in UDG connecting them with at most 3 hops. Graph $VirtG$ is connected. It is natural to form a connected dominating set by finding connectors to connect any pair of dominators u and v if they are connected in $VirtG$. This strategy is also adopted by Wan, *et al.* [68] and Wu and Lou [15]. Notice that, in the approach by Stojmenovic *et al.* [64], they set any dominatee node as the connector if there are two dominators within its 2-hop neighborhood. This approach is very pessimistic and results in very large number of connectors in the worst case [52]. Instead, Wan *et al.* [16] suggested to find only one unique shortest path to connect any two dominators that are at most three hops away.

We briefly review their basic idea of forming a CDS in a distributed manner. Let $\Pi_{UDG}(u,v)$ be the path connecting two nodes u and v in UDG with the smallest number of hops. Let's first consider how to connect two dominators within 3 hops. If the path $\Pi_{UDG}(u,v)$ has two hops, then u finds the dominatee with the smallest ID to connect u and v. If the path $\Pi_{UDG}(u,v)$ has three hops, then u finds the node, say w, with the smallest ID such that w and v are two hops apart. Then node w selects the node with the smallest ID to connect w and v. Wang and Li [48] and Alzoubi *et al.* [68] discussed in detail some approaches to optimize the communication cost and the memory cost.

The graph constructed by this algorithm is called a CDS graph (or *backbone* of the network). If we also add all edges that connect all dominatees to their dominators, the graph is called extended CDS, denoted by CDS'. Let *opt* be the size of the minimum connected dominating set. It was shown [46] that the size of the computed maximal independent set has size at most $4 * opt + 1$. We already showed that the size of the connected dominating set found by the above algorithm is at most $\ell_3 k + k$, where k is the size of the maximal independent set found by the clustering algorithm. It implies that the found connected dominating set has size at most $4(\ell_3 + 1) * opt + \ell_3 + 1$. Consequently, the computed connected dominating set is at most $4(\ell_3 + 1)$ factor of the optimum (with an additional constant $\ell_3 + 1$). It was shown in [16, 48] that the CDS' graph is a sparse spanner in terms of both hops and length, meanwhile CDS has a bounded node degree.

3.1.3 Distributed Weighted CDS

In the previous subsection, we assumed that the power needed by every node is the same. Thus, to find the minimum power structure for broadcast is equivalent to find the minimum connected dominating set. In this subsection, we discussed the scenario when different nodes may have different power to send messages to their neighbors. Then, finding the minimum power structure for broadcast is equivalent to find the minimum cost weighted connected dominating set. It is well-known that, for a general weighted network modelled by an arbitrary graph, we can find a weighted connected dominating set whose cost is no more than $O(\log n)$ times the optimum in a centralized manner. It is unknown how to achieve such in an efficient distributed manner and whether we can achieve better approximation ratio for networks with some special properties such as unit disk graphs, or the networks whose nodes weights are smooth.

Wang *et al.* [69] recently presented the first distributed methods to address this problem and proved that their method has a better worst-case performances than all previous distributed methods. Before we review their methods here, we first give some necessary definitions that will be used in our presentation later.

Let $d_G(u)$ be the degree of node u in a graph G and d be the maximum degree of all wireless nodes. The average node degree is called *density* of the network. We assume that each wireless node u has a cost $c(u)$ of being in the backbone. Here the cost $c(u)$ could be the value computed based on a combination of its remaining battery power, its mobility, its node degree in the communication graph, and so on. We assume that smaller $c(u)$ means that the node is more suitable of being in the backbone. Let $\delta = \max_{i,j \in E} c(i)/c(j)$, where E is the set of communication links in the wireless network G. We call δ the *cost smoothness* of the wireless networks. When δ is bounded by some small constant, we say the node costs are *smooth*.

We call all nodes within a constant k hops of a node u in the communication graph G as the *k-local nodes* or *k-hop neighbors* of u, denoted by $N_k(u)$, which includes u itself. The k-local graph of a node u, denoted by $G_k(u)$, is the induced graph of G on $N_k(u)$, i.e., $G_k(u)$ is defined on vertex set $N_k(u)$, and contains all edges in G with both end-points in $N_k(u)$. The independence number, denoted as $\alpha(G)$, of a graph G is the size of the maximum independent set of G. The *k-local independence number*, denoted by $\alpha^{[k]}(G)$, is defined as $\alpha^{[k]}(G) = \max_{u \in V} \alpha(G_k(u))$. It is well-known that for a unit disk graph, $\alpha^{[1]}(G) \leq 5$ and $\alpha^{[2]}(G) \leq 18$. A subset C of V is a *minimum weighted connected dominating set* (MWCDS) if C is a connected dominating set with minimum total cost among all CDSs.

The method in [69] has the following two phases. The first phase (clustering phase) is to find a set of wireless nodes as the dominators[1]. The second phase is to find a set of nodes to connect these dominators to form the final backbone of the network. Notice that these two phases could interleave in the actual construction method. They separate them just for the sake of easy presentations.

We then first review their method [69] for constructing a connected dominating set whose total cost is comparable with the optimum solution. Their method first constructs a maximal independent set (MIS) using node weights as the selection criterion. Then for each node v in MIS, we run local greedy set cover method on *local neighborhood* $N_2(v)$ to find some nodes $GRDY_v$ to cover all one-hop neighbors of v. If $GRDY_v$ has a total cost smaller than v, then we use $GRDY_v$ to replace v as dominators, which will further reduce the cost of MIS. The method works as follows.

Algorithm 1 ***Constructing Efficient Dominating Set***

1. First assume that all nodes are originally marked WHITE.
2. A node u sends a message ItryDominator to all its one-hop neighbors if it has the smallest cost among all its WHITE neighbors. Node u also marks itself PossibleDominator.
3. When a node v receives a message ItryDominator from its one-hop neighbors, it marks itself Dominatee and sends a message IamDominatee to all its one-hop neighbors.
4. When a node w receives a message IamDominatee from its neighbor v, node w removes node v from its list of WHITE neighbors.
5. Each node u marked with PossibleDominator collects the cost and ID of all of its two hop neighbors $N_2(u)$.
6. Using the greedy method for minimum weighted set cover (like the second method), PossibleDominator node u selects a subset of its two hop-neighbors to cover *all* the one-hop neighbors (including u) of node u. If the cost of the selected subset, denoted by $GRDY_u$, is smaller than the cost of node u, then node u sends a message YouAreDominator(w) to each node w in the selected subset. Otherwise, node u just marks itself Dominator.
7. When a node w received a message YouAreDominator(w), node w marks itself Dominator.

The second step of weighted connected dominating set formation is to find some *connectors* (also called *gateways*) among all the dominatees to connect the dominators. The connectors and the dominators form a *connected dominating set* (or called backbone). Their method forms a connected dominating set by finding connectors to connect any pair of dominators u and v if they are connected in $VirtG$. Their method uses the following data structures and messages.

1. $D_k(v)$ is the list of dominators that are k-hops away from a node v.
2. $P_k(v, u)$ is the least cost path from v to u using at most k-hops. Notice u and v may be less than k-hops away.
3. OneHopDominatorList($v, D_1(v)$): nodes $D_1(v)$ are the 1-hop dominators of node v.
4. TwoHopDominatorList($v, D_2(v)$): nodes $D_2(v)$ are the 2-hop dominators of node v.
5. TwoHopDominator($v, u, w, c(w)$): node u is a 2-hop dominator of node v and the path uwv has the least cost.

Algorithm 2 ***Connector Selection***

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[1] We will interchange the terms cluster-head and dominator. The node that is not a cluster-head is also called *ordinary* node or *dominatee*. A node is called *white* node if its status is yet to be decided by the clustering algorithm. Initially, all nodes are white. The status of a node, after the clustering method finishes, could be *dominator* or *dominatee*.

1. Every dominatee node v broadcasts to its 1-hop neighbors the list of its one-hop dominators $D_1(v)$ using message $\mathsf{OneHopDominatorList}(v, D_1(v))$. When a node w receives $\mathsf{OneHopDominator}(v, D_1(v))$ from one-hop neighbor v, it puts the dominator $u \in D_1(v)$ to $D_2(w)$ if $u \notin D_1(w)$. Update the path $P_3(w, u)$ as uvw if it has a smaller cost.
2. When a dominatee node w receives messages $\mathsf{OneHopDominatorList}$ from *all* its one-hop nodes, for each dominator node $u \in D_2(w)$, node w sends out message $\mathsf{TwoHopDominator}(w, u, x, c(x))$, where wxu is the least cost path $P_2(w, u)$.
3. When a dominator z receives a message $\mathsf{TwoHopDominator}(w, u, x, c(x))$ from its neighbor w, it puts u to $D_3(z)$ if $u \notin D_2(z)$, and updates the path $P_3(w, u)$ as $uwxz$ if $c(w) + c(x)$ has a less cost.
4. Each dominator u builds a virtual edge $\widetilde{uv}$ to connect each neighboring dominator v. The length of $\widetilde{uv}$ is the cost of path $P_3(u, v)$. Notice that here the cost of end-nodes u and v is not included. All virtual edges forms an *edge weighted* virtual graph $VirtG$ in which all dominators are its vertices.
5. Run a distributed algorithm to build a MST on graph $VirtG$. Let $VMST$ denote $MST(VirtG)$.
6. For any virtual edge $e \in VMST$, select each of the dominatees on the path corresponding to e as a connector.

The following theorems are proved to show that the constructed structure is indeed efficient for broadcast and also unicast. Please refer [69] for the detail of the proofs.

Theorem 4 *For any communication graph G, our algorithm constructs a weighted connected dominating set whose total cost is no more than*

$$\min(\alpha^{[2]}(G)\log d, (\alpha^{[1]}(G) - 1)\delta + 1) + 2\alpha^{[1]}(G)$$

times of the optimum.

Specifically, when the network is modelled by a unit disk graph, we have the following corollary.

Corollary 5 *For homogeneous wireless networks, our algorithm constructs a weighted connected dominating set whose total cost is no more than* $\min(18\log d, 4\delta + 1) + 10$ *times of the optimum.*

For unicast, they proved that

Theorem 6 *For any communication graph, the unicast based on the constructed structure has a cost at most* 3 *times the cost on the original communication network .*

3.2 Localized Low Weight Structures

The centralized algorithms do not consider computational and message overheads incurred in collecting global information. Several of them also assume that the network topology does not change between two runs of information exchange. These assumptions may not hold in practice, since the network topology may change from time to time, and the computational and energy overheads incurred in collecting global information may not be negligible. This is especially true for large-scale wireless networks where the topology is changing dynamically due to the changes of position, energy availability, environmental interference, and failures, which implies that centralized algorithms that require global topological information may not be practical.

Santivanez *et al.* [70] show that flooding is a good solution for the sake of scalability and simplicity. Several flooding techniques for wireless networks have been proposed [71, 72, 64], each with respect to certain optimization criterion. However, none of them takes advantage of the feature that the transmission power of a node can be adjusted.

Some distributed heuristics are proposed, such as [73, 74, 75]. Most of them are based on distributed MST method. A possible drawback of these distributed method is that it may not perform well under frequent topological changes as it relies on information that is multiple hops away to construct the MST. Refer to [76] for more detail. The relative neighborhood graph, the Gabriel graph and the Yao graph all have $O(n)$ edges and contain the Euclidean minimum spanning tree. This implies that we can construct the minimum spanning tree using $O(n \log n)$ messages.

Localized minimum energy broadcast algorithms are based on the use of a locally defined geometric structure, such as RNG (relative neighborhood graph), proposed by Toussaint [94]. RNG consists of all edges uv such that uv is not the longest edge in any triangle uvw. That is, uv belongs to RNG if there is no node w such that $uw < uv$ and $vw < uv$.

Cartigny *et al.* [77] proposed a localized algorithm, called RBOP [77] that is built upon the notion of relative neighborhood graph (RNG). In RBOP, the broadcast is initiated at the source and propagated, following the rules of neighbor elimination [64], on the topology represented by RNG. Simulation results show that the energy consumption could be as high as 100% as compared to BIP. However, the communication overhead due to mobility and changes in activity status in BIP are not considered, therefore RBOP is superior to BIP in dynamic ad hoc networks. Li and Hou [76], and Cartigny *et al.* [100] proposed another localized algorithms, which applies LMST (localized minimal spanning tree) instead of RNG as the broadcast topology. In LMST, proposed in [79], each node calculates local minimum spanning tree of itself and its 1-hop neighbors. A node uv is in LMST if and only if u and v select each other in their respective trees. The simulations [76, 100] show that the performance of LMST based schemes is significantly better than the performance of RBOP, and with about 50% more energy consumption than BIP in static scenarios. Cartigny *et al.* [96] demonstrated that, when $c > 0$ in power-attenuation model where energy consumption for transmitting over an edge uv is $\|uv\|^{\beta} + c$, there exists an optimal 'target' transmission radius, so that further energy savings can be obtained if transmission radii are selected near target radius.

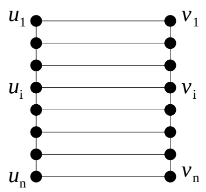


Figure 1: An instance of wireless nodes that every network structures described previously (except MST) have an arbitrarily large total weight.

However, as shown in [21] (also by Figure 1), the total weights of RNG and LMST could still be as large as $O(n)$ times of the total weight of MST. Here, in Figure 1, $\|u_i v_i\| = 1$ and $\|u_i u_{i+1}\| = \|v_i v_{i+1}\| = \epsilon$ for a very small positive real number ϵ. Given a graph G, let $\omega_b(G) = \sum_{e \in G} \|e\|^b$. Then $\omega_1(RNG) = \Theta(n) \cdot \omega_1(MST)$ and $\omega_1(LMST) = \Theta(n) \cdot \omega_1(MST)$.

In [21, 26], the authors described several low weight planar structures that can be constructed by localized methods with total communication costs $O(n)$. The energy consumption of broadcast based on those structures are within $O(n^{\beta-1})$ of the optimum, i.e., $\omega_\beta(H) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$, $\omega_\beta(LMST_2) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$, $\omega_\beta(IMRG) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$ for any $\beta \geq 1$. This improves the previously known "lightest" structure RNG by $O(n)$ factor since in the worst case $\omega(RNG) = \Theta(n) \cdot \omega(MST)$ and $\omega_\beta(RNG) = \Theta(n^\beta) \cdot \omega_\beta(MST)$.

We will now review in detail these three structures.

3.2.1 Structure Based on RNG'

Although RNG is very sparse structure (the average number of neighbors per node is about 2.5), in some degenerate cases a particular node may have arbitrarily large degree. This motivated Stojmenovic [23] to define a modified structure where each node will have degree bounded by 6. The same structure was independently proposed by Li in [21], with an additional motivation. Li proved that the modified RNG is the first localized method to construct a structure H with weight $O(\omega(MST))$ using total $O(n)$ local-broadcast messages. Note that, if each node already knows the positions and IDs of all its neighbors, then no messages are needed to decide which of its edges belong to (modified) RNG. Notice that, traditionally, the relative neighborhood graph will always select an edge uv even if there is some node on the boundary of $lune(u, v)$. Here $lune(u, v)$ is the intersection of two disks centered at nodes u and v with radius $\|uv\|$ respectively. Thus, RNG may have unbounded node degree, e.g., considering $n-1$ points equally distributed on the circle centered at the nth point v, the degree of v is $n-1$. Notice that for the sake of lowing the weight of a structure, the structure should contain as less edges as possible without breaking the connectivity. Li [21] and Stojmenovic [23] then naturally extended the traditional definition of RNG as follows.

We need to make distinct edge lengths. This can be achieved by adding the secondary, and if necessary, the ternary keys for comparing two edges. Each node is assumed to have an unique ID. Then consider the record $(\|uv\|), ID(u), ID(v)$, where $ID(u) < ID(v)$ (otherwise u and v are exchanged for given edge). Two edges compare their lengths first to decide which one is longer. If same, they then compare their secondary key, which is their respective lower endpoint node's ID. If this is also same, then the ternary key resolves the comparison (otherwise we are comparing edge against itself). This simple method for making distinct edge length was proposed in [24, 79]. The edge lengths, so defined, are then used in the regular definition of RNG. It is easy to show that two RNG edges uv and uw going out of the same node must have angle between them at least $\pi/6$, otherwise $vw < uv$ or $vw < uw$, and one of the two edges becomes the longest in the triangle and consequently could not be in RNG. Li denoted modified RNG structure by RNG'. Obviously, RNG' is a subgraph of traditional RNG. It was proved in [21, 23] that RNG' still contains a MST as a subgraph. However, RNG' is still not a low weight structure.

Notice that it is well-known that the communication complexity of constructing a minimum spanning tree of a n-vertex graph G with m edges is $O(m+n\log n)$; while the communication complexity of constructing MST for UDG is $O(n\log n)$ even under the local broadcasting communication model in wireless networks. It was shown in [21] that it is *impossible* to construct a low-weighted structure using only one hop neighbor information.

The localized algorithm given in [21] that constructs a low-weighted structure using only some two hops information is as follows.

Algorithm 3 *Construct Low Weight Structure H*

1. All nodes together construct the graph RNG' in a localized manner.
2. Each node u locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node u listens to the messages from its one-hop neighbors.
3. Assume node u received a message informing existence of edge $xy \in RNG'$ from its neighbor x. For each edge $uv \in RNG'$, if uv is the longest among uv, xy, ux, and vy, node u removes edge uv. Ties are broken by the label of the edges. Here assume that $uvyx$ is the convex hull of u, v, x, and y.
4. Let H be the final structure formed by all remaining edges in RNG'.

Obviously, if an edge uv is kept by node u, then it is also kept by node v. The following theorem was proved in [21].

Theorem 7 *[21] The total edge weight of H is within a constant factor of that of the minimum spanning tree.*

This was proved by showing that the edges in H satisfy the *isolation property* (defined in [78]). They [21] also showed that the final structure contains MST of UDG as a subgraph.

Clearly, the communication cost of Algorithm 3 is at most $7n$: initially each node spends one message to tell its one-hop neighbors its position information, then each node uv tells its one-hop neighbors all its incident edges $uv \in RNG'$ (there are at most total $6n$ such messages since RNG' has at most $3n$ edges). The computational cost of Algorithm 3 could be high since for each link $uv \in RNG'$, node u has to test whether there is an edge $xy \in RNG'$ and $x \in N_1(u)$ such that uv is the longest among uv, xy, ux, and vy. Then [26, 27] present some new algorithms that improve the computational complexity of each node while still maintains low communication costs.

3.2.2 Structure Based on LMST$_k$

The first new method in [26] uses a structure called *local minimum spanning tree*, let us first review its definition. It is first proposed by Li, Hou and Sha [79]. Each node u first collects its one-hop neighbors $N_1(u)$. Node u then computes the minimum spanning tree $MST(N_1(u))$ of the induced unit disk graph on its one-hop neighbors $N_1(u)$. Node u keeps a directed edge uv if and only if uv is an edge in $MST(N_1(u))$. They call the union of all directed edges of all nodes the *local minimum spanning tree*, denoted by $LMST_1$. If only symmetric edges are kept, then the graph is called $LMST_1^-$, i.e., it has an edge uv iff both directed edge uv and directed edge vu exist. If ignoring the directions of the edges in $LMST_1$, they call the graph $LMST_1^+$, i.e., it has an edge uv iff either directed edge uv or directed edge vu exists. They prove that the graph is connected, and has bounded degree 6. In [26], Li *et al.* also showed that graph $LMST_1^-$ and $LMST_1^+$ are actually planar. Then they extend the definition to k-hop neighbors, the union of all edges of all minimum spanning tree $MST(N_k(u))$ is the k *local minimum spanning tree*, denoted by $LMST_k$. For example, the 2 local minimum spanning tree can be constructed by the following algorithm.

Algorithm 4 *Construct Low Weight Structure $LMST_2$ by 2-hop Neighbors*

1. Each node u collects its two hop neighbors information $N_2(u)$ using a communication efficient protocol described in [80].
2. Each node u computes the Euclidean minimum spanning tree $MST(N_2(u))$ of all nodes $N_2(u)$, including u itself.
3. For each edge $uv \in MST(N_2(u))$, node u tells node v about this directed edge.
4. Node u keeps an edge uv if $uv \in MST(N_2(u))$ or $vu \in MST(N_2(v))$. Let $LMST_2^+$ be the final structure formed by all edges kept. It keeps an edge if either node u or node v wants to keep it. Another option is to keep an edge only if both nodes want to keep it. Let $LMST_2^-$ be the structure formed by such edges.

In [26], they prove that structures $LMST_2$ ($LMST_2^+$ and $LMST_2^-$) are connected, planar, low-weighted, and have bounded node degree at most 6. In addition, MST is a subgraph of $LMST_k$ and $LMST_k \subseteq RNG'$. Although the constructed structure $LMST_2$ has several nice properties such as being bounded degree, planar, and low-weighted, the communication cost of Algorithm 4 could be very large to save the computational cost of each node. The large communication costs are from collecting the two hop neighbors information $N_2(u)$ for each node u. Although the total communication of the protocol described in [80] is $O(n)$, the hidden constant is large.

3.2.3 Combining RNG' and LMST$_k$

We could improve the communication cost of collecting $N_2(u)$ by using a subset of two hop information without sacrificing any properties. Define $N_2^{RNG'}(u) = \{w \mid vw \in RNG' \text{ and } v \in N_1(u)\} \cup N_1(u)$. We describe our modified algorithm as follows.

Algorithm 5 *Construct Low Weight Structure IMRG by 2-hop Neighbors in RNG'*

1. Each node u tells its position information to its one-hop neighbors $N_1(u)$ using a local broadcast model. All nodes together construct the graph RNG' in a localized manner.
2. Each node u locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node u listens to the messages from its one-hop neighbors.
3. Each node u computes the Euclidean minimum spanning tree $MST(N_2^{RNG'}(u))$ of all nodes $N_2^{RNG'}(u)$, including u itself.
4. For each edge $uv \in MST(N_2^{RNG'}(u))$, node u tells node v about this directed edge.

5. Node u keeps an edge uv if $uv \in MST(N_2^{RNG'}(u))$ or $vu \in MST(N_2^{RNG'}(v))$. Let $IMRG^+$ be the final structure formed by all edges kept. Similarly, the final structure is called $IMRG^-$ when edge $uv \in RNG'$ is kept iff $uv \in MST(N_2^{RNG'}(u))$ and $uv \in MST(N_2^{RNG'}(v))$. Here IMRG is the abbreviation of *Incident MST and RNG Graph.*

Notice that in the algorithm, node u constructs the local minimum spanning tree $MST(N_2^{RNG'}(u))$ based on the induced UDG of the point sets $N_2^{RNG'}(u)$. It is obvious that the communication cost of Algorithm 5 is at most $7n$.

It was shown that structures $IMRG^+$ and $IMRG^-$ are still connected, planar, bounded degree, and low-weighted. They are obviously planar, and with bounded degree since both structures are still subgraphs of the modified relative neighborhood graph RNG'. Clearly, the constructed structures are supergraphs of the previous structures, i.e., $LSMT_2+ \subseteq IMRG^+$ and $LSMT_2^- \subseteq IMRG^-$, since Algorithm 5 uses less information than Algorithm 4 in constructing the local minimum spanning tree. It is proved in [26] that Algorithm 5 constructs structures $IMRG^-$ or $IMRG^+$ using at most $7n$ messages. The structures $IMRG^-$ or $IMRG^+$ are connected, planar, bounded degree, and low-weighted. Both $IMRG^-$ and $IMRG^+$ have node degree at most 6.

Recall that until now there is no efficient localized algorithm that can achieve all following desirable features: bounded degree, planar, low weight and spanner. It is still an open problem.

3.2.4 A Negative Result

In [21, 26, 27], Li *et al.* proposed several methods to construct structures in a localized manner such that the total edge lengths of these structures are within a constant factor of MST. They also showed that the energy consumption of broadcasting based on those structures are within $O(n^{\beta-1})$ of the optimum, i.e., $\omega_\beta(H) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$, $\omega_\beta(LMST_2) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$, $\omega_\beta(IMRG) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$ for any $\beta \geq 1$.

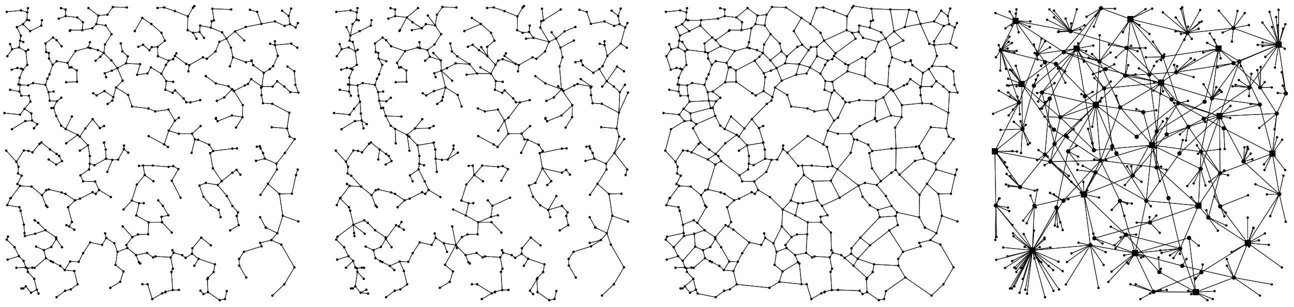
They further showed that it is impossible to design a deterministic localized method that constructs a structure such that the broadcasting based on this structure consumes energy within a factor $o(n^{\beta-1})$ of the optimum. Assume that there is a deterministic localized algorithm to do so: it uses k-hop information of every node u to select the edges incident on u, and the energy consumption is no more than $o(n^{\beta-1})$ times of the optimum. They construct two set of nodes configurations such that the k-hop information collected in a special node u is same for both configurations. In addition, there is an edge uv in both UDGs such that if node u decides to keep edge uv (then edge uv is kept in both configurations), the energy consumption of one configuration is already more than $o(n^{\beta-1})$ times of the optimum; if node u decides to remove edge uv (then edge uv is removed in both configurations), then the structure constructed for another configuration is disconnected. See [27] for more detail. This implies that the low weighted structures are asymptotically optimum in terms the worst case energy consumption for broadcasting among *any* locally constructed topologies when assuming that the energy needed to support a link uv is proportional to $\|uv\|^\beta$.

3.3 Combining Clustering and Low Weight

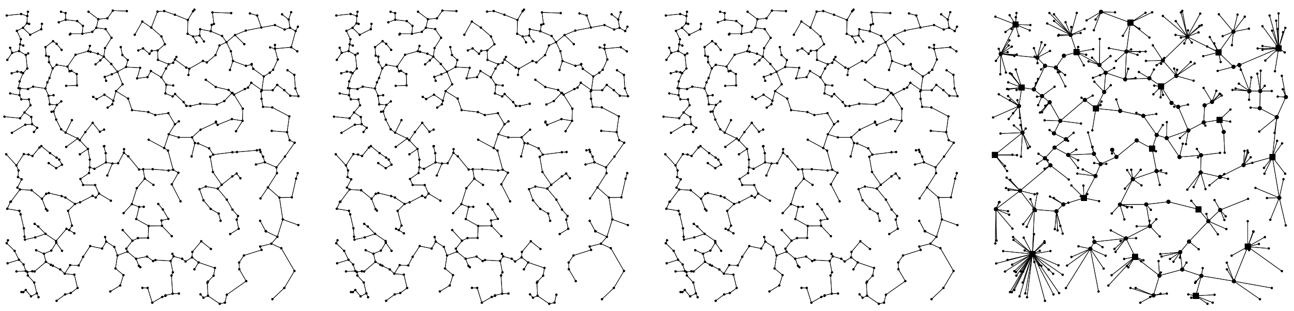
Seddigh, Gonzalez and Stojmenovic [81] specify two more location based broadcasting algorithms that combine RNG and internal node concept (connected dominating set) as follows. PI-broadcast algorithm applies the planar subgraph construction first, and then applies the internal nodes concept on the subgraph. The result is different from the internal nodes applied on the whole graph. IP-broadcast algorithm changes the order of concept application compared to the previous algorithm. Internal nodes are first identified in the whole graph, and then the obtained subgraph (containing only internal nodes) is further reduced to planar one by the RNG construction.

The solution in [81] is for one-to-one communication model, where message sent from one node is received by only the targeted neighbor. In [82], Li *et al.* combine the low-weighted structures and the connected dominating set for energy efficient broadcasting in traditional one-to-many (omnidirectional antenna) networks. Similarly, they proposed two approaches for combining them as in [81]. Notice that the constructed low-weighted structures are a subgraph of RNG, thus, they are still planar graphs. For simplicity, they also call these two combinations, *PI-broadcast* and *IP-broadcast* respectively. They found that the energy consumption of the IP-broadcast schemes in dense networks is significantly less than that of the PI-broadcast schemes. The reason is that, in the IP-broadcast schemes, one retransmission by the internal nodes will be received by many non-internal nodes in dense networks, thus, energy consumption is reduced. Several localized improvement heuristics are also applied after the IP-broadcast or PI-broadcast schemes to further improve the energy consumption. Ingelrest (cf. [96]) in his master thesis also combined LMST with dominating sets, and target radius idea to derive new minimum energy broadcast protocols.

Following Figure 2 illustrates the different structures used for broadcasting. All figures are computed on a set of 500 nodes randomly distributed in a square region with side length $7500m$. Each node has transmission range $500m$. Here the CDS is generated using ID as criterion for selecting dominator/conector; IMRG-CDS is the subgraph constructed by applying IMRG on the induced unit disk graph of all CDS nodes, i.e., dominators and connectors. Our simulation results also confirms that CDS structure and IMRG-CDS consumes the least energy when the node power is non-adjustable, while the IMRG consumes the least energy when each node can adjust its power according to the longest incident link.



MST BIP RNG CDS



LMST1 LMST2 IMRG IMRG-CDS

Figure 2: Different structures generated from a UDG used for broadcasting.

3.4 Flooding Based Methods

3.4.1 Selecting Forwarding Neighbors

The simplest broadcasting mechanism is to let every node retransmit the message to all its one-hop neighbors when receiving the first copy of the message, which is called *flooding* in the literature. Despite its simplicity, flooding is very inefficient and can result in high redundancy, contention, and collision. One approach to reducing the redundancy is to let a node only forward the message to a subset of one-hop neighbors who together can cover the two-hop neighbors. In other words, when a node retransmits a message to its neighbors, it explicitly asks a subset of its neighbors to relay the message.

In [83], Lim and Kim proposed a broadcasting scheme that chooses some or all of its one-hop neighbors as rebroadcasting node. When a node receives a broadcast packet, it uses a Greedy Set Cover algorithm to determine which subset of neighbors should rebroadcast the packet, given knowledge of which neighbors have already been covered by the sender's broadcast. The Greedy Set Cover algorithm recursively chooses 1-hop neighbors which cover the most *uncovered* 2-hop neighbors and recalculates the cover set until all 2-hop neighbors are covered.

Călinescu *et al.* [84] gave two practical heuristics for this problem (they called selecting forwarding neighbors). The first algorithm runs in time $O(n \log n)$ and returns a subset with size at most 6 times of the minimum. The second algorithm has an improved approximation ratio 3, but with running time $O(n^2)$. Here n is the number of total two-hop neighbors of a node. When all two-hop neighbors are in the same quadrant with respect to the source node, they gave an exact solution in time $O(n^2)$ and a solution with approximation factor 2 in time $O(n \log n)$. Their algorithms partition the region surrounding the source node into four quadrants, solve each quadrants using an algorithm with approximation factor α, and then combine these solutions. They proved that the combined solution is at most 3α times of the optimum solution. They then gave two different algorithms for finding a disk cover when the two-hop neighbors are restricted to one quadrant with approximation ratio $\alpha = 2$ amd 1 respectively.

Their approach assumes that every node u can collect its 2-hop neighbors $N_2(u)$ efficiently. Notice that, the 1-hop neighbors of every node u can be collected efficiently by asking each node to broadcast its information to its 1-hop neighbors. Thus all nodes get their 1-hop neighbors information by using total $O(n)$ messages. However, until recently, it was not known how to collect the 2-hop neighbors information with $O(n)$ communications. The simplest broadcasting of 1-hop neighbors $N_1(u)$ to all neighbors u does let all nodes in $N_1(u)$ to collect their corresponding 2-hop neighbors. However, the total communication cost of this approach is $O(m)$, where m is the total number of links in UDG. Recently, Călinescu [80] proposed an efficient approach to collect $N_2(u)$ using the connected dominating set [68] as forwarding nodes. Assume that the node position is known. He proved that the approach takes total communications $O(n)$, which is optimum within a constant factor.

3.4.2 Gossip and Probabilistic Schemes

Probabilistic Scheme: The Probabilistic scheme from [12] is similar to Flooding, except that nodes only rebroadcast with a predetermined probability. In dense networks multiple nodes share similar transmission coverages. Thus, randomly having some nodes not rebroadcast saves node and network resources without harming delivery effectiveness. In sparse networks, there is much less shared coverage; thus, nodes wont receive all the broadcast packets with the Probabilistic scheme unless the probability parameter is high. When the probability is 100%, this scheme is identical to Flooding. Cartigny and Simplot [98] applied probability which is a function of the distance to the transmitting neighbor.

Counter-Based Scheme: Tseng *et al.* [12] show an inverse relationship between the number of times a packet is received at a node and the probability of that node being able to reach additional area on a rebroadcast. This result is the basis of their Counter-Based scheme. Upon reception of a previously unseen packet, the node initiates a counter with a value of one and sets a RAD (which is randomly chosen between 0 and Tmax seconds). During the RAD, the counter is incremented by one for each redundant packet received. If the counter is less than a threshold value when the RAD expires, the packet is rebroadcast. Otherwise, it is simply dropped. From [12], threshold values above six relate to little additional coverage area being reached.

The overriding compelling features of the Counter-Based scheme are its simplicity and its inherent adaptability to local topologies. That is, in a dense area of the network, some nodes won't rebroadcast; in sparse areas of the network, all nodes rebroadcast. The disadvantage of all counter and probabilistic schemes is that delivery is not guaranteed to all nodes even if ideal MAC is provided. In other words, they are not reliable.

3.4.3 Area Based Decision

In either probabilistic schemes or the counter-based schemes a node decides whether to rebroadcast a received packets purely based on its own information. Tseng *et al.* [12] proposed several other criteria based on the additional coverage area to decide whether the node will rebroadcast the packet. These coverage-area based methods are similar to the methods of selecting forwarding neighbors, which tries to select a set of one-hop neighbors sufficient to cover all its two-hop neighbors. While area based methods only consider the coverage area of a transmission; they don't consider whether nodes exist within that area. Two coverage-area based methods are proposed in [12]: *Distance-Based Scheme* and *Location Based Scheme.*

In Distance-Based Scheme, a node compares the distance between itself and each neighbor node that has previously rebroadcast a given packet. Upon reception of a previously unseen packet, a RAD is initiated and redundant packets are cached. When the RAD expires, all source node locations are examined to see if any node is closer than a threshold distance value. If true, the node doesn't rebroadcast.

The Location-Based scheme uses a more precise estimation of expected additional coverage area in the decision to rebroadcast. In this method, each node must have the means to determine its own location, e.g., a GPS. Whenever a node originates or rebroadcasts a packet it adds its own location to the header of the packet. When a node initially receives a packet, it notes the location of the sender and calculates the additional coverage area obtainable were it to rebroadcast. If the additional area is less than a threshold value, the node will not rebroadcast, and all future receptions of the same packet will be ignored. Otherwise, the node assigns a RAD before delivery. If the node receives a redundant packet during the RAD, it recalculates the additional coverage area and compares that value to the threshold. The area calculation and threshold comparison occur with all redundant broadcasts received until the packet reaches either its scheduled send time or is dropped.

We will review also some upcoming work related to dominating sets and broadcasting problem. In [100], a beaconless broadcasting method is proposed. All nodes have the same transmission radius, and nodes are not aware of their neighborhood. That is, no beacons or hello messages are sent in order to discover neighbors prior to broadcasting process. The source transmits the message to all neighbors. Upon receiving the packet (together with geographic coordinates of the sender), each node calculates the portion of its perimeter, along circle of transmission radius, that is not covered by this and previous transmissions of the same packet. Node then sets or updates its timeout interval, which inversely depends on the size of the uncovered perimeter portion. If the perimeter becomes fully covered, the node cancels retransmissions. Otherwise, it retransmits at the end of timeout interval. The method is reliable, as opposed to other area based methods.

3.4.4 Neighbor Coverage Based Decision

The method presented in the previous subsection were based on covering an area where nodes could be located. Instead of covering area, one could simply cover neighboring nodes, assuming their location, or existence of their link to a previous transmitting node, are known. The basic method was independently and almost simultaneously (August 2000) proposed in two articles [85, 86]. The methods were called Neighbor Elimination by Stojmenovic and Seddigh [86], while a similar method, called Scalable Broadcast Algorithm, was proposed by Peng and Lu [85]. Two-hop neighbors information is used to determine whether a node will rebroadcast the packet. Suppose that a node u receives a broadcast data packet from its neighbor node v. Node u knows all the neighbors of node v, and thus all nodes that are common neighbors of them (already received the data from v). If node u has additional neighbors not reached by node v's broadcast, node u schedules the packet for delivery with a RAD. However, if node u receives a redundant broadcast packet from some other neighbors within RAD, node u will recalculate whether it needs rebroadcast the packet. This process is continued until either the RAD expires and the packet is then sent, or the packet is dropped (when all its neighbors are already covered by the broadcasts of some of its neighbors).

Lipman, Boustead and Judge [99] described the following broadcasting protocol. Upon receiving a broadcast message(s) from a node h, each node i (that was determined by h as a forwarding node) determines which of its one-hop neighbors also received the same message. For each of its remaining neighbors j (which did not receive a message yet, based on i's knowledge), node i determines whether j is closer to i than any one-hop neighbors of i (that are also forwarding nodes of h) who received the message already. If so, i is responsible for message transmission to j, otherwise it is not. Node i then determines a transmission range equal to that of the farthest neighbor it is responsible for.

4 Scheduling Active and Sleep Periods

In ad hoc wireless networks, the limitation of power of each host poses a unique challenge for power-aware design. There has been an increasing focus on low cost and reduced node power consumption in ad hoc wireless networks. Even in standard networks such as IEEE 802.11, requirements are included to sacrifice performance in favor of reduced power consumption. In order to prolong the life span of each node and, hence, the network, power consumption should be minimized and balanced among nodes. Unfortunately, nodes in the dominating set in general consume more energy in handling various bypass traffic than nodes outside the set. Therefore, a static selection of dominating nodes will result in a shorter life span for certain nodes, which in turn result in a shorter life span of the whole network.

Wu, Wu, and Stojmenovic [87] study dynamic selection of dominating nodes, also called activity scheduling. Activity scheduling deals with the way to rotate the role of each node among a set of given operation modes. For example, one set of operation modes is sending, receiving, idles, and sleeping. Different modes have different energy consumptions. Activity scheduling judiciously assigns a mode to each node to save overall energy consumptions in the networks and/or to prolong life span of each individual node. Note that saving overall energy consumptions does not necessarily prolong life span of a particular individual node. Specifically, they propose to save overall energy consumptions by allowing only dominating nodes (i.e., gateway nodes) to retransmit the broadcast packet. In addition, in order to maximize the lifetime of all nodes, an activity scheduling method is used that dynamically selects nodes to form a connected dominating set. Specifically, in the selection process of a gateway node, we give preference to a node with a higher energy level. The effectiveness of the proposed method in prolonging the life span of the network is confirmed through simulation. Source dependent forwarding sets appear to be more energy balanced. However, it was experimentally confirmed in [88] that the difference in energy consumption between an idle node and a transmitting node is not major, while the major difference exists between idle and sleep states of nodes. Therefore the most energy efficient methods will select static dominating set for a given round, turning all remaining nodes to a sleep state. Depending on energy left, changes in activity status for the next round will be made. The change can therefore be triggered by changes of power status, in addition to node mobility. From this point of view, internal nodes based dominating sets provide static selection for a given round and more energy efficiency than forwarding set based method that requires all nodes to remain active in all the rounds.

In [89], the key for deciding dominating set status is a combination of remained energy and node degree. Xu, Heidemann, and Estrin [90] discuss the following sensor sleep node schedule. The tradeoff between network lifetime and density for this cell-based schedule was investigated in [91]. The given 2-D space is partitioned into a set of squares (called cells), such as any node within a square can directly communicate with any nodes in an adjacent square. Therefore, one representative node from each cell is sufficient. To prolong the life span of each node, nodes in the cell are selected in a alternative fashion as a representative. The adjacent squares form a 2-D grid and the broadcast process becomes trivial. Note that the selected nodes in [90] make a dominating set, but the size of it is far from optimal, and also it depends on the selected size of squares. On the other hand, the dominating set concept used here has smaller size and is chosen without using any parameter (size of square, which has to be carefully selected and propagated with node relative positioning in solution [90]).

The Span algorithm [92] selects some nodes as coordinators. These nodes form dominating set. A node becomes coordinator if it discovers that two of its neighbors cannot communicate with each other directly or through one or two existing coordinators. Also, a node should withdraw if every pair of its neighbors can reach each other directly or via some other coordinators (they can also withdraw if each pair of neighbors is connected via possibly non-coordinating nodes, to give chance to other nodes to become coordinators). Since coordinators are not necessarily neighbors, three-hop neighboring topology knowledge is required. However, the energy and bandwidth required for maintenance of three-hop neighborhood information is not taken into account in experiments [92]. On the other hand, if the coordinators are restricted to be neighboring nodes, then the dominating set definition [92] becomes equivalent to one given by Wu and Li [62]. Next, protocol [92] heavily relies on proactive periodic beacons for synchronization, even if there is no pending traffic or node movement. The recent research on energy consumption [88] indicates that the use of such periodic beacons or hello messages is an energy expensive mechanism, because of significant start up cost for sending short messages. Finally, [91] observed that the overhead required for coordination with SPAN tends to *explode* with node density, and thus counterbalances the potential savings achieved by the increased density.

5 Conclusion and Future Research Directions

In this chapter, we reviewed several methods for efficient broadcasting for wireless ad hoc networks. There are still many challenging questions left open for further research. So far, all the known theoretically good algorithms either assume that the power needed to support a link uv is proportional to $\|uv\|^\beta$ or is a fixed cost that is independent of the neighboring nodes that it will communicate with. In practice, the energy consumption of a node is neither soly dependent on the distance to its farthest neighbor, nor totally independent of its communication neighbor. For example, a more general power consumption model for a node u would be $c_1 + c_2 \cdot \|uv\|^\beta$ for some constants c_1 and c_2 where v is its farthest communication neighbor in a broadcast structure. No theoretical result is known about the approximation of the optimum broadcast or multicast structure under this model.

Another important aspect of designing energy efficient protocols for broadcast and multicast that is often neglected in the literature is the physical constraints of the wireless communications. In most of the algorithms, it is assumed that the signal sent by a node will be received by all nodes at one shot it intends to send. However, in practice, it is not the case, which will makes designing energy efficient broadcast and multicast protocols much harder. The first difficulty is that the sender needs coordinate with the receivers so *all* receivers are ready to receive. It is often difficult, if not impossible, to do so. Then a natural question is how to set the threshold t such that if the number of receivers that are ready is more than t, node u sends the packets; otherwise node t will not send the packets. Clearly, the setting of the threshold will affect the total actual energy consumption of the broadcast and multicast protocols, in addition to affect the system performance and the stability of the networks. The second difficulty is that, even some receivers are ready to receive, the receivers cannot always decode the signal correctly due to the signal strength fluctuation. In other words, the link between two nodes is often probabilistic. We clearly should take this into account when we design energy efficient broadcast and multicast protocols.

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