

Resonant Andreev Tunneling in Strongly Interacting Quantum Dots

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Abstract

We study resonant Andreev tunneling through a strongly interacting quantum dot connected to a normal and to a superconducting lead. We obtain a formula for the Andreev current and apply it to discuss the linear and non-linear transport in the nonperturbative regime, where the effects of the Kondo resonance on the two particle tunneling arise. In particular we notice an enhancement of the Kondo anomaly in the $I - V$ characteristics due to the superconducting electrode.

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The way a superconducting electrode affects the electrical resistance of a small mesoscopic region via Andreev tunneling, has been the subject of an impressive research activity over the last years [1]. This has led to the discovery of a number of new phenomena. Examples are the zero-bias anomalies in Normal-Insulating-Superconducting (NIS) structures [2] or temperature (and voltage) re-entrant behaviour in the conductance of a small metallic wire in contact with a superconducting lead [3]. Several of these phenomena have been successfully explained by models of non-interacting electrons [4–9]. Electron-electron interaction strongly modifies the two-particle tunneling. In small capacitance junctions, for example, it leads to the Coulomb blockade of Andreev tunneling [10], whereas in a Luttinger liquid-Superconducting structure anomalous $I - V$ characteristics appear due to the excitation of the low lying modes [11].

Electrical transport via resonant tunneling in strongly interacting systems has been intensively investigated because of the possibility to study the behavior of an Anderson-like impurity model. The prototype model is a Quantum Dot (QD), which mimics the impurity level, coupled by tunnel junctions to two non-interacting leads. When the dot level is far below the Fermi energy of the leads, the formation of a spin singlet between the impurity spin and the conduction electrons gives rise to a many-body resonance at the Fermi energy. Such a resonance, known as Abrikosov-Suhl or Kondo resonance, manifests in a peak in the density of states at the Fermi level and leads to a perfect transmission at zero temperature [12]. The two reservoirs, differently from the case of the Anderson impurity in metals, can be kept at different chemical potentials. Nonlinear transport through a QD provides then a beautiful tool to study a Anderson impurity out of equilibrium [13]. The nonlinear $I - V$ characteristics and the Kondo resonance in nonequilibrium conditions have been studied by a variety of methods: the equation of motion for the Green's function [13], variational procedure [14], a newly developed diagrammatic technique [15], and the $1/N$ expansion [16]. Experimental evidences of Kondo-like correlations have been reported in metal point contact [17] and very recently in SET transistors [18]. Zero bias anomalies have also been reported recently in experiments involving superconductor/Anderson-insulator interfaces, though their physical interpretation is still under debate [19].

Remarkably a QD offers the, yet unexplored, possibility to study a quantum impurity coupled to two different types of electronic systems, such as superconductors and normal metals at the same time. The development of superconductor-semiconductor integration technology should allow to study the signatures of the Kondo effect in the Andreev tunneling.

In this Letter we will study Resonant Andreev Tunneling (RAT) in a QD coupled to a normal and to a superconducting reservoirs as shown schematically in Fig. 1. Differently from the conventional problem of a Kondo impurity in a superconductor [20] here the low lying excitations are still present due to the presence of the normal electrode.

The model Hamiltonian for the N-QD-S system under consideration can be written as

$$H = H_N + H_S + H_D + H_{T,N} + H_{T,S} \quad (1)$$

where $H_N = \sum_{k,\sigma} \epsilon_k c_{N,k,\sigma}^\dagger c_{N,k,\sigma}$, $H_S = \sum_{k,\sigma} \epsilon_k c_{S,k,\sigma}^\dagger c_{S,k,\sigma} + \sum_k (\Delta c_{S,k,\uparrow}^\dagger c_{S,-k,\downarrow}^\dagger + c.c.)$ and $H_D = \epsilon_d d_\sigma^\dagger d_\sigma + U n_{d,\uparrow} n_{d,\downarrow}$ are the Hamiltonians of the normal lead, the superconducting lead (Δ is the superconducting gap) and the dot respectively. The single particle energy ϵ_d is double degenerate in the spin index σ and the interaction is included through the on-site repulsion U . The position of the dot level can be modulated by an external gate voltage.

The tunneling between the leads and the dot is described by $H_{T,\eta} = \sum_{k,\sigma} (V_{\eta,k,\sigma} c_{\eta,k,\sigma}^\dagger, d_\sigma + c.c.)$ where $\eta = N, S$ and $V_{\eta,k,\sigma}$ is the tunneling amplitude.

The average current, which for convenience we compute in the normal electrode, is given by $I = 2e \text{Im} \sum_k \langle \Psi_{N,k}^\dagger \hat{\tau}_z \hat{H}_k \Phi \rangle$, where we have adopted the Nambu notation (the hat indicates matrices in the Nambu space): $\Psi_{N,k} = (c_{N,k,\uparrow}, c_{N,-k,\downarrow}^\dagger)$ and $\Phi = (d_\uparrow, d_\downarrow^\dagger)$. \hat{H}_k is a diagonal matrix with elements $H_{11} = V_{N,k,\uparrow}, H_{22} = V_{N,-k,\downarrow}^*$.

By means of the Keldysh technique, as employed in Ref. [21], the current I can be rewritten in the form

$$I = ie \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \Gamma_N \text{Tr} \{ \hat{\tau}_z \hat{G}^R(\epsilon) [\hat{\Sigma}^R(\epsilon) \hat{f}_N(\epsilon) - \hat{f}_N(\epsilon) \hat{\Sigma}^A(\epsilon) + \hat{\Sigma}^<(\epsilon)] \hat{G}^A(\epsilon) \} \quad (2)$$

where $\hat{G}^{R(A)}$, $\hat{G}^<$ are the retarded (advanced) and the lesser Green's functions of the dot (for example $\hat{G}^R(t) = -i\theta(t) \langle \{ \Phi(t), \Phi^\dagger(0) \} \rangle$). In deriving eq.(2), the relation $\hat{G}^< = \hat{G}^R \hat{\Sigma}^< \hat{G}^A$ has been used. Here $\Gamma_N(\epsilon) = 2\pi \sum_k |V_{N,k,\sigma}|^2 \delta(\epsilon - \epsilon_k)$. For the sake of simplicity, from now on we will neglect the spin and energy dependence of the tunneling matrix elements. The diagonal matrix \hat{f}_N has elements $f_{N,11} = f((\epsilon + eV)/T)$ and $f_{N,22} = 1 - f((-\epsilon + eV)/T)$ if the normal electrode is kept at a chemical potential which differs by eV from that of the superconductor ($f(x)$ is the Fermi function and V the voltage drop).

To obtain a suitable expression for the current, we need to evaluate the lesser self-energy. In the non-interacting case, $\hat{\Sigma}^R(\epsilon)$ can be computed exactly and it is expressed in terms of the retarded and advanced self-energies as $\hat{\Sigma}_0^<(\epsilon) = -\sum_{\eta=N,S} (\hat{\Sigma}_{0,\eta}^R(\epsilon) \hat{f}_\eta(\epsilon) - \hat{f}_\eta(\epsilon) \hat{\Sigma}_{0,\eta}^A(\epsilon))$. In the interacting case, we generalize Ng's ansatz to the present case [22]. The lesser and greater self-energies are assumed to be of the form

$$\hat{\Sigma}^< = \hat{\Sigma}_0^< \hat{A} \quad , \quad \hat{\Sigma}^> = \hat{\Sigma}_0^> \hat{A} \quad (3)$$

where \hat{A} is a matrix to be determined by the condition $\hat{\Sigma}^< - \hat{\Sigma}^> = \hat{\Sigma}^R - \hat{\Sigma}^A$. This ansatz is exact both in the non-interacting limit, $U = 0$, and in the absence of superconductivity, $\Delta = 0$. Moreover it guarantees automatically a current conserving scheme for any given approximation procedure to evaluate the retarded Green's function. As a result we obtain $\hat{\Sigma}^< = \hat{\Sigma}_0^< (\hat{\Sigma}_0^R - \hat{\Sigma}_0^A)^{-1} (\hat{\Sigma}^R - \hat{\Sigma}^A)$. The expression for the current can be greatly simplified in the relevant limit $U, \Delta \gg k_B T, V$. In this case the non-interacting self-energy due to the superconducting lead $\hat{\Sigma}_{0,S}^{R(A)}$ is real and purely off-diagonal whereas that due to the normal lead, $\hat{\Sigma}_{0,N}^{R(A)}$, is diagonal. As a consequence, we obtain the following form for the Andreev current through a QD

$$I = ie \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \Gamma_N \text{Tr} \{ \hat{\tau}_z \hat{G}^R(\epsilon) [\hat{\Sigma}^R(\epsilon), \hat{f}_N(\epsilon)] \hat{G}^A(\epsilon) \} \quad (4)$$

This is the first central result of this work. It generalizes to the case of a strongly interacting dot the formula valid in the non interacting case [23] and allows to study the transport through a N-QD-S under non-equilibrium situations and in the nonperturbative regime.

The presence of a superconducting lead introduces, besides U , another energy scale in the problem, the energy gap Δ . One is often interested to voltages and temperatures well below the energy scale set by Δ and could be tempted to take the $\Delta \rightarrow \infty$ from the outset. In an interacting quantum dot, the $\Delta \rightarrow \infty$ limit cannot be taken, in contrast to the non-interacting case. In order to have coherent Andreev scattering, the two electrons enter the

superconductor without double occupying the QD (which is forbidden in the $U \rightarrow \infty$ limit). This can happen only on a time scale of the order $1/\Delta$. We will then consider the case $U \gg \Delta \gg T, V$ [24].

The generalization of the decoupling scheme for \hat{G}^R in the presence of superconductivity is conveniently done by rewriting the superconducting lead Hamiltonian, H_S , in terms of quasiparticles operators by means of a Bogoliubov transformation. The reason for transforming to the quasiparticles basis is dictated by the type of approximations introduced in the equation of motion approach. In fact, one may generalize to the case of a superconducting lead, the decoupling usually introduced in the normal case. Such decoupling, which neglects correlations in the lead, is done in terms of the equilibrium number of quasiparticles. The final expression for the matrix Green's function of the dot is [25,26]

$$\begin{pmatrix} \epsilon - \epsilon_d - \sigma_{N,11}(\epsilon) - \sigma_{S,11}(\epsilon) & \Gamma_S/2 \\ \Gamma_S/2 & \epsilon + \epsilon_d - \sigma_{N,22}(\epsilon) - \sigma_{S,22}(\epsilon) \end{pmatrix} \hat{G} = \begin{pmatrix} 1 - \langle n_\downarrow \rangle & 0 \\ 0 & 1 - \langle n_\uparrow \rangle \end{pmatrix} \quad (5)$$

where $\sigma_{N,11}$ is the $U = \infty$ -limit self-energy considered in Ref. [13]

$$\sigma_{N,11}(\epsilon) = -i \frac{\Gamma_N}{2} + \sum_k |V|^2 \frac{f_N(\epsilon_k)}{\epsilon - \epsilon_k + i0^+} \quad (6)$$

and $\sigma_{S,11}$ is the corresponding self-energy due to the superconducting electrode

$$\sigma_{S,11}(\epsilon) = - \sum_k |V|^2 \frac{v_k^2}{\epsilon + E_k + i0^+} \quad (7)$$

($v_k^2 = 1/2(1 - \epsilon_k/E_k)$ and $E_k = \sqrt{\epsilon_k^2 + |\Delta|^2}$). The self-energy for the hole propagator can be obtained using the property $\sigma_{N(S),22}(\epsilon) = \sigma_{N(S),11}^*(-\epsilon)$. The self-energy $\sigma_{S,11}$ is weakly energy dependent due to the low energy cutoff provided by Δ (in what follows, we consider $\omega_c \gg \Delta \gg T, \epsilon$ with ω_c the bandwidth). At energies much smaller than the gap quasiparticles present in the dot cannot decay by tunneling into the superconductor (as the imaginary part of the diagonal elements of the self-energy $\hat{\sigma}_S$ vanishes). As a result the contribution of the self energy due to the superconducting lead $\sigma_{S,11}$ simply shifts the dot level to the new value $\tilde{\epsilon}_d \approx \epsilon_d + (\Gamma_S/2\pi) \ln \omega_c/\Delta$. The divergence of the level energy renormalization with Δ reveals that the process occurs via a virtual state in which a quasiparticle is created in the superconductor. It is however due to the weak (logarithmic) nature of this divergence that RAT can be observed in any realistic situation. Note also that the off-diagonal element, $\Gamma_S/2$, entering eq.(5), has the same form of the non interacting case apart from an overall phase-shift of π . This π phase-shift comes from a relative cancellation between the non-interacting and interacting off-diagonal self-energies. Substituting the expression of the QD's Green's function in Eq.(2) we get the desired result

$$I(V) = \int_{-\infty}^{\infty} d\epsilon \frac{f(\epsilon - eV) - f(\epsilon + eV)}{2e} G_{NS}(\epsilon) \quad (8)$$

with

$$G_{NS}(\epsilon) = \frac{4e^2}{h} \frac{2(\Gamma_N \Gamma_S)^2 ((\Gamma_{1,N} + \Gamma_{2,N})/2\Gamma_N)}{[4(\epsilon - \epsilon_1)(\epsilon + \epsilon_2) - \Gamma_{1,N}\Gamma_{2,N} - \Gamma_S^2]^2 + 4[\Gamma_{1,N}(\epsilon + \epsilon_2) + \Gamma_{2,N}(\epsilon - \epsilon_1)]^2} \quad (9)$$

where $\epsilon_{1(2)} = \epsilon_d + Re\hat{\sigma}_{N,11(22)} + Re\hat{\sigma}_{S,11(22)}$, $\Gamma_{1(2),N} = -2Im\hat{\sigma}_{N,11(22)}$. Eqs.(4,9) are the main results of the present paper.

The spectral function $G_{NS}(\epsilon)$ associated with the resonant Andreev tunneling is plotted in Fig.2 for various bias voltages (for comparison, the non-interacting case is shown in the inset). Several features are worth noticing. First, two peaks at $\pm\tilde{\epsilon}$ are due to particle and hole bare levels. Note that in the interacting case the bare level energy includes the renormalization due to the superconducting electrode self-energy as discussed above. Second, at low temperatures a Kondo peak develops at the Fermi energy. Quite remarkably, at finite positive (negative) voltages the Kondo peak shifts pinned to the Fermi level of the normal metal while a small kink develops at negative (positive) voltages. At finite voltages hole and particle energies differ by $2eV$, and while the electron (hole) is on resonance for positive (negative) voltage, the Andreev reflected hole (electron) is off resonance with respect to the shifted Fermi level. Compared to the N-QD-N case, here there is no peak splitting. Third, the Kondo peak remains rather pronounced even in the nonequilibrium situation. The differences between the N-QD-N and the N-QD-S cases, can be traced back to the fact that the superconducting electrode acts simply as a boundary condition, even in the nonequilibrium situation, the Kondo resonance being achieved through the tunneling into the normal electrode. For this reason the Kondo peak is shifted but not suppressed.

The differential conductance, for various temperatures, is shown in Fig. 3. In the low temperature regime the zero bias anomaly associated with the Kondo effect is clearly seen. We note that the Kondo peak survives up to temperatures of the order of $\Gamma/4$, about one order of magnitude larger than the N-QD-N case. This effects is shown in the inset of Fig. 3 where the zero bias anomaly for the N-QD-S (present work) is compared with the corresponding quantity in the N-QD-N, Ref. [13]. The anomaly in the RAT clearly survives in a temperature regime where is already absent for normal tunneling. This may be useful for the experimental detection of the effect.

In this Letter we studied the RAT in a Normal metal - Quantum Dot - Superconductor device. An explicit form of the current through the device is obtained. The analysis of the $I - V$ characteristics has been carried out in the limit $U \gg \Delta \gg T, V$, where two electrons tunnel almost independently since the onsite Coulomb repulsion (which is the largest energy scale in the problem) prohibits the double occupancy of the Dot. In this case the Kondo effect enhances the Andreev conductance at low temperatures. In the opposite limit, $\Delta \gg U \gg T, V$, different processes dominate the transport and a suppression of the Andreev tunneling is expected. The results were obtained by generalizing, in the presence of superconductivity, an ansatz due to Ng and by means of the equation of motion method to determine the Green's function of the Dot. While the expression for the current in Eq. (4) holds under general circumstances, the results for the spectral function and the differential conductance are quantitatively valid for temperatures larger than the Kondo temperatures and only qualitatively valid for lower temperatures [13,22]. We then expect that, even though a more refined treatment is necessary for more quantitative results in the Kondo region, all qualitative features of our findings will survive. The combined effect of a finite Δ and U is currently being investigated and it will be a subject of a forthcoming publication.

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- [24] In order to appreciate this point, let us consider the Hamiltonian of an isolated quantum dot, plus a term which models the Andreev scattering at the boundary $H_{T,S} = (t_A d_{\uparrow}^{\dagger} d_{\downarrow}^{\dagger} + c.c.)$. This problem can be solved exactly and the off-diagonal element of the inverse Green's function reads $-t_A (1 + U(\epsilon + \epsilon_d + U)/(\epsilon^2 - (\epsilon_d + U)^2 - t_A^2))$ which vanishes in the $U \rightarrow \infty$ limit. Then, in the limiting sequence $\Delta \rightarrow \infty$ first, $U \rightarrow \infty$ after, the induced superconductivity in the dot is completely suppressed. The ansatz discussed in the paper, Eq. 3, is exact in this soluble case.
- [25] As discussed in Ref. [13], the equation of motion method does not provide quantitative

results in the Kondo region, however it gives the qualitative feature both in and out of equilibrium case.

[26] Details of the derivation will be published elsewhere, R. Fazio and R. Raimondi (unpublished).

FIGURES

FIG. 1. The system under consideration. A Quantum Dot coupled by tunnel barriers to a Normal and to a Superconducting electrodes. The position of the level in the dots can be tuned by means of the gate voltage V_g

FIG. 2. The spectral density for the two particle tunneling $G_{NS}(\epsilon)$ is plotted for various bias voltages ($V = 0$ solid line, $V = \pm 0.005$ dotted line, $V = \pm 0.01$ dashed line, $V = \pm 0.015$ dot-dashed line, $\tilde{\epsilon}_d = -0.07$, $\Gamma_S = \Gamma_N = 0.02$ and $T = 0.0001$, in units of the bandwidth ω_c). In the inset the non interacting case is shown for comparison.

FIG. 3. The differential conductance of the NQDS device, in units of $4e^2/h$, is plotted for different temperatures ($T = 0.0001$ solid line, $T = 0.001$ dotted line, $T = 0.01$ dot-dashed line, $\tilde{\epsilon}_d = -0.04$, $\Gamma_S = \Gamma_N = 0.02$, in units of the bandwidth ω_c). In the inset the zero bias anomaly, detected by measuring the relative change of the differential conductance $G(V)$ between zero and high voltage ($\Delta G/G \equiv (G(0) - G(-0.04))/G(0)$), is shown for the N-QD-S (triangles) and for the N-QD-N (filled circles) [13]





