High-resolution imaging of moving train by ground-based radar with compressive sensing

X.-C. Xie and Y.-H. Zhang

A compressive sensing (CS) based method for ISAR imaging with a stepped-frequency chirp signal (SFCS) is presented. It is outlined that by using the CS technique, the data rate has been remarkably reduced in bandwidth synthesis processing for SFCS to achieve a high range resolution. Imaging results of a moving train demonstrate the effectiveness of the proposed method.

Introduction: It is well known that high range resolution can be achieved by using a train of sub-pulses and each sub-pulse has a relatively narrow bandwidth but increased central frequencies, i.e. stepped-frequency chirp signal (SFCS) [1], so only a low-speed A/D converter is needed. Although the application of SFCS can reduce the requirement on the A/D converter, it cannot decrease the total data rate of radar high resolution imaging. Over the past few years, compressive sensing (CS) theory and techniques have introduced a new approach for reducing the number of data samples beyond the Nyquist theorem, while perfect reconstruction of the original signal can be obtained by signal processing [2]. Several approaches have been carried out for the applying CS technique to high-resolution radar imaging [3]. To achieve high range resolution ISAR imaging with fewer data samples of SCFS, we propose a phase-preserving range compression algorithm based on the compressive sensing (CS) technique. Based on this range compression algorithm, a scheme of 2D ISAR imaging with SFCS is presented. In this scheme the range compression is completed using the CS technique, and the azimuth compression is completed using an ordinary method.

Compressive sensing: CS is a new theory that enables sampling below the Nyquist rate, while the quality of reconstruction is guaranteed [2]. Consider a time-domain signal $x \in C^{N\times 1}$, which has a representation in some basis $\Psi = [\psi_1|\psi_2|\cdots|\psi_N]$,

$$x = \sum_{k=1}^{N} \psi_k \alpha_k = \Psi\alpha \tag{1}$$

where α is $N\times 1$ column vector of weighting coefficients $\alpha_k = \langle x, \psi_k\rangle$. If there is only $K(K<<N)$ of the α_k coefficients are nonzero, x is called sparse in Ψ domain with K sparsity. If the measurement of x is acquired in the time domain also, that is,

$$y = \Phi x + n \tag{2}$$

where $\Phi \in C^{M\times N}$ is the observation matrix, $y \in C^{M\times 1}$ is the measurement, and $n \in C^{M\times 1}$ is the measurement noise. Since $M < N$, x cannot be recovered directly from y. But by substituting x with (1), y can be written as,

$$y = \Phi x + n = \Phi\Psi\alpha + n = \Theta\alpha + n \tag{3}$$

Because α in (3) is K sparsity, and $K < M < N$, sparsity coefficient α could be solved from a optimisation problem,

$$\min_{\alpha} \|\alpha\|_{l_p} \quad s.t. \quad \|y - \Theta\alpha\|_2 \le \varepsilon \tag{4}$$

Thus signal x is also reconstructed. Problem (4) can be solved with different kinds of methods. To guarantee that α can be reconstructed from $M = O(K\log(N/K))$ measurements y, the observation matrix should obey what is known as a uniform uncertainty principle (UUP), which can be expressed by the following inequation,

$$C_1 \frac{M}{N} \le \frac{\|\Phi x\|}{\|x\|} \le C_2 \frac{M}{N}$$

where $C_1 \le 1 \le C_2$.

De-chirping and CS based range compression: The chirp signal can be expressed as follows:

$$s(t) = rect[(t - nT_r)/T_p]\exp^{j2\pi f_c t + j\pi K_r (t-nT_r)^2}$$

where $rect[]$ denotes the rectangular function, f_c is the carrier frequency, $K_r = B/T_p$ is the chirp rate, T_p represents the pulse width, B is the bandwidth and T_r is the pulse repetition interval (PRI) of the chirp signal. Assume a point target positions at a distance of r from the radar, so the radar echo from the target can be expressed as:

$$s_e(t) = As(t - 2r/c)$$

The de-chirping reference function is, correspondingly, to be:

$$s_{ref}(t) = rect[(t - nT_r - 2r_0/c)/T_{ref}] \times \exp^{-j2\pi f_c(t-2r_0/c) - j\pi K_r (t-nT_r-2r_0/c)^2}$$

where T_{ref} is the time duration of the de-chirping reference function, and r_0 is the referenced range. After the de-chirping operation (usually realised by hardware), the signal becomes to be:

$$\begin{aligned} s_{dc}(t) &= s_e(t) \otimes s_{ref}(t) \\ &= Arect[(t - nT_r - 2r/c)/T_p]\exp^{-j4\pi K_r (t-nT_r)(r-r_0)/c} \\ &\quad \times \exp^{-j4\pi f_c (r-r_0)/c - j4\pi K_r (r^2 - r_0^2)/c^2} \end{aligned} \tag{5}$$

where $\otimes$ denotes de-chirping or the mixing operation. After transforming (5) into the frequency domain, the signal can be expressed as follows:

$$S_{dc}(f) = AT_p \exp^{j\Phi_{dc}} \sin c\{T_p[f + 2K_r(r - r_0)/c\} \tag{6}$$

where $\Phi_{dc} = -4\pi[fr/c + f_c(r - r_0)/c + K_r(r^2 - r_0^2)/c^2]$. If the sinc function in (6) is approximated by a Dirac delta, that is

$$S_{dc}(f) \simeq AT_p \exp^{j\Phi_{dc}} \delta[(f + 2K_r(r - r_0)/c] \tag{7}$$

Thus, the de-chirped echo has sparse representation in the frequency domain. At the same time, the peak position $f_{dc} = -2K_r(r - r_0)/c$, amplitude information $A_{dc} = AT_p$ and phase information Φ_{dc} in (6) are all reserved.

Based on the above analysis, we choose the Fourier basis as sparsity basis Ψ, and propose a CS based range compression algorithm, which is composed of measurement and reconstruction processes. In the measurement process, de-chirped echo $s_{dc}(t)$ is processed by an AIC [4] and we get measurement y directly. The echo is demodulated by a pseudorandom wideband signal $p(t)$, and then filtered by a lowpass filter $h(t)$, sampled at sub-Nyquist rate using a traditional ADC finally. So the observation matrix Φ in (3) comprises a downsampling matrix D, a filter matrix H and a random matrix P, that is:

$$\Phi = DHP$$

where H and P are built based on $h(t)$ and $p(t)$, respectively. In the reconstruction process, we choose sparse Bayesian learning [5] as the solution method of (4), and recover the Fourier coefficients α of $S_{dc}(f)$. Thus we get the range profile from the measurement y.

Bandwidth synthesis and 2D imaging with SFCS: SFCS is realised by transmitting a sequence of chirp sub pulses with a step of increased carrier frequency, and the key step in imaging processing of stepped-chirp ISAR is how to compress the pulses in a burst to get a much larger bandwidth and much higher range resolution than that achieved with a single chirp. Here, we modify the measurement process based on the algorithm proposed in [6].

In the following, we briefly outline the process. Assume that $f_{mc} = f_c + m\Delta f \;\; (m = 0, 1, \ldots, M-1)$ denotes the carrier frequency of the nth sub pulse, and Δf is the frequency step. Then, the mth subpulse in one SFCS burst can be expressed as

$$s_m(t) = rect[(t - mT_{sr})/T_{sp}]\exp^{j2\pi f_{mc} t + j\pi K_r (t-mT_{sr})^2} \tag{8}$$

By de-chirping the echo of each subpulse, we get

$$\begin{aligned} s_{mdc}(t) &= Arect[(t - mT_{sr} - 2r/c)/T_{sp}] \\ &\quad \times \exp^{-j4\pi K_r (t-mT_{sr})(r-r_0)/c} \exp^{-j4\pi K_r (r^2-r_0^2)/c^2} \\ &\quad \times \exp^{-j4\pi(f_c + m\Delta f)(r-r_0)/c} \end{aligned} \tag{9}$$

Equation (9) shows that there is a time-shift decided by $\Delta f/K_r$ between any two successive sub-pulses in a burst. If we select appropriate Δf and sampling frequency f_s to let $T_{sp} = \Delta f/K_r$, and $f_s \times T_{sp}$ is a integer, obviously, the phases of any two successive sub-pulses after de-chirping will be continuous. If we delay the received echo of the later sub pulse to $\Delta f/K_r$, and then combine the successive echoes together, it means that we can get longer observation time for each target. Combined with the above process, we can get measurements of the bandwidth-synthesised range profile and get higher resolution after reconstruction.

From (7) we can find that the CS based range compression algorithm reserves the phase information of the point target. The reserved phase information enables us to perform azimuth compression using traditional techniques, including SAR or the ISAR imaging method. Thus, 2D radar imaging with SFCS based on CS method is achieved.

Experimental results: The primary aims of our experiment are to verify the proposed range compression method with SFCS and to obtain high resolution ISAR images of a moving train. The de-chirped echo data is acquired by a ground stationary Ka-band radar system with 2 GHz bandwidth which is obtained by synthesising 20 sub-pulses with carrier frequencies starting from 33 to 34.9 GHz at 100 MHz frequency steps and the bandwidth of each sub-pulse is 120 MHz. Detailed information about the system is described in [6]. In our experiment, CS based range compression is carried on the acquired data with a downsampling rate of 50.

To verify the proposed algorithm, we compare the ISAR images of the train got by the CS and FFT methods. After performing CS based range compression and azimuth compression on the de-chirped echo data, we finally get a high-resolution ISAR image of the moving train as shown in Fig. 1. The ISAR image of the train got by the FFT method and the optical picture of the train is given in Figs. 2 and 3 for reference. Both Figs. 1 and 2 clearly show the train has a six-section structure, and the ventilator of the air-conditioner of each compartment is very easy to identify. Besides the ventilators, the windows, doors, connections between compartments, and the upper body of the train are together shown as a 'straight line'. Comparing with Fig. 3, we see that the CS method represents the same detailed information of the train on azimuth as the FFT method, and sidelobes on the range are much suppressed.

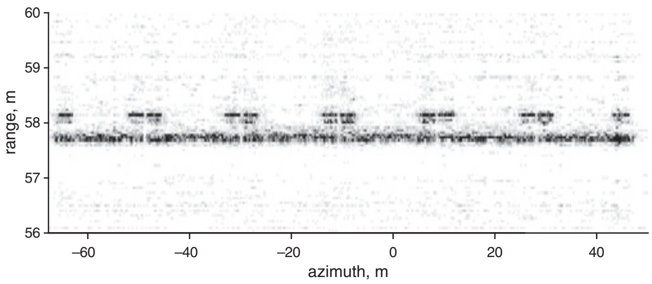

Fig. 1 *ISAR image of train obtained by CS method*

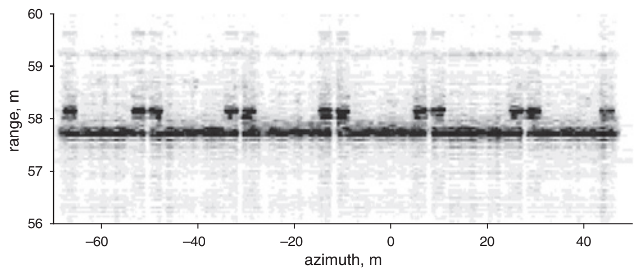

Fig. 2 *ISAR image of train obtained by FFT method*

Fig. 3 *Optical picture of train*

Conclusion: A CS based imaging algorithm with SFCS is proposed and tested by experiment of radar imaging of a moving train. It is shown that only 1/50 of the data used by the FFT based method is required to get almost the same image.

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One or more of the Figures in this Letter are available in colour online.

X.-C. Xie and Y.-H. Zhang (*Center for Space Science and Applied Research, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*)

E-mail: xiexiaochun@ieee.org

X.-C. Xie: Also with the School of Physics and Electronics Information, Gannan Normal University, Ganzhou, People's Republic of China

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