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Global Tracking Controllers for Flexible-joint Manipulators: a Comparative Study*

B. BROGLIATO,† R. ORTEGA‡ and R. LOZANO‡

Using some recent nonlinear controller design techniques, we derive, and compare, several globally stable nonlinear controllers for flexible-joint robots.

Key Words—Flexible-joint manipulators; adaptive control; passivity; nonlinear systems.

Abstract—Several new controller design techniques for global stabilization of nonlinear systems have recently been reported. Typically, these methodologies provide the designer with various degrees of freedom; consequently, their application in specific examples leads to the definition of various control schemes. One question of interest is the relationship between these schemes, or whether one contains the other. Further, since these schemes will, in general, exhibit different transients and possess different robustness properties, another challenging research problem is to establish some common framework to compare their robustness and performance properties. In this paper we investigate these questions for three different controller design techniques as applied to the problem of global tracking of robots with flexible joints. The connection between the various controllers are investigated. Further, they are compared using the following performance indicators: continuity properties *vis-à-vis* the joint stiffness, availability of adaptive implementations when the robot parameters are unknown, and robustness to 'energy-preserving' (i.e. passive) unmodelled effects. Complete stability proofs of all the resulting controllers are given.

1. INTRODUCTION

1.1. *Problem formulation*

A problem that has attracted the attention of researchers for some time is the motion control of robots with flexible joints. This problem has a strong practical motivation for high-performance robots, where elasticity is no longer negligible and has to be explicitly taken into account in the design, typically by adding a linear torsional spring. It is also a very challenging theoretical problem, since the number of degrees of freedom of the system is twice the number of control actions, and the matching property between nonlinearities and inputs is lost.

Throughout this paper, we shall consider the simplified model of an n-link robot proposed by Spong (1987), which assumes that the angular part of the kinetic energy of each rotor is due only to its own rotation (for a model that relaxes this assumption, see e.g. Nicosia & Tomei, 1992), and is given by

$$\begin{aligned} D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) &= K(q_2 - q_1), \\ J\ddot{q}_2 + K(q_2 - q_1) &= u, \end{aligned} \tag{1}$$

where $q_1 \in \mathbb{R}^n$ and $q_2 \in \mathbb{R}^n$ represent the link angles and motor angles respectively, $D(q_1)$ is the $n \times n$ inertia matrix for the rigid links, J is a diagonal matrix of actuator inertias reflected to the link side of the gears, $C(q_1, \dot{q}_1)\dot{q}_1$ represents the Coriolis and centrifugal forces, $g(q_1)$ represents the gravitational terms, and $K > 0$ is a diagonal matrix containing the joint stiffness coefficients. As suggested by Spong and Vidyasagar (1989), we define $C(Q_1, \dot{q}_1)$ via the Christoffel symbol. For ease of reference, we shall refer in the future to the first and second equations above as link dynamics and motor dynamics respectively. Also, to simplify the notation, we shall omit the arguments of $D(\cdot)$, $C(\cdot, \cdot)$, $g(\cdot)$.

We are interested here in the following.

Global tracking problem. For the system (1), define an internally stable control law that, for all $q_{1d} \in \mathscr{C}^4 \cap \mathscr{L}_\infty^n$ and arbitrary initial conditions, ensures

$$\lim_{t\to\infty} \tilde{q}_1 = \lim_{t\to\infty} (q_1 - q_{1d}) = 0.$$

Control laws that solve this problem will be referred as *globally tracking controllers.* To avoid the possible excitation of high-frequency modes—a critical effect in flexible systems—we

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† Laboratoire d'Automatique de Grenoble, URA CNRS 228, BP 46, Saint Martin D'Hères, France.

‡ Université de Technologie de Compiègne, URA CNRS 817, BP 233, 60206 Compiègne, France.

further restrict our attention to smooth controllers that do not inject high gain (e.g. *à la* Corless and Leitmann, 1981) into the loop, as in the schemes of Chen and Fu (1989), Dawson *et al.* (1991) and Stepanenko and Yuan (1992).

1.2. *Literature review*

It is now well known (Spong and Vidyasagar, 1989) that the flexible-joint robot model (1) is globally feedback-linearizable (by static state feedback), and therefore globally stable controllers can be derived using 'classical' geometric techniques. Besides the intrinsic lack of robustness of schemes based on nonlinearity cancellation, the proposed solutions suffer from the additional drawback that the control implementation relies on the availability of link acceleration and jerk. Even though these signals can be derived without differentiation from the systems model, this is not a very desirable procedure, since the accuracy in their calculation will be highly sensitive to uncertainty in the robot parameters. One way to overcome this difficulty is to use parameter adaptation techniques; unfortunately, it is not clear at this point how to make these schemes adaptive while preserving the global stability property.

Lozano and Brogliato (1992) have proposed the first globally stable scheme that is not based on feedback linearization. Availability of link acceleration and jerk is still required, but the sensitivity problems mentioned above are overcome by the adaptive implementation, for which a global convergence proof is given. The controller is a complicated dynamic state feedback that requires the realization of several (apparently *ad hoc*) filtering stages. Also its parametrization should be modified to avoid possible unboundedness of regressor signals.

In Nicosia and Tomei (1992) a Lyapunov-based backstepping technique* (popularized in Kokotovic, 1991) is applied to derive a global tracking controller. However, its adaptive implementation is presented only for a single-link robot.

It should be remarked that, to the best of our knowledge, the global tracking problem for the complete model is as yet open. Some claims concerning dynamic feedback linearization for certain particular robot structures are made in De Luca (1988). Other efforts aimed at solving this problem may be found in Lanari *et al.* (1992).

1.3. *About this paper*

It is well known that systems described by Euler–Lagrange equations possess some nice passivity properties that follow directly from the energy balance principles (see e.g. Nijmeijer and van der Schaft, 1990; Ortega *et al.*, 1994). In particular, robots with flexible joints define a passive operator from applied torques to motor shaft velocities, though it is not passive with respect to the link velocities. Furthermore, because of the block diagonal structure of its inertia matrix, the simplified model (1) can be decomposed into two cascaded subsystems with a suitable change of input coordinates, and link acceleration and jerk are available from the system model and its state without differentiation.

In this paper we use these fundamental properties to define various globally tracking controllers, applying three different stabilization techniques as follows.

- *Decoupling-based schemes.* These use the cascade decomposition property, and are motivated by the result on stability of cascaded connections of stable systems with bounded orbits of Seibert and Suarez (1990). The name 'decoupling-based' stems from the fact that the resulting closed loop is also a cascade connection. Two controllers are explicitly derived.
- *Backstepping-based schemes.* These also use the cascade decomposition property of the model, but combined with the integrator augmentation stabilization of Kokotovic and Sussmann (1989). With this technique, we derive controllers that contain as particular case the one proposed by Nicosia and Tomei (1992).
- *Passivity-based schemes.* These are obtained from the application of the passivity-based technique for stabilization of underactuated Euler–Lagrange systems proposed by Ortega and Espinosa (1993); see also Ortega and Espinosa (1991). Using this technique, we first rederive the controller proposed by Lozano and Brogliato (1992). Further, we prove that a simpler control law, i.e. static state feedback, can be obtained choosing a different desired closed-loop 'potential energy'. The new controller is exponentially stable, and also admits an adaptive implementation. It is interesting to note how the passivity-based methodology creates the possibility of obtaining different controllers proceeding from the fundamental physical notion of closed-loop total energy (Ortega *et al.*, 1994).

It is clear that the resulting controllers, which are derived from apparently unrelated techniques, will exhibit different transients and possess different robustness properties. The main objective of this paper is to establish a common

* See Section 4.8 of Sontag (1990) for a brief history of this technique.

framework to identify the relationships (if any) between these designs, and compare their robustness and performance properties.

The remainder of the paper is organized as follows. Three basic lemmas that motivate our design approaches are presented in Section 2. The various control schemes are described, and their stability analysed* in Section 3. We are interested to know if using the backstepping methodology, which is perhaps the most systematic technique available to date, we can obtain the controllers derived from the other methodologies. Our conclusion here is that there is no unique methodology from which all controllers can be derived, unless significant modifications to the design techniques are introduced. We then carry out three comparison studies for these algorithms, which are given in Section 4. First, we study the continuity properties of the closed loop *vis-à-vis* the joint stiffness, that is, the behaviour of the control law in the transition from flexible- to rigid-joint robot. Specifically, we are interested in knowning whether the loop gain grows unbounded with increasing stiffness, and if the stability analysis still carries on in this practically important case. Secondly, we study the possibility of an adaptive implementation. In particular, we show how the backstepping-based controllers can be made adaptive for the n-dof case. Also, we discuss the limitations in this respect of the passivity-based and decoupling-based controllers. Finally, we provide a decomposition of the various closed loops into passive blocks in feedback interconnection. This analysis is motivated by the well-known fact that passivity is invariant under feedback interconnection. Thus some robustness properties of 'energy-preserving' unmodelled effects may be inferred from such passive decompositions (for further discussion of this point, see e.g. Ortega and Spong, 1989).

Notation. $|\cdot|$ is the Euclidean norm, $\mathscr{L}_2^n$ and $\mathscr{L}_{2e}^n$ are the space of n-dimensional square-integrable functions and its extension, $\|\cdot\|_2$ is the $\mathscr{L}_2^n$ norm, and $\langle\cdot \mid \cdot\rangle$ is the inner product in $\mathscr{L}_2^n$. For further details and definitions see Desoer and Vidyasagar (1975).

2. DESIGN APPROACHES

In this section we present some properties of the model and three technical lemmas that will help us to define in the forthcoming section the various globally stable control laws for tracking of flexible joint manipulators.

2.1. *Cascaded subsystems*

Before presenting the approach, we note the simple fact that with a suitable input change of coordinates, we can transform (1) into two cascaded subsystems with inputs u and q_2 respectively. This change of coordinates is non-unique; for instance, we achieve the objective with $u = Kq_1 + v$ or with $u = Jv + K(q_2 - q_1)$. This freedom will be used later to derive different control laws.

Now, we present a technical result that we shall use in the sequel to obtain the first type of controllers. It pertains to the stability of cascaded interconnections of stable systems when boundedness of the orbits is insured.

Lemma 1. (Seibert and Suarez (1990).) If the systems $\dot{x} = F(x)$ and $\dot{y} = G(0, y)$ are globally asymptotically stable (GAS), and if every orbit of the cascaded system

$$\dot{x} = F(x), \quad \dot{y} = G(x, y)$$

is bounded in the future, then the overall system is also GAS.

The geometrical idea behind this result is that if the reduction $\dot{y} = G(0, y)$ of the system to the invariant manifold $x = 0$ is GAS then the overall system is also GAS, provided that the solutions do not grow unbounded. This is an interesting result, because it shows that if the peaking phenomenon makes two cascaded GAS systems not GAS then it has the disastrous effect of making the solutions grow unbounded.

2.2. *Integrator backstepping*

The following result, which motivates another set of globally tracking controllers, pertains to the problem of stabilizing a nonlinear system in cascade with an integrator chain.

Lemma 2. (Kokotovic and Susşman (1989).) If a system $\dot{x} = f(x, u)$ is smoothly stabilizable then the system

$$\dot{x} = f(x, \xi_1), \quad \dot{\xi}_1 = \xi_2, \ldots, \dot{\xi}_k = v \qquad (2)$$

obtained by cascading the original system with a chain of integrators is smoothly stabilizable as well.

A stabilizing control law for (2) comes as a

* For the sake of brevity, the stability proofs are given with the explicit derivations of the state and the coordinate transformations. They may be found in the conference paper by Brogliato *et al.* (1993a) or in its full version available from the authors.

corollary of the now well-known backstepping method (see e.g. Kokotovic, 1991; Kanellakopoulos *et al.*, 1991) and the passive design of Kokotovic and Sussmann (1989). The essential idea of the backstepping technique is to start from the knowledge of a smooth feedback, say $\xi_{1d}(x)$, such that $\dot{x} = f(x, \xi_{1d}(x))$ is GAS with known Lyapunov function. Then generate an error equation consisting of the GAS system and an error term $\xi_1(x) - \xi_{1d}(x)$, and add to its input an integrator. For the augmented system a new stabilizing feedback law is explicitly designed and shown to be stabilizing for a new Lyapunov function, and so on.

Remark 1. It is interesting to note that with this powerful idea one can 'rederive' some classical control schemes. For instance, a state observer $u = K\hat{x}$ may be derived by setting $u = Kx + K(\hat{x} - x)$ and then adding an integrator for $\hat{x} - x$. The same procedure works for a parameter adaptation scheme $u = \hat{K}x$, where now we add the integrator to $\hat{K} - K$. Actually, as pointed out in Kokotovic and Sussmann (1989), the latter application motivated their developments.

2.3. *Passivity-based control*

The last design approach we shall consider in this section is the passivity-based technique proposed by Ortega and Espinosa (1991, 1993) to control Euler–Lagrange systems with fewer control actions than degrees of freedom. The passivity-based technique is well known in rigid-robot control (Ortega and Spong, 1989; see also Kelly *et al.*, 1989; Brogliato *et al.*, 1991). It was introduced in the seminal paper by Takegaki and Arimoto (1981) to solve the rigid-robot regulation problem via shaping of the system potential energy and addition of the required damping. This simple physically motivated idea has been extended in several directions, yielding designs with enhanced robustness properties that do not need cancellation of nonlinearities (see e.g. Slotine and Li, 1988; Nijmeijer and van der Schaft, 1990; Ortega and Espinosa, 1991, 1993; Lanari and Wen, 1992; Berghuis and Nijmeijer, 1993; Ailon and Ortega, 1993).

The approach is based on the well-known passivity property of the robot model, which is demonstrated here for completeness, and the strict passivity of the 'error equation' presented below. To simplify the notation, we find it convenient to rewrite (1) in compact form as

$$\bar{D}\ddot{q} + \bar{C}\dot{q} + \bar{g} + \bar{K}q = Mu, \tag{3}$$

where

$$q^{\mathrm{T}} = [q_1^{\mathrm{T}} \quad q_2^{\mathrm{T}}],$$

$$\bar{D} = \begin{bmatrix} D & 0 \\ 0 & J \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{g} = \begin{bmatrix} g \\ 0 \end{bmatrix},$$

$$\bar{K} = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ I_n \end{bmatrix}.$$

Lemma 3. (Passivity properties.) The system (3) defines a passive operator $\Sigma: \mathcal{L}_{2e}^n \to \mathcal{L}_{2e}^n : u \mapsto \dot{q}_2$. That is, there exists $\beta \in \mathbb{R}$ such that $\langle u \mid \dot{q}_2 \rangle \geq \beta$, for all $u \in \mathcal{L}_{2e}^n$. Furthermore, for all $q_1, \dot{q}_1 \in \mathcal{L}_{2e}^n$, the system

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\int_0^t s(\tau)\,\mathrm{d}\tau = \psi, \tag{4}$$

with $\bar{B} = \bar{B}^{\mathrm{T}} > 0$, defines an output strictly passive operator $\Sigma_{\mathrm{d}}: \mathcal{L}_{2e}^{2n} \to \mathcal{L}_{2e}^{2n} : \psi \mapsto s$, i.e. there exist $\beta \in \mathbb{R}$ and $\alpha > 0$ such that $\langle \psi \mid s \rangle \geq \alpha \|s\|_2^2 + \beta$ for all $v \in \mathcal{L}_{2e}^{2n}$. Consequently, if $\psi \equiv 0$, we have $s \in \mathcal{L}_2^{2n}$.

Proof. The first passivity property can be easily established by taking the time derivative of the robot total energy function

$$H = \tfrac{1}{2}\dot{q}^{\mathrm{T}}\bar{D}\dot{q} + \tfrac{1}{2}q^{\mathrm{T}}\bar{K}q + V_{\mathrm{g}}, \tag{5}$$

where $g^{\mathrm{T}} = \partial V_{\mathrm{g}}/\partial q_1$, which (using the well-known skew-symmetry property $\dot{D} = C + C^{\mathrm{T}}$) yields $\dot{H} = u^{\mathrm{T}}\dot{q}_2$. The proof is completed by integrating the previous equation and noting that the total energy is bounded from below.

To prove the output strict passivity of $v \mapsto s$, we proceed as above with the 'total energy' function*

$$H_{\mathrm{d}} = \tfrac{1}{2}s^{\mathrm{T}}\bar{D}s + \tfrac{1}{2}\left[\int_0^t s^{\mathrm{T}}(\tau)\,\mathrm{d}\tau\right]\bar{K}\left[\int_0^t s(\tau)\,\mathrm{d}\tau\right], \tag{6}$$

which yields $\dot{H}_{\mathrm{d}} = v^{\mathrm{T}}s + s^{\mathrm{T}}\bar{B}s$. □

The interest of the above lemma for our global tracking problem is easier to appreciate if we recall the energy-shaping interpretation of the controller of Slotine and Li (1988) for rigid robots. To this end, we first define the error signals

$$s_1 = \dot{\tilde{q}}_1 + \Lambda_1\tilde{q}_1, \qquad \tilde{q}_1 = q_1 - q_{1\mathrm{d}}, \tag{7}$$

with $\Lambda_1 > 0$, diagonal. Now, the control law is defined to change the system total energy so as to match the desired 'total energy' $\tfrac{1}{2}s_1^{\mathrm{T}}Ds_1$; we

* It is important to note that, rigorously speaking, (6) is not the total energy of the system (4). *Faute de mieux*, we keep this notation with the quotation marks.

call this step energy shaping. Further, to ensure strict passivity, we add the required damping $B_1 s_1$, $B_1 > 0$. The error equation corresponding to the desired 'total energy' is $D\dot{s}_1 + (C + B_1)s_1 = 0$, which, using $\dot{D} = C + C^T$ as in the above lemma, ensures $s_1 \in \mathscr{L}_2^n$. From the definition of s_1, and invoking the arguments used by Ortega and Spong (1989), we conclude that $\lim_{t\to\infty} \tilde{q}_1 = 0$.

A similar procedure will be followed in the next section to define passivity-based controllers for the flexible-joint case. However, the extension is not straightforward, because the flexible robot has fewer control actions than degrees of freedom, and consequently the 'potential energy' cannot be arbitrarily shaped. See Ortega *et al.* (1994).

3. CONTROL SCHEMES

In this section we present the globally tracking controllers inspired by the three previous lemmas. For ease of presentation, we start with the simplest design, namely that based on triangularization of the closed loop. Then we present the backstepping-based schemes, and wrap up the section with the passivity-based controllers.

3.1. *Decoupling-based schemes*

The decoupling-based controllers use the triangularization property of the model to obtain two GAS cascaded subsystems. The motor dynamics define the unforced subsystem, which drives the link dynamics via q_2. The rationale is then to make q_2 converge to some function (say q_{2d}) that, if applied as input to the link dynamics, will drive q_1 towards q_{1d}. In particular, to be able to invoke Lemma 1, we need the link dynamics to be GAS when $q_2 \equiv q_{2d}$. Several choices are possible for the signal q_{2d}. To unify the presentation, and allow for the possibility of an adaptive implementation, we have adopted here the solution proposed by Slotine and Li (1988). That is, we define

$$q_{2d} = K^{-1} u_R + q_1, \qquad (8)$$

where

$$u_R = D\ddot{q}_{1r} + C\dot{q}_{1r} + g - B_1 s_1 \qquad (9)$$

and $\dot{q}_{1r} = \dot{q}_{1d} - \Lambda_1 \tilde{q}_1$. The signal u_R is the control law of Slotine and Li (1988) designed for the rigid part of the robot model. Note that, with this definition, we can rewrite the first equation in (1) as a GAS system with an input perturbation term $\tilde{q}_2 = q_2 - q_{2d}$ as

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K\tilde{q}_2. \qquad (10)$$

The proof of GAS of the system above with $\tilde{q}_2 \equiv 0$ may be found in Spong *et al.* (1990), where

$$V_R = \tfrac{1}{2} s_1^T D s_1 + \tilde{q}_1^T \Lambda_1^T b_1 \tilde{q}_1$$

is shown to be a strict Lyapunov function.

To complete the design, we now propose a control law that decouples the motor dynamics and makes it GAS as

$$u = K(q_2 - q_1) - K_1 \tilde{q}_2 - K_2 \dot{\tilde{q}}_2 + J\ddot{q}_{2d}, \qquad (11)$$

with K_1, $K_2 > 0$. This control yields a decoupled error equation

$$J\ddot{\tilde{q}}_2 + K_2 \dot{\tilde{q}}_2 + K_1 \tilde{q}_2 = 0. \qquad (12)$$

A very important remark at this point is that $\ddot{q}_{2d}$ is computable without differentiation. This fundamental property of the model (1) is lost in the complete model of Nicosia and Tomei (1992).

We are in position to present our first main result.

Proposition 1. The control law (11), (7)–(9) solves the global tracking problem.

Proof. The closed-loop dynamics are described by (10) and (12). Now consider the Lyapunov function candidate $V_{DB} = V_R + \tfrac{1}{2} z^T P z$, with V_R as defined above, $z = [\tilde{q}_2^T \ \dot{\tilde{q}}_2^T]^T$, and $P = P^T > 0$ the unique solution of

$$A^T P + PA = -I, \quad \text{with } A = \begin{bmatrix} 0 & I_n \\ -K_1 & -K_2 \end{bmatrix}.$$

Taking the derivative of V_{DB} along the trajectories of (10) and (12), we obtain

$$\dot{V}_{DB} = -\dot{\tilde{q}}_1^T B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^T \Lambda_1^T B_1 \Lambda_1 \tilde{q}_1 + \dot{\tilde{q}}_1^T K z_1 + \tilde{q}_1^T \Lambda_1^T K z_1 - z^T z.$$

From the fact that z is bounded, it can be deduced that the whole state vector is bounded, since v_{DB} is strictly negative outside a compact set in the state space. Moreover, since z exponentially decays to zero, it follows that $\dot{V}_{DB}$ can be written as

$$\dot{V}_{DB} \leq -\dot{\tilde{q}}_1^T K_1 \dot{\tilde{q}}_1 - \tilde{q}_1^T \Lambda_1^T K_1 \Lambda_1 \tilde{q}_1 - z^T Q z + \varepsilon$$

where ε is exponentially decaying. Following, for example, the stability proofs in Corless and Leitmann (1981), it can be deduced that the full state asymptotically converges towards zero. □

Remark 2. It is interesting to underscore the simplicity of this controller. However, as discussed in the next section, the scheme does not seem amenable to an adaptive implementation.

Remark 3. From the above derivations, it is clear that other global tracking controllers can easily be derived. For instance, we could have chosen u as

$$u = K(q_{2d} - q_1) - K_1\tilde{q}_2 - K_2\dot{\tilde{q}}_2 + J\ddot{q}_{2d}, \quad (13)$$

where now K_1 and K_2 are such that $\det[J\lambda^2 + K_2\lambda + (K + K_1)]$ is Hurwitz. It is clear from Proposition 1 that, under this condition, the control (13), (7)–(9) is also GAS. Also, inverse dynamics could have been used instead of those of Slotine and Li (1988) in the definition of u_R. To facilitate comparisons between the various schemes, this choice will be adopted in all cases. Another variation that could have been adopted is to use also the ideas of Slotine and Li (1988) to stabilize the motor dynamics, instead of the 'computed torque' scheme used above.

3.2. *Backstepping-based schemes*

This subsection is devoted to showing that the backstepping method can be applied almost directly to the case of flexible-joint manipulators. A similar procedure has been used by Nicosia and Tomei (1992).

To apply the methodology, we need to express the system as a cascade connection of integrators and the link dynamics (see Lemma 2). This is achieved with the inner feedback law

$$u = Jv - K(q_1 - q_2). \quad (14)$$

The flexible-joint manipulator dynamics then reduces to the cascaded form

$$D\ddot{q}_1 + C\dot{q}_1 + g + Kq_1 = Kq_2, \qquad \ddot{q}_2 = v. \quad (15)$$

Step 1. Assume that q_2 is the control input in the first equation of (15), and consider the feedback law $q_2 = q_{2d}$, with q_{2d} defined in (8). Then the closed-loop equation is (10) with $\tilde{q}_2 \equiv 0$, which we know is GAS with Lyapunov function V_R. Now, since q_2 is not the input, we have an error $\tilde{q}_2$. We now add an integrator at the input to get

$$\dot{\tilde{q}}_2 = -\dot{q}_{2d} + \dot{q}_2, \quad (16)$$

where, as pointed out before, $\dot{q}_{2d}$ is computable from position and velocity measurements only.

Step 2. Assume that $\dot{q}_2$ is now the input, and consider $V_2 = V_R + \frac{1}{2}\tilde{q}_2^T\tilde{q}_2$ as a Lyapunov function candidate for the system (10), (16). We have

$$\dot{V}_2 = -\dot{\tilde{q}}_1^T B_1\dot{\tilde{q}}_1 - \tilde{q}_1^T\Lambda_1^T B_1\Lambda_1\tilde{q}_1 + s_1^T K\tilde{q}_2 + \tilde{q}_2^T(\dot{q}_2 - \dot{q}_{2d}).$$

Setting $\dot{q}_2 = -Ks_1 - \tilde{q}_2 + \dot{q}_{2d}$, we could cancel the cross-term $s_1^T K\tilde{q}_2$ and add a quadratic term in $\tilde{q}_2$ to ensure GAS. Since $\dot{q}_2$ is not the input, we look again at the error $e_2 = \dot{q}_2 - e_{2d}$, where

$$e_{2d} = -Ks_1 - \tilde{q}_2 + \dot{q}_{2d}. \quad (17)$$

(It is important to note that e_2 and e_{2d} are not the time derivatives of $\tilde{q}_2$ and q_{2d} respectively.) Adding an integrator, we obtain the overall error equations

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K\tilde{q}_2, \qquad \dot{\tilde{q}}_2 = e_2 - Ks_1 - \tilde{q}_2, \qquad \dot{e}_2 = -\dot{e}_{2d} + v. \quad (18)$$

Note again that $\dot{e}_{2d}$ is available from position and velocity measurements only.

Step 3. This is the last step, since the control v already appears in (18). Consider now the following Lyapunov function candidate $V_{BB} = V_2 + \frac{1}{2}|e_2|^2$. Setting

$$v = -e_2 + \dot{e}_{2d} - \tilde{q}_2$$

and taking the derivative of V_{BB} along the trajectories of (18), we get

$$\dot{V}_{BB} = -\dot{\tilde{q}}_1^T B_1\dot{\tilde{q}}_1 - \tilde{q}_1^T\Lambda_1^T B_1\Lambda_1\tilde{q}_1 - |\tilde{q}_2|^2 - |e_2|^2,$$

which establishes GAS of the closed loop.

The above derivations are summarized in the following proposition, whose proof is established by direct replacement of the previous definitions in u.

Proposition 2. The control law

$$u = K(q_2 - q_1) + J[\ddot{q}_{2d} - 2\dot{\tilde{q}}_2 - 2\tilde{q}_2 - K(\dot{s}_1 + s_1)]$$

together with (8), (9) solves the global tracking problem.

Robustification. We shall show in the next section that a more robust design is obtained if in Step 2 above we use $V_2 = V_R + \frac{1}{2}q_2^T K\tilde{q}_2$ as a Lyapunov function candidate. Note the presence of $K > 0$ in the second right-hand term in (18). Following the same procedure, we arrive at the GAS control

$$u = K(q_2 - q_1) + J[\ddot{q}_{2d} - 2\dot{\tilde{q}}_2 - 2\tilde{q}_2 - (\dot{s}_1 + s_1)],$$

where we note that the presence of K has been removed inside the brackets.

Remark 4. We have adopted here the backstepping technique as a 'step-by-step' methodology precisely to stress the important fact that it is a systematic procedure. Several variations are posssible at some stages of the backstepping design—perhaps leading to a controller with better performances. Nevertheless, it does not seem possible (without a substantial modification of the procedure) to obtain the other controllers. This stems from the fact that intrinsic to this method is the introduction of terms in the control law to cancel the (error-induced) cross-terms in the (partial) Lyapunov function derivative at each integration step, namely $\tilde{q}_2^{\mathrm{T}} K s_1$ and $e_2^{\mathrm{T}} K \dot{s}_1$. This procedure destroys in the closed loop the initial triangular structure, resulting instead in an 'antisymmetric system'.

Remark 5. We have chosen here to define q_2 as the 'hypothetical' input to start the backstepping procedure. Another alternative would have been the signal $q_2 - q_1$, which leads to the first error equation

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K(\tilde{q}_2 - \tilde{q}_1)$$

and the Lyapunov function candidate $V_2 = V_{\mathrm{R}} + \frac{1}{2}\tilde{q}^{\mathrm{T}}\bar{K}\tilde{q}$, and so on. This is the choice made by Nicosia and Tomei (1992), and gives another globally tracking controller. It is worth noting that the form of backstepping procedure used by Nicosia and Tomei (1992), when applied to the rigid-robot problem, leads to the well-known passivity-based controller of Slotine and Li (1988). Although this fact is not observed by the authors, it follows directly from their calculations.

3.3. *Passivity-based schemes*

To apply the passivity-based methodology, we proceed analogously to the rigid case and define $s = \dot{\tilde{q}} + \bar{\Lambda}\tilde{q}$, with $\bar{\Lambda} = \mathrm{diag}(\Lambda_1, \Lambda_2)$, $\Lambda_1, \Lambda_2 > 0$ and diagonal.

In this procedure $q_{2\mathrm{d}}$ will be defined to ensure energy shaping, i.e. such that the closed-loop 'total energy' matches the desired function. This is in contrast with the definition of $q_{2\mathrm{d}}$ in the two previous controllers, where it represents the 'desired input' to the link dynamics; see (8). Another difference with the rigid case is that to define the desired 'energy function', we must take into account the presence of the potential energy term $q^{\mathrm{T}}\bar{K}q$, which cannot be removed. Similarly to the regulation problem (Ortega *et al.*, 1994), we can obtain different controllers by different choices of the desired 'potential energy' term. Two different selections are given below.

3.3.1. *The controller of Lozano and Brogliato.* The above discussion and Lemma 3 motivate us to choose the desired closed-loop 'total energy' as (6). At this point, we find it convenient to write (3) in terms of the error signals as
positive-definite diagonal matrices, is the 'damp-

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\int_0^t s(\tau)\,\mathrm{d}\tau = \psi. \quad (19)$$

Here $\bar{B} = \mathrm{diag}(B_1, B_2)$, where B_1 and B_2 are

ing coefficient' of the closed loop, and ψ plays the role of a perturbation term for the desired error system and is defined by

$$\psi = \bar{u} - (\bar{D}\ddot{q}_{\mathrm{r}} + \bar{C}\dot{q}_{\mathrm{r}} + \bar{K}q_{\mathrm{r}} + \bar{g}) + \bar{B}s - \bar{K}\tilde{q}(0), \quad (20)$$

where $q_{\mathrm{r}} = q_{\mathrm{d}} - \bar{\Lambda}[\int_0^t \tilde{q}(\tau)\,\mathrm{d}\tau]$. Taking the time derivative of H_{d} along the trajectories of (19) we get

$$\dot{H}_{\mathrm{d}} = -s^{\mathrm{T}}\bar{B}s + s^{\mathrm{T}}\psi$$

It is then clear from Lemma 3 that the next step of the design procedure is to calculate, using (20), the control signals u and the functional relations for $q_{2\mathrm{d}}$, required to match the desired 'energy function', that is, to ensure that $\psi \equiv 0$.

We are in position to present the following proposition that summarizes the previous developments and provides a reinterpretation in terms of energy shaping of the controller of Lozano and Brogliato (1992).

Proposition 3. The solution of $\psi \equiv 0$ with ψ given by (20) defines a nonlinear dynamic state feedback globally tracking controller.

Proof. After some lengthy but straightforward calculations, we can show that the solution of $\psi \equiv 0$ from (20) yields

$$q_{2\mathrm{d}} = \mathrm{p}(\mathrm{p}I + \Lambda_2)^{-1}\Big\{K^{-1}u_{\mathrm{R}} + q_{1\mathrm{d}} + K[\tilde{q}_1(0) - \tilde{q}_2(0)] - \int_0^t (\Lambda_1\tilde{q}_1 - \Lambda_2 q_2)\,\mathrm{d}\tau\Big\} \quad (21)$$

and the control law

$$u = -B_2 s_2 + J\ddot{q}_{2\mathrm{r}} - K(q_{1\mathrm{r}} - q_{2\mathrm{r}}), \quad (22)$$

where $\mathrm{p} = \mathrm{d}/\mathrm{d}t$. It is clear that replacing this control in (3) leads to the closed-loop equations (19) with $\psi \equiv 0$, for which Lemma 3 applies. To prove internal stability, we note that if $q_{2\mathrm{d}}$ is bounded then all signals are also bounded. Boundedness of $q_{2\mathrm{d}}$ follows trivially on noting that $q_{2\mathrm{d}}$ is obtained by filtering bounded signals. □

There are three drawbacks of the previous scheme. First, owing to the presence of additional states, it is not possible to establish its Lyapunov stability from this analysis. Note that the analysis of Lemma 3 only ensures global convergence of the error signals—a property weaker than asymptotic stability. Secondly, to yield an easier implementation and a clearer comparison between the various schemes, we should like to simplify the controller by removing the dynamic part. Thirdly, it is apparent from (21) that the initial conditions are constrained. In the next section we show that modifying the desired 'total energy' allows us to remove both shortcomings.

3.3.2. *Simplified passivity-based controller.* Observing that the additional dynamics in the above controller comes from our choice of the desired 'potential energy' term $\frac{1}{2}(\int_0^t s^{\mathrm{T}}\,\mathrm{d}\tau)\bar{K}(\int_0^t s\,\mathrm{d}\tau)$ in (6), we propose here a new desired 'total energy' of the form

$$H_{\mathrm{d}} = \tfrac{1}{2}s^{\mathrm{T}}\bar{D}s + \tfrac{1}{2}\tilde{q}^{\mathrm{T}}\bar{K}\tilde{q},$$

for which we have the perturbed desired error dynamics

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\tilde{q} = \psi$$

with perturbation term

$$\psi = \bar{u} - (\bar{D}\ddot{q}_{\mathrm{r}} + \bar{C}\dot{q}_{\mathrm{r}} + \bar{K}q_{\mathrm{d}} + \bar{g}) + \bar{B}s.$$

Compare this with (19) and (20). This perturbation is set equal to zero, with the control law

$$\begin{aligned} q_{2\mathrm{d}} &= q_{1\mathrm{d}} + K^{-1}u_{\mathrm{R}}, \\ u &= -K_2 s_2 + J(\ddot{q}_{2\mathrm{d}} - \Lambda_2\dot{\tilde{q}}_2) - K(q_{1\mathrm{d}} - q_{2\mathrm{d}}). \end{aligned} \tag{23}$$

Note that this is a static state feedback. The proof of global convergence follows exactly the same lines as above, noting that if we take $\Lambda_1 = \Lambda_2$ then the operator $\psi \to s$ is output strictly passive. Furthermore, we can also prove Lyapunov stability with the Lyapunov function candidate

$$\begin{aligned} V_{\mathrm{PB}} = \tfrac{1}{2}s^{\mathrm{T}}\bar{D}s + \tilde{q}_1\Lambda_1^{\mathrm{T}}k_1\tilde{q}_1 + \tilde{q}_2^{\mathrm{T}}\Lambda_2^{\mathrm{T}}K_2\tilde{q}_2 \\ + \tfrac{1}{2}\tilde{q}^{\mathrm{T}}\bar{K}\tilde{q}, \end{aligned}$$

for which it can be shown that $\dot{V}_{\mathrm{PB}} \leq -\alpha V_{\mathrm{PB}}$ for some $\alpha > 0$. Hence we conclude GAS of the equilibrium.

Remark 6. The derivations above show how different controllers can be obtained by a suitable selection of the desired 'total energy' function. The fact that engineering intuition can be extensively used in the choice of this function makes this a remarkable feature of passivity-based designs. See Ortega *et al.* (1994) for the application of this idea to the regulation problem.

4. COMPARISONS

In the preceding sections we have presented five different global tracking controllers. This section is devoted to emphasizing the differences and similarities between them. In particular, we are interested in the following aspects. First we make a qualitative comparison of the control laws with particular emphasis on their continuity between the flexible-joint and rigid cases. Then we investigate the possible extensions of the four algorithms to the adaptive case. Finally, we examine the passivity properties of the closed-loop error systems, both in the known-parameters and adaptive cases.

4.1. *'Almost-rigid' manipulators*

We are interested here in providing answers to the following questions: What happens in the 'almost-rigid' case, that is, when the joint stiffness K takes infinitely large values? Do the proposed schemes converge to the corresponding 'rigid' ones (i.e. the 'first-stage' input used here to stabilize the rigid part of the dynamic equations u_{R} corresponds in our case to the algorithm proposed by Slotine and Li, 1988)? Do they yield high-gain designs?.

The five control laws are summarized in the following equations:

decoupling-based control

$$\begin{aligned} u &= J\ddot{q}_{2d} - K_1\tilde{q}_2 - K_2\dot{\tilde{q}}_2 + K(q_2 - q_1), \\ q_{2\mathrm{d}} &= K^{-1}u_{\mathrm{R}} + q_1; \end{aligned} \tag{24}$$

backstepping-based control

$$\begin{aligned} u &= J[\ddot{q}_{2\mathrm{d}} - 2\dot{\tilde{q}}_2 - 2\tilde{q}_2 - K(\dot{s}_1 + s_1)] \\ &\quad + K(q_2 - q_1), \\ q_{2\mathrm{d}} &= K^{-1}u_{\mathrm{R}} + q_1; \end{aligned} \tag{25}$$

robustified backstepping-based control

$$\begin{aligned} u &= J[\ddot{q}_{2\mathrm{d}} - 2\dot{\tilde{q}}_2 - 2\tilde{q}_2 - (\dot{s}_1 + s_1)] \\ &\quad + K(q_2 - q_1), \\ q_{2\mathrm{d}} &= K^{-1}u_{\mathrm{R}} + q_1; \end{aligned} \tag{26}$$

passivity-based control

$$\begin{aligned} u &= J\ddot{q}_{2\mathrm{r}} - K\left[q_{1\mathrm{d}} - q_{2\mathrm{d}} - \int_0^t (\Lambda_1\tilde{q}_1 - \Lambda_2\tilde{q}_2)\,\mathrm{d}\tau\right] \\ &\quad - B_2 s_2, \\ q_{2\mathrm{d}} &= \mathrm{p}(\mathrm{p}I + \Lambda_2)^{-1}\Big\{K^{-1}u_{\mathrm{R}} + q_{1\mathrm{d}} \\ &\quad + K[\tilde{q}_1(0) - \tilde{q}_2(0)] - \int_0^t (\Lambda_1\tilde{q}_1 - \Lambda_2 q_2)\,\mathrm{d}\tau\Big\}; \end{aligned} \tag{27}$$

modified passivity-based control

$$\begin{aligned} u &= J\ddot{q}_{2\mathrm{r}} + K(q_{2\mathrm{d}} - q_{1\mathrm{d}}) - B_2 s_2, \\ q_{2\mathrm{d}} &= K^{-1}u_{\mathrm{R}} + q_{1\mathrm{d}}. \end{aligned} \tag{28}$$

The following remarks are in order.

- The backstepping-based controller (25) becomes a high-gain design for increasing values of the joint stiffness, because of the term $K(\dot{s}_1 + s_1)$. Note that this effect does not appear as a consequence of a term $K(q_2 - q_1)$

because of the convergence of $q_2 - q_1$ to zero as $K \to \infty$. This drawback is removed in (24) and (26), and is conspicuous by its absence in the passivity-based designs. Note in particular that the control signal u is independent of the gain K (28): the dependence on K comes only from K^{-1} and $K(q_2 - q_1)$, which remain bounded as $K \to +\infty$.

- As K grows unbounded, the control (28) converges to the Slotine and Li (1988) controller for the completely rigid robot. On the other hand, in the control (13) a term $J\ddot{q}_1$ is added to u_R.
- For large K, all decoupling- and backstepping-based controllers feed directly into the loop the signal $\ddot{q}_1$ that is calculated using (1) through $\ddot{q}_{2d}$, while (28) uses instead the noise-free reference $\ddot{q}_{1d}$. Therefore, it is reasonable to expect better noise-sensitivity properties for the latter.
- The controller (27) converges to (28) when the filter $p(pI + \Lambda_2)^{-1}$ is 'arbitrarily fast'. Therefore all remarks above concerning (28) apply as well to (27), provided that the eigenvalues of Λ_2 are chosen sufficiently large.

It is very difficult to draw a definite conclusion about the performance of the different controllers from the observations made above. This is particularly true since, as we have shown, modifications introduced at various stages of the backstepping and decoupling designs yield significant improvements. In this respect, the passivity-based technique yields robust 'tuning knob free' designs in 'one-shot', provided of course we can come out with the right desired 'total energy'. On final remark is that it is not clear to us how to remove the noise-sensitivity problem of backstepping and decoupling controllers.

4.2. *The adaptive case*

This section is devoted to studying the possible extension of the previously presented algorithms to adaptive schemes, when the dynamic parameters of the manipulator are unknown. For reasons that will appear clear later, we first study the extension of backstepping-based schemes. In the whole section we shall assume that K is a known matrix (we refer the reader to a remark below for more precision about this).

4.2.1. *Backstepping-based algorithms.* We have to solve two main problems:

(i) the input v in (15) must be LP (linear in some set of parameters), so that u is;
(ii) the signals $\tilde{q}_2$ and e_2 have to be available on line, because they will be used in the update laws.

To solve (i), we can use the idea of Lozano and Brogliato (1992), which consists in adding the determinant of the inertia matrix $\det D(q_1)$ to the Lyapunov function V_{BB}. The trick is that the nonlinearity in the unknown parameters comes from the terms containing the inverse of the inertia matrix, $D^{-1}(q_1)$. Premultiplying by $\det(D)$ allows us to retrieve LP terms, since $(\det D)D^{-1}$ is indeed LP (the price to pay is an overparametrization of the controller). Moreover, (ii) implies that q_{2d} and e_{2d} are available on line, and thus do not depend on unknown parameters.

In the following θ_i^* will generically denote a vector of unknown constant parameters, whereas $Y_i(\cdot)$ is a known and computable matrix, both of appropriate dimensions. The analysis is divided into three steps, as in the known-parameters case.

Step 1. The right-hand-side of (8) can be written as $K^{-1}Y_1(q_1, \dot{q}_1, t)\theta_1^*$. Therefore we choose q_{2d} in (8) as

$$Kq_{2d} = Y_1(q_1, \dot{q}_1, t)\hat{\theta}_1, \qquad (29)$$

where $\hat{\theta}_1$ stands for an estimate of θ_1^*. Thus

$$\tilde{q}_2 = q_2 - K^{-1}Y_1(q_1, \dot{q}_1, t)\hat{\theta}_1. \qquad (30)$$

Adding $\pm Y_1(\cdot)\theta_1^*$ to the right-hand side of the first equation in (1) and differentiating (30), we obtain

$$\begin{aligned} &D(q_1)\dot{s}_1 + C(q_1, \dot{q}_1)s_1 + B_1 s_1 = K\tilde{q}_2 + Y_1\tilde{\theta}_1, \\ &\dot{\tilde{q}}_2 = \dot{q}_2 - K^{-1}\frac{d}{dt}(Y_1\hat{\theta}_1) \end{aligned} \qquad (31)$$

(compare with (10)).

Step 2. Now consider e_{2d} in (17). The first two terms are available, but the third term is a function of unknown parameters and it is not LP (it contains D^{-1}). Assume now that V_2 is replaced by

$$V_{2a} = V_R + \tfrac{1}{2}\tilde{\theta}_1^T\tilde{\theta}_1 + \tfrac{1}{2}(\det D)\tilde{q}_2^T\tilde{q}_2, \qquad (32)$$

where $(\tilde{\cdot}) \triangleq (\hat{\cdot}) - (\cdot)$. Setting $\dot{q}_2 = e_{2d} + e_2$, i.e. $\dot{\tilde{q}}_2 = e_{2d} + e_2 - K^{-1}(d/dt)(Y_1\hat{\theta}_1)$, we get, along trajectories of (31),

$$\begin{aligned} \dot{V}_{2a} \le & -\dot{\tilde{q}}_1^T B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^T\Gamma_1^T B_1 \Gamma_1 \tilde{q}_1 + s_1^T Y_1 \tilde{\theta}_1 \\ & + \dot{\hat{\theta}}_1^T\tilde{\theta}_1 + \tilde{q}_2^T K s_1 + \tilde{q}_2^T(\det D)e_2 \\ & + \tilde{q}_2^T(\det D)(e_{2d} - \dot{q}_{2d}) + \tilde{q}_2^T\frac{d}{dt}(\tfrac{1}{2}\det D) \end{aligned} \qquad (33)$$

Let us denote $\det D = Y_2(q_1)\theta_2^*$ and

$$\begin{aligned} & Y_3(q_1, \dot{q}_1, q_2, t)\theta_3^* \\ & = \frac{d}{dt}(\tfrac{1}{2}\det D)\tilde{q}_2 - (\det D)\dot{q}_{2d} + Ks_1. \end{aligned} \qquad (34)$$

Thus (33) can be rewritten as

$$\dot{V}_{2a} \leq -\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Gamma_1^{\mathrm{T}} B_1 \Gamma_1 \tilde{q}_1 + \tilde{q}_2^{\mathrm{T}} (\det D) e_2 + \tilde{q}_2^{\mathrm{T}} (Y_2 \theta_2^* e_{2d} + Y_3 \theta_3^*) + s_1^{\mathrm{T}} y_1 \tilde{\theta}_1 + \dot{\tilde{\theta}}_1 \tilde{\theta}_1 \quad (35)$$

(We drop the arguments for convenience). Now choose

$$Y_2 \hat{\theta}_2 e_{2d} = -Y_3(q_1, \dot{q}_1, q_2, t)\hat{\theta}_3 - \tilde{q}_2, \quad (36)$$

$$\dot{\hat{\theta}}_1 = -Y_1^{\mathrm{T}}(q_1, \dot{q}_1, t) s_1. \quad (37)$$

Introducing $\pm \tilde{q}_2^{\mathrm{T}} Y_2 \hat{\theta}_2 e_{2d}$ on the right-hand side of (35), we obtain

$$\dot{V}_{2a} \leq \dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Gamma_1^{\mathrm{T}} B_1 \Gamma_1 \tilde{q}_1 + \tilde{q}_2^{\mathrm{T}} (\det D) e_2 - \tilde{q}_2^{\mathrm{T}} e_{2d} Y_2 \tilde{\theta}_2 - \tilde{q}_2^{\mathrm{T}} Y_3 \tilde{\theta}_3 - \tilde{q}_2^{\mathrm{T}} \tilde{q}_2. \quad (38)$$

Defining $V_{3a} = V_{2a} + \frac{1}{2}\tilde{\theta}_2^{\mathrm{T}}\tilde{\theta}_2 + \frac{1}{2}\tilde{\theta}_3^{\mathrm{T}}\tilde{\theta}_3$ and setting

$$\dot{\hat{\theta}}_3 = Y_3^{\mathrm{T}} \tilde{q}_2, \quad (39)$$

$$\dot{\hat{\theta}}_2 = Y_2^{\mathrm{T}} e_{2d}^{T} \tilde{q}_2, \quad (40)$$

we therefore obtain

$$\dot{V}_{3a} \leq -\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Gamma_1^{\mathrm{T}} B_1 \Gamma_1 \tilde{q}_1 + \tilde{q}_2^{\mathrm{T}} (\det D) e_2 - \tilde{q}_2^{\mathrm{T}} \tilde{q}_2. \quad (41)$$

Remark 7. In order to avoid any singularity in the control input, the update law in (40) has to be modified using a projection algorithm, assuming that θ_2^* belongs to a known convex domain. We refer the reader to Lozano and Brogliato (1992) and Bridges *et al.* (1995) for details of how this domain may be calculated, and the stability analysis related to the projection. For the sake of clarity of this presentation, we do not introduce this modification here, although we know it is necessary.

At this stage, our goal is partially reached, since we have defined signals $\tilde{q}_2$ and e_2 available on line.

Step 3. Now consider the function

$$V_{4a} = V_{3a} + \tfrac{1}{2}(\det D) e_2^{\mathrm{T}} e_2. \quad (42)$$

Noting that $\dot{e}_2 = v - \dot{e}_{2d}$, we obtain

$$\dot{V}_{4a} \leq \dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Gamma_1^{\mathrm{T}} B_1 \Gamma_1 \tilde{q}_1 + \tilde{q}_2^{\mathrm{T}} (\det D) e_2 - \tilde{q}_2^{\mathrm{T}} \tilde{q}_2 + e_2^{\mathrm{T}} (\det D)(v - \dot{e}_{2d}) + e_2^{\mathrm{T}} \frac{\mathrm{d}}{\mathrm{d}t}(\tfrac{1}{2}\det D) e_2 \quad (43)$$

Note that

$$-(\det D)\dot{e}_{2d} + \frac{\mathrm{d}}{\mathrm{d}t}(\tfrac{1}{2}\det D) e_2 = Y_4(q_1, \dot{q}_1, q_2, \dot{q}_2, t)\theta_4^* \quad (44)$$

for some $Y_4(\cdot)$ and θ_4^* matrices of suitable dimensions.

Let us now denote $\det D = Y_2(q_1)\theta_5^*$ (this is strictly equal to $Y_2(q_1)\theta_2^*$ defined above, but we choose a different notation because the estimate of θ_5^* is going to be chosen differently). Choose $v = -\tilde{q}_2 + w$ and

$$Y_2 \hat{\theta}_5 w = -Y_4 \hat{\theta} - e_2. \quad (45)$$

Introducing $\pm e_2^{\mathrm{T}} Y_2 \hat{\theta}_2 w$ into (43) and from (44) and (45), we obtain

$$\dot{V}_{4a} \leq -\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Gamma_1^{\mathrm{T}} B_1 \Gamma_1 \tilde{q}_1 - \tilde{q}_2^{\mathrm{T}} \tilde{q}_2 - e_2^{\mathrm{T}} w Y_2 \tilde{\theta}_5 - e_2^{\mathrm{T}} Y_4 \tilde{\theta}_4 - e_2^{\mathrm{T}} e_2. \quad (46)$$

Finally we choose as a Lyapunov function for the whole closed-loop system

$$V_{\mathrm{BBa}} = V_{4a} + \tfrac{1}{2}\tilde{\theta}_4^{\mathrm{T}}\tilde{\theta}_4 + \tfrac{1}{2}\tilde{\theta}_5^{\mathrm{T}}\tilde{\theta}_5, \quad (47)$$

and set the update laws

$$\dot{\hat{\theta}}_4 = Y_4^{\mathrm{T}} e_2, \quad (48)$$

$$\dot{\hat{\theta}}_5 = Y_2^{\mathrm{T}} w^{\mathrm{T}} e_2 \quad (49)$$

(a projection algorithm has to be applied to $\hat{\theta}_5$ as well; see the remark above). We obtain

$$\dot{V}_{\mathrm{BBa}} \leq -\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Gamma_1^{\mathrm{T}} B_1 \Gamma_1 \tilde{q}_1 - \tilde{q}_2^{\mathrm{T}} \tilde{q}_2 - e_2^{\mathrm{T}} e_2. \quad (50)$$

We therefore conclude that $\hat{\theta} \in \mathcal{L}_\infty$, $\tilde{q}_2$, e_2, $\tilde{q}_1$ and $s_1 \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $q_2 \in \mathcal{L}_\infty$ (see (30)), $\dot{q}_2 \in \mathcal{L}_\infty$. Finally, from the definition of s_1 and a lemma in Desoer and Vidyasagar (1975), we conclude that $\tilde{q}_1 \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\dot{\tilde{q}}_1 \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\tilde{q}_1 \to 0$ as $t \to +\infty$. From the dynamic equations in (1), it follows that $\ddot{q}_1 \in \mathcal{L}_\infty$. Hence $\dot{\tilde{q}}_1$ is uniformly continuous, and since $\tilde{q}_1$ converges towards a constant value, it follows from Barbalat's lemma that $\dot{\tilde{q}}_1 \to 0$ asymptotically.

The following remarks are in order.

- It is noteworthy that the above procedure can be seen as a modified (because of the linearity in the parameter problem, plus the *a priori* knowledge of K) version of the work of Kanellakopoulos *et al.* (1991). We retrieve the fact that the estimates are functions of the 'previous' estimates (see (36), (48), (49), (39), (40) and (30)), because of the backstepping procedure. It would also be interesting to investigate the possible extension of the work of Seto *et al.* (1994) to flexible-joint maniupulators.
- We have considered for simplicity K as a known matrix. However, in practice it may be

interesting to relax this assumption. The difficulty arises from the fact that the 'intermediate' input q_{2d} in (8) feeds the rigid dynamics through K; i.e. if K is unknown then the interconnection term between the two subsystems in (1) is unknown. This problem has been given a solution in Lozano and Brogliato (1992) by replacing K by an estimate $\hat{K}$ such that $\hat{K}$ remains full-rank and is twice-differentiable. Recently Bridges *et al.* (1995) presented a backstepping adaptive scheme based on ideas in Lozano and Brogliato (1992), without knowledge of K.

- If J is unknown, one can slightly modify $\frac{1}{2}(\det D)e_2^{\mathrm{T}}e_2$ to $\frac{1}{2}(\det D)e_2^{\mathrm{T}}Je_2$ in V_{BBA}, so that it can be incorporated into θ_4^* (see (44) and (43)).
- The adaptive case was considered by Nicosia and Tomei (1992) for the single-arm robot case. Obviously, in this simple case the LP property problem encountered for general manipulators disappears.

4.2.2. *Decoupling-based scheme.* The closed-loop error equation associated with the control law (8), (9) can be seen as a linear autonomous exponentially stable system that feed a nonlinear system; see (10) and (12). The stability of the composite cascaded system can be studied using a two-term positive-definite function $V_{\mathrm{DB}} = V_{\mathrm{R}} + \frac{1}{2}z^{\mathrm{T}}Pz$, where $A^{\mathrm{T}}P + PA = -Q$ for a given $Q = Q^{\mathrm{T}} > 0$. Consider now the input u in (11). Obviously u in (11) is not LP, because $\ddot{q}_{2d}$ contains D^{-1}. Still following the ideas in Lozano and Brogliato (1992), we could consider here multiplying the second term of V_{DB} above by $\det D$, so that some LP properties are recovered in the terms to be compensated by the input. However, it is not clear then how we can retrieve a closed-loop error equation that fits with the theorem in Seibert and Suarez (1990). Indeed, not only shall we have to deal with parameter errors (which we cannot expect to converge towards zero), but it is also not clear how we could choose the input so that the derivative of $\frac{1}{2}z^{\mathrm{T}}P(\det D)z$ results in a negative-definite term. In conclusion, although we do not claim that an adaptive extension of the decoupling-based scheme is impossible, we believe that the required modification will significantly depart from the conceptual simplicity of the known-parameters case.

4.2.3. *Passivity-based schemes.* Both algorithms derived from the energy-shaping method belong to the same family of schemes. We shall not give in this paper a detailed analysis of the adaptive version of the energy-shaping schemes, but rather we focus on a particular problem, namely the choice of the 'first-stage' input q_{2d} in (21) or (23). It is worth noting that q_{2d} in (29) (adaptive backstepping scheme) could have been chosen differently, just as it can be done in the known-parameters case. In particular, the ideas of Berghuis *et al.* (1992) can be used to enhance the robustness of the scheme with respect to velocity measurement noise. In contrast, the design of q_{2d} is much less obvious for the adaptive energy-shaping schemes. Roughly speaking, this is due to the fact that both $\tilde{q}_2$ and $\dot{\tilde{q}}_2$ have to be available on line to update some estimated parameters. This significantly reduces the possible choices for q_{2d}, which has to be chosen such that $\dot{q}_{2d}$ is computable on line; in particular, $\dot{q}_{2d}$ must be a function of positions and velocities only, and must be independent of unknown parameters. It has been shown by Lozano and Brogliato (1992) (and later used in Bridges *et al.*, 1995) that the algorithm presented by Sadegh and Horowitz (1990) can be used successfully to design q_{2d} so that $\dot{q}_{2d}$ can be calculated without acceleration measurements. We refer the reader to Lozano and Brogliato (1992) for details concerning this adaptive algorithm.

4.3. *Closed-loop passivity*

During the last few years, the passivity approach has witnessed a renewed attention. It has proved useful in the study of rigid-manipulator control (Ortega and Spong, 1989; Kelly *et al.*, 1989; Landau and Horowitz, 1989; Brogliato *et al.*, 1991; Berghius and Nijmeijer, 1993) and the global stabilization of nonlinear systems (Kokotovic and Sussman, 1989; Byrnes *et al.*, 1991; Ortega, 1991; Lozano *et al.*, 1992; Brogliato *et al.* 1993b). An underlying motivation for some of these studies is the inherent robustness properties of passive systems. Within our framework, an interesting question is the possibility of representing the closed-loop system equations as two passive operators connected in negative feedback. Such an analysis is carried out in this subsection. For the sake of brevity, we only present the block diagrams associated with the fixed-parameters versions of the backstepping-based, passivity-based and decoupling-based schemes; the passivity proofs are standard, and are therefore omitted.

4.3.1. *Backstepping-based schemes* (known-parameters case). The closed-loop system error equation can be split into two parts

$$(\mathrm{H1})\quad \begin{cases} D(q_1)\dot{s}_1 + C(q_1, \dot{q}_1)s_1 = -K_1s_1 + K\tilde{q}_2, \\ \dot{\tilde{q}}_1 = -\Lambda_1\tilde{q}_2 + s_1 \end{cases} \tag{51}$$

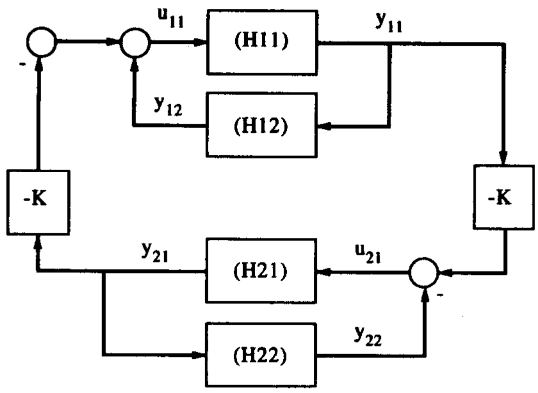

Fig. 1. Block diagram for (51) and (52).

and

$$\text{(H2)} \quad \begin{cases} \dot{\tilde{q}}_2 = -\tilde{q}_2 + e_2 - K s_1, \\ \dot{e}_2 = -e_2 - \tilde{q}_2. \end{cases} \tag{52}$$

We therefore consider the following four subsystems depicted in Fig. 1:

(H11) state vector s_1, input $u_{11} = K\tilde{q}_2 - K_1 s_1$, output $y_{11} = s_1$;
(H12) state vector $\tilde{q}_1$, input $u_{12} = s_1$, output $y_{12} = K_1 s$;
(H21) state vector $\tilde{q}_2$, input $u_{21} = e_2 - K_1 s_1$, output $y_{21} = \tilde{q}_2$;
(H22) state vector e_2, input $u_{22} = \tilde{q}_2$, output $y_{22} = -e_2$.

4.3.2. *Decoupling-based scheme* (known parameters). From the error equation in (10) and (12) and from (8), we know that $K\tilde{q}_2 \in \mathcal{L}_1 \cap \mathcal{L}_\infty$. We can therefore treat $K\tilde{q}_2$ as a $\mathcal{L}_2$-bounded disturbance, denoted by ε. The closed-loop block diagram is depicted in Fig. 2.

4.3.3. *Passivity-based schemes* (known parameters). Let us now consider the scheme proposed by Lozano and Brogliato (1992). It can be shown that the complete error equation is given by

$$\begin{aligned} \dot{\tilde{q}}_1 &= -\Lambda_1 \tilde{q}_1 + s_1, \\ D(q_1)\dot{s}_1 &+ C(Q_1, \dot{q}_1)s_1 \\ &= -K_1 s_1 + J\int_0^t (s_2 - s_1)\,\mathrm{d}t, \\ \dot{\tilde{q}}_2 &= -\Lambda_2 \tilde{q}_2 + s_2, \\ J\dot{s}_2 &= -K_2 s_2 - K\int_0^t (s_2 - s_1)\,\mathrm{d}t. \end{aligned} \tag{53}$$

This error system can be represented as the interconnection of two blocks, (H1) and (H2), where (H1) has

state vector $[\tilde{q}_1 \quad s_1 \quad \tilde{q}_2 \quad s_2]$,

input $\begin{bmatrix} K\int_0^t (s_2 - s_1)\,\mathrm{d}t \\ K\int_0^t (s_1 - s_2)\,\mathrm{d}t \end{bmatrix}$,

output $\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$,

and (H2) has

input $\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, output $\begin{bmatrix} K\int_0^t (s_1 - s_2)\,\mathrm{d}t \\ k\int_0^t (s_2 - s_1)\,\mathrm{d}t \end{bmatrix}$.

state vector $\int_0^t (s_1 - s_2)\,\mathrm{d}t$.

The superscripts (1) and (2) used below refer to the two subsystems $(\text{H1})^{(1)}$ with state $(\tilde{q}_1, s_1)$ and $(\text{H1})^{(2)}$ with state $(\tilde{q}_2, s_2)$ in (53). The block diagram is depicted in Fig. 3.

As can be expected, the only difference between the passivity interpretation of the passivity-based scheme in Lozano and Brogliato (1992) and the modified one lies in the subsystem (H2), which this time has

input $\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, output $\begin{bmatrix} K(\tilde{q}_1 - \tilde{q}_2) \\ -K(\tilde{q}_1 - \tilde{q}_2) \end{bmatrix}$,

state vector $K(\tilde{q}_2 - \tilde{q}_1)$.

Remark 8. From Figs 1 and 2, it is clear that the backstepping-based and decoupling-based schemes can be given very different interpretations from their closed-loop equations: while the first can be interpreted as the interconnection of two passive cascaded systems in closed loop where the 'flexible' error dynamics in (H21) and (H22) feeds back the 'rigid' error dynamics in (H11) and (H12), the second is composed of an autonomous subsystem with output ε that feeds a

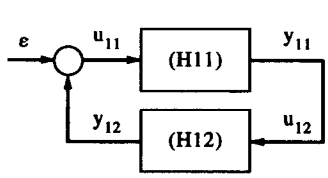

Fig. 2. Block diagram for (10) and (12).

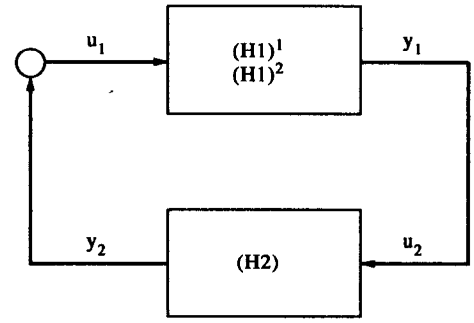

Fig. 3. Block diagram for (53).

passive subsystem. It is also interesting to note that the closed-loop equations for the scheme in Slotine and Li (1988) and for the passivity-based schemes presented in this paper differ only by a 'potential energy'-like term in the (H2) block in Fig. 3.

5. SIMULATION RESULTS

To illustrate some of the conclusions of this paper, we have simulated four algorithms (backstepping-based, robustified backstepping-based, decoupling-based and modified passivity-based) on a two-degree-of-freedom robot. The dynamic characteristics of the manipulator, which is an existing device in the Laboratoire d'Automatique de Grenoble, are described in Pastore (1992). We have considered the known-parameters case for two different sets of stiffness coefficients ($K = \mathrm{diag}\,(1, 4)$ and $K = \mathrm{diag}\,(400, 450)$). The control signals, desired trajectories, actual trajectories and tracking errors for the first joint are depicted in Figs 4 and 5. As can be seen from Figs 4(a–d), the behaviours of both control energy and tracking performance for the small stiffness case are very similar. However, as predicted by our analysis, as the stiffness coefficient increases, the control law of the backstepping-based scheme significantly degrades, behaving almost like a relay controller (see Fig. 5a). Note the smoothness of the passivity-based controller in Fig. 5(d), in accordance with the fact that this is the only controller that actually converges to the rigid controller as the stiffness goes to infinity.

6. CONCLUSIONS

In this paper we have studied different feedback control algorithms for flexible-joint manipulators. In each case we have started from a general stability result available in the literature; then we have shown that flexible-joint robots with dynamic model (1) and known parameters fit within the proposed frameworks. At this stage, we have a variety of controllers guaranteeing global tracking (although the results are not the same from one design philosophy to another) at our disposal. An interesting point is obviously to start a comparison work between those conceptually different fixed-parameters controllers. To this end, we have first examined whether these schemes share common features—that is, whether we can obtain one controller via the application of a different methodology. We have studied what happens when the joint stiffness becomes infinitely large—that is, do the proposed schemes enjoy a 'continuity' property between the rigid and the flexible cases? How does the loop gain behave in the 'almost-rigid' robot case?. We have presented the extension of the fixed-parameters case to the adaptive case. Finally, we have proved that these schemes can be interpreted in closed loop as the interconnection of passive subsystems, thus extending the now well-known results on the passivity of rigid robots.

The Lyapunov and passivity analyses provide us with some measure of the transient performances and robustness with respect to (benign) unmodelled dynamics. The study of the limiting case of 'almost-rigid' robots gives us some insight into the expected behaviour *vis-à-vis* noise rejection or poor knowledge of the stiffness matrix K, although this latter problem deserves further consideration. Complexity of the algorithms (especially in the adaptive case, where an overparametrization is inherent to the employed method) also deserves further work. We believe that this work has to be considered as a first tentative (theoretical) comparison between several control laws, derived from basic tools recently presented in the literature by several authors.

A particularly interesting open problem is the derivation of globally stable schemes for more complete robot models that incorporate cross-terms in the inertia and Coriolis matrices. Although it has been claimed in the literature that these models are feedback-linearizable by dynamic state feedback, and several authors have reported the solution for particular cases, the problem remains open. The main stumbling block to solving it with the techniques presented here is the unavailability (without differentiation) of higher-order derivatives of the joint angles, which is related to the inability to render the system triangular and decompose it into the feedback interconnection of passive subsystems (Ortega *et al.*, 1994).

In closing these conclusions, we should like to remark that we believe that a more practical approach to handling the elasticity problem is to view the flexible robot as a perturbation of the rigid one. This was done for instance by Ghorbel and Spong (1992), who used composite controls based on integral manifold ideas. Even though some very interesting local stability results have been established for these schemes, we restrict ourselves in our comparison to controllers for which global stability is ensured. We also feel that control strategies that rely on high-gain loop properties, like sliding-mode techniques, will tend to behave poorly for this particular problem of elastic mechanical structures (in spite of their somewhat attractive theoretical features).

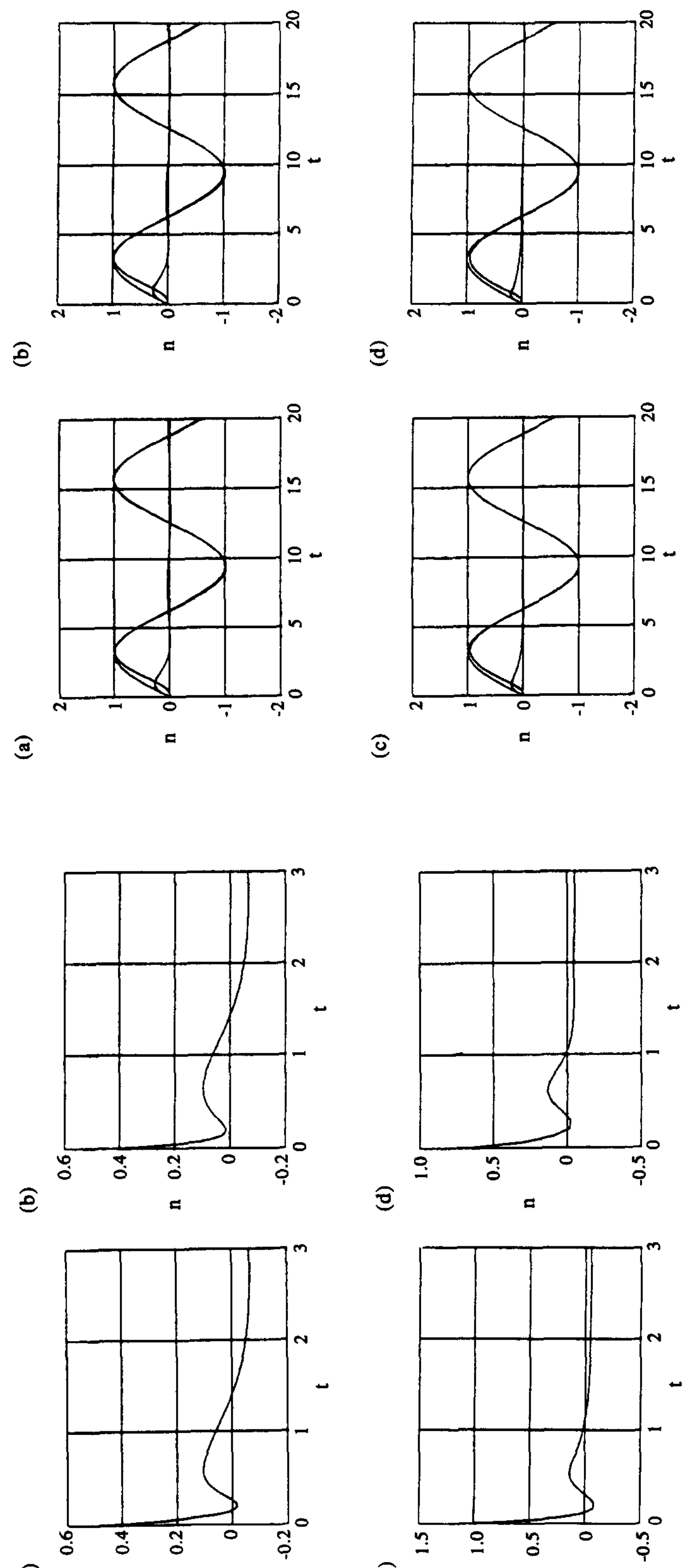

Fig. 4. Control signals and closed-loop trajectories for K-diag (1, 4).

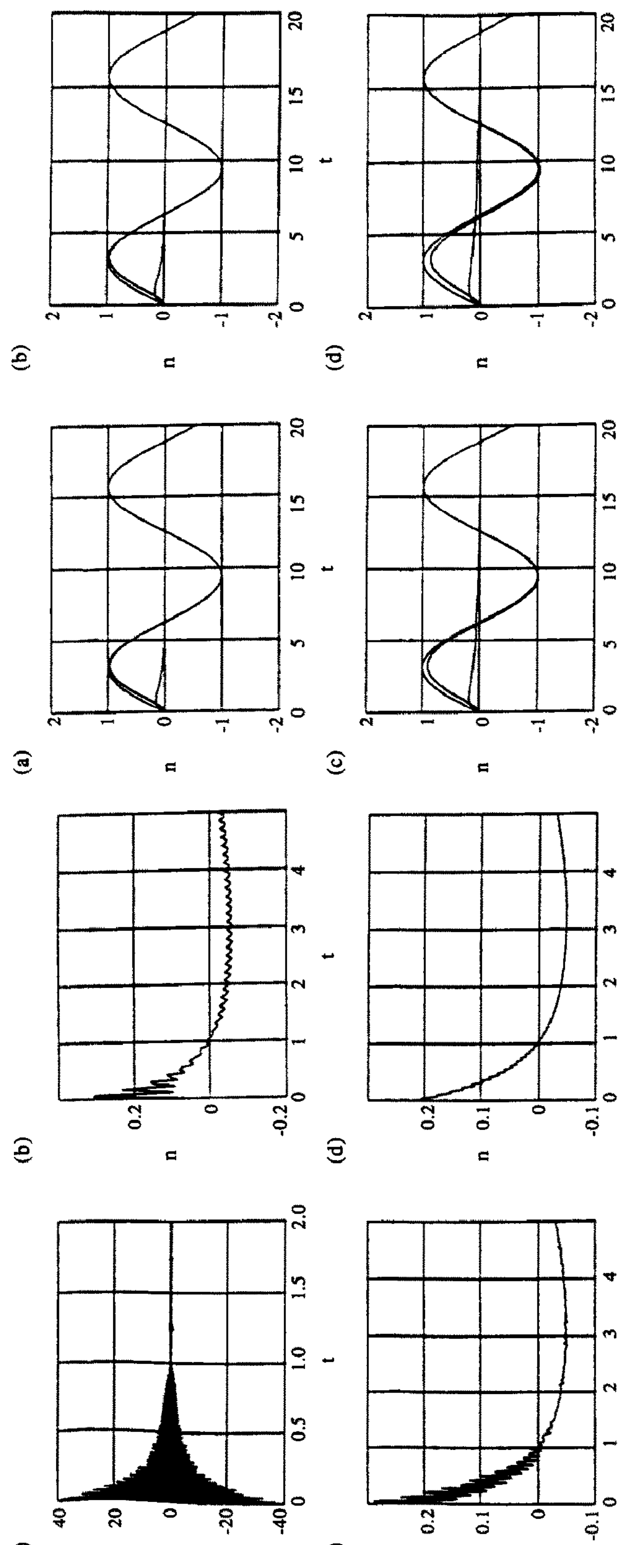

Fig. 5. Control signals and closed-loop trajectories for $K = \mathrm{diag}\,(400, 450)$.

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