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Narrowband Interference Suppression in Spread Spectrum CDMA

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Code-division multiple-access (CDMA) implemented with direct-sequence spread spectrum signaling is among the most promising multiplexing technologies for cellular telecommunications services, such as personal communications, mobile telephony, and indoor wireless networks [1, 2, 3, 4, 5]. The advantages of direct-sequence spread spectrum for these services include superior operation in multipath environments, flexibility in the allocation of channels, the ability to operate asynchronously, privacy, and increased capacity in bursty or fading channels. Also among the attractive features of spread spectrum CDMA is the ability of spread spectrum systems to share bandwidth with narrowband communication systems without undue degradation of either system's performance. In particular, the ability of spread spectrum to provide reliable performance in severe signal-to-noise (SNR) environments and its low energy profile make the sharing of frequency bands by multiple and disparate users a real possibility. This ability provides a means by which to alleviate overcrowding in the radio frequency spectrum, as well as to allow more user flexibility in the choice of modulation format.

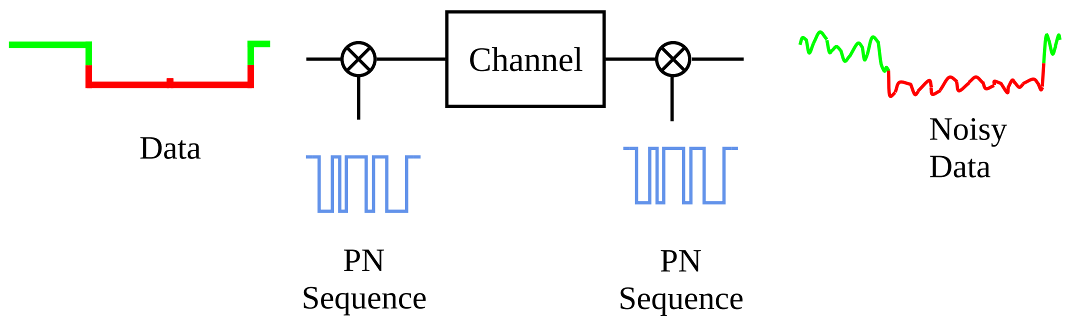

Figure 1: Spreading/Despreading Procedure

In a direct-sequence spread spectrum system, a data signal is modulated with a binary *pseudonoise* (PN) signal having a nearly flat spectrum before transmission; so that the transmission bandwidth is much greater than the message bandwidth. At the receiver, the incoming signal is "despread" by correlating it with the PN signal. This process is illustrated in Fig. 1. The binary pulses comprising the PN signal are known as *chips* to distinguish them from the binary bits of the data signal. The number of chips per data bit, G, is the *spreading ratio* or *coding gain* of the system, and the noise immunity improves with increasing G. Each user in a spread spectrum CDMA system has a distinct PN code that allows the receiver to distinguish it from the other users in the system. Again, increasing the coding gain (and hence the transmitted bandwidth) allows for the accommodation of more users since it allows for lower cross-correlations between the PN signals of the multiple users [6]. For the demodulation of CDMA signals, the despread data signal can be processed via one of several multiuser receiver algorithms, including simple sign extraction, decorrelation [7, 8] or maximum-likelihood sequence detection [9].

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As noted above, the spreading of the data signal's energy over a sufficiently wide bandwidth allows it to co-exist with narrowband signals with only a minimum of interference for either signal. Obviously, the low spectral density of the spread spectrum signal assures that it will cause little damage to the narrowband signal beyond that already caused by the ambient wideband noise in the channel. On the other hand, although the narrowband signal has very high spectral density, this energy is concentrated near one frequency and is of very narrow bandwidth. The despreading operation of the spread spectrum receiver has the effect of spreading this narrowband energy over a wide bandwidth, while at the same time it collapses the energy of the originally spread data signal down to the original data bandwidth. So, after despreading, the situation is reversed between the original narrowband interferer (which is now wideband), and the original data signal (which is now narrowband). A bandpass filter can be employed so that only the interferer power that falls in the bandwidth of the despread signal causes any interference. This will be only a fraction, $1/G$, of the original narrowband interference that could have occupied that same bandwidth before despreading. This process is illustrated in Fig. 2.

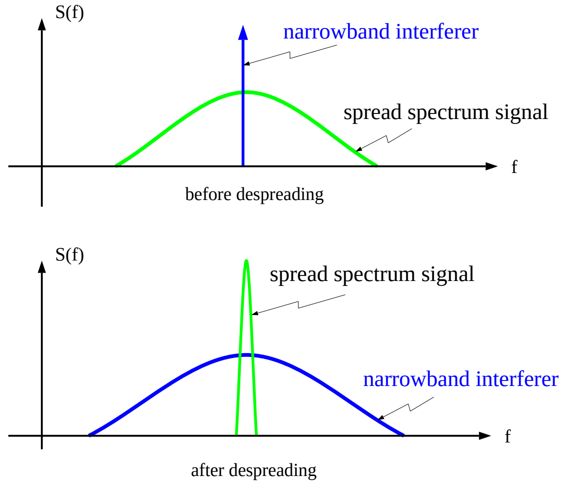

Figure 2: Spectral Effects

Thus, spread spectrum communications is inherently resistant to the narrowband interference (NBI) caused by co-existence with conventional communications. However, it has been demonstrated that the performance of spread spectrum systems in the presence of narrowband signals can be enhanced significantly through the use of active NBI suppression prior to despreading [10, 11, 12]. Not only does active suppression improve error-rate performance [13], but it also leads to increased CDMA cellular system capacity [3] and improved acquisition capability [14].

Over the past two decades, a significant body of research has been concerned with the development of techniques for active NBI suppression in spread spectrum systems, and the purpose of this paper is to provide an overview of these techniques. An excellent review of those methods developed prior to 1988 can be found in a survey paper authored by Milstein [12]. Thus, although we will review all of the methods that have been proposed, we will treat these earlier methods only briefly, and we will focus instead on those that have been developed more recently. It should also be noted that much of the work in this area has been motivated by the application of spread spectrum as an anti-jamming signaling method for military use; however it is equally applicable to the problem of NBI in cellular CDMA. Current research makes a more conscious effort to incorporate methodologies that allow for performance improvements when considering the spread signal as a CDMA signal, as we will discuss later.

We begin, in the following section, with a brief review of NBI mitigation techniques based on the linear signal processing regimes of adaptive linear transversal filtering, and Fourier-domain filtering. These methods represent the original approaches to this problem, and they are quite well developed. Most of this paper, on the other hand, will be concerned with recently developed model-based techniques that employ non-standard signal processing methods to enhance the interference rejection capabilities in CDMA systems beyond those of standard methods. In particular, in the second section we consider techniques based on nonlinear filtering [15] in which the spread spectrum CDMA signal (which is digital) is modeled as a non-Gaussian noise in the interference suppression process. Further, in the third section we consider the situation in which the NBI is also a digital communications signal. In this situation, multiuser detection techniques [16] can be used to give quite significant performance improvement over other methods. Finally, we present some conclusions and a discussion of open issues in this area.

Linear Techniques for Interference Suppression

In this section we will describe NBI suppression methodology based on linear signal processing paradigms, which represents a large part of the work in NBI mitigation. This methodology is quite well-developed, and it has been investigated extensively, from theoretical analyses through implementation studies. Since this work has been reviewed comprehensively by Milstein in [12], we will provide only a broad-brush overview here.

Briefly described, this development has focused on two basic types of techniques: *estimator/subtracter* methods that perform time-domain notch filtering; and *transform-domain* methods that operate to block (or suppress) narrowband energy in the frequency domain. These two types of systems are depicted in Figs. 3 and 4.

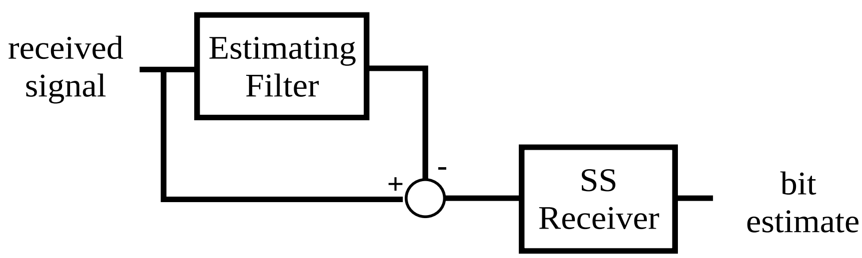

Figure 3: Estimator/Subtracter

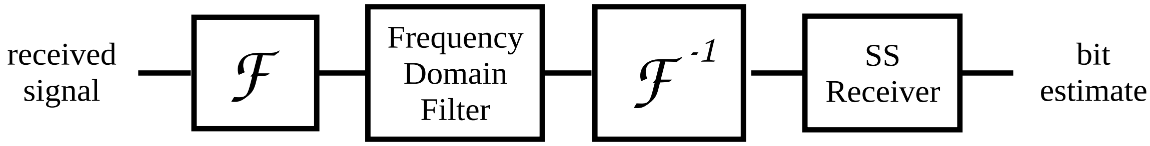

Figure 4: Fourier Transform Technique

The estimator/subtracter implementation shown in Fig. 3 essentially forms a replica of the NBI which can be subtracted from the received signal to enhance the wideband components. Systems of this type are described in [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. One way of forming such a replica is to exploit the disparity in predictability of the NBI and the spread spectrum signal. In particular, since the spread data signal has a nearly flat spectrum, it cannot be predicted accurately from its past values unless, of course, use is made of knowledge of the PN sequence. The interfering signal, being narrowband, can be predicted accurately on the time scale of the PN signal. Hence, a prediction of the received signal based on previously received values will, in effect, be an estimate of the interfering signal provided the prediction horizon is beyond the correlation time of the PN signal. By subtracting predicted values of the received signal obtained in this way from the actual received signal and using the resulting prediction error as the input to the PN correlator, the effect of the interfering signal can be reduced significantly. This procedure is, in effect, performing a *whitening* operation on the received signal.

An alternative estimator/subtracter implementation is formed by replacing the NBI replica formed by prediction with one formed by interpolation [19, 20, 27]. This latter approach has performance advantages over the predictor/subtracter, including enhanced processing gain and a more desirable phase profile. The only disadvantage is delay on the order of a few correlation times of the PN signal, which is easily tolerated since it is much less than a symbol duration.

The estimator/subtracter form of notch filter has primarily involved the use of linear transversal prediction or interpolation filters to create the NBI replica. Such a filter forms a linear prediction of the received signal based on a fixed number of previous samples, or a linear interpolation based on a fixed number of past and future samples. This estimate is subtracted from the appropriately timed received signal to obtain the error signal to be used as input to the PN correlator. The filter tap coefficients are typically updated using a suitable adaptive algorithm, such as the least mean squares (LMS) algorithm. The cost of this processing is the introduction of some distortion into the spread spectrum signal. This distortion is negligible when PN sequences of sufficient lengths are used to spread the desired signal. In particular, the length of the PN sequence must be much greater than the length of the filter, a requirement easily met in any practical spread spectrum system [21]. These approaches will be discussed in further detail in the next section.

In transform-domain NBI suppression techniques (see Fig. 4) the principal approach is to take the Fourier transform of the received signal, to apply a mask in the frequency domain to notch out the NBI, and then to inverse transform the result back to the time domain for correlation with the PN code (see, e.g., [3, 14, 18, 30, 31]). A useful mask to consider is an adaptive one that excises those Fourier components whose energy levels exceed a set threshold [12]. Alternatively, a whitening mask can be used by first applying a nonparametric spectrum estimator to the received signal, from which such a mask can be derived [18]. Depending on the overall system bandwidth and on the consequent processing speed requirements, the Fourier transforms required by this techniques can be performed in hardware such as surface-acoustic-wave (SAW) technology, or in software using the fast Fourier transform (FFT).

Each of these two general types of NBI suppression techniques is effective in improving the performance of spread spectrum systems in the presence of NBI. Neither is uniformly superior to the other, and which is best for a given application depends largely on implementation considerations, some of which are discussed in [12] and [18]. A hybrid system involving both transform-domain and time-adaptive filtering has been considered recently in [32]. In this work, an LMS adaptive filter is used to suppress frequency components that are correlated from FFT frame to FFT frame. By combining favorable features of each approach, this technique yields a lower complexity system for multiple narrowband interferers, when compared to purely time-domain systems with similar performance characteristics.

In the following two sections we describe two more recently developed methodologies for NBI mitigation, both of which make use of signal modeling to yield performance improvements. The first of these, which is described in the following sections, falls within the category of an estimator/subtracter technique, but it takes advantage of the non-Gaussian nature of the spread spectrum signal to improve the estimator performance through nonlinear filtering. The second technique exploits the fact that the NBI signals to be encountered in cellular CDMA applications are likely to be digital communications signals. In prior work, the NBI has typically been modeled either as a sinusoidal signal, or as a narrowband autoregression. Either of these models is well-suited to treatment via standard signal processing methods. However, if the NBI is also a digital communications signal, this structure can be exploited to further improve the performance of active NBI suppression.

Nonlinear Estimation Techniques

In this section we describe nonlinear filtering methods that offer improved suppression capability over linear methods in the estimator/subtracter configuration of Fig. 3. This work was developed in [33, 34, 35], where the narrowband signal is modeled as an *autoregressive (AR)* process, that is, as the output of an all-pole linear filter driven by additive white Gaussian noise. This model is similar to that used in analysis described in the previous section. For the situation in which the statistics of this AR process are known to the receiver, this work prescribes a time-recursive nonlinear filter whose nonlinearity takes the form of soft decision feedback of an estimate of the spread spectrum signal. The much more common situation is that in which the receiver is ignorant of the parameters that characterize the autoregressive signal. For this situation this approach results in an adaptive nonlinear filter using a standard LMS adaptation algorithm to predict the interferer by incorporating the soft decision feedback into this algorithm.

The techniques to be presented in this section result from a fresh examination of the prediction of the interferer. A linear prediction method is optimal in the minimum mean square error (MMSE) sense when trying to predict a Gaussian autoregressive process in the presence of additive white Gaussian noise. When the prediction is done in a non-Gaussian environment, linear methods are no longer optimal and we turn to nonlinear methods. For narrowband interference added to a spread spectrum signal, the prediction of the interferer takes place in the presence of *both* Gaussian and non-Gaussian noise. The non-Gaussian noise is the spread spectrum signal itself.

As is the case for linear filtering, nonlinear filtering techniques exploit the predictability of the narrowband interferer *vis-à-vis* the spread spectrum signal. The prediction is subtracted from the observation and what remains, which in the case of ideal prediction would be the spread spectrum signal plus additive white Gaussian noise, is sent on to a multiuser receiver. The multiuser receiver is selected from the variety of possible receivers noted previously.

In the next section we describe an appropriate system model in which to discuss this methodology and we focus on the case where the statistics of the interferer are known. A nonlinear, time-recursive filter is described along with its performance. We compare and contrast the linear Kalman-Bucy filter with this nonlinear filter. An adaptive version of the nonlinear filter, to be used when the statistics of the NBI are not known, is presented in the subsequent section. These filters (both linear and nonlinear, adaptive and nonadaptive) can be cast into an interpolating, vice predicting, structure for added gain and better phase characteristics. Finally, we discuss open research questions in the analysis of the nonlinear filters.

Known Statistics

System Model

Consider a received signal that is passed through a filter matched to the chip waveform and chipsynchronously sampled once during each chip interval, per Fig. 5. The equivalent discrete time received signal will have components due to the spread spectrum signal, s_k, the narrowband interference, i_k, and the ambient white noise, n_k. The observation at sample k is then given by

$$z_k = i_k + s_k + n_k.$$

The noise can be modeled as being additive white Gaussian noise (AWGN) with variance σ_n^2, and the signal s_k as having the following binomial density function

$$p(s_k) = 2^{-N} \cdot \sum_{j=1}^{N} \binom{N}{j} \delta(s_k - N + 2j)$$

where N is the number of users in the CDMA system. This model assumes equal energies for all users for the sake of simplicity in illustrating the techniques, however, this is not a necessary assumption. As noted in the preceding, the interference is taken to have bandwidth much less than the spread bandwidth. The three signals can be assumed to be mutually independent.

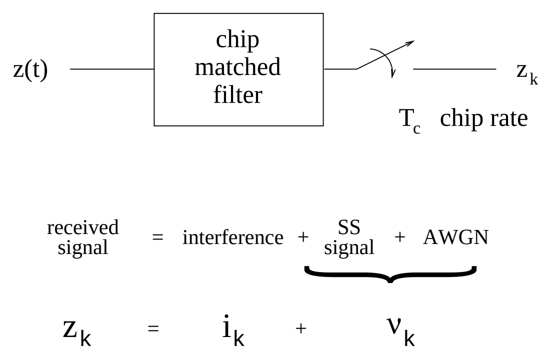

Figure 5: Discrete Time Model

Vijayan and Poor cast the interference suppression problem in state space form for use with the Kalman-Bucy filter [33]. As noted previously, the interference is modeled as an AR process, *i.e.*,

$$i_k = \sum_{j=1}^{p} \phi_j \cdot i_{k-j} + e_k$$

where e_k is a white Gaussian process and $\phi_1, \ldots, \phi_p$ are the coefficients of the regression. A state space representation of this system is given in the following equations and definitions. The state vector, $\underline{x}_k$, consists of i_k and the $p-1$ previous interference samples, where p is the order of the AR model, *i.e.*,

$$\underline{x}_k = [i_k \;\; i_{k-1} \;\; \ldots \;\; i_{k-p+1}]^T \;\; .$$

The state transition matrix Φ is determined by the coefficients in the AR model (*i.e.*, it is the companion matrix of the vector $[\phi_1, \ldots, \phi_p]$), and given by

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \ldots & \phi_{p-1} & \phi_p \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix} .$$

The vector $\underline{w}_k$ is formed from the white Gaussian noise process driving the AR model and is completely characterized by its variance σ_i^2:

$$\underline{w}_k = [e_k \;\; 0 \;\; \ldots \;\; 0]^T .$$

Finally, on defining $H = [1 \;\; 0 \;\; \ldots \;\; 0]$, we can write the state and observation equations as

$$\underline{x}_k = \Phi \underline{x}_{k-1} + \underline{w}_k \tag{1}$$

and,

$$z_k = H \underline{x}_k + \nu_k \;\; . \tag{2}$$

The observation noise, ν_k, is a sum of the white Gaussian measurement noise, n_k, and the spread spectrum signal, s_k, as is clear from Fig. 5. Optimality of linear prediction schemes would require that the observation noise be Gaussian. For spread spectrum signals with power comparable to the AWGN power, this approximation is very poor and nonlinear filters are called for. We first consider the case when the the AR parameters Φ are known and present a nonlinear filter that takes into account the non-Gaussian observation noise. In the subsequent section an adaptive version of this nonlinear filter is described.

Kalman-Bucy and ACM Filters

For the state space system of (1)-(2), the Kalman-Bucy filter (see for example [15]) is the optimal *linear* predictor of the state (and observation), and it is the *optimal* predictor when the observation noise is Gaussian. In this case, however, the observation noise density is the convolution of that of the spread spectrum signal with the Gaussian density. The smaller the measurement noise power in relation to the spread spectrum signal, the more pronounced is this deviation from a Gaussian environment.

Recall that the MMSE estimator of the state (1) at a fixed time k given the previous observations is $E[\underline{x}_k | z_0^{k-1}]$. If the observation noise ν_k were Gaussian, this would imply that the state and observations were jointly Gaussian. In this case, the conditional mean (and hence the MMSE estimator) would also have a Gaussian distribution. The Kalman-Bucy recursions are based on this model of Gaussian observation noise. For the system model used here the measurement noise is clearly not Gaussian, and the optimal filter (that is, the exact conditional mean) is nonlinear with complexity that increases exponentially in the time index [36]. For the general state space filtering formulation with non-Gaussian measurement noise, Masreliez proposed an approximation to this optimal filter [37] that greatly reduces complexity. In particular, Masreliez proposed that some, but not all, of the Gaussian assumptions used in the derivation of the Kalman-Bucy filter be retained in defining a nonlinear recursively updated filter. He abandoned the requirement that the observation noise be Gaussian. However, he retained a Gaussian distribution for the conditional mean, although it is not a consequence of the probability densities of the system (as is the case for Gaussian observation noise); hence the name *approximate conditional mean (ACM)* that is applied to this filter.

Using the Masreliez assumption one can derive a nonlinear ACM filter with recursive updates. The time updates are:

$$\underline{\overline{x}}_{k+1} = \Phi \underline{\hat{x}}_k ,$$

and

$$M_{k+1} = \Phi P_k \Phi^T + Q_k .$$

The measurement updates are given by:

$$\underline{\hat{x}}_k = \underline{\overline{x}}_k + M_k H^T g_k(z_k) ,$$

and

$$P_k = M_k - M_k H^T G_k(z_k) H M_k .$$

The predicted estimate $\underline{\overline{x}}_k$ is the mean of $\underline{x}_k$ conditioned on previous observations, $E[\underline{x}_k | z_0^{k-1}]$, and M_k is its covariance. The vector $\underline{\hat{x}}_k$ is the filtered estimate and its conditional covariance matrix is P_k. Q_k is the covariance matrix of the state input (the AR process).

Note that the time updates of the ACM filter are identical to those of the Kalman-Bucy filter. The terms G_k and g_k denote nonlinearities arising from the (non-Gaussian) distribution of the observation noise and are given as

$$g_k(z_k) = -\left[\frac{\partial p(z_k|z_0^{k-1})}{\partial z_k}\right] \cdot \left[p(z_k|z_0^{k-1})\right]^{-1} ,$$

and

$$G_k(z_k) = \frac{\partial g_k(z_k)}{\partial z_k}$$

where $p(z_k|z_0^{k-1})$ denotes the measurement prediction density. The measurement updates reduce to the standard equations for the Kalman-Bucy filter when the observation noise is Gaussian.

For the sake of illustration, consider a system with one CDMA user. In this case, the observation noise is the sum of a Gaussian random variable and one that takes values of ± 1 with equal probability. Its density is given by the following Gaussian mixture

$$p_{\nu_k}(\nu) = \frac{1}{2}\left[\mathcal{N}_{\sigma_n^2}(\nu-1) + \mathcal{N}_{\sigma_n^2}(\nu+1)\right] \tag{3}$$

where

$$\mathcal{N}_{\sigma^2}(x) \triangleq \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} .$$

Defining ϵ_k as the innovation (or residual) signal and σ_ν^2 as its variance, *i.e.*,

$$\epsilon_k \triangleq z_k - H\underline{\hat{x}}_k \qquad \text{and} \qquad \sigma_\nu^2 \triangleq H M_k H^T + \sigma_n^2 ,$$

we can write the functions g and G in this case as

$$g_k(z_k) = \frac{1}{\sigma_\nu^2}\left[\epsilon_k - \tanh\left(\frac{\epsilon_k}{\sigma_\nu^2}\right) \right] ,$$

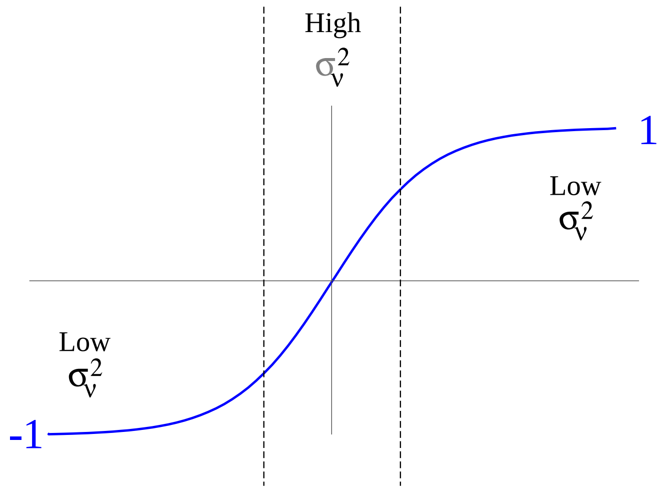

Figure 6: Soft Decision Feedback via *tanh*

and

$$G_k(z_k) \quad = \quad \frac{1}{\sigma_\nu^2}\left[1 - \frac{1}{\sigma_\nu^2}\mathrm{sech}^2\left(\frac{\epsilon_k}{\sigma_\nu^2}\right)\right] \quad .$$

Note that without the nonlinear terms *tanh* and *sech* the ACM recursions reduce to the (linear) Kalman-Bucy recursions [15].

The ACM filter provides decision feedback in the *tanh* term; that is, it corrects the measurement by a factor in the range [-1,1] that estimates the spread spectrum signal. (Recall that this chip estimation is an incidental step in the suppression process.) When the ACM filter is performing well, the variance term in the denominator of the *tanh* is low. This means the argument of the *tanh* is larger, driving the *tanh* into a region where it behaves like the $sgn(\cdot)$ function, as seen in Fig. 6. In this region, the spread spectrum chip is estimated to be +1 if the residual signal ϵ_k is positive, and -1 if the residual is negative. On the other hand, when the filter is not making good estimates, the variance is high and *tanh* is in a linear region of operation. In this region, the filter hedges its bet on the accuracy of $sgn(\epsilon_k)$ as an estimate of the spread spectrum signal. Here the filter behaves essentially like the (linear) Kalman-Bucy filter.

In the preceding all of the power variables (σ_n^2 and σ_i^2) are referenced to the spread spectrum power, which we take to be unity. When the power of the spread spectrum signal is not known at the receiver the *tanh* term must be multiplied by an estimate of the amplitude. An adaptive algorithm for performing this estimate is described in [38, 39]. In [38] it was observed that soft decision feedback systems outperformed hard decision feedback filters developed previously [22, 38].

The previous discussion was for the case of one CDMA user, however systems with N users have a very similar structure. In this case the observation noise is a more complicated Gaussian mixture than that given in (3), leading to a more complicated nonlinearity. For example, the *tanh* function is replaced by a smooth quantizer that returns an estimate of the total spread spectrum signal. Consider the case where all users have the same power which we can take to be unity. When the variance is high, this function is nearly linear between the extreme values of $-N$ and N. When the variance is low, it acts like a step function taking values $\{-N, -N+2, \ldots, N-2, N\}$. Details of this analysis can be found in [35].

As the number of spread spectrum users increases, the total power of the spread spectrum signal also grows. One would expect that as the power increases, the spread spectrum signal is even more easily distinguishable from the noise and the ACM filter will be tracking s_k with greater accuracy. Contrast this with the performance of the Kalman-Bucy (linear) filter where the increased power of the spread spectrum signal causes the measurement noise to be even more highly non-Gaussian. Its performance will degrade as the number of users increases.

The figure of merit in comparing estimator/subtracter methods is the SNR improvement, *i.e.*, the ratio of the SNR at the output of filtering to the SNR at the input. While higher input SNR leads to lower bit error rate, the quantitative improvement in probability of error will be smaller. This is due to the fact that the processing gain of the spread spectrum signal provides some interference suppression in its own right, from which both linear and nonlinear processing will benefit. It is shown in [40, 41] that the reduction in probability of error is not significant, despite larger SNR, when a very small processing gain (for example, length 7) is used. However, for moderate and large processing gain, the SNR is shown to be a useful measure of performance.

In order to assess the performance gains afforded by the nonlinear techniques described in the preceding, Fig. 7 provides the results of simulations for a second order AR interferer with both poles at 0.99 (*i.e.*, ϕ_1=1.98 and ϕ_2=-0.9801). In this simulation, the noise power was held constant at $\sigma_n^2 = 0.01$, while the total of noise plus interference power was varied from -20 dB to 5 dB (all relative to a unity power for a single spread spectrum signal).

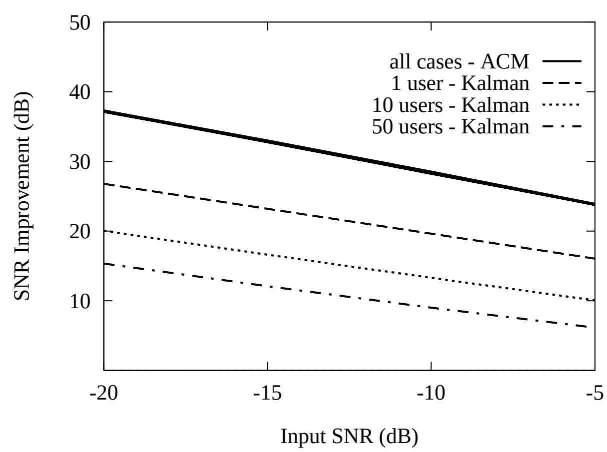

Figure 7: Predictor Performance for Known Statistics - Multiple spread spectrum Users

As expected the Kalman-Bucy filter's performance falls off as the number of users (and hence the spread spectrum power) increases. The ACM filter's performance, in contrast, changes imperceptibly and all curves overlap. The ACM filter exhibits significant improvement in SNR over the Kalman-Bucy filter, more so as the number of users increases.

Adaptive Filtering

When the statistics for the AR process modeling the NBI are not known, an adaptive algorithm must be used in place of the fixed filters described previously. Adaptive versions of the Kalman-Bucy and ACM filters suffer from slow convergence in this application, and thus fixed-length transversal filters are more useful. In this context, the LMS algorithm is one of the simplest adaptive algorithms to analyze and implement. A linear predictor using an LMS filter of length L has the system diagram given in Fig. 8.

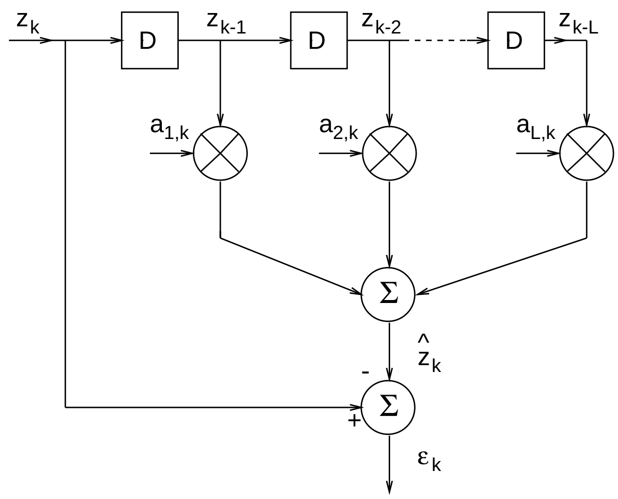

Figure 8: Linear LMS Predictor

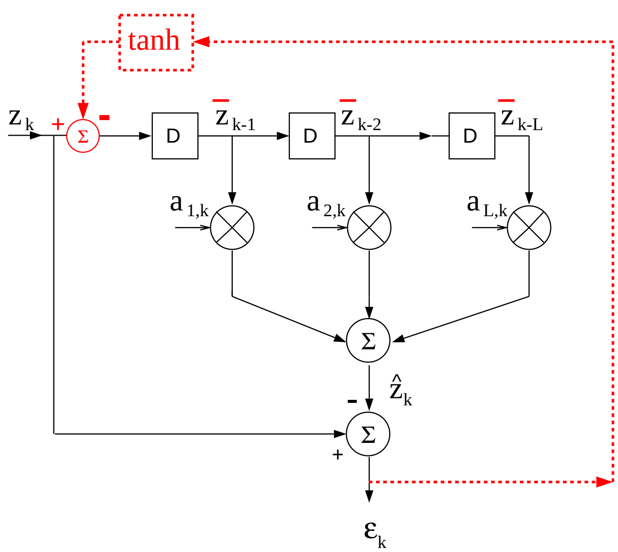

Figure 9: Nonlinear Adaptive Predictor

The residual of the prediction, ϵ_k, is sent to the input of the spread spectrum receiver. The observation vector X_k and the tap weight vector θ_k are defined as follows.

$$X_k \triangleq [z_{k-1}\ z_{k-2}\ \ldots\ z_{k-L}]^T$$
$$\theta_k \triangleq [a_{1,k}\ \ a_{2,k}\ \ \ldots\ \ a_{L,k}]^T$$

The LMS estimate $\hat{z}_k$ is determined by

$$\begin{aligned} \hat{z}_k &= X_k^T \cdot \theta_{k-1} \\ \theta_k &= \theta_{k-1} + \mu_k(z_k - \hat{z}_k) \cdot X_k \end{aligned} \tag{4}$$

where μ_k is a normalized step size.

The noise in this LMS algorithm is the observation noise ν_k which is the sum of Gaussian and non-Gaussian noise. The nonlinearity derived for the ACM filter can be incorporated into this LMS structure and thereby substantially remove the spread spectrum signal from the adaptation process.

Once again for the sake of simplicity we consider the case of one CDMA user, and refer the reader to [35] for the solution of the general system of N CDMA users. The soft decision feedback of the spread spectrum signal is introduced into the LMS algorithm as illustrated in Fig. 9.

Let $\overline{z}_k$ represent the observation less the soft decision on the spread spectrum signal, that is,

$$\overline{z}_k \triangleq z_k - \tanh\left(\frac{\epsilon_k}{\sigma_k^2}\right) .$$

By altering the tap weight update (4) to be based on the residual *less* the soft decision feedback, we have an update equation given by

$$\theta_k = \theta_{k-1} + \mu_k(\overline{z}_k - \hat{z}_k)\,[\,\overline{z}_{k-1}\ \ \overline{z}_{k-2}\ \ \ldots\ \ \overline{z}_{k-L}\,]^T \ .$$

When the decision feedback is accurate, the filter adaptation is indeed being done in essentially Gaussian noise, *i.e.*, without the spread spectrum signal.

To assess this alternate adaptive algorithm, simulations are presented for the same AR model for interference given in the previous section. Results given in Fig. 10 show that this adaptive algorithm achieves the same performance as do the recursive filters that make use of the statistics of the interferer.

Interpolating Filters

As mentioned earlier, linear interpolating filters were found to have good phase characteristics as well as greater SNR improvement for NBI suppression than do linear predicting filters [19, 20, 27]. The nonlinear filtering techniques described previously can also be applied to an interpolating, vice predicting, filter. In the nonlinear interpolators (both for known statistics and the adaptive version) a block of data is processed both forward and backward through the filter equations. The results are combined into one interpolated estimate of the interferer, as described in the following paragraph.

In [35] interpolating versions of the Kalman-Bucy and ACM filters are found. For the nonlinear filter, the assumption is made (analogously to that in the ACM filter derivation) that the densities of both forward and backward estimates are Gaussian. The interpolated estimate at any given time index (*i.e.*, the expected value of the state conditioned on previous and following observations), is then shown to be Gaussian with the following mean and covariance,

$$mean = \mu_f^T \Sigma_f^{-1}[\Sigma_f^{-1} + \Sigma_b^{-1} - \Sigma^{-1}]^{-1} + \mu_b^T \Sigma_b^{-1}[\Sigma_f^{-1} + \Sigma_b^{-1} - \Sigma^{-1}]^{-1}$$

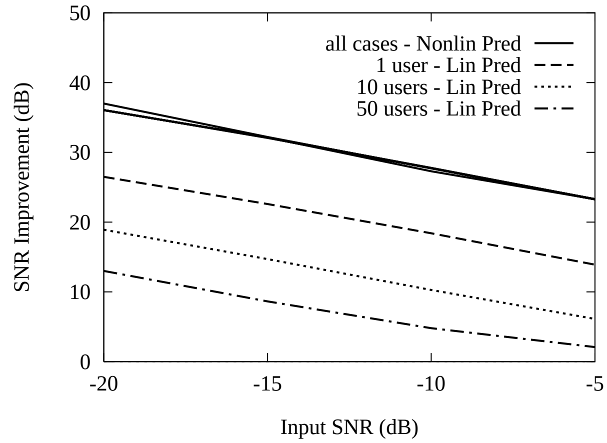

Figure 10: Adaptive Predictor Performance - Multiple spread spectrum Users

$$covariance = [\Sigma_f^{-1} + \Sigma_b^{-1} - \Sigma^{-1}]^{-1}$$

where μ_f and μ_b are the forward and backward estimates, Σ_f and Σ_b their covariances, and Σ is the covariance of the AR process describing the interference. In the case of the nonlinear filter, the forward and backward estimates and covariance matrices are determined by nonlinear filter recursions.

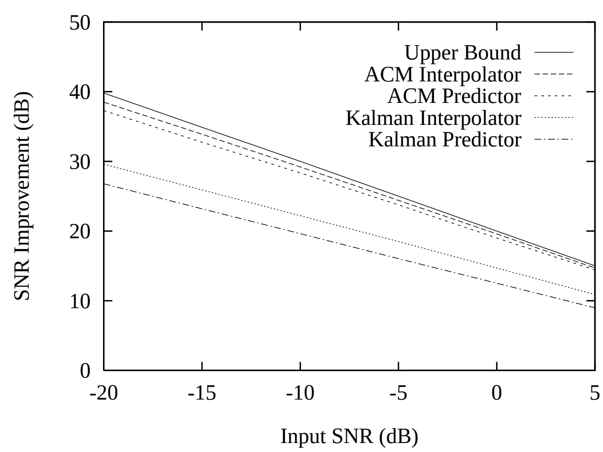

Figure 11: Interpolator Performance for Known Statistics

Simulation results for these interpolators in the case of known statistics, using the system parameters described earlier, are given in Fig. 11. It is clear that for both the Kalman-Bucy and ACM filters the interpolating version outperformed the predictor. The solid line in Fig. 11 gives an upper bound on the SNR improvement when the narrowband interference is predicted with noiseless accuracy, that is, as $\sigma_n \to 0$. This is calculated by setting $E(|\epsilon_k - s_k|^2)$ equal to the power of the AWGN driving the AR process, *i.e.*, the unpredictable portion of the interference. For the system parameters used in these simulations the

ACM predictor already performs well, and there is little margin or improvement via use of an interpolator. The adaptive filter in Fig. 12 shows greater margin for improvement, on which the interpolator capitalizes.

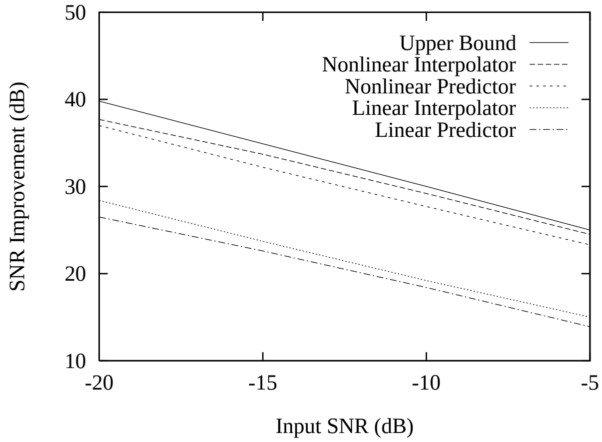

Figure 12: Adaptive Interpolator Performance

However, in either case, the interpolator does offer improved phase characteristics and some performance gain at the cost of additional complexity (less than three times as many operations) and a delay (due to block length) in processing. The delay should be inconsequential in most spread spectrum applications and, depending on the hardware implementation of the ACM filter, the added complexity may also prove to be acceptable.

Analysis

Many fundamental results for the ACM filter remain open questions. In the case of Kalman-Bucy filtering, it is well known that the covariance matrix will reach a steady state value if the eigenvalues of the state transition matrix are less than unity. No similar criterion is know for the ACM filter. For Kalman-Bucy filtering the covariance matrix is determined by the following well known discrete time Riccati equation,

$$\Sigma_{k+1|k} = \Phi \left[\Sigma_{k|k-1} + \Sigma_{k|k-1} H^T K_k H \Sigma_{k|k-1} \right] \Phi^T + Q \quad ,$$

where $K_k = (H \Sigma_{k|k-1} H^T + N + \sigma_n^2)^{-1}$ is independent of the data and is deterministic. This is not the case for the ACM filter, whose covariance is governed by

$$M_{k+1|k} = \Phi \left[M_{k|k-1} + M_{k|k-1} H^T G_k(\epsilon_k) H M_{k|k-1} \right] \Phi^T + Q$$

where G_k is a nonlinear function of the observation. The study of this *nonlinear stochastic difference equation* poses a research topic in its own right. During simulations the ACM covariance can be observed to converge to a value equal to that of a Kalman-Bucy filter with an input composed solely of the narrowband interference and AWGN. That is, in the steady state the ACM performed like a Kalman-Bucy filter acting on observations from which the spread spectrum signal had been removed. Criteria to assure a probabilistic convergence of ACM covariance and a characterization of the rate of convergence are needed, as well as convergence analysis of the adaptive nonlinear filter.

Simulations of nonlinear filtering systems demonstrate that the ACM filter can dramatically outperform the Kalman-Bucy filter. To what extent are these results dependent on the parameters chosen for the simulation? Clearly the nonlinear method is effective because the measurement noise, AWGN and spread spectrum signal, is non-Gaussian. If the spread spectrum signal actually lies below the noise floor, then the Gaussian assumption is more reasonable, and the Kalman-Bucy filter may actually outperform the nonlinear filter. Simulations show that for $\sigma_n^2 > 1$ the Kalman-Bucy filter and the nonlinear filter have virtually the same performance, and for values just below one the Kalman-Bucy sometimes has a slight edge in performance. Note, however, that the ACM filter is never significantly outperformed by the Kalman-Bucy filter since it reduces to the Kalman-Bucy filter in the limit of vanishing signal power.

Similarly, the disparity of bandwidth between the existing users and the spread spectrum signal is essential for the success of this filtering method and altering the AR model parameters (poles) will impact performance. Until criteria are known to guarantee (in a probablistic sense) that the ACM performance will be better than that of the Kalman-Bucy filter, we can only extend these simulation results by appealing to our intuition that the conditions of 1) low background (AWGN) noise relative to the spread spectrum signal and 2) the narrow bandwidth of the interferer relative to the spread spectrum signal are favorable conditions for this scheme to work.

Multiuser Detection Techniques

Most of the methods of interference suppression discussed so far have been based on the estimator/subtracter structure in Fig. 3. Analysis of these filtering techniques has involved modeling the narrowband signal as either a deterministic sinusoidal signal or an *autoregressive signal (AR)*, *i.e.*, the output of a linear filter driven by additive white Gaussian noise. These models greatly simplify analysis and have characteristics that capture the narrowbandedness of the interferer. Consider, however, the situation where the interferer is actually a digital communications signal with a data rate much lower than the spread spectrum chip rate. This signal is indeed a narrowband interferer, but it is poorly modeled as either a sinusoid or an AR process. In such a situation the structure of the digital interferer can be exploited to develop a spread spectrum receiver that optimally rejects the interference. The estimator/subtracter structure is only optimal if the estimation is errorless.

The estimator/subtracter separates the problem of narrowband interference suppression from the detection and estimation of the spread spectrum signal. This is understandable for the interference models proposed, but it is not appropriate for a digital interferer. When the interference is digital it is not unlike the spread spectrum signal itself, also a digital signal. Because of this similarity, techniques from multiuser detection theory that work well in defeating multiple access interference (which represents wideband interference) have also proved to be effective against the narrowband interference. This idea has been explored in [42], and we review this approach here.

In order to apply methods from multiuser detection theory, the single narrowband interferer is treated as a collection of virtual spread spectrum users. The case of one true spread spectrum user and one narrowband interferer has been studied in [42, 43], however, these results can be extended to more general situations. Although such a system of one true spread spectrum user and one digital, narrowband interferer is not a code division multiple access system in the usual sense, in the following section we describe how this can be thought of as a system of $m+1$ spread spectrum users, where m is a function of the relative data rates of the two signals. The techniques of multiuser, CDMA detection have been applied to this model to derive a new receiver for the spread spectrum user that encompass the narrowband suppression function.

Ideally we would like to use a receiver that optimally, either in a maximum likelihood or minimum probability of error sense, detects the desired signal in the presence of both additive white Gaussian noise and multiple access interference. Such a receiver is described by Verdú in [9]. Unfortunately the decision algorithm for this sequence detector has complexity that is exponential in the number of users. For the system under consideration the receiver would have complexity exponential in the number of narrowband data symbols per spread spectrum data symbol, a number that can be quite large. To reduce this computational burden a linear detector known as the *decorrelating detector*, proposed by Lupas and Verdú in [8], was considered for this new application of multiuser detection theory. The decorrelating detector performs as well as the optimal detector in the limit where system errors are due to interference from other signals vice thermal noise, and for large interference power.

Since in general the spread spectrum signal will not be synchronized to the narrowband interference, we will discuss only the asynchronous case here. A detector for a synchronous system is treated in [42]. For an asynchronous system a one-shot approach to bit decisions is not optimal, and decisions are instead made for a frame of bits at a time. The outputs of a bank of matched filters, where each filter is matched to a spreading code of an active user, is known to form a sufficient statistic for detecting the spread spectrum bit. Therefore any asynchronous multiuser detector can be cast in the form illustrated in Fig. 13. As the frame of data becomes infinitely large the decision algorithm of the decorrelating detector approaches an infinite impulse response (IIR) filter [8].

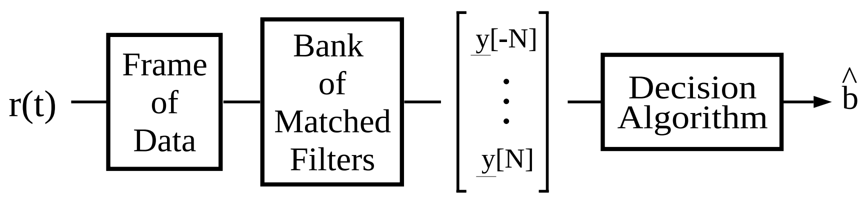

Figure 13: Asynchronous Multiuser Detector

Finally, in addition to the decorrelating detector we will also discuss the *conventional,* or *matched filter, detector*. The decision algorithm for this detector is quite simple. Only the output of the filter matched to the desired user is considered. The algebraic sign of this output is used as the bit estimate. This detector is optimal only in the case of a single spread spectrum user in AWGN. Due to its simplicity it is often adopted despite its poor performance in the presence of strong multiple access interference.

The following section introduces the model used to describe a system of one true spread spectrum user and one digital interferer. The structure of the virtual CDMA system leads to a solution for the IIR filter coefficients of the decorrelating detector. Closed form expressions for the probability of error for the decorrelating and matched filter detectors, as well as a lower bound on the probability of error for a predictor/subtracter are given in the subsequent section. Finally, these curves are plotted for system parameters derived from a field test of a spread spectrum system overlaid on a narrowband service.

System Model

Consider a system with one spread spectrum signal and one narrowband binary communications signal in an otherwise AWGN channel. Each data bit of the spread spectrum user is modulated by a pseudonoise signature sequence (each entry being one *chip*), which spreads the signal in the frequency domain. The transmission rate of the spread spectrum signal is the data rate times the length of the signature sequence.

We assume a relationship between the *data rates* of the two users, *i.e.*, m bits of the narrowband user occur

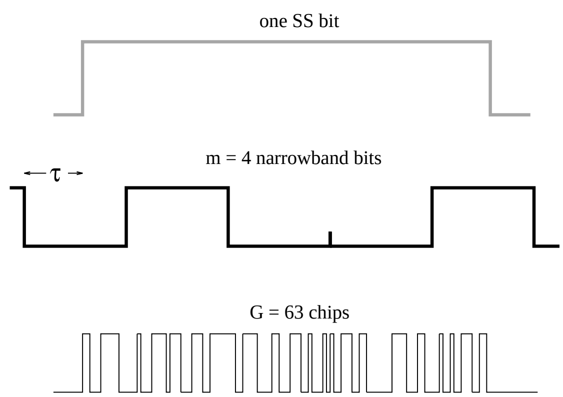

Figure 14: Signal Composition

for each bit of the spread spectrum user. Given that most digital data is sent at rates that are powers of two, it is reasonable to employ an integer relationship between the bit rates; indeed, m is most likely to be a power of two. Let T be the bit duration of the spread spectrum user, which implies T/m is the bit duration of the narrowband user. We call τ the fixed, known time lag between the spread spectrum bit and the nearest previous start of a narrowband bit, *i.e.*, $0 \leq \tau \leq T/m$.

Figure 14 illustrates these ideas for $m = 4$. For the sake of illustration we feature rectangular waveforms at baseband, however all remarks hold for arbitrary waveforms and carrier frequencies that are offset. We can think of the m bits of the narrowband user as arising from m different users. For instance they could be m "virtual" users in a time division multiple access system. We can also consider them to be m users in a CDMA system. The signature waveforms of these users are not PN sequences hence this signal is *not* spread in the frequency domain. These virtual signature waveforms are zero everywhere except one subinterval of the spread spectrum bit.

In Fig. 15 each of the narrowband bits has been separated into a virtual CDMA user. The first virtual user's signature sequence is one during the first "chip" interval and zero everywhere else. Similarly each bit can be thought of as a virtual user with a signature sequence with only one non-zero entry. These form a set of orthogonal users, uncorrelated with one another. However, in general, the i^{th} virtual user will have some cross-correlation with the spread spectrum user. If we call $\underline{\rho}$ the vector of cross-correlations during a referenced spread spectrum bit, defined explicitly in (5), and $\underline{\gamma}$ the vector of cross-correlations during the previous bit interval, defined in (6), we see that the two cross-correlation matrices $R[0]$ and $R[1]$ for this virtual multiuser system have very simple structures,

$$R[0] = \begin{bmatrix} I_m & \underline{\rho} \\ \underline{\rho}^T & 1 \end{bmatrix}$$

where I_m is the $m \times m$ identity matrix, and

$$R[1] = \begin{bmatrix} 0 & \underline{\gamma} \\ 0 & 0 \end{bmatrix} \quad .$$

Let G be the processing gain of the spread spectrum signal. Then the chip interval has length T/G, versus T/m for the narrowband bit. By the assumption that the interferer is narrowband, we have $G \gg m$.

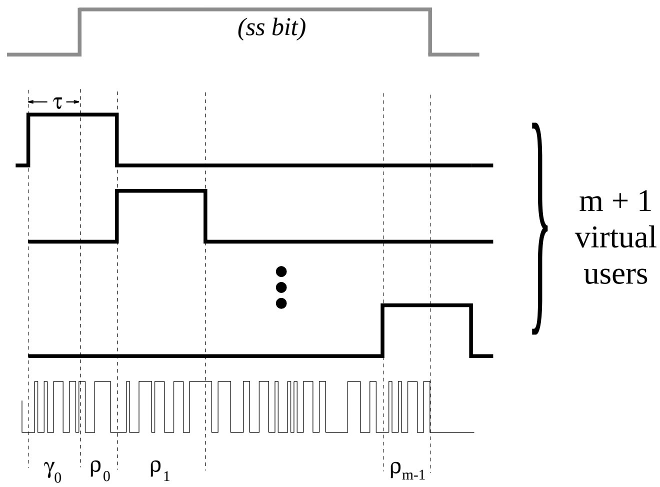

Figure 15: Virtual CDMA System

Let $s_1(t)$ be the normalized bit waveform of the narrowband user and $s_2(t)$ be the normalized chip waveform of the spread spectrum user, that is,

$$s_1(t) = 0 \;\; \forall \;\; t \notin (0, T/m) \qquad \text{and} \qquad \int_0^{T/m} s_1^2(t)dt = 1$$
$$s_2(t) = 0 \;\; \forall \;\; t \notin (0, T/G) \qquad \text{and} \qquad \int_0^{T/G} s_2^2(t)dt = 1 \quad .$$

Also, let σ_n^2 be the power of the AWGN in the shared channel; and, assuming that the received strength of the spread spectrum and narrowband signals remains constant for the larger bit interval, let w_1 be the received energy of the narrowband signal, and w_2 the received energy of the spread spectrum user, including the processing gain. Finally, let β_i be the i^{th} chip value of the spread spectrum signature sequence. The received signal, $r(t)$, is thus

$$\sum_{k=-N}^{N} \left\{ b_m[k]\sqrt{w_2} \sum_{i=0}^{G-1} \beta_i s_2 \left(t - i\frac{T}{G} - \tau - kT \right) + \sqrt{w_1} \sum_{j=0}^{m-1} b_j[k] s_1 \left(t - j\frac{T}{m} - kT \right) \right\} + n(t)$$

where the frame size is $2N+1$, and $b_0[k], \ldots, b_{m-1}[k]$ or $\underline{b}[k]$ is the vector of the narrowband bits, and $b_m[k]$ or $b_{SS}[k]$ is the spread spectrum user's bit, all during the k^{th} bit interval of the frame.

The outputs of the bank of matched filters shown in Fig. 13 form a sufficient statistic for determining the users' bits (see for example [15], pp 408-409). The filter bank consists of filters matched to the spreading codes of the active users. The cross-correlations mentioned earlier are defined by

$$\rho_k \;\; \triangleq \;\; \frac{1}{\sqrt{G}} \sum_{i=0}^{G-1} \beta_i \int_{-\infty}^{\infty} s_1 \left(t - k\frac{T}{m} \right) s_2 \left(t - i\frac{T}{G} - \tau \right) dt \tag{5}$$

and

$$\gamma_k \;\; \triangleq \;\; \frac{1}{\sqrt{G}} \sum_{i=0}^{G-1} \beta_i \int_{-\infty}^{\infty} s_1 \left(t - k\frac{T}{m} \right) s_2 \left(t - i\frac{T}{G} + T - \tau \right) dt \tag{6}$$

Due to the definition of the delay only the first component of $\underline{\gamma}$ will be non-zero; however for ease in some matrix equations we maintain the vector notation. If we form a matrix W of received energies,

$$W^{\frac{1}{2}} = \mathrm{diag}(\sqrt{w_1}, \ldots, \sqrt{w_1}, \sqrt{w_2}) \ ,$$

then the outputs of the matched filters form a vector $\underline{y}$ that can be written as

$$\begin{aligned} & R[1]^T W^{\frac{1}{2}} \begin{bmatrix} \underline{b}[1] \\ b_{SS}[1] \end{bmatrix} + R[0] W^{\frac{1}{2}} \begin{bmatrix} \underline{b}[0] \\ b_{SS}[0] \end{bmatrix} + R[1] W^{\frac{1}{2}} \begin{bmatrix} \underline{b}[-1] \\ b_{SS}[-1] \end{bmatrix} + \underline{n} \\ = & \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \gamma_0 \sqrt{w_1} b_0[1] \end{bmatrix} + R[0] W^{\frac{1}{2}} \begin{bmatrix} \underline{b}[0] \\ b_{SS}[0] \end{bmatrix} + \begin{bmatrix} \gamma_0 \sqrt{w_2} b_{SS}[-1] \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \underline{n} \end{aligned} \tag{7}$$

where $\underline{n}$ is a Gaussian vector with zero mean and covariance matrix $\sigma_n^2 R[0]$.

Decorrelating Detector

Because of the complexity of the maximum likelihood and minimum probability of error receiver, several other performance criteria have been proposed to gauge the effectiveness of suboptimal multiuser detectors [44]. One such quantity is the *efficiency* of a receiver which is determined by the ratio of *effective* signal-to-noise ratio to actual SNR. The effective SNR is defined as the signal level required in the presence of multiple access noise to achieve the same probability of error when the multiple access noise is removed.

The *asymptotic efficiency* is the limit of the efficiency as we allow the background AWGN to go to zero. This asymptotic quantity is not only more tractable analytically than the efficiency, but it also gives a better sense of how the detector performs in the stated environment, *i.e.*, when the dominant source of signal corruption is not AWGN but rather the narrowband interferer.

Consider the limiting form of the receiver in Fig. 13. As the size of the frame of data grows infinitely long *on either side* of the spread spectrum bit interval of interest, the decision algorithm for the decorrelating detector approaches an IIR filter [8]. The coefficients of this filter are a function of the asymptotic efficiency. If we were to look at the entire frame of data covering N spread spectrum bits and consider a concatenated vector of all the matched filter outputs for the entire frame, we could form one cross-correlation matrix defined by

$$\mathcal{R}_N = \begin{bmatrix} R[0] & R^T[1] & 0 & \ldots & 0 \\ R[1] & R[0] & R^T[1] & \ddots & 0 \\ 0 & R[1] & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & R^T[1] \\ 0 & \ldots & 0 & R[1] & R[0] \end{bmatrix} \quad \leftarrow N \quad \text{diagonal} \quad \text{submatrices}$$

In [8] it was found that the asymptotic efficiency of the decorrelating detector is determined by inverting this cross-correlation matrix $\mathcal{R}_N$.

It is shown in [42] that as $N \to \infty$ the asymptotic efficiency of a decorrelating detector in a system of one true spread spectrum user and one digital interferer is given by

$$\eta = \sqrt{\left(1 - \underline{\rho}^T \underline{\rho} - \underline{\gamma}^T \underline{\gamma}\right)^2 - 4(\underline{\rho}^T \underline{\gamma})^2} \tag{8}$$

and the filter coefficients are given by

$$f(n) = \frac{\xi^{|n|}}{\eta} \quad \text{where} \quad \xi = \frac{1 - \underline{\rho}^T\underline{\rho} - \underline{\gamma}^T\underline{\gamma} - \eta}{2\underline{\rho}^T\underline{\gamma}} \ .$$

Performance

It is of interest to compare the performance of the decorrelating detector to that of the two most common receivers to date, the matched filter and the predictor/subtracter followed by a matched filter. Exact expressions for the probability of error for the matched filter and the decorrelating detector are known; however there is no such closed form expression for the predictor/subtracter. Therefore we consider a crude lower bound on the probability of error for the predictor/subtracter, representing the theoretical limit on predictability of the narrowband signal. At the conclusion of this section, the exact probability of error for these detectors are plotted for a specific PN sequence. These results demonstrate how the multiuser detectors can outperform the conventional detector and the idealized predictor/subtracter detector.

Ideal Predictor/Subtracter

The predictor/subtracter receivers are effective because the disparity in bandwidth means that, when sampled at the chip rate of the spread spectrum signal, the interference is the only predictable portion of the total received signal. To obtain a crude lower bound on the probability of error resulting from the prediction process, assume for the moment that only the binary narrowband signal is present, and that all information is available about the waveform of this signal. We assume the interferer is matched filtered as appropriate to obtain an equivalent square pulse binary signal. This square pulse is sampled at the chip rate.

Because of the assumed perfect knowledge of the narrowband signal waveform, it is known when a sample is interior to the narrowband bit, and when a transition has occurred between samples. For an interior point perfect prediction can be achieved, and hence the only errors occur during samples with bit transitions, as illustrated in Fig. 16. In plots presented later, the curves labeled "Ideal Predictor" represent

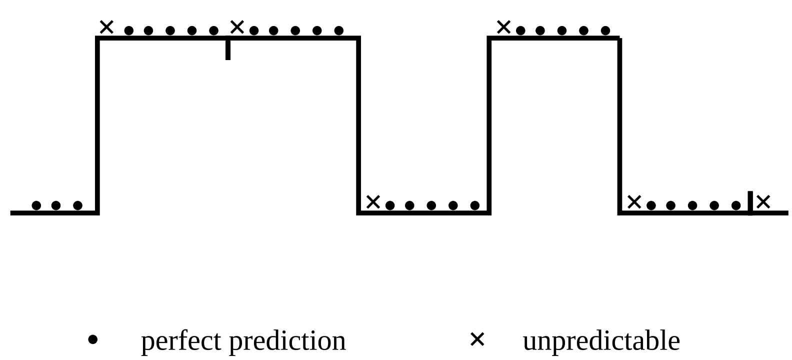

Figure 16: Ideal Predictor

a calculation of the probability of error under these assumptions.

Conventional and Decorrelating Detectors

The estimate for the spread spectrum bit given by the matched filter is $\hat{b}_{SS} = \text{sgn}(y_m[0])$, where $y_m[0]$

is computed using (7) to be

$$\begin{aligned} y_m[0] &= \sqrt{w_1}\gamma_0 b_0[1] + \sqrt{w_1}\underline{\rho}^T \underline{b}[0] + \sqrt{w_2} b_{SS}[0] + n_m \\ &= \sqrt{w_1}[\underline{\rho}^T \quad \gamma_0]\underline{\tilde{b}} + \sqrt{w_2} b_{SS}[0] + n_m \end{aligned}$$

where $\underline{\tilde{b}}$ is an $(m{+}1) \times 1$ vector of possible bit values. The probability of error is

$$\begin{aligned} Pr(\hat{b}_{SS} = 1 | b_{SS} = -1) &= Pr(n_m > \sqrt{w_2} - \sqrt{w_1}[\underline{\rho}^T \quad \gamma_0]\underline{\tilde{b}}) \\ &= \frac{1}{2^{m+1}} \sum_{i=0}^{2^{m+1}-1} Q\left(\frac{\sqrt{w_2} - \sqrt{w_1}[\underline{\rho}^T \quad \gamma_0]\underline{\tilde{b}}^i}{\sigma_n} \right) \end{aligned}$$

where $\{\underline{\tilde{b}}^i\}$ is an ordering of the 2^{m+1} possible vectors of narrowband bits.[2]

The probability of error for the decorrelating detector as the number of spread spectrum bits in a frame goes to infinity is determined by the asymptotic efficiency calculated in (8). Application of the limiting IIR filter yields the spread spectrum bit with power w_2 plus Gaussian noise with variance σ_n^2/η. Therefore the probability of error is

$$Q\left(\frac{\sqrt{w_2 \eta}}{\sigma_n}\right).$$

Simulations

Data provided in [11] for an overlaid CDMA system was used to select system parameters to study the performance of these detectors. The noise variance is chosen to be $\sigma_n^2 = 4$, as required for the spread spectrum signal to be 6 dB down from the ambient noise and to ensure no degradation in performance of the pre-existing narrowband user's communications system. The narrowband user's power is allowed to vary from parity with the spread spectrum signal, to 40 dB above it. All powers are referenced to a unity spread spectrum signal, before despreading. A processing gain of 63 is adequate to get reasonable system performance for these parameters. An m-sequence of length 63 is used as the spreading code and a square pulse for both $s_1(t)$ and $s_2(t)$. The narrowband signal is allowed to have 1, 2, 4, and 8 bits relative to the spread spectrum signal. Larger values make the probabilities of error very difficult to compute and they also conflict with the assumption of a narrowband signal (m/G is a measure of the narrowbandedness of the signal).

The performance of all three detectors (the matched filter, ideal predictor/subtracter, and decorrelating detector) depends on the fixed time delay τ; therefore Figs. 17 and 18 plot the range of probabilities of error for all delays, with the average performance indicated by the discrete points. In Fig. 17, $m = 1$ and $m = 2$, representing the most narrowband interferer, a regime where we would expect the predictor/subtracter to have its best performance. In Fig. 18, $m = 4$ and $m = 8$, and the interferer is, relatively speaking, less narrowband.

Consider first the performance of the matched filter detector. It is only for interference powers below the AWGN (6dB) that this detector performs well. It is near-far limited, by which we mean that as the interferer's power becomes arbitrarily large, the probability of error approaches one half.

As expected, the ideal predictor/subtracter does best for small values of m. As m becomes larger, the difference in bandwidth between the narrowband signal and the spread spectrum signal becomes less

[2] $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-v^2/2} dv$

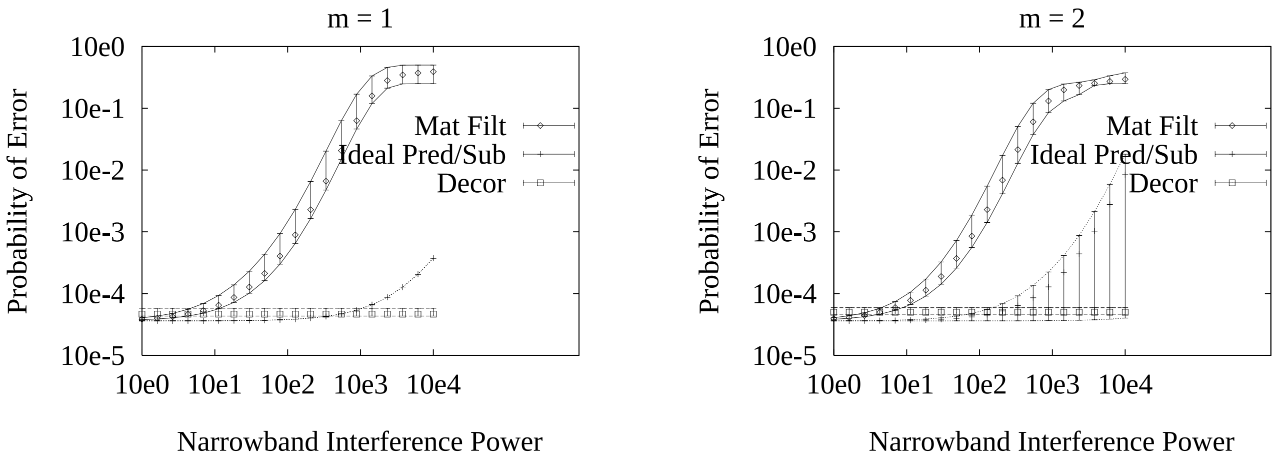

Figure 17: More-narrowband interferer

pronounced. The predictor/subtracter relies on this disparity in bandwidth to predict only the narrowband signal and not the spread spectrum signal. Though the ideal predictor/subtracter outperforms the decorrelating detector for low and moderate values of interference power, Monte Carlo simulations of the synchronous case show that actual predictor/subtracter performance will have much greater error probability.

Finally, consider the performance of the decorrelating detector. One characteristic of these detectors is that performance is independent of the interference power, as is reflected in these curves. The decorrelating detector outperforms the matched filter detector for all scenarios of interest, that is a strong interferer. For very strong interference powers, and for (relatively) less narrowband signals (*i.e.*, $m = 8$), the decorrelating detector also offers a significant increase in performance even over the idealized version of the predictor/subtracter performance. The decorrelating detector offers good performance over a large spectrum of interference powers and bandwidths.

In Fig. 19 we illustrate a causal implementation of the decorrelating detector for an asynchronous interferer. The IIR filter has been truncated and the output delayed by the length of the truncated filter. The computational burden of the proposed system is not excessive, and is on a par with (same order of magnitude) most predictor/subtracters. Implicit in this detector, however, is knowledge of the interfering signal's shape and timing. In essence we require the same information as the intended receiver of the narrowband signal. Given the premise that the existing user enjoys reliable communications over the channel, these parameters should be recoverable with reasonable hardware requirements.

Future research is needed to examine the feasibility of this approach in a system with more than one spread spectrum user and more than one narrowband interferer. In such a case, one would expect the size of the system to grow (matched filters), while the structure remains little changed. Also, it should be noted that the decorrelating detector is only one of several possibilities for use in this application. For example, an estimator/subtracter in which the estimator is a demodulator for the narrowband data signal would be very similar in philosophy to the iterative multiuser detectors of [45, 46, 47]. Unlike the decorrelating detector, such a detector would require amplitude estimates of the NBI which could be problematic in fading channels. Nevertheless, the results presented here indicate that the general class of

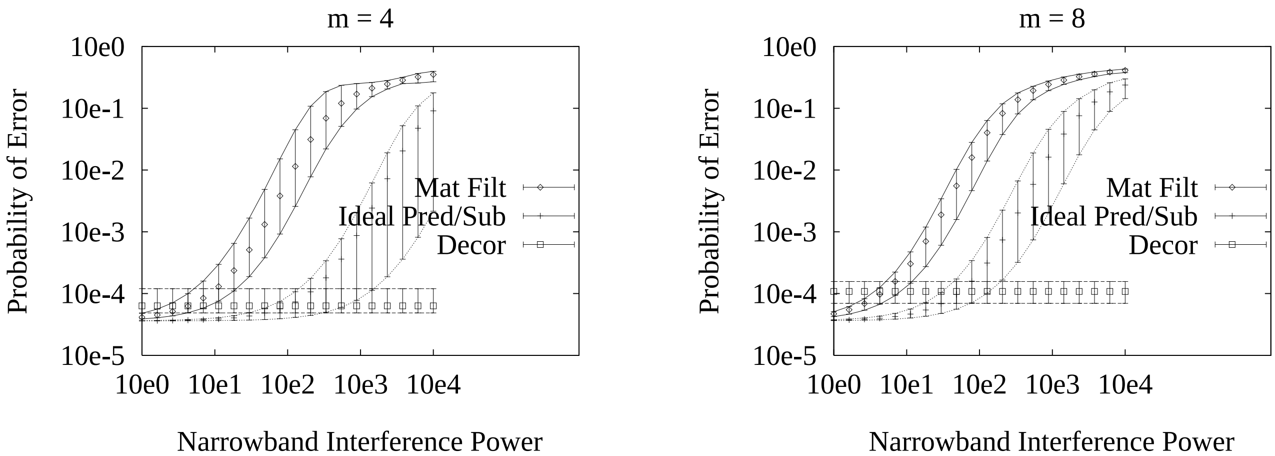

Figure 18: Less-narrowband interferer

multiuser detectors (see [16]) warrants consideration for the task of NBI suppression in spread spectrum systems.

Conclusions

In this paper we have described methods for mitigation of narrowband interference in spread spectrum systems. These methods can be divided into two general groups. The first group of techniques comprises systems based on the linear signal processing paradigms of adaptive transversal filtering and Fourier-domain filtering. These techniques are quite well-developed and have been studied thoroughly, in terms of both theoretical properties and the more practical issues of implementation. The second group of methods discussed herein are the model-based techniques of nonlinear estimator/subtracters and multiuser detectors. These techniques are of relatively recent vintage, and their development is not as complete as is that of the first group. In particular, the focus of work on these latter techniques has been theoretical, and much work remains to determine whether these methods are practical. However, the theoretical analyses point to potentially significant performance improvement through the use of these methods. In view of the increasing importance of spread spectrum methods and of the likelihood of exponential growth of wireless services, any methods for getting further efficiency and capacity from CDMA systems warrant consideration. Thus, these methods represent promising possibilities whose practical features warrant investigation.

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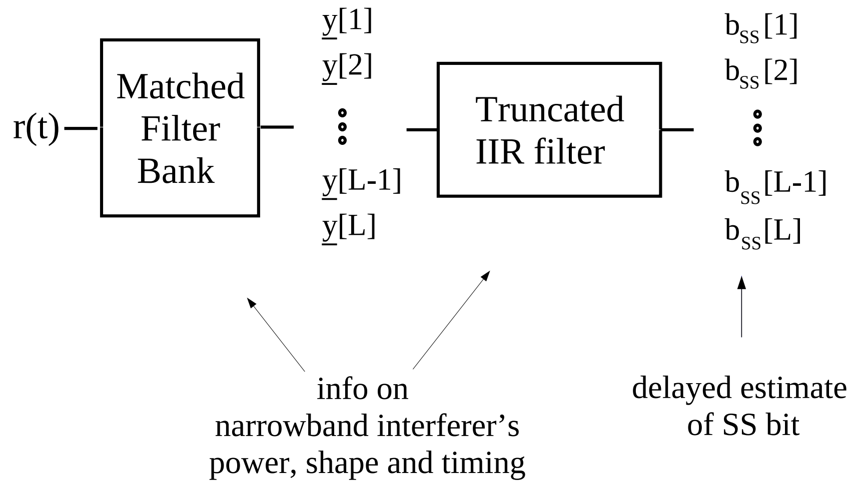

Figure 19: Causal Asynchronous Decorrelating Detector

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