

# **Sensor deployment strategy for detection of targets traversing a region**

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## **Abstract**

*Sensing devices can be deployed to form a network for monitoring a region of interest. This paper investigates detection of a target traversing the region being monitored by using collaborative target detection algorithms among the sensors. The objective of the study is to develop a low cost sensor deployment strategy to meet a performance criteria. The paper defines a path exposure metric as a measure of goodness of deployment. It then gives a problem formulation for the random sensor deployment and defines cost functions that take into account the cost of single sensors and the cost of deployment. A sequential sensor deployment approach is then developed. The paper illustrates that the overall cost of deployment can be minimized to achieve the desired detection performance by appropriately choosing the number of sensors deployed in each*

*step of the sequential deployment strategy.*

Keywords: collaborative target detection, deployment, exposure, sensor networks, value fusion.

## 1 Introduction

Recent advances in computing hardware and software are responsible for the emergence of sensor networks capable of observing the environment, processing the data and making decisions based on the observations. Such a network can be used to monitor the environment, detect, classify and locate specific events, and track targets over a specific region. Examples of such systems are in surveillance, monitoring of pollution, traffic, agriculture or civil infrastructures [10]. The deployment of sensor networks varies with the application considered. It can be predetermined when the environment is sufficiently known and under control, in which case the sensors can be strategically hand placed. In some other applications when the environment is unknown or hostile, the deployment cannot be a priori determined, for example if the sensors are air-dropped from an aircraft or deployed by other means, generally resulting in a random placement.

This paper investigates deployment strategies for sensor networks performing target detection over a region of interest. In order to detect a target moving through the region, sensors have to make local observations of the environment and collaborate to produce a global decision that reflects the status of the region covered [2]. This collaboration requires local processing of the observations, communication between different nodes, and information fusion [11]. Since the local observations made by the sensors depend on their position, the performance of the detection algorithm is a function of the deployment. One possible measure of the goodness of deployment for target detection is called *path exposure*. It is a measure of the likelihood of detecting a target traversing the region using a given path. The higher the path exposure, the better the deployment. The set of paths to be considered may be constrained by the environment. For example, if the target is expected to be following a road, only the paths consisting of the roads need to be considered.

In this study, the deployment is assumed to be random which corresponds to many practical applications where the region to be monitored is not accessible for precise placement of sensors. The focus of this paper is to determine the number of sensors to be deployed to carry out target detection in a region of interest. The tradeoffs lie between the network performance, the cost of the sensors deployed, and the cost of deploying the sensors. This paper is organized as follows. In section 2, a definition for path exposure is proposed and a method to evaluate the exposure of a given path is developed. In section 3, the

problem of random deployment is formulated and several solutions are presented. An analytical study of these solutions is given in section 4 and section 5 presents simulation results that are used in section 6 to determine the optimum solution for a given environment. The paper concludes with section 7.

## 2 Path exposure

Before formulating the deployment problem, a model for sensor network target detection, a definition of path exposure, and expressions for evaluating this path exposure are developed.

### 2.1 Model

Consider a rectangular sensor field with  $n$  sensors deployed at locations  $s_i, i = 1, \dots, n$ . A target at location  $u$  emits a signal which is measured by the sensors. The signal from the target decays as a polynomial of the distance. If the decay coefficient is  $k$ , the signal energy of a target at location  $u$  measured by the sensor at  $s_i$  is given by

$$S_i(u) = \frac{K}{\|u - s_i\|^k} \quad (1)$$

where  $K$  is the energy emitted by the target and  $\|u - s_i\|$  is the geometric distance between the target and the sensor. Depending on the environment the value  $k$  typically ranges from 2.0 to 5.0 [6].

Energy measurements at a sensor are usually corrupted by noise. If  $N_i^2$  denotes the noise energy at sensor  $i$  during a particular measurement, then the total energy measured at sensor  $i$ , when the target is at location  $u$ , is

$$E_i(u) = S_i(u) + N_i^2 = \frac{K}{\|u - s_i\|^k} + N_i^2. \quad (2)$$

The sensors collaborate to arrive at a consensus decision as to whether a target is present in the region. We consider two basic approaches for reaching this consensus: Value fusion and Decision fusion [4]. In value fusion, one of the sensors gathers the energy measurements from the other sensors, totals up the energy and compares the sum to a threshold ( $\eta$ ) to decide whether a target is present. If the sum exceeds the threshold, then the consensus decision is that a target is present. In contrast, in decision fusion, each individual sensor compares its energy measurement to a threshold ( $\eta_1$ ) to arrive at a local decision as to whether a target is present. The local decisions (1 for target present and 0 otherwise) from the sensors are totaled at a sensor and the sum is compared to another threshold ( $\eta_2$ ) to arrive at the consensus decision. In some situations, value fusion outperforms decision fusion and vice versa.

### 2.1.1 Value Fusion.

The probability of consensus target detection when the target is at location  $u$  is

$$\begin{aligned} D_v(u) &= \text{Prob} \left[ \sum_{i=1}^n \left( \frac{K}{\|u - s_i\|^k} + N_i^2 \right) \geq \eta \right] \\ &= \text{Prob} \left[ \sum_{i=1}^n N_i^2 \geq \eta - \sum_{i=1}^n \frac{K}{\|u - s_i\|^k} \right], \end{aligned} \quad (3)$$

where  $\eta$  is the value fusion threshold. If the noise processes at the sensors are independent, then the probability density function of  $\sum_{i=1}^n N_i^2$  equals the convolution of the probability density function of  $N_i^2$ ,  $i = 1, \dots, n$ . In particular, if the noise process  $N_i$  at each sensor is Additive White Gaussian Noise (AWGN), then  $\sum_{i=1}^n N_i^2$  has a Chi-square distribution of degree  $n$ .

Due to the presence of noise, the sensors may incorrectly decide that a target is present even though there is no target in the field. The probability of a consensus false target detection is

$$F_v = \text{Prob} \left[ \sum_{i=1}^n N_i^2 \geq \eta \right]. \quad (4)$$

As above, if the noise processes at the sensors are independent and AWGN, then the false alarm probability can be computed from the Chi-square distribution of degree  $n$ .

### 2.1.2 Decision Fusion.

For decision fusion, let  $h_{d,i}$  be the local decision at sensor  $i$  in the presence of a target in the region. The probability of consensus target detection when the target is located at  $u$  is

$$\begin{aligned} D_d(u) &= \text{Prob} \left[ \sum_{i=1}^n h_{d,i}(u) \geq \eta_2 \right] \\ &= \sum_{j=\eta_2}^n \sum_{\text{permutations } \sigma} \prod_{l=1}^j P_{1,\sigma(l)} \cdot \prod_{l=j+1}^n P_{0,\sigma(l)} \end{aligned}$$

where  $\sigma(j)$  denotes the  $j$ -th element of a permutations of sequence  $\{1,2,\dots,n\}$ ,  $\eta_1$  is the threshold used to make the local decision at each sensor,  $\eta_2$  is the threshold used to fuse the local decisions, and

$$\begin{aligned} P_{1,i} &= \text{Prob}[h_{d,i}(u) = 1] \\ &= \text{Prob}\left[N_i^2 \geq \eta_1 - \frac{K}{\|u - s_i\|^k}\right], \text{ and} \\ P_{0,i} &= \text{Prob}[h_{d,i}(u) = 0] \\ &= 1 - \text{Prob}[h_{d,i}(u) = 1]. \end{aligned}$$

can be computed from Chi-square distribution of degree 1 for AWGN noise process.

Let  $g_{d,i}$  be the local decision at sensor  $i$  in the absence of target in the region. The probability of false target detection at sensor  $i$  is

$$\begin{aligned} \text{Prob}[g_{d,i} = 1] &= \text{Prob}[N_i^2 \geq \eta_1] \text{ and} \\ \text{Prob}[g_{d,i} = 0] &= 1 - \text{Prob}[g_{d,i} = 1]. \end{aligned}$$

Therefore, the probability of consensus false target detection is

$$\begin{aligned} F_d &= \text{Prob}\left[\sum_{i=1}^n g_{d,i} \geq \eta_2\right] \\ &= \sum_{j=\eta_2}^n \binom{n}{j} \cdot P_1^j \cdot P_0^{(n-j)} \end{aligned}$$

where  $P_1$  and  $P_0$  are not function of the position of the sensor or the target, and are given by

$$\begin{aligned} P_1 &= \text{Prob}[g_{d,i} = 1] \\ P_0 &= \text{Prob}[g_{d,i} = 0] \end{aligned}$$

The above equations serve as an analytic basis for evaluating exposure as defined in the following subsection.

Note that in value and decision fusion the knowledge of the sensors' locations can be used when making the detection decision. For example, a sensor can report values that differ substantially from its neighbors values. This discrepancy can be analyzed to reduce false alarms or misses and thus improve the detection performance. However, such algorithms are not considered in this paper.

## 2.2 Definition of exposure

*Figure 1 goes here*

We define *exposure* to be the probability of detecting the target or an intruder carrying out the unauthorized activity, where the activity depends on the problem under consideration. In this paper, the activity considered is the Unauthorized Traversal (UT) as defined below.

**Unauthorized Traversal (UT) Problem:** Consider a sensor field with  $n$  sensors at locations  $s_1, s_2, \dots, s_n$  (see Figure 1).

Further, we assume that the stochastic characterization of the noise at each sensor and a tolerable bound,  $\alpha$ , on the false alarm probability are known. Let  $P$  denote a path from the west to the east periphery of the sensor field. A target traversing the sensor field using path  $P$  is detected if it is detected at some point  $u \in P$ . We define the *exposure* of path  $P$  to be the net probability of detecting a target that traverses the field using  $P$ . The target is assumed to be able to follow any path through the field and the problem is to find the path  $P$  with the least exposure.

The notion of *exposure* was initially introduced in [7, 8, 9] to evaluate the coverage of a region by a set of sensors performing target detection. The exposure as defined in these studies evaluates the detectability of a target traveling through a path, but is not a direct measure of it. In [7], the authors relate the exposure of a path  $P$  to the distance from  $P$  to the sensors and point out two paths of interest. The maximal breach path is defined as a path where its closest distance to any of the sensors is as large as possible, which is the least exposed path. And the maximal support path is defined as a path where its farthest distance from the closest sensor is as small as possible, which is the most exposed path. The authors also describe algorithms for efficiently determining the maximal breach and maximal support path for a given sensor field using Voronoi diagrams and Delaunay triangulation.

In [8], exposure of a path  $P$  is defined as the total energy that the sensors will gather from the target moving through the field following  $P$ . The smaller this energy the lesser the likelihood of detecting the target and the coverage provided by a deployment is measured by the minimum exposure when considering all possible paths. An algorithm for determining a path with the least exposure in this sense is developed in [8]. The algorithms presented in [7, 8] assume that all the sensors will collaborate in performing detection. A similar algorithm for finding minimum exposure is developed in [9] when assuming

that detection is run locally by only part of all the sensors.

Finally, studies [3, 5, 12] also define metrics to measure the quality of deployment. However, they consider that the sensors have to fully cover the region in which they are deployed instead of considering the detection of target following paths as in unauthorized traversal. These studies consider different models for the coverage of each sensor, a probabilistic model in [5, 12] that accounts for the possible presence of obstacles in the region, and a sensing range model in [3]. The problem is to reach a desired average coverage as well as maximizing the coverage of the most vulnerable points of the region.

### 2.3 Solution to the UT problem

Let  $P$  denote a path from the west to the east periphery through the sensor field. A target that traverses the field using  $P$  is not detected if and only if it is not detected at any time while it is on that path. Since detection attempts by the sensor network occur at a fixed frequency, we can associate each detection attempt with a point  $u \in P$  when assuming that the target traverses the field at a constant speed. The detection attempts are based on energy measured over a period of time  $T$  during which the target is moving. Therefore, the detection probability associated with each point  $u$  reflects the measurements performed during time  $T$ . Considering the path, the net probability of not detecting a target traversing the field using  $P$  is the product of the probabilities of no detection at each point  $u \in P$ . That is, if  $G(P)$  denotes the net probability of not detecting a target as it traverses over path  $P$ , then,

$$\log G(P) = \sum_{u \in P} \log(1 - D(u)),$$

where  $D(u)$  is either  $D_v(u)$  or  $D_d(u)$  depending on whether the sensors use value or decision fusion to arrive at a consensus decision. Note that  $D(u)$  is the probability to detect the target during a time period  $T$  where the target is moving. Therefore, the energy values used to find  $D(u)$  are the average energies measured at each sensor over period  $T$ . Since the exposure of  $P$  is  $(1 - G(P))$ , the problem is to find the path which minimizes  $(1 - G(P))$  or equivalently the path that minimizes  $|\log G(P)|$  (note that,  $G(P)$  lies between 0 and 1 and thus  $\log G(P)$  is negative).

In general, the path  $P$  that minimizes  $|\log G(P)|$  can be fairly arbitrary in shape. The proposed solution does not exactly compute this path. Instead, we rely on the following approximation. We first divide the sensor field into a fine grid and then assume that the target only moves along this grid. The problem then is to find the path  $P$  on this grid that minimizes  $|\log G(P)|$ . Note that, the finer the grid the closer the approximation. Also, one can use higher order grids [8] instead of

the rectangular grid as used in this paper. The higher order grids change the runtime of the algorithm but the approach is the same as with the rectangular grid.

For the target not to be detected while traversing along path  $P$ , it must not be detected at any point  $u$  lying between any two adjacent grid points of  $P$ . We therefore subdivide any path  $P$  as a chain of grid segments. Let us consider two adjacent points, say  $v_1$  and  $v_2$  on the grid. Let  $l$  denote the line segment between  $v_1$  and  $v_2$ . Also, let  $m_l$  denote the probability of not detecting a target traveling between  $v_1$  and  $v_2$  on the line segment  $l$ . Then, from the discussion above, the probability  $m_l$  can be evaluated by finding the detection probability  $D(u)$  at each point  $u \in l$ , and it is given by

$$\log m_l = \sum_{u \in l} \log(1 - D(u)) \tag{5}$$

Note that,  $m_l$  lies between 0 and 1 and, therefore,  $\log m_l$  is negative.

To find the least exposed path, a non-negative weight equal to  $|\log m_l|$  is assigned to each segment  $l$  on the grid. Also, a fictitious point  $a$  is created and a line segment is added from  $a$  to each grid point on the west periphery of the sensor field. A weight of 0 is assigned to each of these line segments. Similarly, a fictitious point  $b$  is created and a line segment is added from  $b$  to each grid point on the east periphery of the sensor field. A weight of 0 is assigned to each of these line segments.

*Figure 2 goes here*

The problem of finding the least exposed path from west periphery to east periphery is then equivalent to the problem of finding the least weight path from  $a$  to  $b$  on this grid. Such a path can be efficiently identified using the Dijkstra's shortest path algorithm [1]. A pseudo-code of the overall algorithm is shown in Figure 2.

**Example:** Figure 3 shows a sensor field with eight sensors at locations marked by dark circles.

*Figure 3 goes here*

Assume the noise process at each sensor is Additive White Gaussian with mean 0 and variance 1. Further assume that the



sensors use value fusion to arrive at a consensus decision. Then, from Equation 4, we chose a threshold  $\eta = 3.0$  to achieve a false alarm probability of 0.187%. The field has been divided into a  $10 \times 10$  grid. The target emits an energy  $K = 12$  and the energy decay factor is 2. The figure shows the weight assigned to each line segment in the grid as described above. The least exposure path found by the Dijkstra's algorithm for this weighted grid is highlighted. The probability of detecting the target traversing the field using the highlighted path is 0.926.

### 3 Deployment

In this section, the problem of sensor deployment for unauthorized traversal detection is formulated and solutions are identified.

#### 3.1 Problem formulation

Consider a region to be monitored for unauthorized traversal using a sensor network. The energy level  $K$  emitted by a target of interest and the noise statistics in the region are known. The sensors are to be deployed over the region in a random fashion where the sensors locations in the region cannot a priori be determined and only the number or density of sensors can be chosen. The problem is to find a deployment strategy that results in a desired performance level in unauthorized traversal monitoring of the region.

The performance is measured by the false alarm probability and the path exposure defined in section 2. The false alarm probability is independent of the sensor placement and is only determined by the number of sensors deployed and the thresholds used in the detection algorithms. It is assumed to be fixed in this study so that the problem consists of maximizing the exposure at constant false alarm rate. Since targets can traverse the region through any path, the goal of deployment is to maximize the exposure of the least exposed path in the region.

Obviously, the minimum exposure in the region increases (if false alarm rate is kept constant) as more sensors are deployed in the region. However, since the deployment is random, there are no guarantees that the desired exposure level is achieved with a given number of sensors. Indeed some sensor placements can result in very poor detection ability, for example when the sensors are all deployed in the same vicinity. A study of the statistical distribution of exposure for varying sensor placement for a given number of sensors can provide a confidence level that the desired detection level is achieved. In practical situations, only a limited number of sensors are available for deployment and only a limited detection level with

associated confidence level is achievable for a fixed false alarm rate.

### 3.2 Solution

Based on the above discussion, we develop a solution method to the deployment problem when a maximum of  $M$  sensors can be used. Deploying the  $M$  sensors results in the maximum achievable detection level but this is not optimal when considering the cost of sensors. To reduce the number of sensors deployed, only part of the available sensors should be deployed first and the sensors can then report their position. The random sensor placement obtained can be analyzed to determine if it satisfies the desired performance level. If it does not, additional sensors can be deployed until the desired exposure level is reached or until all  $M$  available sensors are deployed.

The number of sensors used in this strategy can be minimized by deploying one sensor at a time. However, a cost is usually associated with each deployment of sensors and deploying one sensor at a time may not be most cost effective if the cost of deployment is sufficiently large with respect to the cost of single sensors. By assigning distinct costs to both single sensors and deployment, the optimal number of sensors to be deployed at first and thereafter can be determined. In the next section, we develop analytical expressions for finding the optimal solution. In general, the optimal cost solution is neither deploying one sensor at a time nor deploying all the sensors at once.

## 4 Analytical solution

In this section, we derive an analytical model for the cost of deployment. Let  $e_d$  be the desired minimum exposure for the sensor network to be deployed when a maximum of  $M$  sensors are available for deployment. The position of sensors are random in the region of interest  $R$  and for a given number of sensors  $n$ , the least exposure  $e$  is a random variable. Let  $F_n(x)$  denote the cumulative distribution function of  $e$ , i.e. the probability that  $e$  is less than  $x$ , when  $n$  sensors are deployed.

As mentioned in the previous section, there is no a priori guarantee that the exposure  $e_d$  will be obtained when deploying the sensors. If  $M$  is the maximum number of sensors available, the confidence of obtaining a least exposure of  $e_d$  or more is  $1 - F_M(e_d)$ . For the proposed solution, we assume that the position of the sensors obtained after a deployment is known so that additional sensors can be deployed if the minimum exposure  $e_d$  is not reached. To evaluate the probability that the exposure  $e_d$  is reached after additional sensor deployment, we make the following approximation: the distribution of exposure for  $n$  sensors is independent of the exposure corresponding to  $k$  of these  $n$  sensors,  $1 \leq k \leq n - 1$ . This is an

approximation since the exposure obtained with  $n$  sensors is always higher than the exposure obtained with only  $k$  of these  $n$  sensors,  $1 \leq k \leq n - 1$ . We observed that the re-deployment of just a few sensors can substantially modify the coverage of the region, for example when filling empty spaces. The approximation is also justified by the loose relation between exposure and sensors positions. Indeed, a given minimum exposure can correspond to many different deployment positions, some of which can be easily improved by deploying a few additional sensors (e.g. when there is a empty space in the region coverage), some of which can only be improved by deploying many additional sensors (e.g. when the sensors are evenly distributed on the region).

As the minimum exposure  $e$  is a random variable, the cost of deploying the sensors in steps until the desired exposure is reached is also a random variable  $C$ . We now derive the expression for the expected value of  $C$ . Let  $n_i$  be the total number of sensors deployed after step  $i$ . Let  $S$  be the maximum number of steps so that  $n_S = M$ . Note that  $n_i - n_{i-1}$  is the number of sensors deployed at step  $i$ . Also let  $C_d$  be the cost of deploying the sensors at each step and  $C_s$  be the cost of each sensor. If the desired exposure is obtained after the first step, the total cost of deployment is  $C_d + n_1 C_s$ , and this event happens with probability  $1 - F_{n_1}(e_d)$ . Considering all the possible events, the expected cost is given by

$$E\{C\} = \sum_{i=1}^{S-1} (i.C_d + n_i.C_s) \left( \prod_{j=1}^{i-1} F_{n_j}(e_d) \right) (1 - F_{n_i}(e_d)) + (S.C_d + M.C_s) \prod_{j=1}^{S-1} F_{n_j}(e_d) \quad (6)$$

Note that a different expression is needed for the cost of step  $S$  since no additional sensors are deployed after this step even when the desired exposure is not achieved.

## 5 Simulation

In this section, we present results of simulations that were conducted to collect the exposure distribution function of the number of sensors deployed.

### 5.1 Method

*Figure 4 goes here*

The exposure distribution is obtained by collecting statistics on the exposure when deploying sensors randomly in a predefined region. The region monitored is of size  $20 \times 20$  and the random deployment is assumed to be Gaussian, centered at the center of the region monitored, i.e. the point of coordinates (10,10), and standard deviation 10. Only sensors lying within the bounds of a predefined deployment region are assumed to collaborate to detect targets. In general, that deployment region needs to be larger than the region to monitor. Indeed, if sensors are deployed merely in region  $R$ , the edges of the region are not well covered. This is undesirable when trying to detect a target traversing the region since paths lying on the edge of the region have low exposure. Therefore, adequately covering the region  $R$  to prevent undetected traversal requires deploying sensors beyond the edges of  $R$  as shown in Figure 4. In the simulation conducted, the deployment region has dimensions 33% larger than the monitored region, i.e. size  $26.6 \times 26.6$ .

For every deployment, the minimum exposure is found using a simulator implementing the algorithm presented in section 2. A decay factor of  $k = 2$  and maximum energy of  $K = 30$  are chosen to model the energy emitted by targets (cf Equation 1). The noise in the time series data is assumed to be normal with variance 1, so that the signal coming from the target is covered by noise when the target is 6 or more unit lengths away from a sensor. The sensors use value fusion, as presented in section 2.1.1, to collaborate when making a common decision on the presence of a target in the region. The noise  $\sum_{i=1}^n N_i^2$  in the sum of energies from  $n$  sensors, computed in value fusion and appearing in equation 3, is from a Chi-square distribution with  $n$  degrees of freedom. The threshold for detection is chosen as a function of the number of sensors to give a constant false alarm probability. The false alarm probability for each detection attempt is chosen so that the expected number of false alarms is one per hour, assuming that detection attempts occur every 2 seconds.

## 5.2 Distribution of minimum exposure

*Figure 5 goes here*

The distribution of minimum exposure were found for the number of sensor deployed varying from 1 to 100. To illustrate our results, the probability density functions for 15, 30 and 50 sensors are shown in Figure 5.

We observe that for 15 sensors deployed, the minimum exposure has zero density for values less than the false alarm probability of .02. The highest density is obtained for values around .08 and then drops exponentially towards zero for higher values of exposure. For deployment of 30 sensors, we find again that the minimum exposure has zero density for values below .02, then increases and has the shape of a bell curve centered around .4. For deployment of 50 sensors, densities start at zero for small values and remain very small for most values of minimum exposure. The density slowly increases and has a large peak for minimum exposure of 1.

As expected, the minimum exposure increases on average as the number of sensors deployed increases. When randomly deploying 15 sensors, it is very unlikely to obtain a placement providing a desirable minimum exposure. When deploying 30 sensors, most of the exposure levels are equally likely and only poor confidence is given to obtain a desirable exposure level. When deploying 50 sensors, it is very likely that the sensor placement will give good exposure and this likelihood keeps increasing with the number of sensors deployed.

*Figure 6 goes here*

We use the cumulative distribution function obtained from the statistics collected to evaluate the likelihood that the desired level of exposure  $e_d$  is obtained for varying number of sensors. The graph of Figure 6 shows the probability that the minimum exposure is above  $e_d$  as a function of the number of sensors deployed for  $e_d = 80\%, 85\%, 90\%$  and  $95\%$ . These values can be used to evaluate the cost expressed in Equation 6. The graph shows that the confidence level to obtain a given minimum exposure level  $e_d$  increases with the number of sensors deployed. The confidence for  $e_d$  when deploying 100 sensors is above .999, which is sufficient for most applications, and therefore we did not evaluate the distribution of exposure when deploying more than 100 sensors.

## **6 Results**

In this section, we evaluate the expected cost of deploying sensors using the simulation results. The optimal number of sensor to deploy at first and in the succeeding steps can be derived from these results.

For this cost analysis, the region parameters and signal model are the same as specified in section 5. We further assume that the number of sensors deployed at every step is constant so that  $n_i - n_{i-1} = n$  for all  $1 \leq i \leq S$ . In this case, Equation 6 reduces to

$$E\{C\} = (C_d + n.C_s) \sum_{i=1}^{S-1} i. \left( \prod_{j=1}^{i-1} F_{j,n}(e_d) \right) (1 - F_{i,n}(e_d)) + (S.C_d + M.C_s) \prod_{j=1}^{S-1} F_{j,n}(e_d) \quad (7)$$

*Figure 7 goes here*

We evaluated the expected cost as a function of  $n$  for three different cost assignments with a desired exposure of  $e_d = 90\%$ . The three corresponding graphs are shown in Figure 7. The first cost assignment is ( $C_d = 0, C_s = 1$ ) so that the expected cost is the expected number of sensors to be used to achieve an exposure of 90%. Since  $C_d = 0$ , the number of steps used to deploy the sensors doesn't affect the cost and it is therefore optimal to deploy one sensor at a time until the minimum exposure  $e_d$  is reached, as we observe on the graph. Overall, the expected number of sensor to be deployed increases with  $n$  but we observe a local minimum for  $n = 55$  that can be explained by the following analysis. The expected number of sensors is a weighted sum of  $i, n, 1 \leq i \leq S$  that are the different number of sensors than can be deployed at a time when deploying  $n$  sensors at each step. For  $n$  around 50, the probability that the minimum exposure is above  $e_d$  varies a lot as shown in Figure 6 and the weight associated with the first term of the sum ( $n$ ) increases rapidly while the weights associated with higher number of sensors decrease. This is the cause of the local minimum and the cost starts to increase again when the increase in  $n$  compensates for the decrease in weights. In other words, the probability to achieve the desired exposure is much higher when deploying 55 sensors randomly than when deploying 40 sensors randomly. Therefore, it is less costly to deploy 55 sensors at every step since one deployment is likely to be sufficient whereas two or more deployments, and thus a total of 80 or more sensors, are most likely to be needed when deploying 40 sensors at every step.

The second cost assignment is ( $C_d = 5, C_s = 1$ ) so that the cost of a deployment is equal to the cost of five sensors (note that only the relative cost of  $C_d/C_s$  determines the shape of the graphs). In this case, deploying one sensor at a time is prohibited by the cost of deployment and the optimal number of sensors to deploy at every step is 22. Again, we find

that the curve presents a local minimum for  $n = 55$  that is due to the variations in weights. The last cost assignment is  $(C_d = 100, C_s = 1)$  and the minimum cost is achieved when deploying 65 sensors at every step.

These results are specific to the region and the parameters characterizing the signal emitted by the target that were chosen for the simulation. Similar results can be derived for other parameters, most of the effort residing in finding the exposure distributions through simulation.

## 7 Conclusion

This paper addresses the problem of sensor deployment in a region to be monitored for target intrusion. A mechanism for sensor collaboration to perform target detection is proposed and analyzed to evaluate the exposure of paths through the region. The minimum exposure is used as a measure of the goodness of deployment, the goal being to maximize the exposure of the least exposed path in the region.

In the case where sensors are randomly placed in a region to be monitored, a mechanism for sequential deployment in steps is developed. The strategy consists of deploying a limited number of sensors at a time until the desired minimum exposure is achieved. The cost function used in this study depends on the number of sensors deployed in each step and the cost of each deployment. Through simulation, the distribution of minimum exposure obtained by random deployment was evaluated for varying number of sensors deployed. These results were used to evaluate the cost of deployment for varying number of sensors deployed in each step.

We found that the optimal number of sensors deployed in each step varies with the relative cost assigned to deployment and sensors. The results of this study can be extended to larger regions with different target parameters. The solution proposed in this paper can also be improved by considering deploying variable number of sensors at each step and this multiple variables problem requires further investigation.

## 8 Acknowledgments

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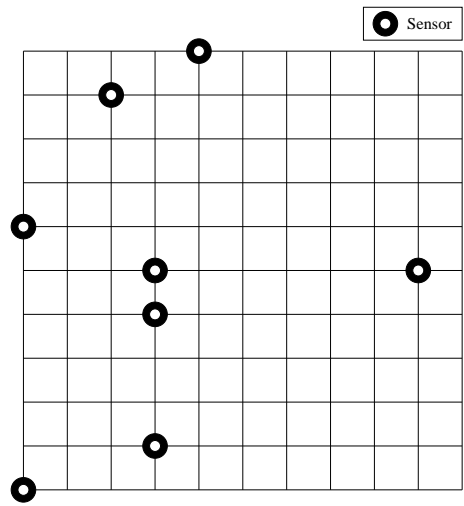


Figure 1. Example sensor fields for UT problem.

- 
1. **Generate** a suitably fine rectangular grid.
  2. **For each** line segment  $l$  between adjacent grid points
  3. **Compute**  $|\log m_l|$  using Equation 5
  4. **Assign**  $l$  a weight equal to  $|\log m_l|$
  5. **Endfor**
  6. **Add** a link from virtual point  $a$  to each grid point on the west
  7. **Add** a link from virtual point  $b$  to each grid point on the east
  8. **Assign** a weight of 0 to all the line segments from  $a$  and  $b$
  9. **Compute** the least weight path  $P$  from  $a$  to  $b$  using Dijkstra's algorithm
  10. **Let**  $w$  equal the total weight of  $P$ .
  11. **Return**  $P$  as the least exposure path with an exposure equal to  $1 - 10^{-w}$ .
- 

**Figure 2. Pseudo-code of the proposed solution for the UT problem.**

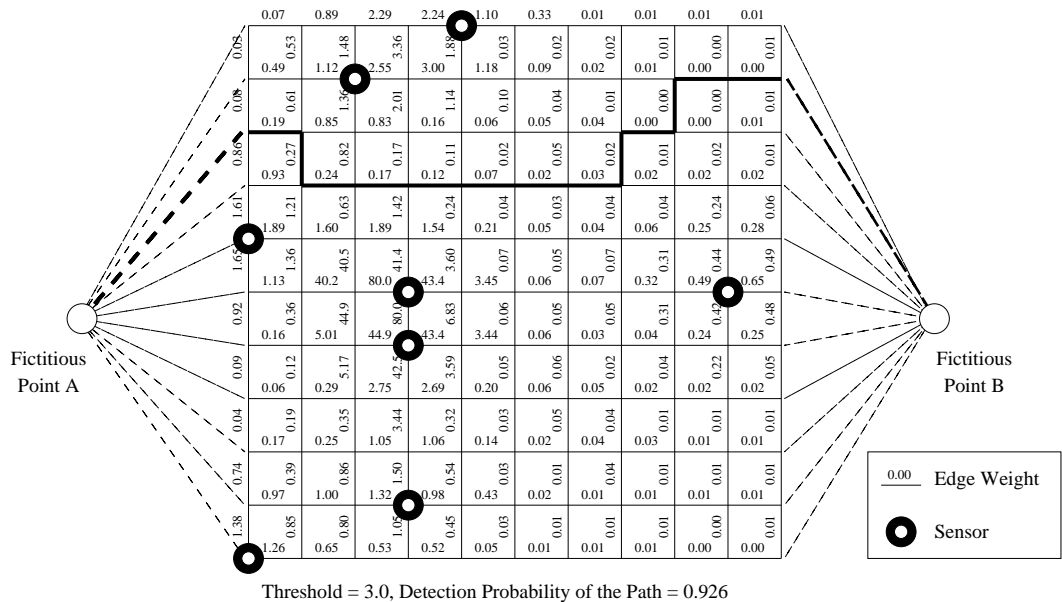
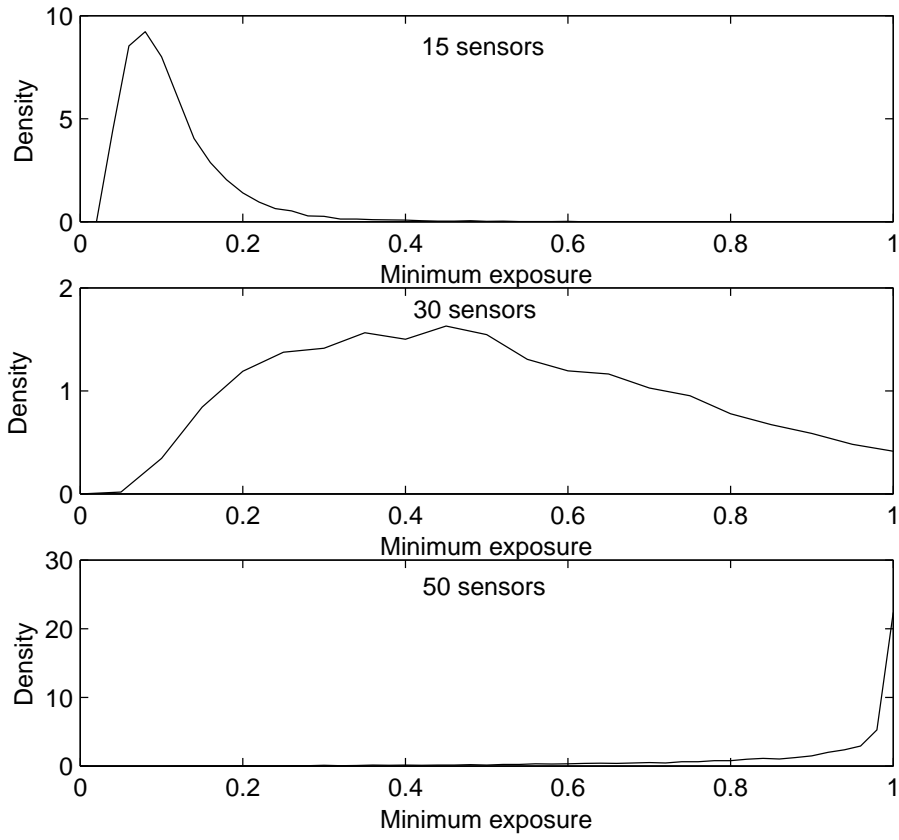


Figure 3. Illustration of the proposed solution for an example UT problem.





**Figure 5. Probability density function for the distribution of minimum exposure for deployments of 15, 30 and 50 sensors.**

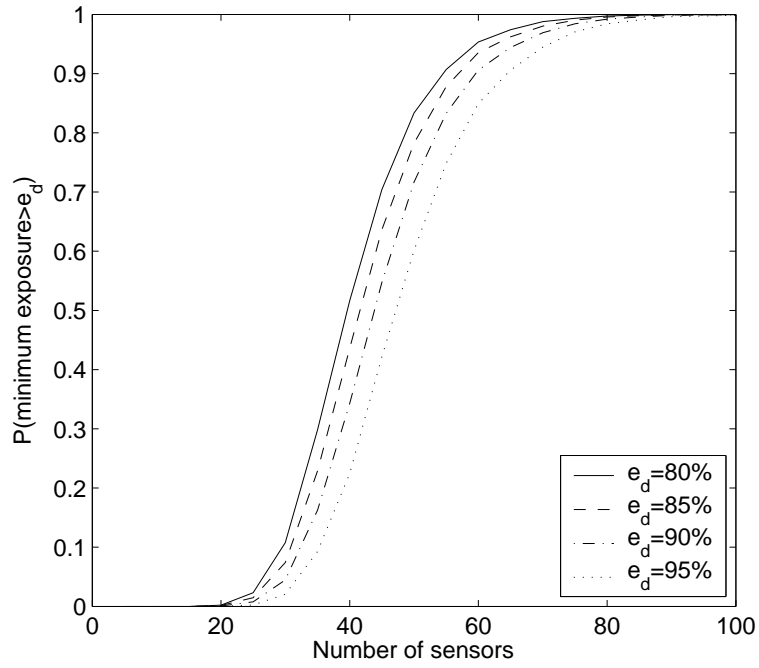
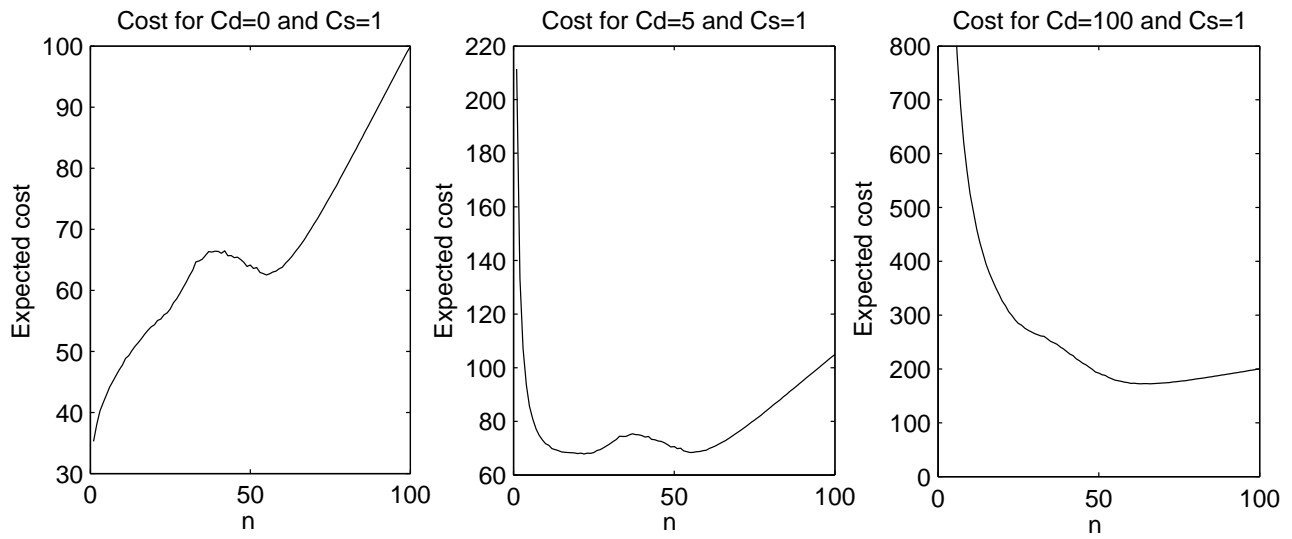


Figure 6. Probability that the minimum exposure is above  $e_d$  for varying number of sensors and  $e_d=80\%,85\%,90\%$  and  $95\%$ .



**Figure 7. Expected cost of achieving minimum exposure of 90% as function of the number of sensors for three different cost assignments.**

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